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Evaluating the Effect of Distributed Generation on Power Supply Capacity in Active Distribution System Based on Sensitivity Analysis

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Abstract: In active distribution system (ADS), the access of distributed generation (DG) can effectively improve the power supply capacity (PSC). In order to explore the effect of DG on the PSC, the influence of accessed DG on the power supply of ADS has been studied based on generalized sensitivity analysis (SA). On the basis of deriving and obtaining the sensitivity of the evaluation indexes of the PSC to the parameters of connected DG, seeking for the DG access instruction for the purpose of improving the PSC, PSC evaluation model with inserted DG is established based on SA. The change degrees and trends of the PSC and its evaluation indexes caused by the slight increase of DG are calculated rapidly, which provides reference for the planning and operation of ADS. Finally, the feasibility and validity of the proposed theory are validated via IEEE 14-node case study.

Keywords: active distribution system; distributed generation; power supply capacity evaluation; sensitivity analysis

1. Introduction

The power supply capacity (PSC) of distribution network is the maximum load that the network can provide on the condition of meeting the branch power and node voltage constraints, which is determined by the operation mode of the distribution network and the growth mode of the load. Thus, the PSC is one of the most significant indexes that reflect the reliability and safety of active distribution system (ADS). The PSC evaluation is supposed to be performed in accurate and reliable system, which is not only able to provide the effective guidance for the grid optimization and economic operation, but also the feasible foundation for the system planning and construction [1–6].

ADS is assembled with the ability of active load control and power flow interaction. In ADS, with the progressive increase of the permeability of distributed generation (DG), flexible load, energy storage components and other distributed energy resources, the PSC of distribution network has been effectively improved [7–9]. Among the DG connected to the grid, those which are installed on the user side are widely distributed and usually in small capacities, easy to install and take usage of. The widespread application of DG in the distribution network can not only promote the local consumption of electricity power and reduce the cost of purchase, but also can effectively alleviate the power shortage during peak load and improve the PSC. Nevertheless, in certain distribution networks, the flexibility of DG provides multiple options for its location distribution, type matching and capacity allocation. The possibility of DG real-time adjustment closely affects the assessment of the PSC. Moreover, the influence on the PSC which is exerted by randomness and volatility of DG output cannot be ignored [10–14]. Therefore, in the background of ADS, the quantitative study of DG changes on the PSC is significant and indispensable to the accurate assessment.

In the field of the PSC evaluation in ADS, plenty of research works have been completed by domestic and foreign scholars. In [15], in view of the interaction of main transformers and the N-1 guideline, the adjustment of PSC calculation is investigated on the condition of the tie-line capacity and the main transformer's overload. Considering the connection of DG and distributed storage system, an N-1 reconstruction model for calculating maximum PSC was mentioned in [16]. Aiming for the characteristics of uncertainties of DG output and periodic fluctuation of grid load, [17] provided ADS with a real-time evaluation method of the PSC.

In all of the investments above, the access of DG is taken into consideration, implying its positive impact on PSC improvement. However, the development of the electricity market puts forward higher requirements for the accuracy and efficiency of the PSC evaluation. The PSC assessment provides real-time capacity and available capacity basis for the electricity market transactions of different time scales, which is an important guarantee for reliable power transaction. In the market environment, when the factors of accessed DG change, such as capacity, output and location, the accurate and rapid re-evaluation of PSC is required necessarily. In order to describe the change of the dependent variable caused by the fluctuation of the independent variable, the generalized sensitivity, one of the most significant means, has been widely used by scholars in multiple fields [18,19]. Drawing on this idea, the PSC evaluation system in ADS is established in this paper.

In order to improve the accuracy of PSC evaluation and explore the DG distribution guidance to enhance the PSC level, and further put forward an evaluation system which is applicable to general situations, an investigation of PSC evaluation in ADS based on sensitivity analysis (SA) is developed. In the proposed model, aiming to analyze the impact on PSC by accessed DG, the SA of indexes to DG factors which includes type, output and location has been carried out referring to the existing PSC evaluation indexes based on the idea of generalized sensitivity. On that basis, the PSC evaluation model is established with DG taken into account. On the foundation of the sensitivity formulas achieved by SA, combined with the PSC evaluation model, the fluctuation of PSC upon the changes of DG can be calculated reliably and rapidly, providing vital reference for the dispatching and operation of ADS.

In the following papers, the PSC evaluation indexes are selected and the sensitivity formulas are deduced in Section 2; then PSC evaluation models are established in Section 3; on that basis, Section 4 is a generalization of the whole method and case study is carried out in Section 5.

2. SA of PSC Evaluation Indexes to DG

2.1. Selection of PSC Evaluation Indexes for SA

In order to evaluate the effect of accessed DG on the PSC of the distribution system comprehensively, [20] put forward a series of general evaluation indexes from the aspects of numerical size, fluctuation situation and contribution degree, including the expectation, the shortage and shortage rate of PSC, the contribution amount and the contribution rate of DG, etc. Here four indexes are chosen as follows:

(1) Expectation of PSC (E_{PSC}):

$$E_{\mathrm{PSC}} = \sum_{i=1}^{M} S_{\mathrm{PSC}}(i)p(i) \tag{1}$$

where M is the number of the total probabilistic scenario; $S_{\mathrm{PSC}}(i)$ is the PSC of the i-th scenario, which is a discrete variable calculated using the parameters of the corresponding scenario; $p(i)$ is the probability of occurrence of the i-th scenario.

(2) Dissatisfied Amount of PSC (DA_{PSC}), representing the value of E_{PSC} that is less than its allowable value:

$$DA_{\mathrm{PSC}} = A - E_{\mathrm{PSC}} \tag{2}$$

where A is the allowed value of E_{PSC}.

(3) Contribution Rate of DG to Expectation of PSC (CR_{DTE}), representing the relative promotion level of E_{PSC} after the connection of DG:

$$CR_{\mathrm{DTE}} = \frac{E_{\mathrm{PSC}}}{E_{\mathrm{PSC,N}}} - 1 \tag{3}$$

where $E_{\mathrm{PSC,N}}(t)$ is the E_{PSC} without the connection of DG.

(4) Contribution Rate of DG to Dissatisfied Amount of PSC (CR_{DTDA}), representing the relative decrease level of DA_{PSC} after the connection of DG:

$$CR_{\mathrm{DTDA}} = 1 - \frac{DA_{\mathrm{PSC}}}{DA_{\mathrm{PSC,N}}} \tag{4}$$

where $DA_{\mathrm{PSC,N}}$ is the DA_{PSC} without the connection of DG.

2.2. Deduction of Sensitivity Formulas of PSC Evaluation Indexes to DG

2.2.1. Sensitivity of the Expectation of PSC to DG output

Calculate the sensitivity of E_{PSC} to DG output at node j suitable for any distribution system, which is shown in Equation (5):

$$\frac{\partial E_{\mathrm{PSC}}}{\partial P_{\mathrm{DG}}(j)} = \sum_{i=1}^{M} p(i) \frac{\partial S_{\mathrm{PSC}}(i)}{\partial P_{\mathrm{DG}}(j)} = \sum_{i=1}^{M} p(i) \sum_{n=1}^{N_l} f\left(\frac{\partial P_{\mathrm{L}}(i,n)}{\partial P_{\mathrm{DG}}(j)}\right) \frac{\partial P_{\mathrm{L}}(i,n)}{\partial P_{\mathrm{DG}}(j)} \tag{5}$$

where $P_{\mathrm{DG}}(j)$ is the DG output at point j; N_l is the sum of the quantities of the branches where loads locate in; $f(\cdot)$ is the empirical formula of the change amount of load carrying capacity on the change amount of the line capacity; $P_{\mathrm{L}}(i,n)$ is the active power flow to the n-th load node in the i-th scenario.

The solution of Equation (5) requires calculating the partial derivative of $P_{\mathrm{L}}(i,n)$ to $P_{\mathrm{DG}}(j)$, which is equivalent to the partial derivative of branch power to node injection power. When the grid is injected by unit power at node j, the active power of branch l can be deduced as follows [21].

Ignore the branch grounding impedance, and the branch power of the system can be presented as Equation (6):

$$P_{jk} = -G_{jk}U_j^2 + G_{jk}U_jU_k \cos\delta_{jk} + B_{jk}U_jU_k \sin\delta_{jk} \tag{6}$$

where P_{jk}, G_{jk} and B_{jk} are active power, conductance and susceptance, respectively; U_j and δ_j are the amplitude and phase angle of node j.

The branch power flow is not only related to the amplitude and phase angle of the node voltage, but also to the size of the power injected at each node, so there is Equation (7):

$$P_{jk}(U_j, \delta_j) = P_j(P_j, Q_j) \tag{7}$$

where P_j and Q_j are the active and reactive power injected at node j.

After performing the first-order Taylor series expansion on both sides, and combining with $P_{jk}(U_{j0}, \delta_{j0}) = P_{jk}(P_{j0}, Q_{j0})$, the Equation (7) can be presented as:

$$\begin{bmatrix} \frac{\partial P_{jk}}{\partial \delta_j} & \frac{\partial P_{jk}}{\partial U_j} \end{bmatrix} \begin{bmatrix} \Delta\delta_j \\ \Delta U_j \end{bmatrix} = \begin{bmatrix} \frac{\partial P_{jk}}{\partial P_j} & \frac{\partial P_{jk}}{\partial Q_j} \end{bmatrix} \begin{bmatrix} \Delta P_j \\ \Delta Q_j \end{bmatrix} \tag{8}$$

when the power flow is calculated using the Newton–Raphson method, the modified equation can be expressed as:

$$\begin{bmatrix} \Delta P_j \\ \Delta Q_j \end{bmatrix} = -\boldsymbol{J}\begin{bmatrix} \Delta\delta \\ \Delta U/U \end{bmatrix} = -\begin{bmatrix} H & N \\ J & L \end{bmatrix}\begin{bmatrix} \Delta\delta \\ \Delta U/U \end{bmatrix} \tag{9}$$

where $\boldsymbol{J}$ is the Jacobi matrix in the modified equation of the Newton–Raphson method; $H_{jk} = \frac{\partial \Delta P_j}{\partial \delta_k}$; $H_{jj} = \frac{\partial \Delta P_j}{\partial \delta_j}$; $N_{jk} = \frac{\partial \Delta P_j}{\partial U_k}U_k$; $N_{jj} = \frac{\partial \Delta P_j}{\partial U_j}U_j$; $J_{jk} = \frac{\partial \Delta Q_j}{\partial \delta_k}$; $J_{jj} = \frac{\partial \Delta Q_j}{\partial \delta_j}$; $L_{jk} = \frac{\partial \Delta Q_j}{\partial U_k}$; $L_{jj} = \frac{\partial \Delta Q_j}{\partial U_j}U_k$.

Substitute Equation (9) into Equation (8), and it gives:

$$\begin{bmatrix} \frac{\partial P_{jk}}{\partial \delta_j} & \frac{\partial P_{jk}}{\partial U_j}U_j \end{bmatrix}\begin{bmatrix} \Delta\delta_j \\ \frac{\Delta U_j}{U_j} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_{jk}}{\partial P_j} & \frac{\partial P_{jk}}{\partial Q_j} \end{bmatrix}(-\boldsymbol{J})\begin{bmatrix} \Delta\delta_j \\ \frac{\Delta U_j}{U_j} \end{bmatrix} \tag{10}$$

when $\Delta\delta_j$ and $\Delta U_j/U_j$ can be arbitrarily small, it gives:

$$\begin{bmatrix} \frac{\partial P_{jk}}{\partial \delta_j} & \frac{\partial P_{jk}}{\partial U_j}U_j \end{bmatrix} = \begin{bmatrix} \frac{\partial P_{jk}}{\partial P_j} & \frac{\partial P_{jk}}{\partial Q_j} \end{bmatrix}(-\boldsymbol{J}) \tag{11}$$

Transpose both sides of the equation at the same time, and Equation (12) can be obtained:

$$\begin{bmatrix} \frac{\partial P_{jk}}{\partial P_j} \\ \frac{\partial P_{jk}}{\partial Q_j} \end{bmatrix} = \left(-\boldsymbol{J}^{\mathrm{T}}\right)^{-1}\begin{bmatrix} \frac{\partial P_{jk}}{\partial \delta_j} \\ \frac{\partial P_{jk}}{\partial U_j}U_j \end{bmatrix} = \begin{bmatrix} \hat{H} & \hat{N} \\ \hat{J} & \hat{L} \end{bmatrix}\begin{bmatrix} \frac{\partial P_{jk}}{\partial \delta_j} \\ \frac{\partial P_{jk}}{\partial U_j}U_j \end{bmatrix} \tag{12}$$

Conduct further deduction of Equation (12) and it gives:

$$\frac{\partial P_{jk}}{\partial P_j} = \hat{H}\frac{\partial P_{jk}}{\partial \delta_j} + \hat{N}\frac{\partial P_{jk}}{\partial U_j}U_j \tag{13}$$

Substitute Equation (13) into Equation (5), and the general sensitivity formula of E_{PSC} to DG output at node j can be achieved:

$$\begin{aligned} \frac{\partial E_{\mathrm{PSC}}}{\partial P_{\mathrm{DG}}(j)} &= \sum_{i=1}^{M} p(i)\sum_{n=1}^{N_l} f\left(\frac{\partial P_{\mathrm{L}}(i,n)}{\partial P_{\mathrm{DG}}(j)}\right)\frac{\partial P_{\mathrm{L}}(i,n)}{\partial P_{\mathrm{DG}}(j)} \\ &= \sum_{i=1}^{M} p(i)\sum_{n=1}^{N_l} f(A)A \end{aligned} \tag{14}$$

where $A = \hat{H}\frac{\partial P_{\mathrm{L}}(i,n)}{\partial \delta_j} + \hat{N}\frac{\partial P_{\mathrm{L}}(i,n)}{\partial U_j}U_j$.

2.2.2. Sensitivity of Other Evaluation Indexes of PSC to DG output

According to the main idea of the section above, the sensitivity formulas of other PSC evaluation indexes to DG output can be derived.

(1) Sensitivity formula of DA_{PSC} to DG output

Deduce the sensitivity of DA_{PSC} to DG output at node j, which is shown in Equation (15):

$$\frac{\partial DA_{\mathrm{PSC}}}{\partial P_{\mathrm{DG}}(j)} = \frac{\partial[A - E_{\mathrm{PSC}}]}{\partial P_{\mathrm{DG}}(j)} = -\frac{\partial E_{\mathrm{PSC}}}{\partial P_{\mathrm{DG}}(j)} \tag{15}$$

(2) Sensitivity formula of CR_{DTE} to DG output

Deduce the sensitivity of CR_{DTE} to DG output at node j, which is shown in Equation (16):

$$\begin{aligned} \frac{\partial CR_{\mathrm{PSC}}}{\partial P_{\mathrm{DG}}(j)} &= \frac{\partial [E_{\mathrm{PSC}}/E_{\mathrm{PSC,N}}-1]}{\partial P_{\mathrm{DG}}(j)} \\ &= \frac{\partial E_{\mathrm{PSC}}/\partial P_{\mathrm{DG}}(j).E_{\mathrm{PSC,N}}-\partial E_{\mathrm{PSC,N}}/\partial P_{\mathrm{DG}}(j).E_{\mathrm{PSC}}}{[E_{\mathrm{PSC,N}}]^2} \end{aligned} \tag{16}$$

Because $E_{\mathrm{PSC,N}}$ is irrelevant with $P_{\mathrm{DG}}(j)$, and $\partial E_{\mathrm{PSC,N}}/\partial P_{\mathrm{DG}}(j) = 0$, Equation (16) can be further simplified as:

$$\begin{aligned} \frac{\partial CR_{\mathrm{PSC}}}{\partial P_{\mathrm{DG}}(j)} &= \frac{\partial E_{\mathrm{PSC}}}{\partial P_{\mathrm{DG}}(j)}\cdot\frac{E_{\mathrm{PSC,N}}}{[E_{\mathrm{PSC,N}}]^2} \\ &= \frac{1}{E_{\mathrm{PSC,N}}}\cdot\frac{\partial E_{\mathrm{PSC}}}{\partial P_{\mathrm{DG}}(j)} \end{aligned} \tag{17}$$

(3) Sensitivity formula of CR_{DTDA} to DG output

Deduce the sensitivity of $\mathrm{CR}_{\mathrm{DTDA}}$ to DG output at node j, which is shown in Equation (18):

$$\begin{aligned} \frac{\partial CR_{\mathrm{DTDA}}}{\partial P_{\mathrm{DG}}(j)} &= \frac{\partial [1-DA_{\mathrm{PSC}}/DA_{\mathrm{PSC,N}}]}{\partial P_{\mathrm{DG}}(j)} \\ &= -\frac{\partial DA_{\mathrm{PSC}}/\partial P_{\mathrm{DG}}(j).DA_{\mathrm{PSC,N}}-\partial DA_{\mathrm{PSC,N}}/\partial P_{\mathrm{DG}}(j).DA_{\mathrm{PSC}}}{[DA_{\mathrm{PSC,N}}]^2} \end{aligned} \tag{18}$$

Likewise, $\partial DA_{\mathrm{PSC,N}}/\partial P_{\mathrm{DG}}(j) = 0$, and Equation (18) can be further simplified as:

$$\begin{aligned} \frac{\partial CR_{\mathrm{DTDA}}}{\partial P_{\mathrm{DG}}(j)} &= -\frac{\partial DA_{\mathrm{PSC}}}{\partial P_{\mathrm{DG}}(j)}\cdot\frac{DA_{\mathrm{PSC,N}}}{[DA_{\mathrm{PSC,N}}]^2} \\ &= -\frac{1}{DA_{\mathrm{PSC,N}}}\cdot\frac{\partial DA_{\mathrm{PSC}}}{\partial P_{\mathrm{DG}}(j)} \\ &= \frac{1}{DA_{\mathrm{PSC,N}}}\cdot\frac{\partial E_{\mathrm{PSC}}}{\partial P_{\mathrm{DG}}(j)} \end{aligned} \tag{19}$$

2.2.3. Sensitivity of Evaluation Indexes of PSC to DG type

The effects of different types of DG on the PSC upgrading mainly reflect on their corresponding output. Thus, the sensitivity of PSC evaluation indexes to DG type can be expressed as below:

(1) Sensitivity formula of E_{PSC} to DG type

$$\frac{\partial E_{\mathrm{PSC}}}{\partial S_{\mathrm{PV}}(j)} = \frac{\partial E_{\mathrm{PSC}}}{\partial P_{\mathrm{DG}}(j)}\cdot P_{\mathrm{PV}}(j\,) = P_{\mathrm{PV}}(j)\sum_{i=1}^{M} p(i)\sum_{n=1}^{N}\left(\hat{H}\frac{\partial P_{\mathrm{L}}(i,n)}{\partial \delta_j} + \hat{N}\frac{\partial P_{\mathrm{L}}(i,n)}{\partial U_j}U_j\right) \tag{20}$$

$$\frac{\partial E_{\mathrm{PSC}}}{\partial S_{\mathrm{WG}}(j)} = \frac{\partial E_{\mathrm{PSC}}}{\partial P_{\mathrm{DG}}(j)}\cdot P_{\mathrm{WG}}(j\,) = P_{\mathrm{WG}}(j)\sum_{i=1}^{M} p(i)\sum_{n=1}^{N}\left(\hat{H}\frac{\partial P_{\mathrm{L}}(i,n)}{\partial \delta_j} + \hat{N}\frac{\partial P_{\mathrm{L}}(i,n)}{\partial U_j}U_j\right) \tag{21}$$

where $S_{\mathrm{PV}}(j)$ and $S_{\mathrm{WG}}(j)$ are the capacity of photovoltaic (PV) power and wind generation (WG) connected at node j, respectively; $P_{\mathrm{PV}}(j)$ and $P_{\mathrm{WG}}(j)$ are the output of PV and WG at node j, respectively.

(2) Sensitivity formula of DA_{PSC} to DG type

$$\frac{\partial DA_{\mathrm{PSC}}}{\partial S_{\mathrm{PV}}(j)} = \frac{\partial [A - E_{\mathrm{PSC}}]}{\partial S_{\mathrm{PV}}(j)} = -\frac{\partial E_{\mathrm{PSC}}}{\partial S_{\mathrm{PV}}(j)} \tag{22}$$

$$\frac{\partial DA_{\mathrm{PSC}}}{\partial S_{\mathrm{WG}}(j)} = \frac{\partial [A - E_{\mathrm{PSC}}]}{\partial S_{\mathrm{WG}}(j)} = -\frac{\partial E_{\mathrm{PSC}}}{\partial S_{\mathrm{WG}}(j)} \tag{23}$$

(3) Sensitivity formula of CR_{DTE} to DG type

$$\frac{\partial CR_{\text{PSC}}}{\partial S_{\text{PV}}(j)} = \frac{\partial [E_{\text{PSC}}/E_{\text{PSC,N}} - 1]}{\partial S_{\text{PV}}(j)} = \frac{1}{E_{\text{PSC,N}}} \cdot \frac{\partial E_{\text{PSC}}}{\partial S_{\text{PV}}(j)} \tag{24}$$

$$\frac{\partial CR_{\text{PSC}}}{\partial S_{\text{WG}}(j)} = \frac{\partial [E_{\text{PSC}}/E_{\text{PSC,N}} - 1]}{\partial S_{\text{WG}}(j)} = \frac{1}{E_{\text{PSC,N}}} \cdot \frac{\partial E_{\text{PSC}}}{\partial S_{\text{WG}}(j)} \tag{25}$$

(4) Sensitivity formula of CR_{DTDA} to DG type

$$\frac{\partial CR_{\text{DTDA}}}{\partial S_{\text{PV}}(j)} = \frac{\partial [1 - DA_{\text{PSC}}/DA_{\text{PSC,N}}]}{\partial S_{\text{PV}}(j)} = \frac{1}{DA_{\text{PSC,N}}} \cdot \frac{\partial E_{\text{PSC}}}{\partial S_{\text{PV}}(j)} \tag{26}$$

$$\frac{\partial CR_{\text{DTDA}}}{\partial S_{\text{WG}}(j)} = \frac{\partial [1 - DA_{\text{PSC}}/DA_{\text{PSC,N}}]}{\partial S_{\text{WG}}(j)} = \frac{1}{DA_{\text{PSC,N}}} \cdot \frac{\partial E_{\text{PSC}}}{\partial S_{\text{WG}}(j)} \tag{27}$$

3. PSC Evaluation Model Based on the SA of DG

Due to the sensitivity formulas of PSC evaluation indexes to DG which is deduced above, the PSC evaluation model is established as follows. On the basis of this evaluation model, the PSC change degrees, change trends as well as evaluation indexes brought by the DG fluctuation can be derived quickly and accurately in the same topology. Moreover, the optimized distribution of DG is able to be obtained aiming to improve the PSC.

3.1. Calculation Model of PSC

(1) Objective function

$$\max P_{\text{L}} = \sum_{i=1}^{N} P_{\text{L}i} \tag{28}$$

where P_{L} is the maximum active load which is supplied by the distribution network; N is the number of the load node; P_{Li} is the active load at load node i.

(2) Constraint condition

$$P_{\text{G}i} + P_{\text{DG}i} - P_{\text{L}i} = U_i \sum_{j=1}^{N} U_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) \tag{29}$$

$$Q_{\text{G}i} + Q_{\text{DG}i} - Q_{\text{L}i} = U_i \sum_{j=1}^{N} U_j (G_{ij} \cos\theta_{ij} - B_{ij} \sin\theta_{ij}) \tag{30}$$

$$U_i^{\min} \le U_i \le U_i^{\max} \tag{31}$$

$$I_l \le I_l^{\max} \tag{32}$$

$$S_T \le S_T^{\max} \tag{33}$$

where $P_{\text{G}i}$, $P_{\text{DG}i}$ and $P_{\text{L}i}$ are the active power of generator, DG and load at node i, respectively; $Q_{\text{G}i}$, $Q_{\text{DG}i}$ and $Q_{\text{L}i}$ are the reactive power of generator, DG and load at node i, respectively; G_{ij} and B_{ij} are the conductance and susceptance of branch i-j; θ_{ij} is the power factor angle between node i and j; U_i, $U_{\max_i}$ and $U_{\min\ i}$ are the upper limit and lower limit of the voltage at node i; I_l and $I_{\max\ l}$ are the current and its upper limit at branch l; S_T and $S_{\max\ T}$ are the power and its upper limit of the transformer T.

3.2. The Non-Parametric Kernel Density Probability Model of DG

In this paper, the non-parametric kernel density probability model of DG is employed to establish the multiple-scenario of SA for PSC evaluation.

Completely rooted from the data samples, non-parametric kernel density estimation method does not need any prior knowledge, which makes it one of the most effective methods to consider the characteristics of DG output. Furthermore, the method has been applied successfully in various areas, including load modeling, wind speed modeling and reliability index calculation [22,23]. In the PSC evaluation based on SA, the non-parametric kernel density estimation method is adopted to model the output of the DG accessed to the system.

Let the n samples of DG output be $p_1, p_2, \ldots, p_n$, and the probability density function of DG output $f_k(p)$ can be obtained based on the theory, which is shown in Equation (34):

$$f_k(p) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{p-p_i}{h}\right) \tag{34}$$

where h is the bandwidth; n is the number of the sample; $K(\bullet)$ is the kernel function.

Select the commonly used Gaussian function as the kernel function, which is expressed in Equation (35).

$$K(u) = 1/\sqrt{2\pi}e^{-u^2/2} \tag{35}$$

On the foundation of the Latin hyper-cube sampling technique [24], the non-parametric probabilistic model of DG output is taken to extract samples with the length of M, and the multiple-scenario probability model of DG output is established.

3.3. PSC Reckoning Based on SA

Based on the deduction and solution of the sensitivity formulas of PSC evaluation indexes, the amount of PSC change is supposed to be calculated when the location, capacity or type of DG differs. According the change amount, the PSC is able to be reckoned when the connection situation of DG changes, which is shown in Equation (36).

$$S'_{\mathrm{PSC}}(t) = S_{\mathrm{PSC}}(t) + \sum_{j=1}^{N}\sum_{k=1}^{K} SA(k)C_{\mathrm{DG}}(j,k) \tag{36}$$

where $S'_{\mathrm{PSC}}(t)$ is the reckoning value of PSC at time point t; $S_{\mathrm{PSC}}(t)$ is the evaluation value of PSC before the DG connection changes at time point t; $SA(k)$ is the sensitivity of PSC to the k-th DG change (when k = 1, $SA(k)$ represents the change of DG output; when k = 2, $SA(k)$ represents the change of DG type); $C_{\mathrm{DG}}(j,k)$ is the parameter which represents the k-th DG change at node j.

Similarly, the reckoning formula of other PSC evaluation indexes to DG is given in Equation (37).

$$S'_{\mathrm{PSC},EI}(t) = S_{\mathrm{PSC},EI}(t) + \sum_{j=1}^{N}\sum_{k=1}^{K} SA_{EI}(k)C_{\mathrm{DG}}(j,k) \tag{37}$$

where $S'_{\mathrm{PSC,EI}}(t)$ is the reckoning value of the EI-th PSC evaluation index at time point t; $S_{\mathrm{PSC,EI}}(t)$ is the evaluation value of the EI-th PSC evaluation index before the DG connection changes at time point t; $SA_{EI}(k)$ is the sensitivity of the EI-th PSC evaluation index to the k-th DG change.

4. Steps of PSC Evaluation Based on SA

The steps PSC evaluation based on SA can be summarized as follows and presented in Figure 1:

(1) Base on the demands of PSC evaluation from various aspects, multiple PSC evaluation indexes are selected according to the consideration of numerical size, adequacy, and contribution degree of DG.

(2) Based on the generalized sensitivity formulas, the sensitivity formulas of position, output and type of accessed DG are defined and deduced.

(3) Taking constraint conditions into consideration, including the active and reactive power flow, branch capacity and node voltage, the basic model of PSC evaluation is established.
(4) On the basis of the non-parametric kernel density estimation theory, the uncertainty of DG output is simulated and the probability model is established. The output samples are extracted from the model by Latin hyper-cube sampling technique to form multiple scenarios with different probabilities.
(5) Due to the traditional PSC evaluation methods, the PSC values are calculated when the type and capacity of the connected DG differs.
(6) Calculate the sensitivity value corresponding to the PSC and its evaluation indexes.
(7) Based on the SA results of step 6, the PSC evaluation results in ADS can be reckoned when the accessed DG are of the same type.
(8) Based on the SA results of step 6, the PSC evaluation results in ADS can be reckoned and compared when the types of the accessed DG are different.
(9) Comparing with the PSC evaluation results in step 5, and the PSC reckoning results based on SA in steps 7 and 8, the feasibility and effectiveness of the PSC evaluation based upon the SA can be verified.

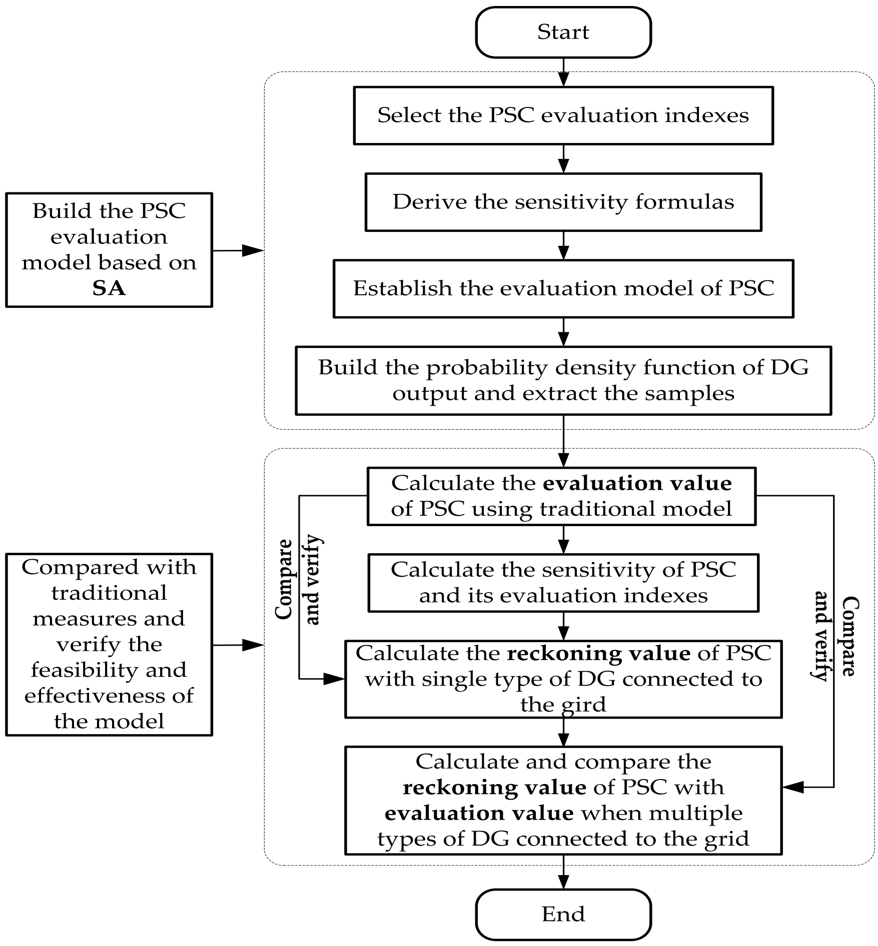

Figure 1. Flow chart of PSC evaluation based on SA.

5. Case Study

5.1. General Situation of the System

The modified IEEE 14-node system is employed to the PSC evaluation in ADS based on SA as in Figure 2, which is simulated in the platform of MATLAB.

In Figure 2, node 1 is the balance node and the rest are all load nodes. The reference capacity of the system is 100 MVA, and the reference voltage of the system is 23 kV. Set the power limit of the branches as its thermal stability limit capacity, and voltage allowable range as $1 \pm 5\%$.

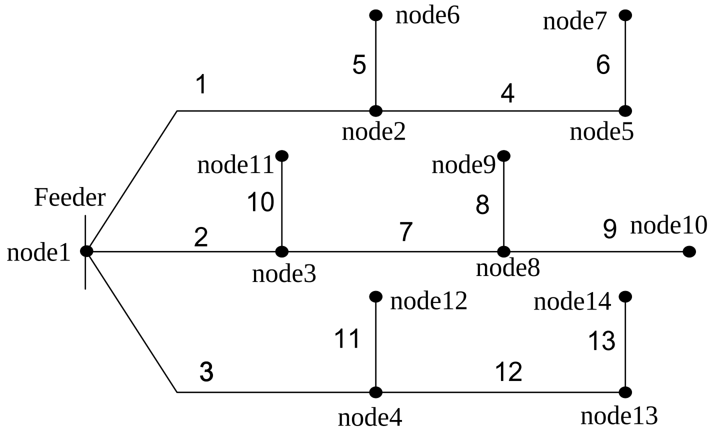

Figure 2. IEEE 14-node system structure.

5.2. General Situation of the Distribution System

Based on the principle of non-parametric kernel density estimation, the probability model of DG output is established. On that basis, applying the Latin hyper-cube technology, the probability samples at the length of 10 are extracted from the output intervals of PV power and WG with the rated power of 1 MW, which is shown in Table 1. Due to the non-sequential sampling results, the weighted sums of the corresponding DG samples are taken as their probabilistic output.

Table 1. Sampling results of the probabilistic output of DG.

Number	PV		WG	
	Sampling Power/MW	**Probability**	**Sampling Power/MW**	**Probability**
1	0.0385	0.0905	0.0550	0.0682
2	0.0955	0.1377	0.1554	0.1393
3	0.1568	0.1271	0.1997	0.1428
4	0.2922	0.0890	0.2486	0.1378
5	0.3924	0.0828	0.3039	0.1285
6	0.5458	0.0946	0.3242	0.1261
7	0.6238	0.1051	0.4320	0.0837
8	0.7073	0.1072	0.4412	0.0799
9	0.7888	0.0791	0.5830	0.0367
10	0.8525	0.0869	0.7700	0.0570

5.3. PSC Evaluation Based on SA

5.3.1. Sensitivity Calculation of PSC Evaluation

Based on the sensitivity formulas of PSC evaluation indexes to DG, which is derived from the previous theory, the evaluation indexes and the sensitivity values of the case are calculated, taking node 3 as an example. The results are expressed in Table 2.

Table 2. Calculation results of PSC evaluation indexes and corresponding sensitivities at node 3.

Calculation Results	PSC Evaluation Indexes			
	E_{PSC}	DA_{PSC}	CR_{DTE}	CR_{DTDA}
PSC evaluation indexes	42.5819/MW	17.4181/MW	0/MW	0/MW
Sensitivity of DG output	1.5384	−1.5384	0.03613	0.08832
Sensitivity of PV capacity	0.6532	−0.6532	0.01534	0.03750
Sensitivity of WG capacity	0.4689	−0.4689	0.01101	0.02692

5.3.2. Reckoning of the PSC Evaluation Indexes Based on SA

(1) Reckoning of the expectation of PSC based on SA

On the condition of single type of DG (take PV power for instance) accessed to the system, let the capacity of DG increase at the speed of 1 MW/time. The E_{PSC} values were reckoned and the comparison of reckoning and evaluation values are completed based on sensitivity, which are illustrated in Figures 3 and 4, respectively. According to Figure 3, the smaller the DG capacity accessed to the grid, the more concise the SA can be. In Figure 4, it is obvious that the relative error increases as the DG capacity grows. It can be seen that, in order to ensure the accuracy of the results obtained, the reckoning of E_{PSC} according to SA is preferably within a small range.

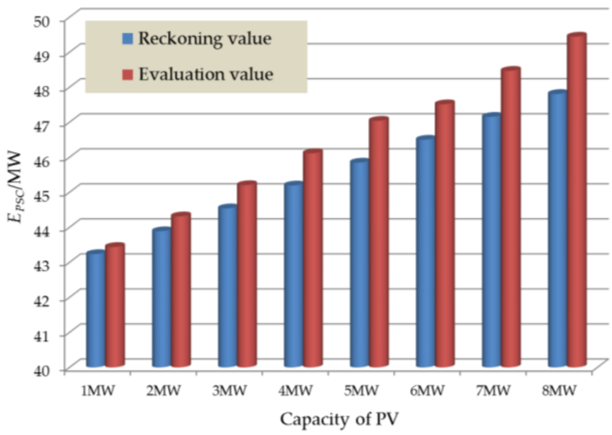

Figure 3. The fluctuation situation of the reckoning value and evaluation value of E_{PSC} with single type of DG connected.

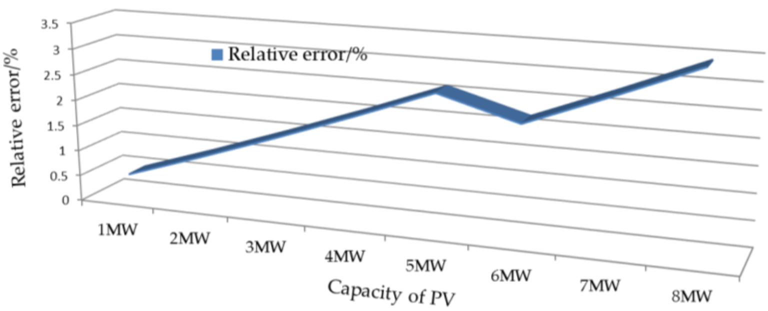

Figure 4. The fluctuation situation of the relative error of reckoning value to evaluation value of E_{PSC} with single type of DG connected.

(2) Reckoning of the other PSC evaluation indexes based on SA

Except for E_{PSC}, sensitivities of the rest of the PSC evaluation indexes to DG mainly contribute to calculate the corresponding evaluation indexes. Here, the sensitivity of each index to the DG output is calculated as an example. Figure 5 presents the comparison between the evaluation results and reckoning results of the four PSC indexes based on sensitivity. The contrast results show that the PSC evaluation indexes calculation can basically rely on the corresponding sensitivities, and the results of which can provide certain reference value. In addition, with the enlargement of the reckoning radius, the prediction accuracy is basically decreasing. It means that the application of SA is conditional, which is suitable for the calculation in a small range.

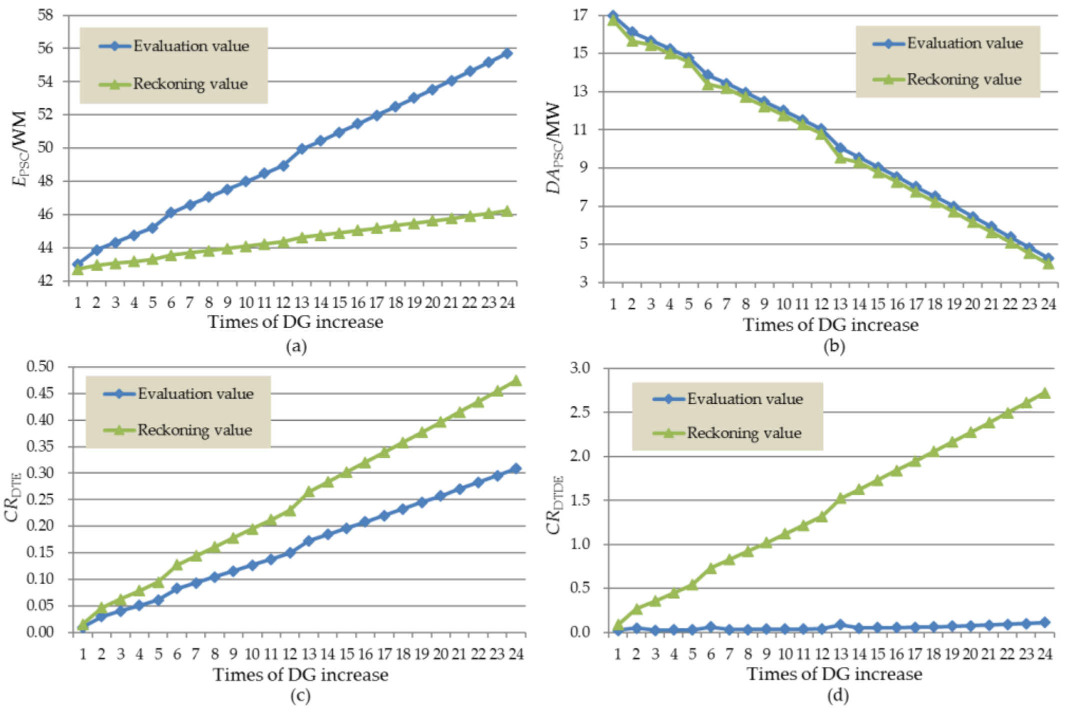

Figure 5. (**a**) Reckoning results of the E_{PSC} based on SA; (**b**) Reckoning results of the DA_{PSC} based on SA; (**c**) Reckoning results of the CR_{DTE} based on SA; (**d**) Reckoning results of the CR_{DTDE} based on SA.

(3) Ratio selection of DG Based on SA

In order to investigate the influence of different types of DG on the PSC, the connection of pure PV, pure WG and the combination of PV and WG are considered, respectively. In each case, E_{PSC} is reckoned using sensitivity index and compared with the evaluation results, as shown in Figure 6. The differences between PSC calculation results are mainly attributed to the type ratios of DG. The E_{PSC} change is mostly due to the actual DG output, in this example, the output is represented by probabilistic average of sampling.

Figure 6 provides users the optimal choice of DG capacity ratio. In this case, it suggests that with the same capacity, the PSC value is the most satisfying when pure PV is accessed, and the one with the connection of PV and WG in a same-size ratio ranked second, which makes the third system all installed with WG the last choice. It follows that the PSC improvement and the increase of the load level of the ADS are more relevant to PV power than WG generation here. This phenomenon may be attributed to the significant contribution of PV power during the day time, which is capable of meeting the peak electricity demand better.

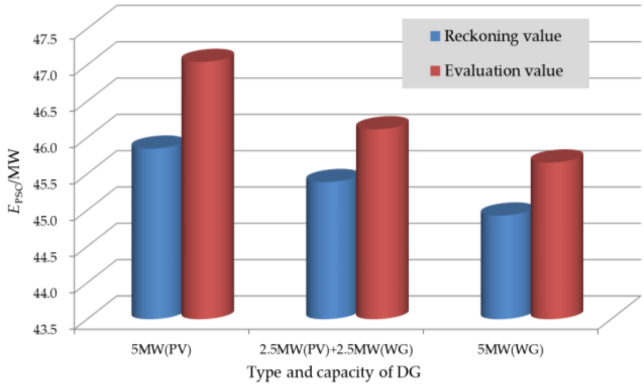

Figure 6. The comparison of the reckoning value and the evaluation value of E_{PSC} with different types of DG accessed.

6. Conclusions

The PSC has a direct bearing on the load capacity of the ADS. For the sake of improving the accuracy of the PSC evaluation and exploring the optimal ratio of PSC to DG, the PSC evaluation indexes are selected and the sensitivity formulas are deduced in view of the idea of the generalized sensitivity. On that basis, the PSC and its evaluation indexes are reckoned precisely and rapidly, providing important references for the operation and dispatching of the ADS. The results of the case study indicate that:

(1) On the foundation of the generalized sensitivity, the sensitivity formulas applicable to PSC evaluation are deduced and calculated, the analysis results of which are capable of providing the sensitivity of PSC evaluation indexes to the type, output and capacity of DG at specific node directly and effectively.
(2) In view of the sensitivity results obtained above, the fluctuation of PSC and its evaluation indexes are supposed to be reckoned at specific node when the type and capacity of added DG varied, offering vigorous evidence for the PSC evaluation of multiple perspectives.
(3) Considering the connection of various types of DG at the same node, the optimal DG proportion of type and capacity can be presented, as well as its corresponding PSC numerical result, which contributes to lowering the investigation costs and optimizing the PSC.
(4) The application of the PSC evaluation based on SA is limited by the DG, the capacity of which is supposed to be within a certain small boundary. Furthermore, the calculation precision is in inverse proportion to the distance between reference point and reckoning point. On one hand, the PSC evaluation based on SA is supposed to meet the demand of the operation and scheduling of ADS basically; on the other hand, starting from the point of view of upgrading and planning, it may be necessary to build up a new sensitivity criteria to provide guidance for the component renewal and the further expansion of the power grid.

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Abbreviations

The following abbreviations are used in this manuscript:

PSC	Power supply capacity
ADS	Active distribution system
DG	Distributed generation
SA	Sensitivity analysis
PV	Photovoltaic
WG	Wind generation
E_{PSC}	Expectation of PSC
DA_{PSC}	Dissatisfied Amount of PSC
CR_{DTE}	Contribution Rate of DG to Expectation of PSC
CR_{DTDA}	Contribution Rate of DG to Dissatisfied Amount of PSC

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