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New Performance Indices for Voltage Stability Analysis in a Power System

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Abstract: The frequent occurrence of voltage instability in a modern power system is alarming and thus, has been of great concern to power system utilities. In this paper, a new performance index based on the power flow solutions for voltage stability assessment of a power system is presented. First, the voltage deviation with respect to reactive power load variation at each load bus is found. Thereafter, the performance voltage bus index for each load bus is computed. An improved modal analysis technique (IMAT) is used to identify weak nodes that are liable to voltage instability in a power system. This technique uses a submatrix of the full Jacobian matrix for voltage stability analysis. Comparison of the proposed method is done with existing voltage stability indices and the conventional modal analysis technique (CMAT). The effectiveness of all the approaches presented are tested on both Western system coordinating Council (WSCC) 9-bus, IEEE 30 bus and IEEE 57 bus test systems. Results obtained show that the proposed techniques can serve as an alternative tool to other conventional techniques for voltage stability assessment in a power system and can be of tremendous benefits in the planning and operation of a power system by system operators.

Keywords: voltage deviation; power flow; voltage collapse; proximity; power system

1. Introduction

Voltage stability is said to be the ability of a power system to maintain acceptable voltages at all network buses of the system under normal operating conditions and after being subjected to a disturbance [1]. The power system utilities have been much more concerned about the incessant occurrence of voltage instability as a result of continuous increase in load demands and lack of transmission capability. Recently, the problem of voltage instability has been observed as the main cause of numerous major network blackouts experienced in various countries such as France, Sweden, Belgium, Sweden, Germany, Japan, Iran and USA [2]. Thus, the need for a reliable method for voltage stability assessment in a power system [3]. Voltage stability is believed to be a dynamic phenomenon. If analysis targets system critical bus identification, reactive power compensation or load margin, then the use of static model is adequate [4]. A considerable portion of research studies has concentrated on the static model aspects of voltage stability. While steady state analysis is simple, it yet gives some practical benefits over dynamic analysis, providing results with satisfactory accuracy and slight computational effort [4]. A system is said to enter a state of voltage instability when a disturbance, an increase in load demand and/or a variation of system conditions cause a continuous and uncontrollable drop in voltage. The key factor, which contributes to voltage instability, is the incapability of a power system to meet the reactive power demand [5].

The importance of reactive power cannot be overemphasized since it affects the transmission system reliability and efficiency with which real power is delivered to end users. If the problems related to the reactive power are ignored, it may eventually result in voltage collapse phenomenon which has been responsible for several blackout incidents throughout the globe [6,7]. Voltage collapse problems may basically be described as the system's inability to supply the reactive power or by an excessive absorption of reactive power by the power system itself [8]. Therefore, it is becoming more and more imperative for power system operators and engineers to perform a comprehensive voltage stability analysis of the power systems.

To achieve this, a considerable number of techniques has been reported in the literature [9,10]. Some of these methods include, but are not limited to the use of real power-voltage (PV) and reactive power-voltage (QV) curves, continuation power flow, multiple power flow solutions, modal analysis and optimization-based techniques [11–15]. These methods have their own benefits and shortcomings, which were presented in [16]. For instance, as reported in [11], although, PV and QV curves and multiple power flow solution techniques have been reported as being useful, especially in the analysis of voltage stability in a power system, these techniques are time-consuming and they require a lot of computational efforts, especially for a large power system network [14]. To overcome the difficulties posed by the use of PV and QV curves, the continuation power flow (CPF) method was proposed for steady state voltage stability analysis [12] in a power system. Although the use of the CPF method has proved to be insightful and meaningful, unfortunately, it is also time-consuming as several loading conditions have to be specified before the critical bus, which is susceptible to voltage collapse is identified. In addition to the aforementioned power flow-based traditional methods, voltage stability indices have gained much attention recently and they include voltage stability index (L-index), voltage collapse proximity index (*VCPI*), fast voltage stability index (FVSI), line stability index (LSI), Full Sum dQ/dV (FSQV) index and so on [8,17–19]. Some of these methods are insightful and useful in the analysis of voltage stability, while some are not practical since their computation cost is high for large power system networks.

This paper proposes a new performance index which is based on the voltage deviation of each load bus of the system for voltage stability analysis. Detailed information regarding the maximum loadability of the load buses and the total step size for the loadability of each load bus can be known from this index. This index can be used as an alternative tool to the existing voltage stability indices to solve voltage stability related issues in a power system. This approach, which made use of only the submatrix of the Jacobian matrix to determine the weak bus of the system, was reported in [19]. However, it is not practical due to its high computation cost. In this study, we try to propose an improved modal analysis technique which is based on the submatrix of the Jacobian matrix instead of either the full matrix or the reduced Jacobian matrix.

The main contributions of this study are thus summarized as follows: (1) a performance index which takes into account the maximum loadability of the buses, the voltage deviations and the total number of steps taken to reach a minimum permissible reactive power load of each load bus is proposed. This information is imperative and could be of significance to power system operators in the analysis of voltage stability of a power system; (2) computational burden usually encountered in the course of the study of voltage stability is also a growing concern to the system utilities. In the case of voltage collapse occurrence, the aftermath of this may be adverse as the end users of electricity may often be left in a total blackout for longer periods of time. Using a full Jacobian matrix for voltage stability analysis may aggravate this problem. Thus, an approach which considers a submatrix of the Jacobian matrix and which also focuses on the important factors that affect voltage stability in a power system is proposed.

The remainder of this paper is organized as follows. Section 2 gives the mathematical formulation of the traditional power-flow-based voltage collapse proximity index, the proposed performance index and the suggested IMAT. Results of the simulation obtained for all the test cases used are presented in Section 3. Discussion of the results is presented in Section 4. The conclusion of the work is presented in Section 5.

2. Power Voltage Performance Indexes

This section presents the mathematical formulations of the conventional index and the proposed methods.

2.1. Mathematical Formulation of the VCPI [17]

Consider a single diagram shown in Figure 1.

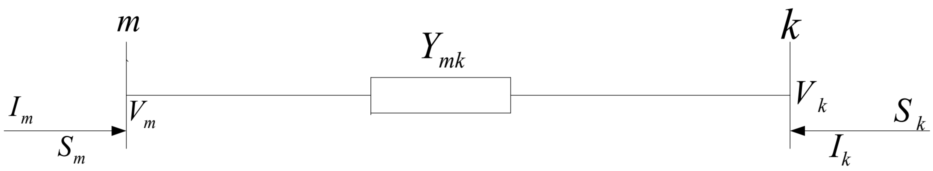

Figure 1. Single line diagram of a two bus power system.

The injected current I_m at node m is given as:

$$I_m = V_m \sum_{\substack{k=1 \\ k \neq 1}}^{n} Y_{mk} - \sum_{\substack{k=1 \\ k \neq 1}}^{n} V_k Y_{mk} \tag{1}$$

where,

- I_m = Current injected at node m
- V_m = Voltage at m^{th} node
- V_k = Voltage at k^{th} node
- Y_{mk} = Mutual admittance between m^{th} and k^{th} nodes

The apparent or complex power injection at m^{th} node is given as:

$$S_m = V_m \, I_m{}^* \tag{2}$$

By substituting (2) into (1):

$$S_m{}^* = |V_m|^2 \sum_{\substack{k=1 \\ k \neq m}}^{n} Y_{mk} - V_m{}^* \sum_{\substack{k=1 \\ k \neq m}}^{n} V_k Y_{mk} \tag{3}$$

If we define:

$$Y_{mm} = \sum_{\substack{k=1 \\ k \neq m}}^{n} Y_{mk} \tag{4}$$

one obtains:

$$S_m{}^* = |V_m|^2 \, Y_{mm} - V_m{}^* \sum_{\substack{k=1 \\ k \neq m}}^{n} V_k{}' Y_{mm} \tag{5}$$

where:

$$V_k{}' = \frac{Y_{mk}}{\sum\limits_{\substack{k=1 \\ k \neq m}}^{n} Y_{mi}} V_k = |V_k{}'| \, \delta_k{}' \tag{6}$$

Re-arranging (5) and (6) gives:

$$\frac{S_m{}^*}{Y_{mm}} = |V_m|^2 - V_m{}^* \sum_{\substack{k=1\\k\neq m}}^{n} V_k{}' \tag{7}$$

$$\frac{S_m{}^*}{Y_{mm}} = |V_m|^2 - (|V_m| \cos \delta_m - j\,|V_m| \sin \delta_m)\; X \left[\sum_{\substack{k=1\\k\neq m}}^{n} \left(\left|V_k{}'\right| \cos \delta_k{}' + j\left|V_k{}'\right| \sin \delta_k{}'\right) \right] \tag{8}$$

where δ_m is the voltage angle at m^{th} node.

Algebraic manipulation of (8) yields:

$$J = \begin{vmatrix} 2|V_m| - \sum\limits_{\substack{k=1\\k\neq m}}^{n} |V_k{}'| \cos \delta & |V_m| \sum\limits_{\substack{k=1\\k\neq m}}^{n} |V_k{}'| \sin \delta \\ \sum\limits_{\substack{k=1\\k\neq m}}^{n} |V_k{}'| \sin \delta & |V_m| \sum\limits_{\substack{k=1\\k\neq m}}^{n} |V_k{}'| \cos \delta \end{vmatrix} \tag{9}$$

At the voltage collapse point, the determinant of (9) will always equal to zero. This means that the matrix (9) becomes singular at the voltage collapse point.

If J = 0, we have:

$$\frac{|V_m| \cos \delta}{\sum\limits_{\substack{k=1\\k\neq m}}^{n} |V_k{}'|} = \frac{1}{2} \tag{10}$$

Re-writing (10), one can obtain:

$$\frac{V_m}{\sum\limits_{\substack{k=1\\k\neq m}}^{n} |V_k{}'|} = \frac{1}{2} + j\,m \tag{11}$$

where m in (11) is a real constant.

Using the complex number identities, (11) can be written as:

$$\left| 1 - \frac{\sum\limits_{\substack{k=1\\k\neq m}}^{n} V_k{}'}{V_m} \right| = 1 \tag{12}$$

Thus, the *VCPI* at m^{th} is given as:

$$VCPI_{m^{th}bus} = \left| 1 - \frac{\sum\limits_{\substack{k=1 \\ k \neq m}}^{n} V_k{}'}{V_m} \right| \tag{13}$$

Equation (13) implies that, the bus voltage is stable if the *VCPI* index is 0 and if it is 1 the bus voltage reaches a collapse point.

2.2. Proposed Performance Voltage Stability Index (PVSI)

Voltage stability analysis in a power system usually begins with a power flow solution to determine the voltage magnitude at each node. Also, using Figure 1, the current injected into a node m in terms of real and reactive power is given as:

$$I_m = \left(\frac{P_m + jQ_m}{V_m} \right)^* \tag{14}$$

The details of the power flow equations by Newton Raphson iterative technique is reported in [20]. In a compact form, a linearized power flow equation may be written as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \tag{15}$$

where J_1, J_2, J_3 and J_4 are the Jacobian matrices of the system.

The new voltage magnitude at a bus m can be updated as:

$$\delta_m{}^{(r+1)} = \delta_m{}^{(r)} + \Delta\delta_m{}^{(r)} \tag{16}$$

$$\left| V_m{}^{(r+1)} \right| = \left| V_m{}^{(r)} \right| + \Delta \left| V_m{}^{(r)} \right| \tag{17}$$

where r is the number of step size (iterations).

By using the bus voltage magnitudes obtained from Equation (17), the absolute values are taken and changed into per unit.

Thus, the performance voltage deviation index (*PVDI*) with respective to reactive power load variations at each load node is given as:

$$(PVDI_m)_{k=1\ldots r} = \sum_{m=1}^{n} \left(\frac{U_{Normi} - U_m}{U_{Normi}} \right)^2_{k=1\ldots r} \tag{18}$$

where n represents the total number of load nodes;

U_{nom} is the nominal voltage magnitude;
U_m is the voltage magnitude at m node;
k is the reactive power loading condition.

Assuming that the change in the reactive power deviation at each load node of the system is ΔQ, with R_T being the total number of step sizes taken by each load node to reach the maximum loadability. Thus, the proposed performance voltage bus index (*PVBI*) can be formulated as:

$$(PVBI_m)_{k=1\ldots r} = \frac{\sum\limits_{m=1}^{n} \left(\frac{U_{Normi} - U_m}{U_{Normi}}\right)^2_{k=1\ldots r}}{R_T\, \Delta Q_m} \quad (19)$$

Equation (19) may be expressed in terms of the maximum loadability of a load node m. That is, the total summation of individual reactive power load changes at each iteration step of a load bus m.

Thus,

$$(PVBI_m)_{k=1\ldots r} = \frac{\sum\limits_{m=1}^{n} \left(\frac{U_{Normi} - U_m}{U_{Normi}}\right)^2_{k=1\ldots r}}{(R_T)_m \left(\sum\limits_{m=1}^{n} (\Delta Q_m)\right)_{k=1\ldots r}} \quad (20)$$

where $\left(\sum\limits_{m=1}^{n} (\Delta Q_m)\right)$ = maximum loadability of load node m.

Let:

$$\left(\sum_{m=1}^{n} (\Delta Q_m)\right) = (Q_{\max})_m \quad (21)$$

Substituting Equation (21) into (20), we have:

$$(PVBI_m)_{k=1\ldots r} = \frac{\sum\limits_{m=1}^{n} \left(\frac{U_{Normi} - U_m}{U_{Normi}}\right)^2_{k=1\ldots r}}{(R_T)_m\,(Q_{\max})_m} \quad (22)$$

Equation (22) implies that, the load node that has maximum value of the proposed *PVBI* is taken as the voltage collapse node due to the inverse relationship that exists with the product of the total step size (R_T) and the maximum loadability ($Q_{\max}$) of a load node m.

Equation (22) may therefore be expressed as:

$$(PVBI_m)_{k=1\ldots r} = \max \frac{\sum\limits_{m=1}^{n} \left(\frac{U_{Normi} - U_m}{U_{Normi}}\right)^2_{k=1\ldots r}}{(R_T)_m\,(Q_{\max})_m} \quad (23)$$

There are two main benefits associated with the proposed *PVBI*: (1) both weak and voltage collapse nodes can be identified using the proposed method. (2) information relating to the maximum loadability of each load node and the total number of step sizes taken to reach the maximum loadability of each load bus can easily be obtained.

2.3. Conventional Modal Analysis Technique (CMAT)

Details of the mathematical formulations involved in the CMAT are presented in [14,21]. This conventional method makes use of a reduced Jacobian matrix, modes of the system and the participation factor of each load bus to study voltage stability in a power system. Although, significant contributions were made by the authors through the proposed CMAT, the approach could still be further simplified and explored to ensure less computational efforts and also focus on the key factors that affect voltage stability in a power system.

2.4. Improved Modal Analysis Technique (IMAT)

The suggested IMAT is also based on the linearized power flow model as follows:

$$\begin{bmatrix} \Delta\ P \\ \Delta\ Q \end{bmatrix} = [J] \begin{bmatrix} \Delta\ \delta \\ \Delta\ V\ /V \end{bmatrix} \tag{24}$$

where $[J]$ is the Jacobian matrix of the system and is given as:

$$[J] = \begin{bmatrix} J_{P\delta} & J_{PV} \\ J_{Q\delta} & J_{QV} \end{bmatrix} \tag{25}$$

Since most voltage instability experienced in the past was caused by insufficient reactive power to meet the power demand, it is believed that information contained in the incremental relationship between the reactive power Q and system voltage magnitude V could be sufficient to evaluate voltage stability in a power system. The suggested modified modal analysis method focuses mainly on the Jacobian element J_{QV} for voltage stability analysis against the use of reduced Jacobian matrix as proposed by [14].

Therefore, based on these assumptions, we may express the relationship that exists between the reactive power Q and voltage magnitude V with respect to J_{QV} as:

$$\Delta Q = \left[J_{QV}\right] \Delta V \tag{26}$$

where $\left[J_{QV}\right]$ comprises of the product of the partial derivatives of the reactive power equation with respect to the voltage magnitude:

$$\Delta V = \left[J_{QV}\right]^{-1} \Delta Q \tag{27}$$

The proposed IMAT also makes use of the eigen-analysis to determine the critical mode. The contribution of each bus to the critical mode identified using eigenvalue decomposition method is determined. This method is more advantageous in that it focuses majorly on the key factor that affect voltage stability in a power system. Analysis with the full Jacobian matrix could be computationally expensive. Thus, the computational efforts involved could be reduced significantly with the use Jacobian element J_{QV}.

2.4.1. Determination of the Modes of a Power Network

The modes of a power system can be determined by finding the eigenvalues and eigenvectors of the Jacobian matrix J_{QV}.

Let:

$$J_{QV}\ = \xi\, \lambda\, \tau \tag{28}$$

where

- ξ is the right eigenvector of the Jacobian matrix J_{QV};
- τ represents the left eigenvector of the Jacobian matrix J_{QV} and
- λ represents the diagonal eigenvalues of the Jacobian matrix J_{QV}.

From Equation (28), we have:

$${J_{QV}}^{-1} = \xi\, \lambda^{-1}\, \tau \tag{29}$$

Substituting Equation (29) in (27), we obtain:

$$\Delta V = \xi \lambda^{-1}\, \tau\, \Delta Q \tag{30}$$

Equation (30) may be re-written as follows:

$$\Delta V = \sum_i \frac{\xi_i \, \tau_i}{\lambda_i} \, \Delta Q \tag{31}$$

Equation (31) shows that large eigenvalues implies small changes in the modal voltage and vice versa, due to the inverse relationship that exists between them. However, as the power system is stressed due to continuous increase in reactive power demand, the i^{th} eigenvalue tends to be smaller, and thus, results in system voltage drop. Voltage collapse may occur if the magnitude of the eigenvalue becomes zero as this may undergo many changes in the reactive power variation. We can therefore infer that the mode which has the smallest eigenvalues is the critical mode of the system. Identification of this mode is very important in voltage stability assessment.

2.4.2. Bus Participation Factor

The left and right eigenvectors of the matrix J_{QV} associated with the critical mode of the system can also provide useful information regarding the voltage instability of the system. Identification of various elements participating in the modes is also of great importance to the planning and operation of the system.

Thus, the bus participation factor measuring the participation of z^{th} node to the i^{th} mode may be expressed in terms of the left and right eigenvectors as follows:

$$PF_{zi} = \xi_{zi} \times \tau_{iz} \tag{32}$$

This we termed as Improved Modal analysis Technique (IMAT) in this paper. Thus,

$$\mathrm{IMAT_{zi}} = PF_{zi} = \xi_{zi} \times \tau_{iz} \tag{33}$$

The load nodes with large participation factor to the critical mode is considered as the voltage unstable buses which are susceptible to voltage collapse. The IMAT is synonymous with the CMAT. This is because both techniques involve determination of the critical mode of the system and the bus participation factor that measures the contributions of each node to the critical mode identified. However, the difference is that the IMAT makes use of the diagonal eigenvalues of the Jacobian matrix J_{QV} whereas the CMAT uses the reduced Jacobian matrix in the analysis.

3. Simulation Results and Discussion

The effectiveness of all the approaches presented are tested on the WSCC 9-bus, IEEE 30 bus and IEEE 57 bus power systems. The detailed descriptions of the WSCC 9-bus and IEEE 30 bus power systems are presented in [22,23], respectively. In this work, load buses 5, 6, 8 and 23, 24, 27, 29 and 30 of the WSCC 9-bus and IEEE 30-bus test systems, respectively, are randomly selected for the purpose of voltage stability analysis. The WSCC 9-bus test system has three (3) loads connected at buses 5, 6 and 8 respectively. Similarly, the IEEE 30 bus system whose single line diagram is shown in Figure 2 consists of six (6) generator nodes, twenty four load nodes and forty one transmission lines with four (4) tap ratios. Also, the IEEE 57 bus test system has seven (7) generator nodes, fifty (50) load nodes and eighty (80) transmission lines. Nodes 12, 25, 27, 30, 31, 32, 33 and 57 of the IEEE 57 bus power system are selected at random in this paper for the purpose of analysis. These nodes were found to have the least allowable reactive power loads among all the load nodes of the system.

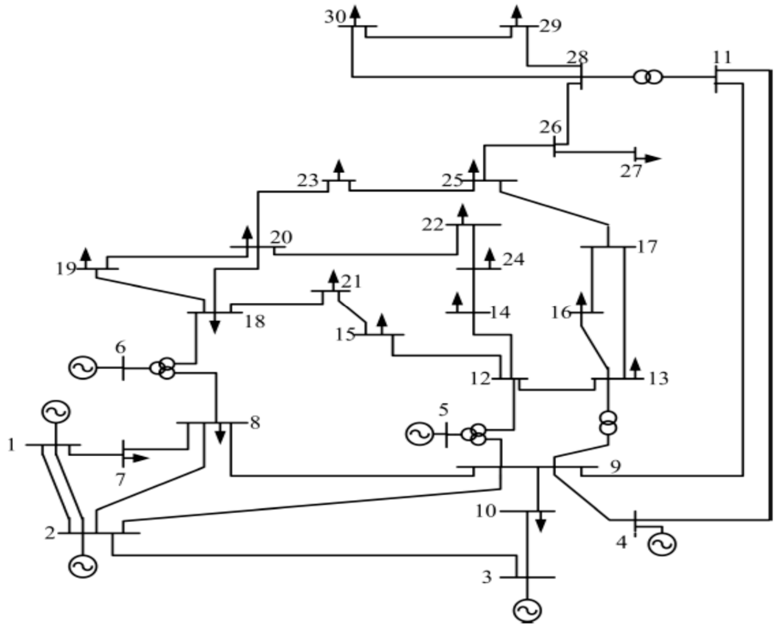

Figure 2. Single line diagram of the IEEE 30 bus system.

Simulation Results of the VCPI, CMAT and the Proposed PVBI and IMAT

All the simulations were done using MATLAB software (R2014a, MathWorks, Natick, MA, USA). Windows 7, HP, 64 bit operating system, with 500 GB hard disc and 4 GB random access memory laptop are used. Results of the simulation obtained are presented in the form of test cases. Test cases A, B and C show the results obtained for the WSCC-9 bus, IEEE 30 bus and IEEE 57 bus test systems, respectively. Tables 1 and 2 and Figure 3 of Test Case A, show the results obtained for the traditional *VCPI*, proposed *PVBI* and voltage magnitude with respect to reactive power load variation at each load bus of the 9-bus test system, respectively. Simulation results of the CMAT and the proposed IMAT are also shown in Tables 3 and 4, respectively. Similarly, the results of *VCPI*, proposed *PVBI* and the voltage magnitude of the IEEE 30 bus system are also presented in Tables 5 and 6, and Figure 4, respectively. We have also presented the results obtained for the CMAT and the proposed IMAT for the IEEE 30 bus system in Tables 7 and 8, respectively. The proposed approaches were also tested on a large-scale IEEE 57 bus test system. Results of the *VCPI* and the proposed *PVBI* for the selected nodes of the IEEE 57 bus test system are as presented in Tables 9 and 10, respectively. The voltage magnitudes obtained at the minimum allowable load of the randomly selected nodes of the IEEE 57 bus system, are also presented alongside with both Tables 9 and 10. We have also shown results of the CMAT and the proposed IMAT for the IEEE 57 bus test system in Tables 11 and 12, respectively.

Test Case A: Results of the *VCPI*, *PVBI*, Voltage magnitude, CMAT and IMAT for the WSCC 9-bus test system.

Table 1. Results of the traditional voltage collapse proximity index (*VCPI*) for the 9-bus system.

Qmax (MVar)	Bus No.	Traditional *VCPI* Method	Total Computation Time (s)	Ranking Order
40	5	0.5235	3.396875	1st
256	6	0.4752	23.886528	3rd
240	8	0.5063	22.758630	2nd

Table 2. Results of the proposed performance voltage bus index (*PVBI*) for the 9-bus test system. PVDBI: performance voltage deviation index.

Bus No.	Qmax (MVar)	Number of Step Size	Proposed PVDBI	Proposed *PVBI* 10^{-4}	Total Computation Time (s)	Ranking Order
5	40	5	0.4060	20.3000	2.569802	1st
6	256	32	0.7933	0.9684	20.109783	3rd
8	240	30	0.8286	1.1508	18.094562	2nd

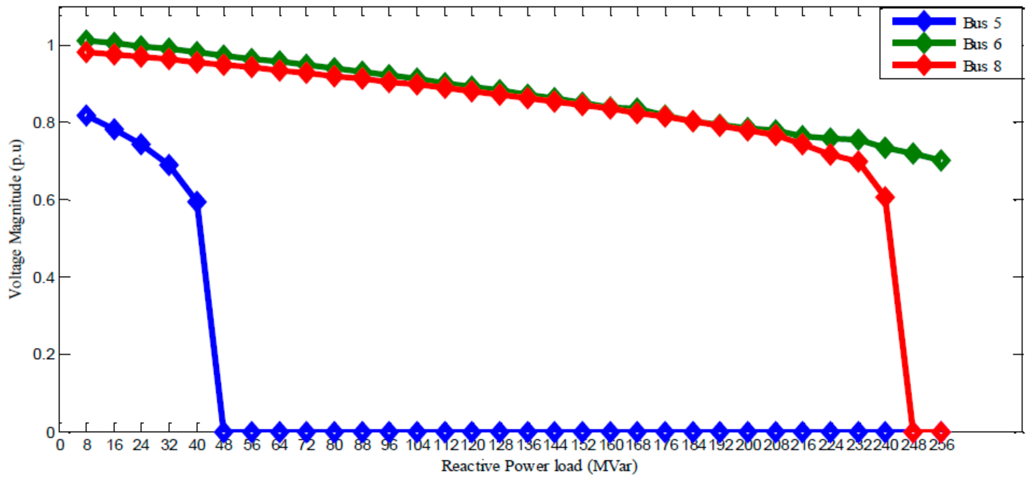

Figure 3. Voltage magnitudes of the western system coordinating council (WSCC) 9-bus power system.

Table 3. Results of the conventional modal analysis technique (CMAT) of the WSCC 9-bus test system.

Load Bus	Eigenvalues	Mode	CMAT	Rank	Computational Time (s)
4	38.3118	1	0.0002	6th	
5	34.7552	2	0.8535	1st	
6	29.8090	3	0.0016	5th	0.559172
7	9.1099	4	0.0631	3rd	
8	5.2841	5	0.0731	2nd	
9	0.1919	6	0.0067	4th	

Table 4. Results of the Proposed improved modal analysis technique (IMAT) of the WSCC 9-bus power system.

Load Bus	Eigenvalues	Mode	Proposed IMAT	Rank	Computational Time (s)
4	37.7712	1	0.0004	6th	
5	34.3277	2	0.8558	1st	
6	29.7026	3	0.0024	5th	0.490854
7	9.0171	4	0.0533	3rd	
8	5.3754	5	0.0806	2nd	
9	1.4946	6	0.0067	4th	

Test Case B: Results of the IEEE 30-bus power system.

Table 5. Results of the *VCPI* for the IEEE 30-bus power system.

Qmax (MVar)	Bus No.	Traditional *VCPI* Method	Total Computation Time (s)	Ranking Order
102	23	0.2470	21.549021	4th
108	24	0.0826	24.209167	5th
25	27	0.4030	15.709843	1st
35	29	0.2779	19.089312	3rd
31	30	0.3225	18.563290	2nd

Table 6. Results of the proposed *PVBI* of the IEEE 30-bus power system.

Bus No.	Qmax (MVar)	Number of Step Size	Proposed PVDBI	Proposed *PVBI* 10^{-4}	Total Computation Time (s)	Ranking Order
23	102	55	1.253418	2.234257	20.870932	4th
24	108	58	0.234238	0.373943	22.760321	5th
27	25	14	0.751936	21.48388	13.039880	1st
29	35	21	0.800705	10.893945	17.903762	3rd
30	31	18	0.61050	10.940860	16.409832	2nd

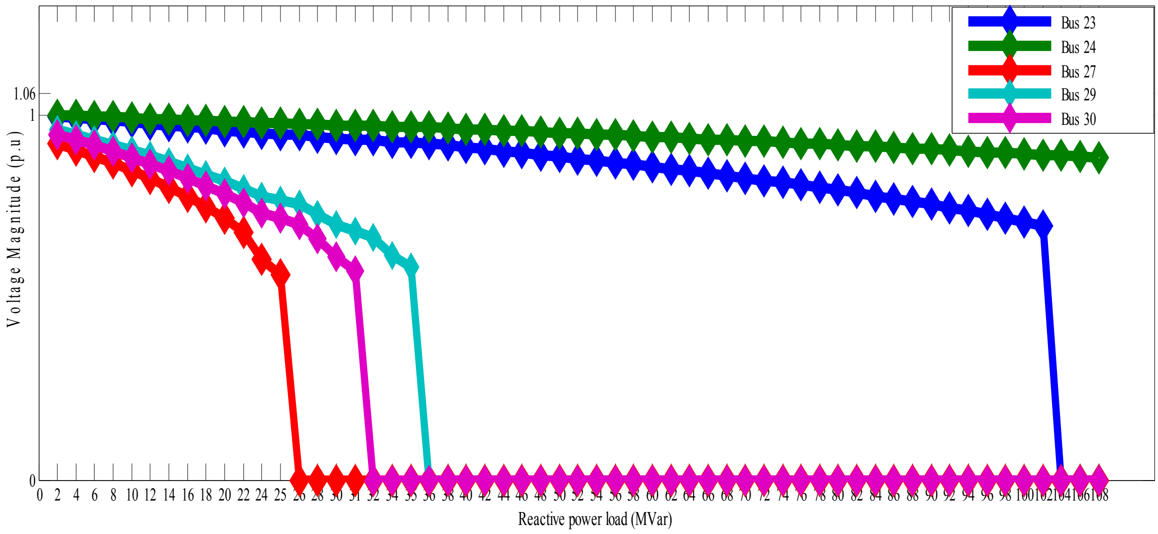

Figure 4. Voltage magnitudes of the IEEE 30-bus power system.

Table 7. Results of the CMAT for the IEEE 30-bus power system.

Load Bus	Eigenvalues	Mode	CMAT	Rank	Computational Time (s)
7	109.7610	1	0.0003	23rd	
8	98.2275	2	0.0004	22nd	
9	65.8089	3	0.0005	21st	0.923476
10	48.9036	4	0.0002	24th	
11	37.9613	5	0.0026	20th	
12	35.1885	6	0.0040	17th	
13	33.3827	7	0.0107	10th	
14	23.0648	8	0.0072	13th	
15	22.9861	9	0.0041	16th	
16	18.8202	10	0.0159	9th	
17	17.0175	11	0.0172	8th	
18	15.5751	12	0.0022	19th	
19	13.6245	13	0.0050	15th	
20	12.8474	14	0.0071	14th	
21	11.2777	15	0.0034	18th	
22	0.4921	16	0.0083	11th	
23	1.1468	17	0.0193	7th	
24	1.6633	18	0.0080	12th	
25	2.8717	19	0.0376	6th	
26	7.9392	20	0.1140	5th	
27	4.0336	21	0.2045	2nd	
28	6.9975	22	0.1099	4th	
29	5.1742	23	0.1997	3rd	
30	6.0380	24	0.2178	1st	

Table 8. Results of the suggested IMAT for the IEEE 30-bus power system.

Load Bus	Eigenvalues	Mode	Proposed IMAT	Rank	Computational Time (s)
7	101.5484	1	0.0002	23rd	
8	79.0143	2	0.0003	22nd	
9	59.9206	3	0.0005	21st	0.809821
10	44.7144	4	0.0001	24th	
11	30.8457	5	0.0027	18th	
12	29.7287	6	0.0030	17th	
13	28.1175	7	0.0080	10th	
14	21.2441	8	0.0061	14th	
15	17.7133	9	0.0031	16th	
16	16.8427	10	0.0125	9th	
17	13.5606	11	0.0137	8th	
18	13.1216	12	0.0016	20th	
19	11.3142	13	0.0037	15th	
20	10.2906	14	0.0062	13th	
21	9.7484	15	0.0026	19th	
22	0.4481	16	0.0072	11th	
23	1.0261	17	0.0180	7th	
24	1.2531	18	0.0068	12th	
25	2.4551	19	0.0354	6th	
26	3.3246	20	0.1133	4th	
27	4.1938	21	0.2458	1st	
28	4.7525	22	0.0971	5th	
29	5.9044	23	0.1960	3rd	
30	5.5165	24	0.2161	2nd	

Test Case C: Results of the IEEE 57-bus power system.

Table 9. Results of the *VCPI* for the IEEE 57-bus power system.

Qmax (MVar)	Bus No.	Traditional *VCPI* Method	Voltage Mag. (p.u.)	Total Computation Time (s)	Ranking Order
180	12	0.3584	0.6734	83.525160	8th
24	25	0.4826	0.5601	14.947784	5th
114	27	0.4004	0.5794	68.497318	7th
21	30	0.5002	0.5491	12.975753	4th
17	31	0.7400	0.5069	7.010215	1st
20	32	0.5021	0.5226	10.208482	3rd
19	33	0.6327	0.5176	9.462271	2nd
36	57	0.4203	0.5758	21.478248	6th

Table 10. Results of the proposed *PVBI* of the IEEE 57-bus power system.

Bus No.	Qmax (MVar)	Number of Step Size	Proposed PVDBI	Proposed *PVBI* 10^{-4}	Voltage Mag. (p.u.)	Total Computation Time (s)	Ranking Order
12	180	60	1.9853	1.83824	0.6734	76.987023	8th
25	24	11	0.6402	24.25000	0.5601	12.690832	5th
27	114	41	1.7494	3.74283	0.5794	56.89705	7th
30	21	10	0.8453	40.25238	0.5491	11.098412	4th
31	17	6	0.5300	51.960784	0.5069	5.809732	1st
32	20	9	0.8774	48.744444	0.5226	8.892109	3rd
33	19	8	0.7607	50.046053	0.5176	7.091207	2nd

Table 11. Results of the CMAT for the IEEE 57-bus power system.

Load Bus	Eigenvalues	Mode	CMAT	Rank	Computational Time (s)
8	167.0587	1	0.00028	46th	
9	117.2839	2	0.00012	49th	1.709213
10	100.1632	3	0.00049	43rd	
11	96.9489	4	0.00090	36th	
12	83.4685	5	0.00089	37th	
13	81.8242	6	0.00082	38th	
14	63.5906	7	0.0010	35th	

Table 11. *Cont.*

Load Bus	Eigenvalues	Mode	CMAT	Rank	Computational Time (s)
15	59.2406	8	0.00038	45th	
16	57.7710	9	0.00026	47th	
17	52.9183	10	0.000071	50th	
18	51.8406	11	0.00018	48th	
19	43.5320	12	0.0025	30th	
20	43.1314	13	0.0048	24th	
21	36.4075	14	0.0083	19th	
22	35.8421	15	0.0084	18th	
23	32.3515	16	0.0089	17th	
24	32.4130	17	0.0173	11th	
25	28.7774	18	0.0940	5th	
26	25.5944	19	0.0144	14th	
27	25.0823	20	0.0041	26th	
28	21.7844	21	0.0017	33rd	
29	17.7898	22	0.00073	41st	
30	16.4373	23	0.1233	4th	
31	15.4034	24	0.1687	1st	
32	15.1964	25	0.1582	3rd	
33	14.0792	26	0.1618	2nd	
34	0.2184	27	0.0328	6th	
35	0.5586	28	0.0255	7th	
36	0.8936	29	0.0202	10th	
37	1.0088	30	0.0165	13th	
38	1.1925	31	0.0074	20th	
39	1.5095	32	0.0167	12th	
40	2.2811	33	0.0204	9th	
41	2.5580	34	0.0063	21st	
42	3.3895	35	0.0105	16th	
43	3.7053	36	0.0020	32nd	
44	4.1351	37	0.0054	22nd	
45	4.5815	38	0.0021	31st	
46	5.4765	39	0.0026	29th	
47	5.8164	40	0.0046	25th	
48	6.6397	41	0.0053	23rd	
49	7.3517	42	0.0039	27th	
50	7.8873	43	0.0031	28th	
51	8.6792	44	0.0013	34th	
52	9.1597	45	0.00081	39th	
53	12.3263	46	0.0008	40th	
54	11.5016	47	0.00060	42nd	
55	10.7510	48	0.00039	44th	
56	10.9852	49	0.0129	15th	
57	10.9553	50	0.0254	8th	

Table 12. Results of the suggested IMAT for the IEEE 57-bus power system

Load Bus	Eigenvalues	Mode	Proposed IMAT	Rank	Computational Time (s)
8	117.6642	1	0.00021	47th	
9	90.2499	2	0.00085	35th	1.035109
10	79.5325	3	0.00037	44th	
11	76.8631	4	0.00073	37th	
12	66.0945	5	0.00063	39th	
13	60.5387	6	0.00066	38th	
14	58.9733	7	0.00082	36th	
15	50.1497	8	0.00031	45th	
16	52.2588	9	0.00019	48th	
17	42.3857	10	0.000053	50th	
18	40.9070	11	0.00012	49th	
19	39.8208	12	0.0029	28th	
20	35.3923	13	0.0055	21st	
21	28.2427	14	0.0083	18th	
22	27.6232	15	0.0082	19th	
23	23.6718	16	0.0088	17th	
24	24.2669	17	0.0191	11th	

Table 12. *Cont.*

Load Bus	Eigenvalues	Mode	Proposed IMAT	Rank	Computational Time (s)
25	22.1195	18	0.0814	5th	
26	21.0441	19	0.0164	14th	
27	18.7207	20	0.0042	25th	
28	16.0007	21	0.0014	33rd	
29	14.5588	22	0.0005	42nd	
30	13.1264	23	0.1143	4th	
31	12.5574	24	0.1669	1st	
32	11.2184	25	0.1539	3rd	
33	11.0863	26	0.1602	2nd	
34	10.6553	27	0.0423	6th	
35	9.8133	28	0.0320	7th	
36	9.3805	29	0.0237	10th	
37	0.1920	30	0.0184	13th	
38	0.5041	31	0.0071	20th	
39	0.7385	32	0.0187	12th	
40	0.8344	33	0.0238	9th	
41	1.1019	34	0.0051	22nd	
42	1.2206	35	0.0109	16th	
43	1.8570	36	0.0016	31st	
44	2.0165	37	0.0050	23rd	
45	7.6179	38	0.0015	32nd	
46	8.1450	39	0.0020	30th	
47	7.9238	40	0.0037	26th	
48	2.6906	41	0.0046	24th	
49	2.8162	42	0.0031	27th	
50	3.3008	43	0.0023	29th	
51	3.9684	44	0.00094	34th	
52	4.3339	45	0.0006	41st	
53	4.8112	46	0.00061	40th	
54	6.3361	47	0.00046	43rd	
55	6.1087	48	0.00028	46th	
56	5.3311	49	0.0157	15th	
57	5.4760	50	0.0291	8th	

4. Discussion of Results

The discussion of the results obtained for all the approaches considered are presented in this section. To ensure clarity of presentation, results of the conventional methods as well as the suggested techniques are discussed separately in subsections of this paper.

4.1. Results of the Traditional VCPI

To identify voltage collapse buses using the traditional power-flow-based approach *VCPI*, the system is subjected to the contingencies of gradual reactive power load increase at each load bus. Power flow is performed for each reactive power load variation at every load bus. The load bus that has maximum value of *VCPI* is considered as the critical bus of the system that is liable to voltage collapse. For this traditional method, load buses 5, 6 and 8 of the WSCC 9-bus system are randomly selected based on the loadability of each bus.

Similarly, for the IEEE 30 bus test system, load buses 23, 24, 27, 29 and 30 have been reported as the most critical buses of the IEEE 30 bus system [23]. Thus, these buses are selected for further study of voltage stability analysis. The outputs of the power flow solutions and the admittance matrix of the network were used by this method to evaluate *VCPIs* for various operating scenarios, which involve gradual reactive power load variation at each load bus.

From the simulation results presented in Tables 1 and 5 of the test cases A and B, respectively, load buses 5 and 27 of the WSCC 9-bus and IEEE 30 bus test systems have maximum values of *VCPI* and are considered as the critical buses of the WSCC 9 bus and the IEEE 30 bus systems, respectively.

Their *VCPI* values are calculated as 0.5235 (bus 5) and 0.4030 (bus 27) for the WSCC 9 bus and IEEE 30 bus systems, respectively. For a reactive power load of 40 MVar at bus 5, the *VCPI* value was calculated to be 0.5235 and the voltage magnitude was reduced to 0.5677 p.u. from an initial value of

0.8476 p.u. This is shown in Figure 3. Similarly, for the IEEE 30 bus system, we observed that load bus 27 has the maximum loadability of 25 MVar and least voltage magnitude of 0.5623 p.u. as shown in Table 5 and Figure 4, respectively. With the *VCPI* method, it takes the total computational time of 50.042033 s and 99.120633 s to identify the critical buses 5 and 27 of the WSCC 9 bus and the IEEE 30 bus test systems, respectively. Results of the *VCPI* for each load node of the IEEE 57 bus power system presented in Table 9 show that, bus 31 is the most susceptible bus to voltage collapse when compared with the results obtained for other load buses. The *VCPI* values of the selected buses 12, 25, 27, 30, 31, 32, 33 and 57 are 0.3584, 0.4826, 0.4004, 0.5002, 0.7400, 0.5021, 0.6327 and 0.4203, respectively, from which bus 31 has the highest *VCPI* value. Thus, with the traditional approach of *VCPI*, bus 31 is considered as the critical bus of the IEEE 57 bus test system. Besides, of all the load nodes of the IEEE 57 bus test system, bus 31 has the lowest voltage magnitude of 0.5069 p.u. at the minimum reactive power load of 17 MVar as shown in Table 9. It takes the total computational time of 228.105231 s to identify this bus.

4.2. Proposed Performance Voltage Bus Index (PVBI)

For the proposed *PVBI* method, first, the performance voltage deviation index (PVDBI) for each load bus with respect to reactive power load variation was calculated using Equation (18). Power flow solution was performed on every load bus at each reactive power load increase. We then estimated the *PVBI* for each load bus using Equation (24) to monitor system voltage stability. The load bus that has maximum value of *PVBI* is taken as a critical bus which is susceptible to voltage collapse. The stability of a bus using the suggested *PVBI* also depends on the maximum loadability value of that bus and the total number of steps taken to reach the collapse point.

The performance of this proposed method is tested on the WSCC 9 bus, IEEE 30 bus and IEEE 57 bus test systems, and results obtained are shown in Tables 2, 6 and 10, respectively. Buses 5, 8 and 6 are considered weak load buses of the WSCC 9 bus system based on the bus ranking order which was done by considering the maximum loadability of each load bus, the total number of steps taken to attain a collapse point and the value of *PVBI* at each bus. However, as can be observed in the case of WSCC 9 bus system, load bus 5 has the maximum value of *PVBI* (20.3000×10^{-4}), minimum allowable load (40 MVar) and least voltage magnitude (0.5677 p.u.) as shown in Table 2 and Figure 3. We also observed a total voltage collapse on this bus, as the power flow solution did not converge for any additional load beyond the minimum permissible reactive power load of 40 MVar. Thus, of all the load buses of the WSCC 9 bus system, bus 5 is the critical bus liable to voltage collapse. The total computational time taken to identify the critical load bus 5 of the 9 bus system is 40.774147 s.

Similar procedures were also followed for the IEEE 30 bus system, to identify a voltage collapse bus in the system. Buses 27, 30, 29, 23 and 24 are identified as the weak buses of the system in accordance with the ranking order as shown in Table 6. However, load bus 27 is found to be the weakest of them all, having the maximum *PVBI* value of 21.48388×10^{-4}, least sustainable load of 25 MVar, lowest total number of step size of 14 and lowest voltage magnitude (0.5677 p.u.). The voltage stability analysis of the IEEE 30 bus system took up to the total computational time of 90.984727 s to attain a solution. This amounts to 8.2% of time saving compared with the conventional modal analysis technique. Also, for a large scale IEEE 57 bus system, bus 31 has the least permissible reactive power load of 17 MVar, maximum *PVBI* value of 51.960784, lowest total number of step size of 6 and least voltage magnitude (0.5069 p.u.) at the minimum permissible load of 17 MVar. Thus, with the proposed *PVBI*, bus 31 is considered the weakest load bus of the IEEE 57 bus test system. It takes the total computational time of 198.559777 s to attain this solution. This amounts to 12.95% of time saving compared with the traditional approach of *VCPI*.

4.3. Conventional Modal Analysis (CMAT) and the Proposed IMAT

The simulation results of the CMAT for the WSCC 9-bus, IEEE 30-bus and the IEEE 57 bus test systems are as shown in Tables 3, 7 and 11, respectively. Eigenvalue decomposition technique was applied on the reduced Jacobian matrix to compute the modes of the system. This enables us to obtain the relative proximity of the system to voltage collapse. The participation factors of each load bus were then computed based on the critical mode to predict voltage collapse bus in the system. The bus that has highest value of the participation factor or CMAT was taken as the critical bus of the system. For the WSCC 9-bus system, the minimum eigenvalue is 0.1919 which corresponds to mode 6 of the system. Mode 6 is then identified as the critical mode as shown in Table 3. The bus participation factors were then computed based on this critical mode. Results of the participation factors obtained for all the load buses show that bus 5, being the bus with the highest value of the participation factor (0.8535), is the most critical bus of the 9-bus power system.

The total computational time taken to achieve this solution is estimated to be 0.559172 s. In the same vein, still for the conventional modal analysis technique, as shown in Table 7, bus 30 of the IEEE 30 bus system has the highest value of the participation factor (0.2178), and thus, it is identified as the critical bus closest to voltage instability. For the IEEE 30 bus system, using the CMAT, it takes the total computational time of 0.923476 s to reach a solution. Result of the CMAT for the IEEE 57 bus power system is shown in Table 11. The least eigenvalue of this system is 0.2184 and this corresponds to mode 27 as shown in Table 11. This mode is then taken as the critical mode and the corresponding participation factors for each load bus of the IEEE 57 bus power system were computed. As can be seen from Table 11, bus 31 has the highest value (0.1687) of CMAT, and thus, it is ranked as the critical bus of the IEEE 57 bus system. It takes the total computational time of 1.709213 s to reach a solution.

For the proposed IMAT, information contained in the Jacobian matrix which relates the reactive power and the voltage magnitude of the system together was further explored for voltage stability assessment. Unlike the CMAT which makes use of the reduced Jacobian matrix to determine the critical mode of the system, the suggested IMAT uses only the submatrix of the full Jacobian matrix to determine the critical mode (smallest eigenvalues) of the system. Tables 4, 8 and 12 show the simulation results obtained using the IMAT for the WSCC 9-bus, IEEE 30 bus and IEEE 57 bus test systems, respectively. For the WSCC 9-bus system, the minimum eigenvalue is 1.4946 and this corresponds to mode 6. Bus participation factors (PFs) to this critical mode are then computed. Results of this computation shows bus 5 as the critical bus of the system. This is due to its highest value of PF (0.8558) as shown in Table 4. The table also shows that the proposed IMAT predicts the critical buses in exactly the same ranking order with the CMAT. Although, the computational time for each technique differs. The proposed IMAT saves time by 6.8% compared with the conventional approach.

Similarly, for the IEEE 30 bus test system, with the IMAT, the smallest eigenvalues of this system is 0.4481 and corresponds to mode 16 as shown in Table 8. Thus, this mode (mode 16) is then considered as the critical mode. The bus PFs for each load bus are then generated based on the critical mode of the system to predict the proximity of it to voltage collapse. Unlike the CMAT, which identified bus 30 as the critical bus of the IEEE 30 bus system, we observed that, for the suggested IMAT, bus 27 was found to have the maximum value of PF (0.2458). Therefore, this bus is taken as the weakest bus of the IEEE 30 bus system. However, it must be stated that, when compared with the traditional power-flow-based *VCPI* and the proposed *PVBI*, bus 27 appears to have the minimum permissible reactive power loading (25 MVar) compared with other load buses of the IEEE 30 bus system. Thus, bus 27, being the critical bus of the IEEE 30 bus system, as identified by the performance voltage stability indices is also in agreement with the result obtained using the proposed IMAT. For the IEEE 30 bus system, the proposed IMAT also saves time by 11.4%. By following same procedures involved in the identification of both critical mode and node using the proposed IMAT, as in both WSCC 9 bus and the IEEE 30 bus test systems, also, for the IEEE 57 bus test system, bus 31 is ranked as the most critical bus of the system being the bus with the highest PF value of 0.1669 as shown in Table 12. When compared with the CMAT, the proposed IMAT saves time by 39.4%.

4.4. A Brief Comparison of all the Techniques Presented

The technique based on the use of *VCPI* takes longer period of time to identify the critical bus of the system compared to the proposed *PVBI*. The proposed *PVBI* also brings out, very clearly, the details information on the voltage deviations of each load bus, the maximum loadability and the total number of steps taken by each bus to reach a point of voltage collapse. This, without doubt, will be of tremendous advantage to the power system utilities, especially, in the planning and operation of the system. Both CMA and IMA techniques have proven to be more significant in voltage stability analysis compared with the power-flow-based performance indices presented. This is because with the modal analysis, solutions are attained in just a single computational step. Nonetheless, the use of the suggested IMAT will go a long way to assist the system engineers in the analysis of voltage stability. This is because computational burden in involved may still be reduced further using the proposed IMAT, as it does not depend on the entire Jacobian matrix. A considerable time is also saved in the course of computation using the IMAT compared with the traditional approach.

5. Conclusions

Performance indices and techniques for predicting voltage collapse in a power system were presented in this paper. The effectiveness of all the approaches presented are tested on the WSCC 9-bus, IEEE 30 bus and IEEE 57 bus power systems. Power flow solution was performed to arrive at a solution in all the cases considered. Comparison of the proposed methods is done with exiting power-flow-based voltage collapse proximity index (*VCPI*) and the conventional modal analysis techniques. Results of the simulation obtained show that the suggested *PVBI* could be of tremendous benefit when compared to the conventional *VCPI* in the analysis of voltage stability. This is because detailed information that may be required by the power system operators can be easily obtained using the technique. Also, the suggested improved modal analysis could serve as an alternative tool to the conventional method for voltage stability assessment in a power system.

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