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A States of Matter Search-Based Approach for Solving the Problem of Intelligent Power Allocation in Plug-in Hybrid Electric Vehicles

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Abstract: Recently, many researchers have proved that the electrification of the transport sector is a key for reducing both the emissions of green-house pollutants and the dependence on oil for transportation. As a result, Plug-in Hybrid Electric Vehicles (or PHEVs) are receiving never before seen increased attention. Consequently, large-scale penetration of PHEVs into the market is expected to take place in the near future, however, an unattended increase in the PHEVs needs may cause several technical problems which could potentially compromise the stability of power systems. As a result of the growing necessity for addressing such issues, topics related to the optimization of PHEVs' charging infrastructures have captured the attention of many researchers. Related to this, several state-of-the-art swarm optimization methods (such as the well-known Particle Swarm Optimization (PSO) or the recently proposed Gravitational Search Algorithm (GSA) approach) have been successfully applied in the optimization of the average State of Charge (*SoC*), which represents one of the most important performance indicators in the context of PHEVs' intelligent power allocation. Many of these swarm optimization methods, however, are known to be subject to several critical flaws, including premature convergence and a lack of balance between the exploration and exploitation of solutions. Such problems are usually related to the evolutionary operators employed by each of the methods on the exploration and exploitation of new solutions. In this paper, the recently proposed States of Matter Search (SMS) swarm optimization method is proposed for maximizing the average State of Charge of PHEVs within a charging station. In our experiments, several different scenarios consisting on different numbers of PHEVs were considered. To test the feasibility of the proposed approach, comparative experiments were performed against other popular PHEVs' State of Charge maximization approaches based on swarm optimization methods. The results obtained on our experimental setup show that the proposed SMS-based *SoC* maximization approach has an outstanding performance in comparison to that of the other compared methods, and as such, proves to be superior for tackling the challenging problem of PHEVs' smart charging.

Keywords: Plug-in Hybrid Electric Vehicles (PHEV); smart grid; particle swarm optimization (PSO); intelligent management; gravitational search (GSA); state of matter search (SMS); nature-inspired

1. Introduction

Carbon dioxide (CO_2) is the main pollutant gas emitted as a result of several human activities related to the combustion of fossil fuels (such as coal, natural gas, and oil). The use of fossil fuel for transportation accounts for about a 25% of the CO_2 emissions around the world, while also accounting for over a 55% of the world's oil consumption [1]. However, in recent years, the traditional view of

power systems has been reshaped as a result of technological advances, as well several economic and environmental incentives. It has been proved by several researches that significant reductions in both green-house gas emissions and the dependence on fossil fuels could be accomplished by the electrification of the transport sector [2]. As a result of this, Plug-in Hybrid Electric Vehicles (or PHEVs) have received never before seen increased attention related to their low emission of pollutants and overall low energy cost. Recent PHEVs technologies promise to increase the overall fuel efficiency by implementing specialized battery systems, which could allow such hybrid vehicles to be charged from traditional power grid systems. In this sense, the currently growing PHEVs tendency promises to shift the current energy demand from fossil fuel to electricity [3].

Statistics provided by the Electric Power Research Institute (EPRI) suggest that by the year 2050 about 62% of the United States vehicle fleet will be comprised of PHEVs [4]. However, an unattended increase of PHEV needs may threaten the stability of power systems, and as such, there is a growing necessity for addressing the technological implications. Furthermore, the power demand patterns related to differences in the needs of multiple PHEVs within a charging station have a naturally significant impact on the electric power market, and therefore, it must also be considered [5].

There is a growing need for more efficient algorithms and mechanisms which could allow smart grid technologies to handle complex problems (such as energy management, efficient charging infrastructure, cost reduction, etc.), subject to a wide variety of different objectives and system constraints. In order to both maximize customer satisfaction and minimize the burdens and disturbances to the power grid, an specialized control approach is required to properly handle multiple PHEV battery loads [6]. These control mechanisms must consider several real-world constraints, such as variations on the infrastructure and communication among individual vehicles. They must also be able to adapt to differences in times between arrivals and departures, as well as the number of PHEVs within the charging station. A crucial constraint for accurate charging of PHEVs is related their battery State-of-Charge. The State-of-Charge (or *SoC*) is a parameter which measures the amount of electrical energy stored in a vehicle's battery. In fact, the performance of PHEVs is highly dependent on the proper management of electric power, which is solely dependent on the battery's *SoC*.

Recently, many researchers have focused their efforts on improving the interaction between PHEVs and the electric power grid by proposing a wide variety of intelligent power allocation frameworks. One of the most popular approaches to solve these power allocation problems involves the use optimization techniques known as swarm-optimization methods [7]. In [8,9], respectively, the Particle Swarm Optimization (PSO) and Gravitational Search Algorithm (GSA) methods have been successfully applied for solving the problem of smart power management among a given number of PHEVs under different *SoC* conditions. While the PSO and GSA methods are known for providing acceptable results, such techniques have a tendency to produce suboptimal solutions related to a lack of balance between the exploration and exploitation of solutions [10,11].

In this paper, the recently developed swarm optimization method known as State of Matter Search (SMS) is proposed for solving the problem of intelligent power allocation on PHEVs. In the SMS method, individual particles emulate a set of molecules which interact in correspondence to a set of evolutionary operators based on the principles of thermal-energy motion [12]. The SMS combines the use of such operators with a unique control strategy based on the states of matter transitions which modifies the parameter setting of each operation during the evolutionary process. In contrast to other swarm optimization techniques, the computational procedures incorporated in the SMS approach yield a better balance between the exploration and exploitation of solutions, typically leading to better results [12]. In order to prove its feasibility for solving the proposed optimization problem, the SMS was compared in terms of performance with several other state-of-the-art swarm optimization methods such as PSO and GSA. The rest of this paper is organized as follows: in Section 2, the Intelligent PHEVs charging optimization problem foundations are presented. In Section 3, the main traits of the SMS optimization method are highlighted. In Section 4, we present our experimental setup and results. Finally, in Section 5, conclusions are drawn.

2. Problem Formulation

The main motivation behind a smart charging scheme is to charge a vehicle whenever it is most beneficial, such as when the price of electricity and the total power demand are low, or when the generated power reaches an excess capacity [13]. In order for the system to be effective, it should allow most PHEVs to leave the charging station prior to their expected charging time.

Suppose there is a charging station with a total power capacity P specifically designed to charge Plug-in Hybrid Vehicles (PHEVs). The main objective is to allocate power intelligently for each individual PHEV that comes into the charging station. In order to do so, it is necessary to consider each PHEV's State-of-Charge (*SoC*). Each vehicle's *SoC* changes constantly as a result of the charging process, so, it is necessary to keep track of such changes in order to allocate power effectively. Therefore, each vehicle's current *SoC* represents the main parameter which has to be maximized in order to ensure a proper power allocation. To do so, an objective function which takes into account several constraints (such as charging time, current *SoC*, and price of the energy) is considered for the maximizing the of average State-of-Charge [14].

In order to illustrate this, let N stand for the number of PHEVs that need to be served on a regular 24-h day. Let k denote a fixed time step for which we aim to maximize the average State-of-Charge $J(k)$. An objective function which models such a maximization case may be defined as follows:

$$J(k) = \max\left[\sum_{i=1}^{N} w_i(k) \cdot SoC_i(k+1)\right] \tag{1}$$

where $w_i(k)$ denotes a weighting term associated to a given PHEV i at time step k, $SoC_i(k+1)$ stands for the vehicle's State-of-Charge at time step $k+1$. In the proposed approach, it is assumed that the charging current remains constant for every instant of time. That is:

$$[SoC_i(k+1) - SoC_i(k)] \cdot C_i = Q = I_i(k)\Delta t \tag{2}$$

It follows that:

$$SoC_i(k+1) = SoC_i(k) + \frac{I_i(k)\Delta t}{C_i} \tag{3}$$

where the sample time Δt is defined by the charging station operators and $I_i(k)$ is the charging current over Δt. Furthermore, the PHEV's battery unit is modeled as a capacitor circuit, where the battery's power capacity C_i is represented by its respective capacitance value (in Farad).

$$C_i \frac{dV_i}{dt} = I_i \tag{4}$$

Therefore, over a small time interval, the change of voltage could be assumed to be linear, such that:

$$C_i \frac{(V_i(k+1) - V_i(k))}{\Delta t} = I_i(k) \tag{5}$$

from which it follows that:

$$V_i(k+1) - V_i(k) = \frac{I_i \Delta t}{C_i} \tag{6}$$

Also, since our decision variable is the power allocated to each individual PHEV, we replace the current term $I_i(k)$ as follows:

$$I_i(k) = \frac{P_i(k)}{V_i'(k)} \tag{7}$$

where $P_i(k)$ represents the amount of electric power assigned to the i-th vehicle on the charging station, while $V_i'(k) = 0.5 \cdot (V_i(k+1) + V_i(k))$ denotes the average voltage between the voltage values at time steps k and $k+1$. By replacing $I_i(k)$ on Equation (3), it follows that:

$$SoC_i(k+1) = SoC_i(k) + \frac{P_i(k)\Delta t}{0.5 \cdot C_i \cdot (V_i(k+1) + V_i(k))} \tag{8}$$

where $V_i(k+1)$ is obtained by replacing $I_i(k)$ in Equation (6). This yields:

$$V_i(k+1) = \sqrt{\frac{2P_i(k)\Delta t}{C_i} + V_i^2(k)} \tag{9}$$

We may finally represent the objective function $J(k)$ as follows:

$$J(k) = \max_{P_i(k)\in\mathbb{R}} \left[\sum_{i=1}^{N} w_i(k) \cdot \left(SoC_i(k) + \frac{P_i(k) \cdot \Delta t}{0.5 \cdot C_i \cdot \left(\sqrt{\frac{2 \cdot P_i(k) \cdot \Delta t}{C_i} + V_i^2(k)} + V_i(k) \right)} \right) \right] \tag{10}$$

which is subject to:

$$\sum_i P_i(k) \leq P_{\text{utility}} \cdot \eta \tag{11}$$

$$0 \leq P_i(k) \leq P_{i,\max} \tag{12}$$

$$0 \leq SoC_i(k) \leq SoC_{i,\max} \tag{13}$$

where $P_{\text{utility}} = P_{i,\max} \cdot N$ denotes the maximum power that can be provided by a charging station with capacity for N PHEVs [15], while $P_{i,\max}$ stands for the maximum power that can be absorbed by a specific PHEV. Also, η denotes the efficiency of the charging station (typically of 90%) and it is assumed to be constant. Furthermore, $SoC_{i,\max} = 0.8$ stands for SoC limit related to the i-th PHEV [9] (see Table 1). In practice, it is not recommended to fully charge a PHEV's battery due to the risk of a possible overload. As such, the value $SoC_{i,\max}$ is set as a security measure in order to prevent damage to the battery [16].

Finally, the charging weight $w_i(k)$ is expressed as a function of three particular parameters, defined as follows:

$$w_i(k) = f(c_i(k), t_i(k), d_i(k)) \tag{14}$$

where $c_i(k) = C_i \cdot (1 - SoC_i(k))$ denotes the proportion of the i-th PHEV's rated battery capacity C_i that remains to be filled at a given time step k. Furthermore, $t_i(k)$ stands for the i-th PHEV's remaining charging time at time step k, and $d_i(k)$ represents the difference between the price of the real-time energy and the price that a specific customer at the i-th PHEV charger is willing to pay at such time step k [17]. In this case, the weighting term $w_i(k)$ gives a degree of preference which is proportional to certain specific attributes of each individual PHEV; i.e., if a given PHEV has both, a lower initial State-of-Charge and less remaining charging time, but the driver is eager to pay a higher price, the system will provide more power to this particular PHEV battery charger:

$$w_i(k) \propto \left[c_i(k) + d_i(k) + \frac{1}{t_i(k)} \right] \tag{15}$$

It is worth noting that the terms $c_i(k)$, $d_i(k)$ and $t_i(k)$ are not of the same scale, and as such all terms must be normalized to assign similar relevance to each of them. Furthermore, the charging station operators may also manifest several different interests, which could be influential when assigning an importance factor to each these terms. With that being said, we may express each weighting term as follows:

$$w_i(k) = \alpha_1 c_i(k) + \alpha_2 t_i(k) + \alpha_3 d_i(k) \tag{16}$$

where α_1, α_2 and α_3 denotes the importance factors assigned to the terms $c_i(k)$, $d_i(k)$ and $t_i(k)$ respectively, and such that $\sum_j \alpha_j = 1$. In the proposed approach, it is assumed that $\alpha_1 = \alpha_2 = \alpha_3$ at every time step k.

Table 1. Parameter settings for the PHEV's smart power allocation objective function.

Parameters	Description	Values
Fixed parameters	PHEV's maximum power absorption	$P_{i,\max} = 6.7$ kW
	Charging station efficiency	$\eta = 0.9$
	Total charging time (time step length)	$\Delta t = 20$ min (1200 s)
Variables	PHEV's State of Charge (SoC)	$0.2 \leq SoC_i \leq 0.8$
	PHEV's battery capacity	16 kWh $\leq C_i \leq$ 40 kWh
Constraints	PHEVs' total absorbed power	$\sum_i P_i(k) \leq P_{utility}(k) \cdot \eta$
	PHEV's State of Charge (SoC)	$0 \leq SoC_i(k) \leq SoC_{i,\max}$
	PHEV's power absorption	$0 \leq P_i(k) \leq P_{i,\max}$

3. The States of Matter Search (SMS) Algorithm

The States of Matter Search (SMS) is a swarm optimization method which emulates the states of matter phenomenon. In this optimization approach, individuals within a population of search agents are represented as molecules which interact with each other by computing a set of unique evolutionary operators based on the physical principles of the thermal-energy motion mechanism [18].

The SMS's evolutionary process is divided in three sequential stages: (1) a gas state in which the molecules experience severe displacements and collisions; (2) a liquid state in which there is a significant reduction of molecular movement; and (3) a solid state in which the force among particles becomes so strong that molecular movement is almost completely inhibited. As the SMS evolutionary process transitions from one stage to another, different movement behaviors are exhibited by the molecules within a given search space. Such behaviors allow the SMS method to preserve a better balance between the exploration and exploitation of solutions, allowing the evolutionary process to find potentially better solutions [19].

3.1. States of Matter Transition

As previously stated, in the States of Matter Search (SMS) approach the whole optimization process is divided into three different stages: (1) gas state; (2) liquid state; and (3) solid state. The gas state comprises the first stage of the SMS method. In this stage, molecules experience severe displacements and collisions. The gas state lasts for 50% of the total iterations which comprise the whole SMS's optimization process. The next stage in the SMS optimization process is the liquid state. In this stage, the motion and collisions exhibited by the molecules within the search space are more restricted in comparison to the gas state. The liquid state lasts for 40% of the total iterations of the SMS evolution process. The third and last stage of the SMS optimization method is represented by the solid state. In this stage, forces among particles are much stronger in comparison to the previous SMS stages, which in turn prevents particles from moving freely. The solid state lasts for only the remaining 10% of total SMS's iterations. The overall transition of the SMS optimization process is described in Figure 1.

During each SMS stage, a series of parameters α, β and γ (which are employed on the molecular movement operators described in Section 3.2) are all modified, allowing SMS to control the way in which the molecules move on each of such stages. Table 2 shows the SMS parameters setup corresponding to each particular SMS stage, as given by its own reference [12].

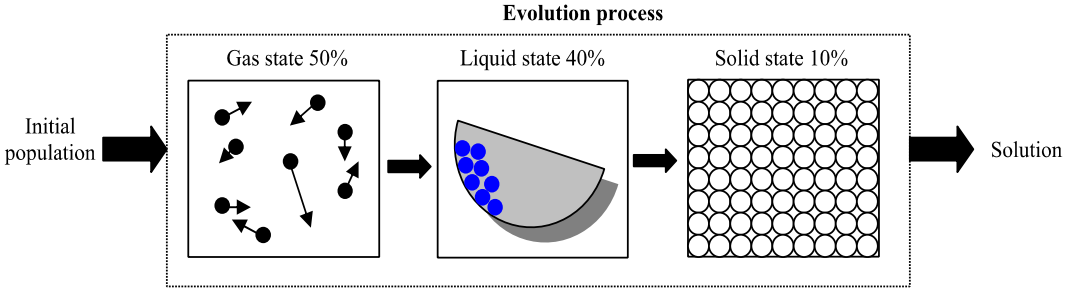


Figure 1. States of Matter Search (SMS) evolution process.

Table 2. States of Matter Search (SMS) parameters setup.

Stage	Duration	ff	fi	fl	$\mathcal{P}$
Gas	50%	0.8	0.8	[0.8, 1.0]	0.9
Liquid	40%	0.4	0.2	[0.0, 0.6]	0.2
Solid	10%	0.1	0.0	[0.0, 0.1]	0.0

3.2. Molecule Movement Operators

In the SMS approach, search agents are treated as molecules whose positions within a multi-dimensional space are changed as the algorithm evolves. In the SMS, the movement of such molecules is analogous to the principles which govern the motion of thermal-energy. The movement of every molecule is determined by considering: (1) the attraction force among the molecules; (2) a series of collision phenomenon; and (3) some random phenomena experienced by the molecules [18].

3.2.1. Direction of Movement

The direction vector operator indicates the way in which molecules will change their positions as the SMS's evolution process develops. For each n-dimensional molecule $\mathbf{p}_i$ within the total population of molecules $\mathbf{P}$, an n-dimensional direction vector $\mathbf{d}_i$ is assigned to each particle. During the SMS's initialization process, all direction vectors ($\mathbf{D} = \{\mathbf{d}_1, \mathbf{d}_2, \ldots \mathbf{d}_N\}$) are randomly initialized with values within the range $[-1,\ 1]$.

As the system develops, the molecules within the search space experience several attraction forces. This attraction force is modeled as a movement towards the best solution found so far by the evolutionary process. Therefore, at each iteration of the evolutionary process, the direction vector update is computed as follows:

$$\mathbf{d}_i^{k+1} = \mathbf{d}_i^k \cdot \left(1 - \frac{it}{itern}\right) \cdot 0.5 + \mathbf{a}_i \tag{17}$$

where $\mathbf{a}_i = \left(\mathbf{p}^{best} - \mathbf{p}_i\right) / \|\mathbf{p}^{best} - \mathbf{p}_i\|$ denotes a unitary attraction vector toward the best individual seen so-far ($\mathbf{p}^{best}$), k represent the current iteration number, and $itern$ stand for the total iterations number which constitute the entire evolution process.

Once a movement direction has been assigned to a given molecule $\mathbf{p}_i$ its respective velocity vector $\mathbf{v}_i$ is then computed and assigned to the particle. The magnitude is computed as follows:

$$\mathbf{v}_i = v_{init} \cdot \mathbf{d}_i \tag{18}$$

where v_{init} denotes a velocity magnitude which is calculated as follows:

$$v_{init} = \alpha \cdot \frac{\sum_{j=1}^{n} \left(b_j^{high} - b_j^{low}\right)}{n} \tag{19}$$

where b_j^{low} and b_j^{high} denote the lower and upper j-th parameter bounds respectively, while n stands for the total number of decision variables (dimensions). Furthermore, $\alpha \in [0,\ 1]$ denotes a scalar factor whose value depends on the current SMS stage (see Table 2).

Finally, the position of each molecule for each molecule $\mathbf{p}_i$ is updated as follows:

$$p_{i,j}^{k+1} = p_{i,j}^{k} + v_j \cdot \text{rand}(0,1) \cdot \left(b_d^{high} - b_d^{low}\right) \cdot \gamma \tag{20}$$

where $\text{rand}(0,1)$ stand for a random number generated within the range of $[0,1]$, while $\gamma \in [0,\ 1]$ stands for a scalar factor whose value depends on the current SMS stage (see Table 2).

3.2.2. Collisions

The collision operator emulates the collision phenomenon that molecules suffer when they interact with each other. The collision operator is only calculated if the distance between two different molecules $\mathbf{p}_i$ and $\mathbf{p}_q$ is smaller than a given collision radius r, calculated as follows:

$$r = \beta \cdot \frac{\sum_{j=1}^{n}\left(b_j^{high} - b_j^{low}\right)}{n} \tag{21}$$

where b_j^{low} and b_j^{high} represent the lower and upper j-th parameter bounds respectively, while n stands for the total number of decision variables (dimensions). Furthermore, $\beta \in [0,1]$ denotes a scalar factor whose value depends on the current SMS stage (see Table 2).

In other words, if $\|\mathbf{p}_i - \mathbf{p}_q\| < r$, it is assumed that molecules $\mathbf{p}_i$ and $\mathbf{p}_q$ have entered into collision; in such a situation, the direction vectors $\mathbf{d}_i$ and $\mathbf{d}_q$ corresponding to each involved particle are exchanged, such that:

$$\mathbf{d}_i = \mathbf{d}_q \text{ and } \mathbf{d}_q = \mathbf{d}_i \tag{22}$$

The collision operator provides SMS the ability to control the diversity of solutions by forcing molecules to change their directions whenever they get close to each other, which prevents them from prematurely overcrowding a given region within the search space.

3.2.3. Random Behavior

In order to simulate the random behavior commonly demonstrated by molecules during their transition from one matter state to another, the SMS method integrates an operator which, by following a probabilistic criterion, allows it to randomly change the positions of molecules within a given search space. Under such operation, each molecule within the set of positions $\mathbf{P}^{k+1} = \left\{\mathbf{p}_1^{k+1}, \mathbf{p}_2^{k+1}, \ldots, \mathbf{p}_N^{k+1}\right\}$ (which corresponds to the set of positions generated by computing the movement operators described in Section 3.2.1) is assigned a probability of changing its current position with that of a randomly generated molecule within the feasible search space. This mechanism may be modeled as follows:

$$p_{i,j}^{k+1} = \begin{cases} b_j^{low} + \text{rand}(0,1) \cdot \left(b_j^{high} - b_j^{low}\right) \text{ with probability } \mathcal{P} \\ p_{i,j}^{k+1} \text{ with probability } (1-\mathcal{P}) \end{cases} \tag{23}$$

where b_j^{low} and b_j^{high} denote the lower and upper j-th parameter bounds respectively, while $\text{rand}(0,1)$ stand for a random number within the range $[0,\ 1]$. Furthermore, $\mathcal{P}$ represents the probability that a given particle has to change its current position and its value (as mentioned in Section 3.2) depending on the current SMS stage (see Table 2).

4. SMS-Based Smart Power Allocation for PHEVs

In this paper, the States of Matter Search (SMS) algorithm (as described in Section 3) is proposed to solve the problem of smart power allocation for PHEVs. As illustrated in Section 2, the main objective

of a PHEV's smart power allocation algorithm may be described as follows: For a given time step k, find an optimal power allocation configuration $[P_1(k), P_2(k), \ldots, P_N(k)]$ which leads to the average State-of Charge $J(k)$ (as given by Equation (10)) yielding a maximum value.

In the proposed approach, the SMS algorithm starts by generating a set of N molecules $\mathbf{P}(k) = \{\mathbf{p}_1(k), \mathbf{p}_2(k), \ldots, \mathbf{p}_N(k)\}$ within a feasible search space. In the context of a PHEV's smart power allocation problem, the positions occupied by each molecule $\mathbf{p}_j(k) \in \mathbf{P}(k)$ (with k denoting a specific time step) represents a specific power allocation vector, given as follows:

$$\mathbf{p}_j(k) = \left[p_{j,1}(k), p_{j,2}(k), \ldots, p_{j,n}(k)\right] \tag{24}$$

where the elements $\mathbf{p}_{j,i}(k)$ represent the total power allocated to the i-th PHEV at a given time step k, and where $j \in \{1, 2, \ldots, N\}$ represents the index of the j-th molecule.

Guided by the SMS's unique evolutionary operators, each molecule $\mathbf{p}_j(k)$ moves around a feasible search space while looking for an optimal power allocation vector configuration. The quality (fitness) of each of molecule $\mathbf{p}_j(k)$ is evaluated by considering the average State-of-Charge function given by Equation (10), such that:

$$J\left(\mathbf{p}_j(k)\right) = \sum_{i=1}^{N} w_i(k) \cdot \left(SoC_i(k) + \frac{p_{j,i}(k) \cdot \Delta t}{0.5 \cdot C_i \cdot \left(\sqrt{\frac{2 \cdot p_{j,i}(k) \cdot \Delta t}{C_i} + V_i^2(k)} + V_i(k) \right)} \right) \tag{25}$$

where $J\left(\mathbf{p}_j(k)\right)$ represents the average State-of-Charge computed with regard to the power allocation vector corresponding to the molecule $\mathbf{p}_j(k)$ at a given time step k. Furthermore, since $J\left(\mathbf{p}_j(k)\right)$ is given for a specific time step k (each defined by charging time of length Δt = 20 min, as given in Table 1) within a regular 24-h day, an optimal power allocation vector must be found for each finite time period. Figure 2 illustrates an example of several power allocation configurations, assigned to 50 PHEVs during a regular 24-h day.

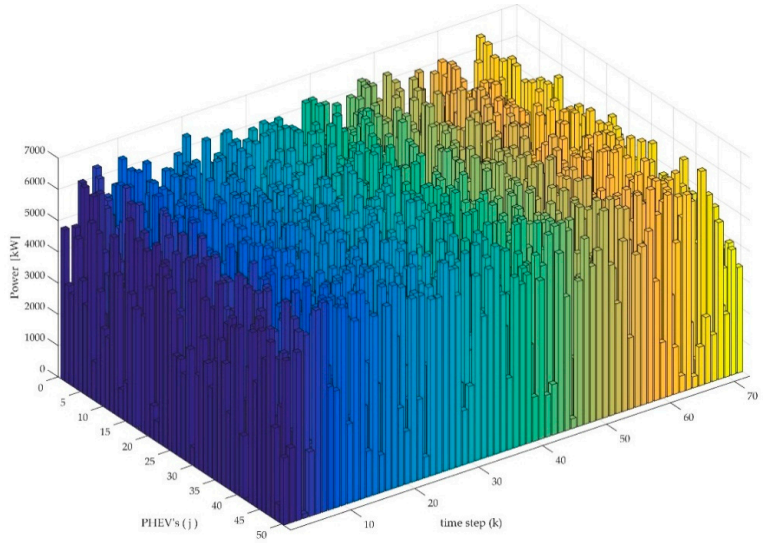


Figure 2. Illustration of different power allocation configurations assigned to 50 PHEVs during a regular 24-h day.

5. Experimental Results

In order to verify the feasibility and effectiveness of the proposed approach, a series of comparative experiments were performed against the following two state-of-the-art swarm optimization methods:

(1) Particle Swarm Optimization (PSO) method [20]; (2) the Gravitational Search Algorithm (GSA) approach [21]; (3) the Firefly Algorithm (FA) [22]; and (4) the Genetic Algorithms (GA) approach [23]. The parameter settings for each implemented method is described as follows:

(1) PSO: The Standard Particle Swarm Optimization (SPSO-2011) proposed in [20] was implemented. The algorithm's learning factors were set to $c_1 = 2$ and $c_2 = 2$.
(2) GSA: The initial gravitation constant value has been set to $G_o = 100$, while the constant parameter alpha has been set to $\alpha = 20$, as given in [21].
(3) FA: The parameters setup for the randomness factor and the light absorption coefficient are set to $\alpha = 0.2$ and $\gamma = 1.0$ respectively, as illustrated on its own reference [22].
(4) GA: The crossover and mutation probabilities are both set to $c_p = 0.8$ and $m_p = 0.2$ respectively [24].
(5) SMS: This algorithm was implemented by considering the parameter setup illustrated in Section 3 (see Table 2), as recommended in [12].

In addition, a randomized PHEV charging algorithm (referred as RCA in this paper) in which the power allocation vectors $\mathbf{p}_j(k)$ at each time step k are randomly assigned was also implemented.

For our experiments, several PHEVs' smart charging scenarios were simulated. As illustrated in Section 4, the general procedure consists of finding an optimal power allocation vector at each finite time step k (where $k = 1, 2, \ldots, 72$ when $\Delta t = 20$ min) within a regular 24-h day. Each of the compared methods are required to run a total of 72 times to complete a single simulation. For our purposes, five different PHEV power allocation scenarios were considered. For each of the scenarios, a fixed number of PHEVs at a time (50, 100, 300, 500 and 1000) is taken into account when performing a simulation. Each scenario was simulated a total of 50 times, by considering a population size of 50 individuals (search agents) and a maximum of 100 iterations for each simulation. All calculations were performed on an AMD (R) A6-5400k CPU 3.60 GHz, 4.0 GB RAM, Microsoft 64 bit Windows 7 OS, and MATLAB© R2015A.

The experimental setup aims to compare the proposed approach's performance against those of PSO and GSA. In each approach, performance is evaluated by averaging the values obtained by computing the objective function $J(k)$ (as given by Equation (25)) at each time step k of an individual 24-h day simulation (see Figure 3). The results for 50 individual runs are reported in Table 3, where the best outcome for each particular PHEVs' smart charging scenario is boldfaced. The reported results consider the following performance indexes: The Average Best-so-far (AB) solution, the Median Best-so-far (MB) and the Standard Deviation (SD) of the best-so-far solution. According to this table, the SMS algorithm performance is superior to those of the other compared methods. Such a large difference in performance is intuitively related the SMS method´s better trade-off between exploration and exploitation. Furthermore, as illustrated in Figure 4, which represents the evolution curves for each particular smart charging scenarios, GSA has a slower convergence rate in comparison to the other methods. On the other hand, PSO and SMS show to have the fastest convergence rate, with them finding their best solutions in less than 20 iterations on average. However, it is still clear that the SMS algorithm surpasses all of the compared methods, in terms of solution quality. Finally, in Table 4, the computational times (in seconds) corresponding to each of the compared methods are shown. As evidenced in the table, the computational times corresponding to the SMS method for each given PHEVs' smart charging scenario are in between those of the other compared methods, with the GSA algorithm taking the most time to complete a given simulation. While PSO has a clear advantage against SMS and GSA in terms of computational time, the superior performance and quality of our proposed approach more than makes up for such differences.

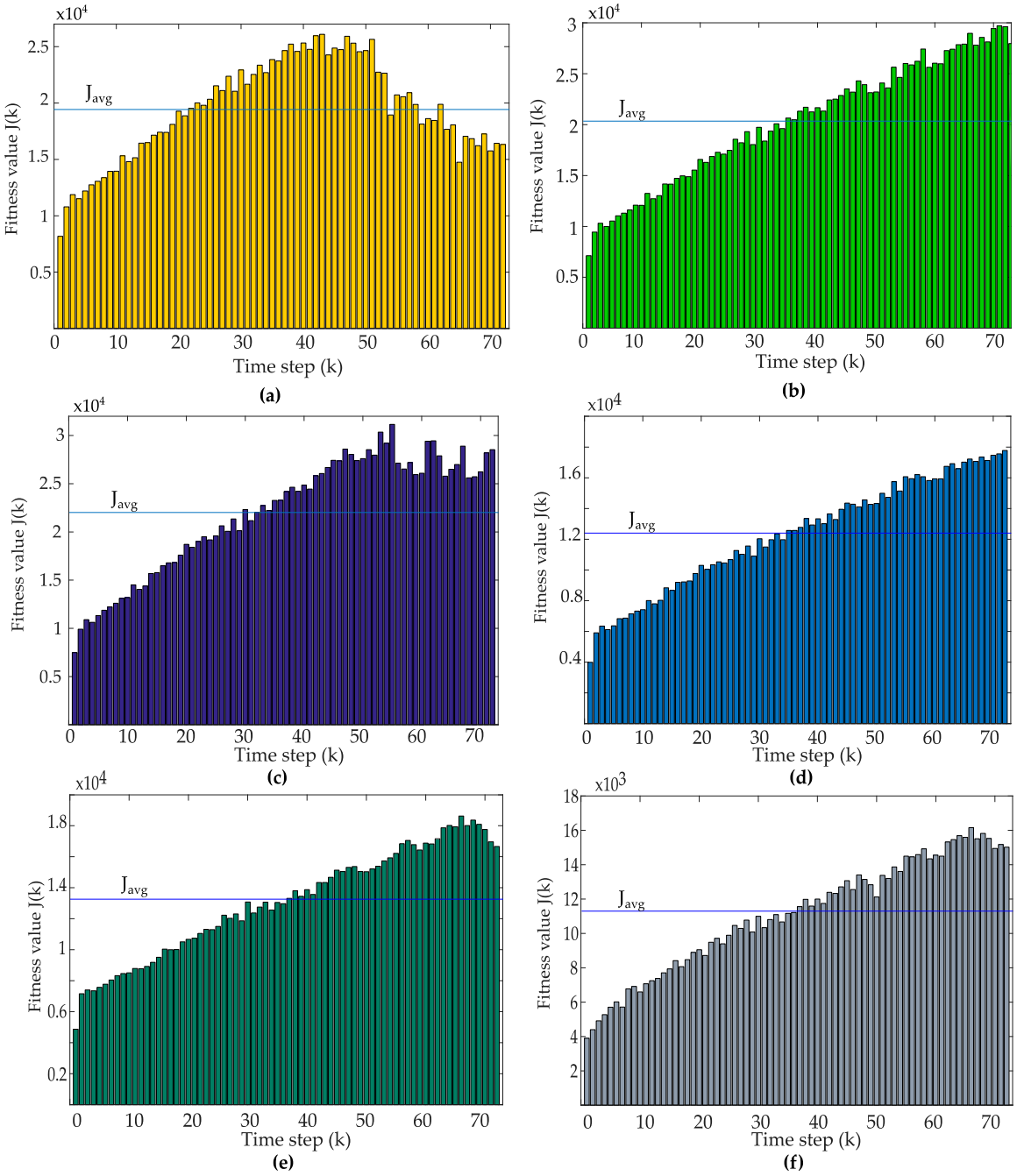


Figure 3. Cost function values $J(k)$ obtained for a simulated PHEVs' smart charging scenario consisting of 50 PHEVs, corresponding to: (**a**) PSO; (**b**) GSA; (**c**) SMS; (**d**) GA; (**e**) FA and (**f**) RCA. In each case, the average values J_{avg}, corresponding to the average of all $J(k)$ values obtained during a single simulation, are indicated.

Table 3. Maximization results for several PHEVs' smart charging scenarios, considering $n = 50$ individual runs and maximum number of iterations $itern = 100$.

Fitness $J(k)$		PHEVs				
		50	**100**	**300**	**500**	**1000**
RCA	AB	1.489×10^4	2.634×10^4	9.077×10^4	1.501×10^5	3.392×10^5
	MB	1.483×10^4	2.486×10^4	9.272×10^4	1.470×10^5	3.678×10^5
	SD	4.025×10^3	6.016×10^3	1.713×10^4	3.320×10^4	9.788×10^3
FA	AB	1.827×10^4	3.401×10^4	1.163×10^5	1.923×10^5	1.764×10^5
	MB	1.845×10^4	3.433×10^4	1.174×10^5	1.964×10^5	2.195×10^5
	SD	1.033×10^3	1.850×10^3	5.077×10^3	5.646×10^3	7.724×10^4
PSO	AB	1.615×10^4	3.271×10^4	7.632×10^4	1.848×10^5	2.624×10^5
	MB	1.770×10^4	3.615×10^4	7.993×10^4	2.179×10^5	2.781×10^5
	SD	2.068×10^3	3.911×10^3	4.154×10^3	2.961×10^4	8.367×10^4
GSA	AB	1.648×10^4	3.367×10^4	1.156×10^5	1.886×10^5	2.899×10^5
	MB	1.834×10^4	3.791×10^4	1.271×10^5	2.011×10^5	2.906×10^5
	SD	3.787×10^3	8.139×10^3	2.055×10^4	2.907×10^4	5.559×10^4
GA	AB	1.777×10^4	3.428×10^4	1.130×10^5	1.912×10^5	3.438×10^5
	MB	1.788×10^4	3.532×10^4	1.140×10^5	1.952×10^5	4.424×10^5
	SD	9.499×10^2	1.780×10^3	4.680×10^3	5.893×10^3	1.352×10^4
SMS	AB	1.864×10^4	3.939×10^4	1.214×10^5	2.067×10^5	3.892×10^5
	MB	1.873×10^4	4.109×10^4	1.219×10^5	2.099×10^5	3.944×10^5
	SD	9.144×10^2	1.881×10^3	4.629×10^3	6.204×10^3	1.163×10^4

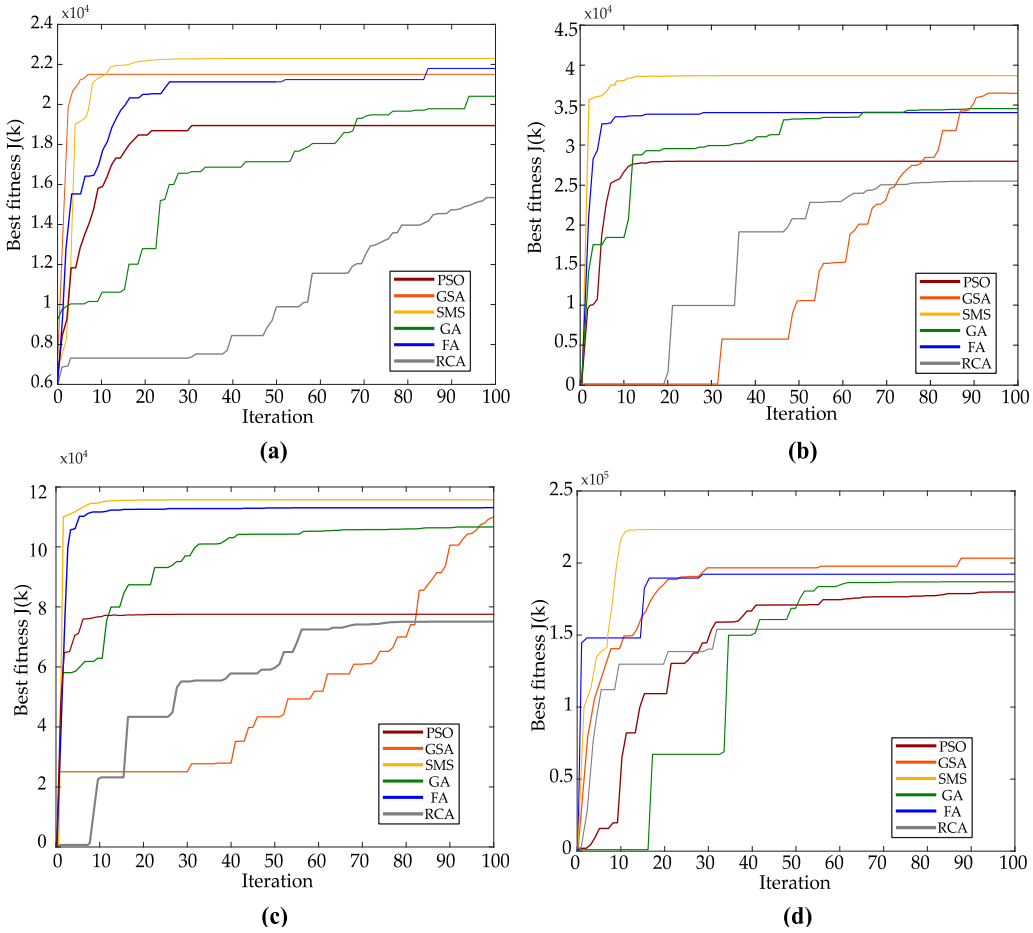
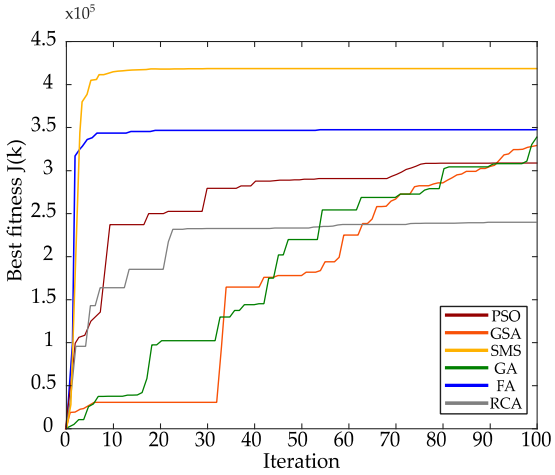


Figure 4. *Cont.*



(e)

Figure 4. Evolution curves for PSO, GSA and SMS, considering several PHEVs' smart charging scenarios, consisting of (**a**) 50 PHEVs; (**b**) 100 PHEVs; (**c**) 300 PHEV; (**d**) 500 PHEVs; and (**e**) 1000 PHEVs.

Table 4. Computation times for several different PHEVs' smart charging scenarios. Al reported times are expressed in seconds (s).

Number of PHEVs	Computational Time (s)					
	SMS	**GSA**	**PSO**	**GA**	**FA**	**RCA**
50	60.868	130.825	45.158	80.372	57.438	12.950
100	69.177	149.800	48.880	101.528	74.495	14.151
300	129.290	237.898	60.616	143.652	112.866	22.177
500	163.656	317.460	70.735	196.335	149.650	27.128
1000	268.833	578.800	99.389	282.747	186.129	36.579

The non-parametric statistical significance proof known as the Wilcoxon's rank sum test for independent samples [25,26] was conducted over the best fitness values found by each of the compared method on 50 independent test runs (50 samples per set). Table 5 reports the p-values produced by the Wilcoxon's test for the pair-wise comparison over two independent fitness samples (SMS vs. PSO, SMS vs. GSA, SMS vs. GA, SMS vs. FA and SMS vs. RCA), by considering a 5% significance level. As a null hypothesis, it is assumed that there is a significant difference between mean values of two algorithms. On the other hand, the alternative hypothesis (rejection of the null hypothesis) considers that the difference between the mean values of both approaches is insignificant. As shown by all of the p-values reported in Table 5 there is enough evidence to reject the null hypothesis (this is that all values are less than 0.05, and as such satisfy the 5% significance level criteria). Such evidence indicates that the proposed method's results are statistically significant and that they had not occurred by coincidence (i.e. due to common noise contained in the process).

Table 5. p-values produced by the Wilcoxon test. The table shows the comparisons for SMS vs. PSO, SMS vs. GSA, SMS vs. GA, SMS vs. FA and SMS vs. RCA over the "averaged best fitness values".

PHEVs' Charging Scenario	SMS vs. PSO	SMS vs. GSA	SMS vs. GA	SMS vs. FA	SMS vs. RCA
50	6.301×10^{-17}	7.713×10^{-18}	7.066×10^{-16}	7.066×10^{-17}	2.852×10^{-10}
100	7.504×10^{-15}	7.504×10^{-17}	1.617×10^{-16}	5.025×10^{-15}	7.713×10^{-10}
300	4.253×10^{-13}	3.946×10^{-14}	2.084×10^{-13}	9.148×10^{-13}	9.726×10^{-8}
500	2.907×10^{-10}	2.449×10^{-13}	8.238×10^{-10}	2.823×10^{-12}	5.628×10^{-6}
1000	3.293×10^{-10}	1.318×10^{-10}	6.821×10^{-8}	4.259×10^{-10}	4.713×10^{-6}

6. Conclusions

In this paper, the swarm optimization method known as States of Matter Search (SMS) was applied to solve the problem of smart power allocation for PHEVs. In the SMS approach, individual molecules that move around a given search space guided by unique evolutionary operators based on the principles of motion of thermal energy. The mechanisms and operators employed by SMS provide a better balance between the exploration and exploitation of new solutions, which in turn prevents several issues commonly found in other swarm optimization methods, such as those related to premature convergence.

The performance of the proposed method has been compared to other similar approaches in terms of performance and solution quality. Such comparisons were made by a cost function which takes into account the average state of charge of several hybrid vehicles within a charging station at different time steps of a regular 24-h day. Furthermore, several different PHEVs smart charging scenarios were also considered while performing the experimental comparisons. Experimental results show that, compared to the other compared methods, the proposed SMS-based PHEVs' smart power allocation approach yields significantly better results in terms of both performance and solution quality, which further proves the proficiency of the proposed approach for solving the complex problem of smart power allocation for PHEVs.

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Conflicts of Interest: The authors declare no conflict of interest.

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