

Hydropower Bidding Using Linearized Start-Ups

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Abstract: Hydropower producers must submit bids to electricity market auctions where they state their willingness to produce power. These bids may be determined using a mixed-integer linear stochastic program. However, for large interconnected river systems, this program may be too complex to be solved within the time limits set by current market rules. This paper investigates whether a linear approximation to start-ups can be used to reduce the computational burden without significantly degrading the solution quality. In order to investigate the trade-off of time versus solution quality, linear approximation is compared to a formulation that uses binary variables in a case study that simulates the operation of a reservoir system over time.

Keywords: electricity auctions; mixed-integer programming; stochastic programming; unit commitment

1. Introduction

How much computing time can be saved by approximating hydro generator startups with linear relationships and continuous variables? What is the accompanying loss in solution quality? In this paper, we quantify this trade-off in the context of bidding on hydropower in electricity market auctions.

In modern power systems, electricity is traded through auctions where producers participate by submitting bids that indicate their willingness to produce power. The format of these auctions varies between countries, but the norm is that most of the power is traded in day-ahead or hour-ahead markets. The problem of determining bids for such markets has been investigated by many researchers; see for instance [1,2] for reviews. Works that describe a similar setting as ours can be found in [3–5].

At the time of bidding, prices and inflows for the coming day are not yet known, and stochastic programming is one of the suggested methods for the optimization of bids [6]. In addition, the on/off decisions of the generating units have a binary nature that requires integer variables. For large producers, the bidding problem, as formulated by a stochastic mixed-integer linear program, may be too computationally demanding to be solved within the time frame set by current market rules, and heuristic methods are currently used in the industry.

The bidding problem is closely related to the unit-commitment problem [7,8] that determines optimal schedules for generating units. There are, however, some important differences—the bidding problem determines market bids, whereas the unit commitment or scheduling problem determines optimal production schedules by allocating production among the units in the system.

In this paper, we investigate the extent to which it is necessary to have a very detailed representation of the production system when determining bids. Bids are determined based on possible future schedules. However, as these schedules are never actually implemented, it might not be necessary to calculate them in detail. Specifically, we look at the on/off-decision of the generation units. In the unit commitment problem, it is obviously important to include the binary nature of start-ups; units have to be either on or off in the final production schedule. To determine bid curves, however, it might be sufficient to model start-ups by a linear relationship.

We therefore analyse the loss in solution quality when using linearized start-ups in a stochastic programming model that determines the day-ahead market bids for hydropower. The motivation for using a linear approximation is the reduction in computational complexity obtainable when solving linear programs (LPs) rather than mixed-integer programs (MILPs). Our results indicate that a substantial reduction in calculation time can be obtained without significantly degrading the solution. In Section 2, background and literature on the hydropower bidding problem are presented. How to evaluate the performance of the linearized formulation is discussed in Section 3. Sections 4 and 5 present the modelling, before results from a case study are reported in Section 6. Section 7 gives some conclusions.

2. The Hydropower Bidding Problem

In the rest of this paper, we focus on the day-ahead market at NordPool, which is the most important market place for power producers in the Nordic area with 391 TWh traded in 2016 [9]. The framework for day-ahead trading for Nordic hydropower producers is explained in [6], which also presents a model for optimal bidding based on mixed-integer linear stochastic programming. Every day, producers must determine the bids for the next day's operation as a set of price-volume pairs for each hour of the day. The points describe a piecewise linear supply curve. Figure 3 in Section 6.1 shows examples of bid curves. Bids are based on the current system state and expectations of future prices and inflows, which at the time of bidding can only be forecasted.

If producers are assumed to be price takers operating in a perfect market, it is optimal to bid according to the marginal costs of production. The marginal costs of hydropower are given by the opportunity cost of releasing water now that could have been stored for production in the future. In this work, focus is on short-term operations, that is, the optimization of the hydropower resources within a horizon of about one week. At the end of the week, linking to longer-term scheduling is accomplished through water value curves. In the simplest case, water should be produced (saved) whenever the price is higher (lower) than the water value. Even within the short-term horizon, the marginal costs will be given by opportunity costs—production in this hour may deplete the resources (amount of water available, pressure height) for production in a future hour. This temporal aspect of marginal cost is an intrinsic property of hydropower production scheduling.

The task at hand is to calculate (an approximation to) the marginal costs in order to determine the bid curves. Of specific importance are the on/off decisions of the generating units, as these link decisions in time. If a unit is started this hour, it may be cheaper to keep the unit running for a few consecutive hours even if the comparison between prices and water values for the individual hours indicate that the unit should be turned off. This is the core of the well-known unit commitment problem, which is of paramount importance in thermal systems due to high costs and long start-up times [8]. Hydropower units typically have lower costs for start-ups and units that are offline can come online in about 15 min, often much faster. Even so, most hydropower units have minimum production levels or forbidden operating zones that require consideration. According to [10,11], start-up costs for hydropower are related to wear on equipment that may increase the risk of failure and eventually lead to increased maintenance or unavailability costs. Start-up costs of multiple production technologies are described in a linear model in [12].

Most large Nordic hydropower producers solve the unit commitment problem by successive linear programming, where mixed-integer linear programs are solved in iterations to account for nonlinearities and state-dependencies [13]. This method has been extended to a stochastic programming model for optimization of bids in [14]. For the current topic, it is important to emphasize that the method involves solving stochastic MILPs in iterations and that each model instance thus should be solved within a few minutes.

3. Simulation

One idea to speed up calculations is to use linearized start-ups instead of the binary formulation. The question is then how much of an error it is in order to approximate the on/off state of the generators by using continuous variables. Use of continuous variables means that it is possible to turn a generator on by a fractional amount. This does not correspond to any feasible operating point, but it might be a reasonable approximation when calculating marginal costs, especially since it is currently not possible to use the full MILP formulation due to time constraints [14]. In [15], it is found that using a linear approximation to start-ups gives an average deviation of 0.01% from the objective function value of the equivalent binary formulation. This indicates that the profits obtained when using linear approximation are not significantly less than when using the MILP model. Linear approximation is thus useful as it obtains nearly the same profits with reduced computational burden. However, the results in [15] are based on testing individual days. The usefulness of linear approximation should rather be based on how the bids perform over time. In this work, we apply a rolling-horizon simulation framework to validate the performance of linear approximation.

In [16], formal optimization of bids based on stochastic programming is compared to heuristic methods currently used by the industry by simulating the operation of a reservoir system over time. The results indicate that formal optimization gives a 0.61% improvement in objective function value as well as a reduced risk of spillage due to a more robust reservoir management strategy. A similar simulation framework is used in this paper to evaluate the performance of the binary and linearized version of the stochastic formulation. The simulation set-up and results are described in Section 6.2.

Other aspects than profits are also important in the operation of hydro systems, such as environmental considerations concerning minimum and maximum reservoir levels or river flows, or technical aspects such as limiting the number of start-ups to reduce tear and maintenance costs. Also, it is important to comply with all legislation set by the system operator. This includes being able to deliver on the commitments from the day-ahead auction, i.e., *to plan in balance*, which means that producers bid based on the expected generation in the day-ahead market and use the intra-day and balancing markets only for adjustments. This may change in the future if larger shares of intermittent renewable production force a shift in priority and/or volumes away from the day-ahead market. Interesting works on multi-market bidding can be found in [17–21], but this is not pursued further in this paper.

It is important that the linear relaxation does not make it harder for the producer to plan in balance, specifically that the fractional start-ups allowed in the bid optimization do not cause commitments in forbidden regions of the generating units. After market clearing, when prices and commitments for the coming day are known, producers have the opportunity to reschedule to cover the commitments in an optimal (cost-minimizing) way. This will allow the producer to avoid forbidden zones, with the intraday and balancing markets as an opportunity for adjusting unacceptable commitments. Even if the bid curve represents fractional start-ups, only full on/off states are allowed in operations. By simulating the operation of the hydropower system, we can investigate whether the linear relaxation is able to provide commitments that comply with all constraints and legislation, as well as give performance measures in terms of profits obtained.

4. MILP Formulation

For a given system of reservoirs and hydropower plants, the aim is to optimize the operation of the installed resources by determining the profit-maximizing bids to be submitted to the day-ahead market. This may be formulated as a two-stage mixed-integer linear stochastic program. The first-stage decision is to determine optimal hourly bid volumes x_{bt} for a set of fixed bid prices, P_b^{Bid} , $b \in B$. The second-stage decisions are related to the production of hydropower in order to supply the volumes sold to the market. The following presentation explains a basic formulation of the hydropower bidding problem, based on [6].

The objective is to maximize the revenues from selling power, less the costs related to start-ups, shut-downs and other costs related to releasing the scarce water resources for production. The objective is expressed in Equation (1).

$$\max \sum_{st} \Phi_s(P_{st}y_{st} - \sum_{gst} (C^{start}v_{gst} + C^{stop}w_{gst}) - C^{penalty} \sum_{st} (z_{st}^+ + z_{st}^-) + \sum_{rs} \Pi_{rs}\sigma_{rst=T}) \quad (1)$$

The first term in Equation (1) is the power price in the day-ahead market times the committed (sold) volume. The next term represents start-up and shut-down costs for all the generators in the system. The third term is a penalty incurred if production differs from the committed volume. The final term is the value of water stored in the reservoirs at the end of the short-term horizon. The value of storage is given by long-term scheduling models and is a known input to the bidding model.

The committed volume is found in the market clearing as an interpolation between the market price, P_{st} , and the bid curve of each producer. Following [6], the bid curve is formulated as a piecewise linear curve given by price-volume points that determine the optimal offer curve of the producer. This is expressed in Equation (2), where volume bids x_{bt} are found for a set of fixed bid prices, $P_b^{Bid}, b \in B$.

$$y_{st} = \frac{P_{st} - P_{b-1}^{Bid}}{P_b^{Bid} - P_{b-1}^{Bid}} x_{bt} + \frac{P_b^{Bid} - P_{st}}{P_b^{Bid} - P_{b-1}^{Bid}} x_{b-1t}, \quad \text{if } P_{b-1}^{Bid} \leq P_{st} < P_b^{Bid} \quad (2)$$

The committed volume in each hour has to equal the produced volume in that hour, as given by Equation (3). If it is not possible to deliver on the commitments, power can be bought or sold in the intraday or balancing markets. These volumes are penalized in the objective function to reflect that the producer is obliged to plan in balance in the day-ahead market. In this formulation, any unbalance between the committed volume and the produced volumes are assigned to the slack variables z_{st}^+ or z_{st}^- which may be thought of as the producer's recourse action.

$$y_{st} = \sum_g p_{gst} + z_{st}^+ - z_{st}^- \quad (3)$$

The release of water is linked to power production by a piecewise linear concave production function for each generator; see Figure 1. The production function has I line segments, each represented with a discharge volume q_{gsti} limited by an upper bound Q_{gi}^{max} and a power output rate E_{gi} . For each unit, the sum of water released from all segments is the total discharge through the unit, as in Equation (4). Similarly, the sum of power produced from all segments is equal to the total power produced from the unit. This is expressed by Equation (5).

$$\sum_i q_{gsti} = q_{gst} \quad (4)$$

$$\sum_i E_{gi} q_{gsti} = p_{gst} \quad (5)$$

If the unit is on, it should produce between given minimum and maximum levels. This is where the on/off state comes into play. Minimum and maximum production levels are considered by Equations (6) and (7).

$$p_{gst} \geq P_g^{min} u_{gst} \quad (6)$$

$$p_{gst} \leq P_g^{max} u_{gst} \quad (7)$$

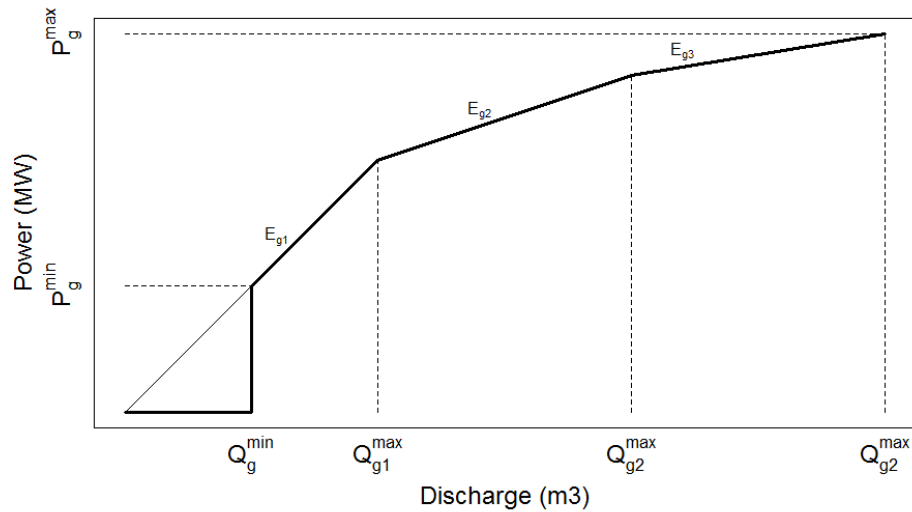


Figure 1. An example of a production function for a generator with three segments. In the mixed-integer program (MILP) formulation, production needs to follow the thick black line, and can either be 0 or above minimum generation. Above minimum generation, production is free to vary over all segments up to P^{max} . In the linearized formulation, production is allowed to follow the thin line between P^{min} and 0, which corresponds to a fractional start.

In addition to the equations that describe how water is transformed into power, a mass balance for water stored in the reservoirs must also be maintained for every time step. This is expressed in Equation (8).

$$s_{rst} = s_{rst-1} - \sum_{gr} q_{gst} + I_{rst} \quad (8)$$

In the MILP formulation, the state variables u_{gst} that represent the on/off state of the generators are binary. Start-up and shut-down variables are linked to the state-variables through the logical restrictions in Equations (9) and (10), where u_{gst} is binary and $0 \leq v_{gst}, w_{gst} \leq 1$.

$$v_{gst} \geq u_{gst} - u_{gst-1} \quad (9)$$

$$w_{gst} \leq u_{gst} - u_{gst-1} \quad (10)$$

Equation (8) states that the amount of water stored at the end of this time step is the amount of water stored at the end of the last time step, less any water released for production from the generators that belong to the reservoir, plus any inflows to the reservoir. Inflows may be stochastic and are therefore denoted with scenario index s . The reservoir level at the end of the short-term horizon is valued by the water value in the objective function in Equation (1). Equation (8) does not consider connections between reservoirs which are important for multi-reservoir systems.

To summarize, the MILP formulation of the bidding problem is to optimize the objective in Equation (1) subject to the constraints in Equations (2)–(8). Input for prices, P_{st} , is given in the form a scenario tree. If inflows are also stochastic, the scenario tree must specify values for both prices and inflows. The size and complexity of the problem depends on the number of scenarios as well as the size and characteristics of the river system(s) to be optimized. Details about the bidding model may be found in [6,14].

5. LP Formulation

If the restriction that u_{gst} is binary is relaxed, the formulation is equivalent to [22], which describes how start-ups for thermal units can be included in a linear formulation of a long-term hydrothermal scheduling model. This is the same method as applied in the models that are used to find boundary conditions (water values) for the bidding problem. Referring to Figure 1, relaxing the binary restriction means that production is free to vary between 0 and P_g^{max} . How well the real production function is approximated by the linear relaxation will depend on the type and specifications of generating units installed. By avoiding integer variables in the formulation, faster run-times may be achieved but this comes at the expense of less accurate modelling. In this case, there is a trade-off between decreased calculation time and more accurate modelling which in turn gives a better assessment of marginal costs.

6. Case Study

The MILP and LP formulation are implemented in the existing framework for short-term scheduling that is used by the Nordic hydropower industry [13,14]. This model is implemented in C and can be solved with CPLEX, Gurobi or COIN solvers. We have chosen to work with the existing framework to assess the reduction in calculation time actually obtainable in operations, and to be able to use real data for testing. The two formulations are within a simulation framework to assess the reduction in solution quality when using linear approximation.

We test the two formulations on data for a river system located in the south of Norway, consisting of seven reservoirs and six hydropower plants. There are nine generating units in the system, with a total maximum production of 390 MW. A sketch of the topology is given in Figure 2. It is assumed that inflow is known for the week ahead, so the scenario tree only contains values for prices. Water values are given by longer-term scheduling models and are considered known input parameters to the bidding problem in the case studies.

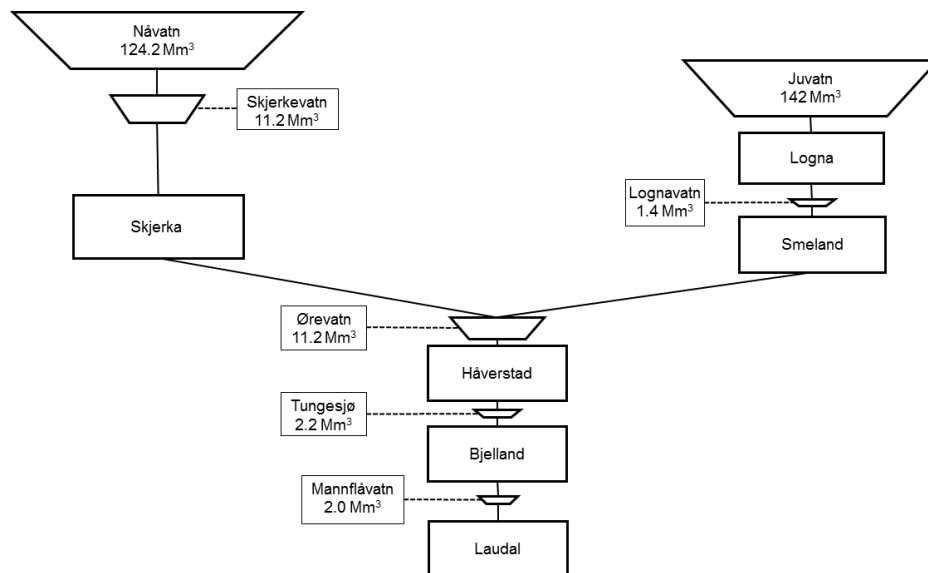


Figure 2. A sketch of the water course topology used in the case study. There is large storage capacity in the upstream reservoirs, while the downstream parts are similar to run-of-river systems due to small reservoir sizes. The challenge is to schedule releases from the upstream reservoirs in order to keep the pressure height in the smaller downstream reservoirs high while at the same time having enough free capacity in the reservoirs to absorb the inflows coming in. River-flow time delay between reservoirs further complicates this challenge.

We have chosen to work with this reservoir system as it represents a typical problem instance in terms of size and flexibility of a river system. Large hydropower producers must determine bids for their total portfolio, i.e., for all their river systems within a market area. The bidding problem can be solved for each river separately or as a larger problem for the portfolio. The latter approach is most common because it increases flexibility. The portfolio may consist of several river systems with generating units of varying size. Even though each individual river system may have limited flexibility and bottlenecks, the producers have quite a lot of flexibility when considering the total portfolio. For producers with smaller portfolios, with a single river or even a single generating unit, there is less flexibility when determining production schedules and unfortunate commitments may lead to extensive use of penalties, which represent trading on intraday or balancing markets. In this work, deviations between the committed volume and the producer volume are considered a pure loss. It may therefore be more important for small producers to get the bids right in order to avoid inefficient production commitments. For small producers, it might therefore be necessary to use the MILP formulation, which will also be feasible because the portfolio and thus the problem size is smaller. However, as most hydropower producers that operate on NordPool hold extensive portfolios, the MILP formulation will, in general, not be feasible in terms of calculation time, and the simplification to the LP formulation is needed.

6.1. Characteristics of Bid Curves

For each day, the model is run for a 1-week horizon with hourly resolution. Bidding is done for hour 25–48, i.e., for the next day's operation, as the production schedule for today is already known at the time of bidding for the day-ahead market. The longer horizon is kept for consistent coupling to long-term models via the water value. The number of scenarios may vary between days depending on the volatility of forecasted prices. Usually, between 15 and 30 scenarios are used. Numbers for problem size and calculation time for an instance with 27 scenarios are given in Table 1.

Table 1. Example numbers for problem size and calculation time for the mixed-integer and linear model version for a 27-scenario, 168 h instance.

	Variables	Binaries	Constraints	MIP Gap	Objective (€)	Time (s)
MILP	590,142	27,216	243,666	0.001%	5,396,444	612.43
MILP	590,142	27,216	243,666	1%	5,401,629	481.89
LP	590,142	-	243,666	-	5,416,653	9.12

From the numbers in Table 1, it is evident that the speed-up when using the LP formulation is substantial, as expected. The calculation time of the MILP model can be reduced by increasing the allowed MIP gap, and this will give a trade-off between optimality and calculation time. However, as both the reduction in computational burden and the accuracy of the modelling are more severely affected by linear approximation than by increasing the MIP gap, we only analyze the effect of linear start-ups in the following. On average, when using linear approximation, we obtain speed-ups of about 25 times for the instances tested. If the differences in obtained results are small, this shorter calculation time is a major improvement.

Figure 3 shows the obtained bid curves for the MILP and LP formulations for one of the days tested. Comparable results are obtained for other days. The curves show the marginal uploading of generating units in the system—cheaper units are committed first, then more and more production is offered as the price increases. The overall view in Figure 3a shows that the two formulations give similar results. However, Figure 3b,c show that there are some differences between the curves. For instance, the MILP model offers larger volumes for low prices for all hours shown. This is because the MILP model has to produce above the minimum generation level of each unit, or keep the unit turned off. For most hours during the day in question, this difference in volume for low prices is the only visible difference between the bid curves. These results indicate that it might be more important

to use binaries for expensive plants than for cheap plants. Cheap plants will most often be committed to producing well above the minimum, so start-up costs are not that relevant, while for the more expensive plants it is important to accurately represent the step from 0 to minimum production. From Figure 3c, another difference is observable: the curve for the MILP model seems to have a more step-like shape. The steps correspond to starting up a generator from 0 to minimum production. In the linear relaxation, generators are allowed to vary continuously from 0, which gives a slightly smoother curve.

The examples for this day indicate that the two formulations give very similar results, at least in the most likely range of prices. Using linear approximation, it looks like we are able to obtain comparable results with a significantly lower running time. This is highly relevant from a practical perspective, since it will reduce the running times of optimization models used for hydropower scheduling.

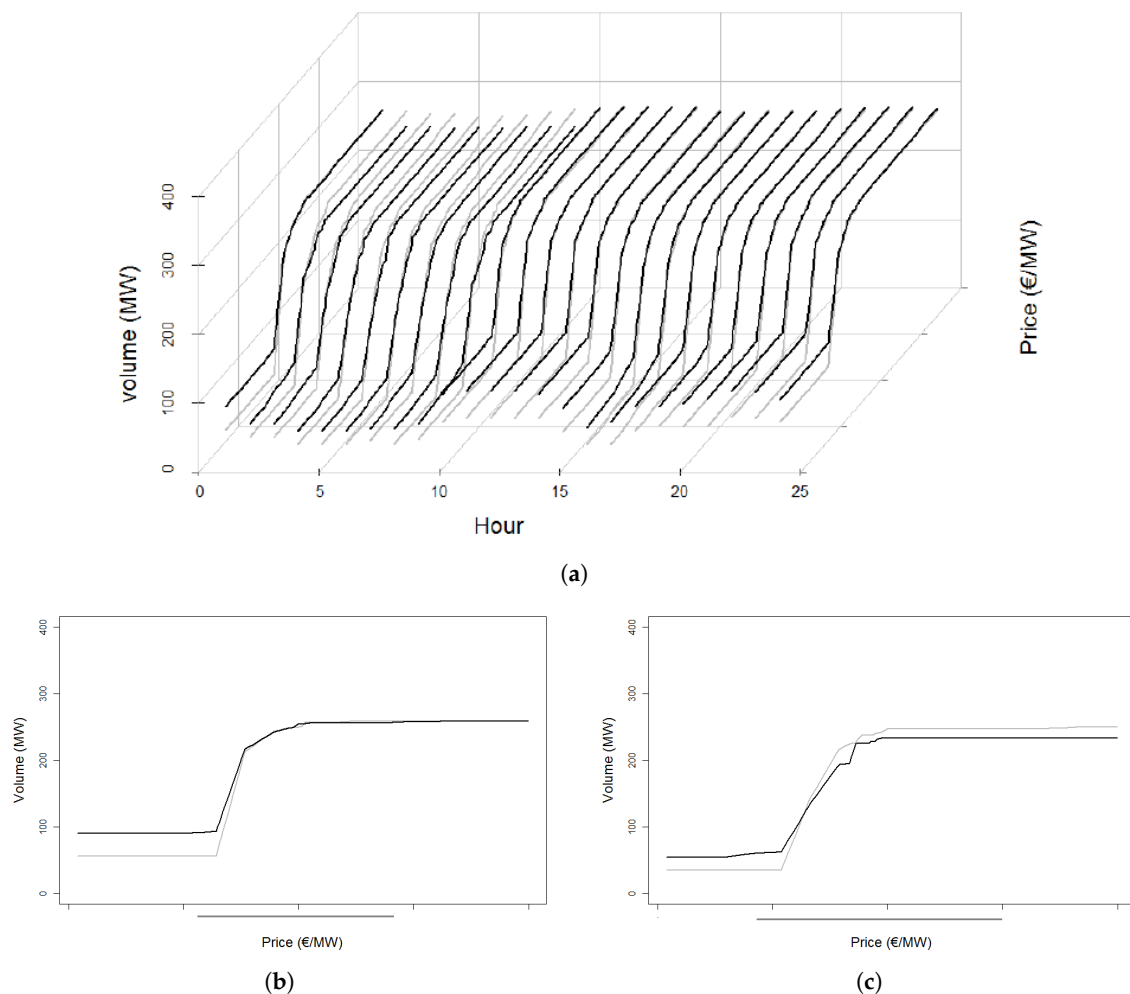


Figure 3. The bid curves for the next day's operation when using the MILP model (black) and linear relaxation (grey) (a). The bid curves for Hour 1 (b) and Hour 5 (c). The prices have been taken away due to confidentiality, but the grey lines below the price-axis indicate the most likely range of prices for the given hour.

6.2. Simulation Results

Section 6.1 presented some examples of how the bid curves from the MILP and LP model differ. How important are these differences in operations? To answer this question, the performance of the two formulations is compared in a simulation study. The simulation mimics the operation of the hydropower system according to how operators at the hydropower company use optimization tools in their daily scheduling.

The simulation procedure is depicted in Figure 4 and consists of a daily and a weekly loop. The daily loop involves bid optimization, market clearing and production scheduling. The weekly loop involves calculation of water values. The daily loop starts with updated forecasts of inflows and prices to generate scenario trees. Both forecasting and scenario tree generation are outside the scope of this text, but it should be noted that all data used in the simulation study is collected from operations at a cooperating hydropower company. Specifications for the river system may also change depending on the day, due to, for instance, maintenance or other time- or state-dependent restrictions.

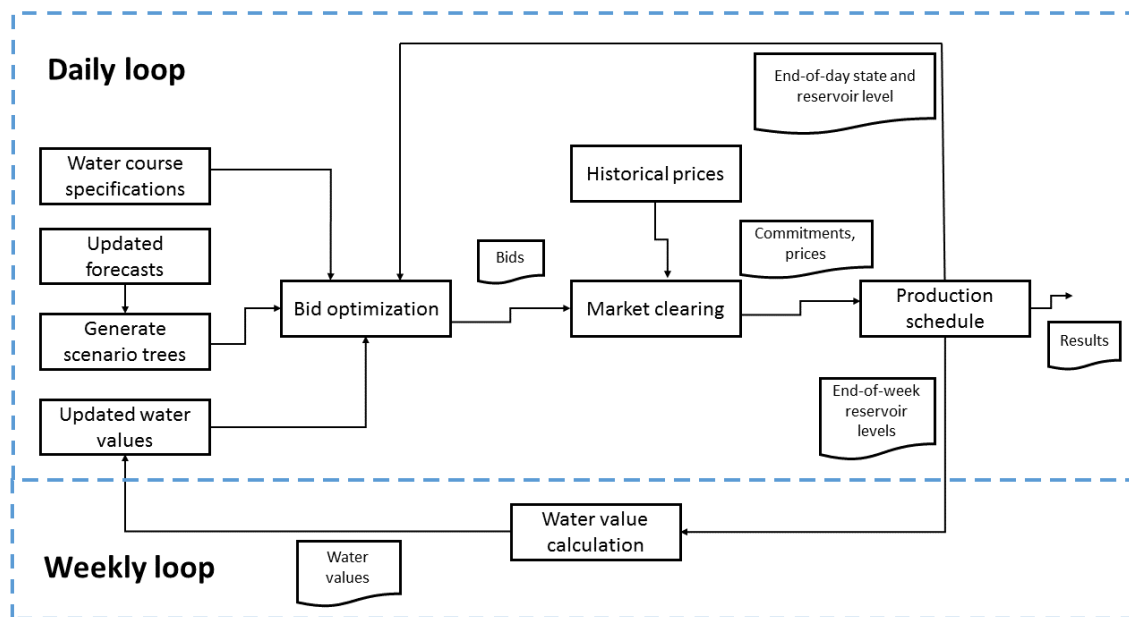


Figure 4. Flow-chart for the simulation procedure.

Input for water values is obtained from a simple calculation instead of the complex longer-term models which are the norm in the industry. We use the historic water values obtained from the company as the value at 50% reservoir level and interpolate with a water value of 0 at full reservoir capacity. This gives a water value curve which is linear in the reservoir level. Due to the simple water value calculation, our simulated operation will not correspond to the historic operation of the system. The comparison between the MILP and LP formulation is still valid, as the two simulations only differ in the method used for bid optimization.

The MILP and LP bid models are run to obtain bid curves similar to the ones shown in Figure 3. As it is assumed that producers are price takers, market-clearing prices are simply taken to be the historical prices corresponding to the day under study. Commitments for each hour are found by interpolation where the market price intersects the bid curves as in Equation (2). Production can now be re-optimized to find the cost-minimizing schedule to cover commitments. This step involves running a deterministic optimization where the known commitments are matched to production. For both simulation runs, this is accomplished by a MILP model to reflect the fact that fractional start-ups are not allowed in real operations. Specifically, this means that regardless of whether the MILP or LP formulation is used for bid optimization, only binary on/off decisions are allowed in the final production schedules. The resulting revenues, costs, penalties and all other results from operating the system according to the final production schedules are saved, and the state and reservoir levels at the end of the day are used as input for the next day. At the end of each week, storage levels are used to calculate water values for the next week. The total simulation is run for 8 weeks corresponding to data from August and September 2012.

A summary of results for the simulation can be found in Table 2. The total difference in objective function value over the 8 weeks is 0.002%, which indicates that linearized start-ups are

a good alternative to the true MILP formulation that, in its current form, takes too long to solve. Both formulations also obtain nearly the same average price. However, even if the objective function is similar for the two formulations, there might be other differences in the results that are important in operations.

Table 2. Results from the simulation. The total objective is Revenues–Penalty–Start-up costs–Cost of water used. The obtained average price is found by dividing the total income by the amount produced. Numbers are given in €, prices in €/MWh.

	MILP	LIN	Difference (%)
Revenue	7,507,349	7,528,399	−0.28
Penalty	18,221	24,334	−33.55
Start-up cost	57,404	64,953	−13.15
Cost of water used	1,118,777	1,126,270	−0.67
Total objective	6,312,947	6,312,842	0.002
Obtained average price	20.29	20.23	0.30

The differences in obtained commitments, revenues and penalties between the two formulations can be seen on the left in Figure 5. Differences in commitments and revenues have the same profile, as the revenue is the committed volume times the price and the price is the same for both formulations. The differences in commitments are significant in quite a few hours, as total generation capacity is around 390 MW and differences are observed to be of 300 MW in size. The plot shows that the largest differences occur when volumes are lower for the MILP formulation. This is due to the fractional starts allowed in the LP model, which makes it too cheap to start up a unit which in turn may result in an excessively high bid from the LP model compared to the MILP model. This leads to higher commitments from the LP model, which sometimes also provides higher revenues. The total committed volume for the MILP formulation is 311,113 MW while it is 312,012 MW for the LP formulation.

If the producer cannot deliver on the day-ahead market commitments, a penalty is incurred. This reflects that the producer should plan in balance and only use the balancing and intra-day markets for adjustments. This may, in fact, not be the case in real operations where it might be beneficial for the producer to participate in other markets. For the purpose of testing linear approximation, the penalty approach is used to only consider operations in the day-ahead market. Figure 5f shows that both formulations sometimes incur penalties from being in unbalance. Linear approximation does, in fact, incur higher penalties than the MILP model, which was the expected result. However, compared to the total objective function, penalty values are small for both formulations. This means that, in general, commitments are rarely so “bad” that they cannot be covered within the allowed operating range of a combination of the generating units.

Reservoir management is also an important aspect of hydropower scheduling, and the right part of Figure 5 shows the reservoir levels over the 8-week simulation period for three of the reservoirs in the river. Reservoir levels for the MILP and LP formulations are similar, but some differences occur particularly for the small reservoirs downstream of the two large upstream reservoirs on either side of the system. This is shown in Figure 5 for N vatn and Skjerkevatn, where the LP model chooses to release more water from the upstream reservoir in weeks 4 and 5. This extra water is used for increased production to cover the sometimes higher commitments for the LP formulation.

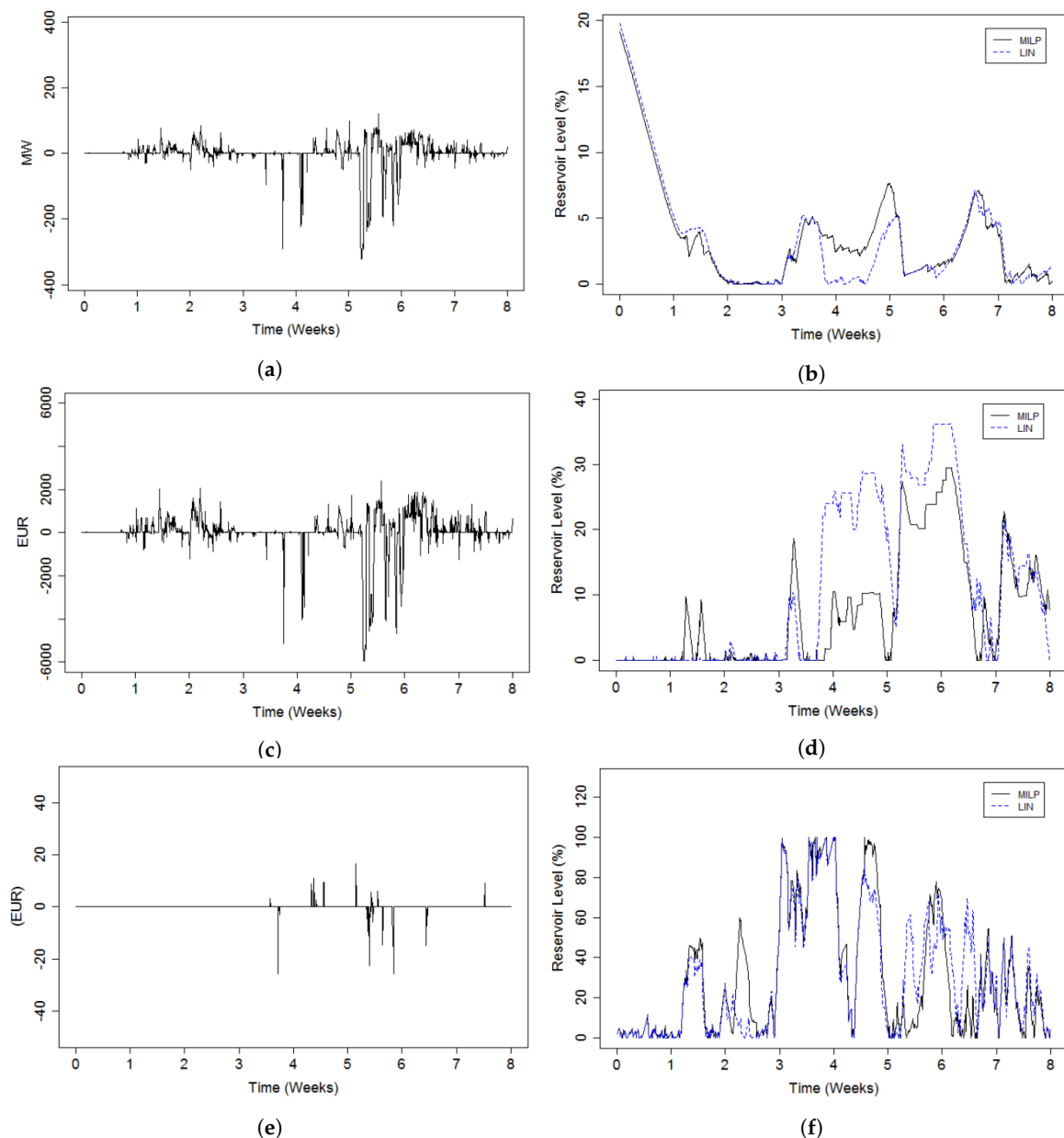


Figure 5. Hourly differences (MILP-LP) in committed volumes (a), revenue (c) and penalties (e). Reservoir profiles for reservoirs in the upper, left part of Figure 2, namely Nâvatn (b), Skjerkevatn (d) and Ørevatn (f).

To summarize, our results show that while the objective function values for the two formulations are similar, there are some qualitative differences regarding the actual production schedules. This might be due to the relative flat objective function in hydropower scheduling problems, which again is due to flexibility in the river system. For more constrained river systems or for smaller systems with fewer combinations of generating units to cover commitments, the differences between the MILP and LP formulation might increase.

7. Conclusions

Two formulations for determining the optimal bids for a price-taking hydropower producer have been compared. The first formulation used binary variables to describe the on/off state of the generating units, while the second formulation used a linear approximation for the state variables.

The motivation for using linear approximation was to reduce the computational burden of the stochastic program that must be solved to find the bids. The two formulations are compared in a simulation setup where the operation of a system of reservoirs and hydropower plants is investigated over an 8-week period. The results indicate that using linear approximation is useful, as the difference in objective function over the simulation period is only 0.002% and both models obtain nearly the same average price.

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Nomenclature

Sets and indices

$b \in B$	Set of price points for which to bid
$g \in G$	Generating units in the hydropower production system
$g_r \in G_r$	Generating units belonging to reservoir r
$i \in I$	Segments for production function
$r \in R$	Reservoirs in the hydropower production system
$s \in S$	Scenarios
$t \in T$	Time steps

Variables

p_{gst}	Production from unit g in scenario s , time step t (MW)
q_{gst}	Discharge from unit g in scenario s , time step t (m ³)
q_{gsti}	Discharge in segment i for unit g in scenario s , time step t (m ³)
s_{rst}	Storage level in reservoir r in scenario s , time step t (m ³)
u_{gst}	On (1) or off (0) for unit g in scenario s , time step t
v_{gst}	Start-up for unit g in scenario s , time step t
w_{gst}	Shut-down for unit g in scenario s , time step t
x_{bt}	Volume offered for bid price b in time step t (MW)
y_{st}	Committed volume in scenario s , time step t (MW)
z_{st}^+	Positive imbalance in scenario s , time step t (MW)
z_{st}^-	Negative imbalance in scenario s , time step t (MW)

Parameters

$C^{penalty}$	Penalty for imbalance (€/MW)
C_g^{start}	Start-up cost for unit g (€)
C_g^{stop}	Shut-down cost for unit g (€)
E_{gi}	Power output (slope) of segment i for unit g (MW/m ³)
I_{rst}	Inflow to reservoir r in scenario s , time step t (m ³)
P_{st}	Market price in scenario s , time step t (€)
p_b^{bid}	Bid price for bid point b (€)
p_g^{max}	Maximum production for unit g (MW)
p_g^{min}	Minimum production for unit g (MW)
Π_{sr}	Value of water in reservoir r in scenario s , when $t = T$ (€/MW)
Φ_s	Probability of scenario s
Q_{gi}^{max}	Upper bound for discharge in segment i for unit g (m ³)

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