



MDPI

Scaling Criteria for Axial Piston Machines Based on Thermo-Elastohydrodynamic Effects in the Tribological Interfaces

Lizhi Shang * and Monika Ivantysynova

Maha Fluid Power Research Center, Purdue University, 1500 Kepner Dr., Lafayette, IN 47905, USA; mivantys@purdue.edu

* Correspondence: shangl@purdue.edu

Received: 25 October 2018; Accepted: 14 November 2018; Published: 19 November 2018



Abstract: In lieu of reliable scaling rules, hydraulic pump and motor manufacturers pay a high monetary and temporal price for attempting to expand their production lines by scaling their existing units to other sizes. The challenge is that the lubricating interfaces, which are the key elements in determining the performance of a positive displacement machine, are not easily scalable. This article includes an analysis of the size-dependence of these units with regard to the significant physical phenomena describing the behavior of their three most critical lubricating interfaces. These phenomena include the non-isothermal elastohydrodynamic effects in the fluid domain, and the heat transfer and thermal elastic deflection in the solid domain. The performance change due to size variation is found to be unavoidable and explained through fundamental physics. The results are demonstrated using a numerical fluid–structure–thermal interaction model over a wide range of unit sizes. Based on the findings, a guide to scaling swashplate-type axial piston machines such as to uphold their efficiency is proposed.

Keywords: axial piston machine; hydraulic pump; hydraulic motor; scaling; size-dependence; lubricating interface; tribology; elastohydrodynamics; heat transfer

1. Introduction

From the electro-hydraulic actuator (EHA) system in aviation to the hydraulic shovel system in the mining industry, the size of swashplate-type axial piston machines varies from under one cc (cubic centimeter) to over a thousand cc. For manufacturers of hydraulic pumps and motors, this generates a demand for wide production lines spanning vastly different unit sizes. Even though axial piston machines of different sizes share the same working principle, finding rules for scaling an existing pump/motor design to a different size while retaining its efficiency has proven difficult.

Series of pumps and motors that differ in size but share the same design do exist in the axial piston machine categories. Designers and manufacturers found that the maximum shaft speed of the pumps and motors is restricted by viscous shear, which is generated by the relative sliding velocity in the tribological interfaces. Ivantysyn and Ivantysynova [1] suggested a relative speed of 3~5 m/s between the cylinder block and valve plate for any size of pumps and motors in the case of an open circuit, and twice the value for a closed circuit. Via this rule of thumb, well-designed units are often scaled to other sizes in order to extend the market range using the following scaling rule:

$$\lambda = \left(\frac{V}{V_0}\right)^{\frac{1}{3}} \quad n = n_0 \cdot \lambda^{-1} \tag{1}$$

where the linear scaling factor λ describes the linear dimensional ratio between the scaled and pre-scaled unit, and is also used to define the operating rotational speed of the scaled unit. For example, when scaling a unit by the linear scaling factor of two, the displacement volume *V* becomes eight times larger than the original one, and the operating speed *n* becomes half that of the original one.

However, to achieve the pre-scaled performance in terms of the energy efficiency and the service lifetime, the required trial-and-error design process is both financially and temporally expensive.

The performance of a swashplate-type axial piston machine is determined by the sealing function and the bearing function of its three lubricating interfaces: the piston/cylinder interface, the cylinder block/valve plate interface, and the slipper/swashplate interface, as shown in Figure 1. Merritt [2] parameterized the energy efficiency of hydraulic pumps and motors through an empirical model, which uses the operating pressure and operating speed as input variables. However, the coefficients used in his model vary with different unit designs and unit sizes, and are therefore not scalable. The friction and leakage losses of the three major lubricating interfaces of such units were calculated analytically under the assumption of a rigid body and fixed gap height by Manring, Ivantysyn, and Ivantysynova [1,3,4]. On the basis of these analyses, their work also suggested an optimal gap height for these interfaces with regard to minimizing losses [1]. However, the scaling rule that was derived from these loss models is limited by its lack of accounting for fundamental physical phenomena, including the non-isothermal fluid properties, hydrodynamic effects, elastic deformations, heat transfer, and the interactions between all of the above. In order to find a more effective scaling rule for swashplate-type axial piston machines, a detailed model is required—one that allows for analyzing the performance of the three lubricating interfaces.



Figure 1. Three lubricating interfaces.

The first numerical analysis of the piston/cylinder interface was presented by van der Kolk in 1972 [5]. In this model, the piston/cylinder interface is modeled as a tilt journal bearing. Ivantysynova [6] advanced the piston/cylinder interface model by considering the non-isothermal nature of the fluid. The piston/cylinder interface modeling approach then was further developed by taking into consideration piston micro-motion [7], elastic deformation [8], and solid body heat transfer [9,10]. The simulation model was further improved and validated against measurements [11,12].

The cylinder block/valve plate interface has also been modeled numerically; groundwork toward this was done by Sartchenko [13], who first studied the force balance condition in this interface, and Franco [14], who first derived the hydrostatic pressure in the cylinder block/valve plate interface. Wieczorek and Ivantysynova first solved the force balance in the cylinder block/valve plate interface with the cylinder block micro-motion considered [7]. Recently, the elastic deformation and solid body heat transfer were added into the model [15,16]. That model was then used to predict the

temperature field in the valve plate, and was compared with a measured temperature distribution [17]. The comparison shows a good agreement between the measurement and the simulation.

Lastly, the slipper/swashplate interface was analytically studied as a pure hydrostatic bearing by Shute and Turnbull [18,19]. Hooke and Li [20,21] first studied the pressure distribution in the slipper/swashplate interface by solving the polar form of the Reynolds equation numerically. Recently, Schenk and Ivantysynova developed a more advanced slipper/swashplate interface model [22–24]. That model was validated by comparing the simulated slipper/swashplate film thickness to measurements taken during operation using eddy current sensors embedded in the swashplate [25].

Together with a pump flow temperature prediction model [26], Shang and Ivantysynova proposed a concept for nonlinearly scaling a number of key design parameters in order to compensate for the performance loss due to the size variation; the concept is based on a full factory simulation study [27,28]. According to their research, nonlinearly scaled design parameters are able to bring the scaled pump and motor performance closer to the pre-scaled one.

The goal of the work presented in this article is to explain the tribological interfaces' performance change due to size variation from fundamental physics, including non-isothermal elastohydrodynamic effects in the fluid domain, and heat transfer and elastic deformation in the solid domain. Furthermore, a general guide to designing the swashplate-type axial piston machine based on the size-dependence of the physical phenomena will be proposed. To accomplish this goal, the paper presents the scaling rules that are derived from a simplified loss model (Section 2), followed by a summary of the detailed model for the three lubricating interfaces (Section 3). The analytical analysis of the fundamental physics and the simulation results are presented in Section 4 of this article. The findings and a scaling guide derived from them are summarized in Section 5, and, finally, the overarching conclusions of the analysis and sizing studies done are presented in Section 6.

2. Scaling of Swashplate-Type Axial Piston Machines

As the core component of fluid power systems, the swashplate-type axial piston machine converts the mechanical power to fluid power and vice versa, depending on its working mode. In the pumping mode, the shaft torque rotates the cylinder block against the opposing torque due to the pressure force in the displacement chambers, which is generated by the piston/slipper assembly through the use of an inclined swashplate. This allows the fluid to be displaced from the low-pressure input port to the high-pressure output port. In the motoring mode, the pressure force in the displacement chambers is applied to the inclined swashplate as a driving torque, thus driving the cylinder block, together with the shaft rotating against the torque load imposed on it.

Parameters that are pertinent to the kinematics of a swashplate-type axial piston machine are shown in Figure 2. A global Cartesian coordinate system is used in Figure 2; its origin is at where the shaft axis crosses the virtual plane on which all the ball joint centers lie. The positive *z*-axis points along the shaft axis toward the swashplate, the positive *y*-axis points toward the outer dead center (ODC), and the positive x-direction is defined according to the right-hand rule. According to Ivantysyn and Ivantysynova [1], the maximum stroke H_K , i.e., the piston's maximum axial travel between the ODC (outer dead center) and the IDC (inner dead center) is dependent on the pitch diameter d_B , and the swash plate angle β :

$$H_K = d_B \cdot \tan\beta \tag{2}$$

The piston stroke and the piston axial velocity are dependent not only on the pitch diameter d_B and the swashplate angle β , but also on the piston angular position φ :

$$s_{K} = -1 \cdot d_{B} \cdot \tan \beta \cdot \frac{1 - \cos \varphi}{2}$$

$$v_{K} = \frac{ds_{K}}{d\varphi} \omega = -\frac{1}{2} \cdot d_{B} \cdot \tan \beta \cdot \sin \varphi \cdot \omega$$
 (3)



Figure 2. Swashplate-type axial piston machine.

Scaling of Lubricating Interfaces in Swashplate-Type Axial Piston Machines

There are three lubricating interfaces separating the sliding solid parts, as shown in Figure 1. Namely, the piston/cylinder interface, the cylinder block/valve plate interface, and the slipper/swashplate interface. The total power loss of a swashplate-type axial piston machine is dominated by the energy dissipation due to the viscous shear of the working fluid in these three tribological interfaces, which happens in two forms.

- The pressure difference across the gap pushes the fluid through the interface, generating fluid shear and energy dissipation. In this way, the energy dissipation has a positive correlation with gap height, as well as with the gap flow rate.
- The relative motion of the solid boundaries of the lubricating gap causes the fluid to shear, and dissipates energy into heat. In this way, the energy dissipation has a negative correlation with gap height.

Unfortunately, due to the nature of these two opposing physical phenomena, a design that benefits one can be harmful to the other. According to Ivantysyn and Ivantysynova [1], the energy efficiency of the three lubricating interfaces is determined by the power loss due to the leakage losses and friction losses. As described by Shang and Ivantysynova [27], the power loss due to the leakage and friction in a parallel gap as shown in Figure 3a,b yield:

$$P_{SQ} = \frac{1}{12\mu} \cdot \frac{\Delta p^2}{l} \cdot b \cdot h^3 \quad P_{ST} = \mu \cdot \frac{v^2}{h} \cdot b \cdot l \tag{4}$$



Figure 3. (a) Poiseuille flow in a paralleled gap; (b) Viscous friction in a paralleled gap.

To minimize the total power loss by varying the gap height:

$$\frac{d(P_{SQ} + P_{ST})}{dh} = 0 \Rightarrow h_{opt} = \sqrt{\frac{2\mu \cdot v \cdot l}{\Delta p}}$$
(5)

When scaling the lubricating interfaces in the swashplate-type axial piston machines, by keeping the same sliding velocity as mentioned before, the gap height should be scaled as:

$$h_{opt} = \sqrt{\frac{2\mu \cdot v \cdot l}{\Delta p}} = \sqrt{\frac{2\mu \cdot v \cdot \lambda \cdot l_0}{\Delta p}} = \sqrt{\lambda} \cdot h_{opt_0}$$
(6)

However, the gap height in the lubricating interfaces of these pumps and motors is determined by the force balance, the elastic deformation, and the position of the solid bodies bounding the three lubricating interfaces. Therefore, the gap height of the lubricating interface is not directly controllable through nominal design parameters, but rather is determined from a series of complicated fluid–structure and thermal interactions.

Not only are the three interfaces required to fulfill a sealing function; they also have a bearing function. The hydrostatic and hydrodynamic pressures generated in the gap have to bear the external load. A design failing to carry the load will result in unfavorable fluid film behavior, extreme gap heights, mixed or solid friction, or even damage to the parts.

The displacement chamber pressure force, which is parallel to the cylinder bore axis, can be decomposed into two forces. One is perpendicular to the swashplate, and is applied on the slipper socket as the main external load of the slipper/swashplate interface. This load is balanced by the fluid's hydrostatic and hydrodynamic pressure force in the slipper/swashplate lubricating gap. The other force component acts on the center of the piston ball joint, pointing in the y-direction and behaving as a bending moment. This is the main external load of the piston/cylinder interface, which is balanced by the hydrodynamic pressure distribution in the piston/cylinder interface. The displacement chamber pressure is applied in the other direction, on the cylinder block, pushing the cylinder block against the valve plate. The normal force due to the displacement chamber pressure, and the moments due to the displacement chamber pressure difference between the suction stroke and discharge stroke, are the main external loads of the cylinder block/valve plate interface, which is balanced hydrostatically and hydrodynamically.

The force balance between the external loads and the hydrostatic–hydrodynamic pressure field in the lubricating gaps is impossible to be calculated analytically due to the complicated motions and the irregular gap shapes caused by elastic deformation.

Therefore, in order to find a more effective scaling rule for swashplate axial piston machines, a detailed lubricating interface model that allows for an understanding of the fluid behavior in the lubricating gap is required. To simulate the behavior of the main interfaces of these units, this article draws on the fluid–structure–thermal interaction model for each of the described three interfaces.

3. Fluid-Structure-Thermal Interaction Model

The fluid–structure–thermal interface model for the piston/cylinder interface [9], for cylinder block/valve plate interface [16], and for slipper/swashplate interface [23] share a similar structure, as shown in Figure 4. The pressure in the lubricating gap is calculated by solving the Reynolds equation in a discretized finite volume fluid grid from the pump kinematics, the shape and deformation of the lubricating gap surfaces, the fluid properties, and the normal squeeze motion. The fluid temperature is solved taking into account convection, conduction, and heat generation in the fluid. The pressure deformation is solved using an offline influence matrix method, assuming a linear relationship between the pressure and the deformation. The fluid pressure, fluid temperature, and the solid body pressure deformation is solved in an inner iterative loop, which is also called the fluid–structure interaction (FSI)

loop. This FSI loop is run until convergence for each time step. There are 360 time steps per revolution for the cylinder block/valve plate interface and the slipper/swashplate interface. There are 1500 time steps for each revolution in the piston/cylinder interface simulation. At the end of each revolution, the three-dimensional heat transfer model solves for the temperature distribution in the solid bodies. It is further used as a thermal load for the thermal elastic deformations. The resulting temperature field and thermal deformations are used in the FSI loop as boundary conditions. The outer iterative loop takes the thermal behavior of the solids into consideration. The simulation concludes when the outer FSTI loop converges.



Figure 4. The fluid-structure and thermal interaction model.

The main output of the simulation is the viscous energy dissipation, the leakage flow rate, and the torque loss. The viscous energy dissipation, assuming no fluid velocity in the direction of the gap height, is described as:

$$\Phi = \int_{V} \mu(\frac{\partial v}{\partial h})^{2} dV \tag{7}$$

4. Size-Dependence of Physical Phenomena in Tribological Interfaces

4.1. Linear Scaling Method (Conventional Approach)

The tribological interfaces in an axial piston machine do not perform the same before and after linear scaling. Before explaining the reason, an example is given to demonstrate the performance difference of the three interfaces, scaled according to the linear scaling law. The linear scaling law scales the axial piston machine proportionally, such that the ratio of one design parameter to another is maintained. The operating speed is scaled to keep the relative sliding speed between the parts that form the tribological interfaces constant. For this example, a Sauer-Danfoss S90 75cc unit is selected as the baseline unit. This baseline is linearly scaled to two different sizes. The smaller one is scaled based on the linear factor λ of 0.5, and the bigger one is scaled based on the linear factor λ of 2. All three units have been simulated using the fluid–structure–thermal interaction model described in the previous section. The piston/cylinder interface fluid domain is discretized into 96,000 finite volumes; the cylinder block/valve plate interface fluid domain is discretized into 360,000 finite volumes; and the slipper/swashplate interface is discretized into 141,600 finite volumes. The meshes of the solid bodies are shown in Figure 5, which indicates the mixed thermal boundaries in white and the Neumann

thermal boundaries in red. As Table 1 shows, the biggest unit has a displacement volume that is 64 times larger than the smallest unit.



Figure 5. Solid body mesh and thermal boundaries.

	Α	В	С
Linear factor λ	0.5	1	2
Unit size [cc]	9.375	75	600
Pressure [bar]	400	400	400
Displacement [%]	100	100	100
Speed [rpm]	7200	3600	1800

Table 1. Summary of unit sizes and operating conditions.

In order to keep the sliding velocity the same, the rotational speed is scaled with the reciprocal of the linear factor λ . The sizes of the three units and their respective operating conditions are summarized in Table 1, with the labels A, B, and C. These three sets of size/operating condition combinations are used repeatedly through this chapter.

Since A, B, and C share the same pressure, the same swashplate angle, and the same sliding velocity, the normalized energy dissipation, the normalized leakage flow rate, and the normalized torque loss of these three cases are comparable for each of the lubricating interfaces. The normalized energy dissipation uses the theoretical output power as reference, the normalized leakage flow rate uses the theoretical outlet flow rate as reference, and the normalized torque loss uses the theoretical input torque as reference. The power loss, the volumetric loss, and the torque loss represent the fluid behavior and the performance of each interface.

Figures 5–7 show the simulated performance of the three lubricating interfaces using the previously described fluid–structure–thermal interaction model, which takes into consideration the non-isothermal fluid, as well as both pressure and thermal solid body deformation.

Figure 6 shows the performance comparison of the piston/cylinder interface for A, B, and C. The normalized energy dissipation, normalized leakage flow, and normalized torque loss correspond to a single piston/cylinder interface. According to the simulation results, the normalized energy dissipation and the normalized torque loss for different unit sizes are relatively close (they differ by less than 20%). However, the normalized leakage exhibits significant size-dependence. To be exact, the piston/cylinder interface C produces more than twice the simulated normalized leakage compared to its smallest counterpart, A.

Figure 7 shows the performance comparison of the cylinder block/valve plate interface of the three cases, A, B, and C. According to the results, the cylinder block/valve plate interface behaves

very differently for different sizes. The larger unit produces more normalized leakage than the smaller ones, while the smaller unit produces more normalized torque loss than the larger ones. This results in a similar normalized energy dissipation between the mid-size B and large-size C, but a greater normalized power loss in the small unit A.



Figure 7. Normalized cylinder block/valve plate interface performance for different sizes.

Figure 8 shows the normalized energy dissipation, normalized leakage, and the normalized torque loss for a single slipper/swashplate interface corresponding to each of the simulated unit sizes. According to the results, similar to the piston/cylinder interface and the cylinder block/valve plate interface, the large unit C produces more leakage than the smaller units. Furthermore, the smaller unit A produces more torque loss than the larger units. At the selected operating conditions simulated for the three units, the energy dissipation in the slipper/swashplate interface is dominated by viscous shear. Therefore, the energy dissipation shows similar trends to the torque loss.



Figure 8. Normalized slipper/swashplate interface performance for different sizes.

linearly scaled piston/cylinder interface, cylinder block/valve plate interface, and slipper/swashplate interface do not perform the same as their pre-scaled counterparts without design modifications. The undesired size-dependence of the lubricating interface performance leads to the demand for an understanding of the physics behind the scaling process. In the following sections, the physical phenomena that contribute to the tribological performance of the three lubricating interfaces are studied independently.

4.2. Analysis of the Governing Equations Describing the Fundamental Physical Phenomena in Scaled Tribological Interfaces

The physical phenomena that are studied in this section include the viscous fluid pressure distribution (Reynolds equation), the elastic solid body deformation under both pressure and thermal loads, the heat dissipation and heat transfer in the fluid domain, and the heat transfer in the solid bodies. For each physical phenomenon, the governing equations are derived using scalable dimensions in order to examine their size-dependence.

4.2.1. Fluid Pressure Distribution

The Reynolds equation [29], which is derived from the Navier–Stokes equation and the conservation of mass, is the fundamental governing equation of a compressible, Newtonian, laminar flow pressure distribution. A general form of the Reynolds equation of an arbitrary lubricating gap, e.g., the one shown in Figure 9, can be written as:

$$\nabla \cdot \left(-\frac{\rho h^3}{12\mu} \nabla p\right) + \frac{\left(\vec{v}_t + \vec{v}_b\right)}{2} \cdot \nabla(\rho h) - \rho \vec{v}_t \cdot \nabla h_t + \rho \vec{v}_b \cdot \nabla h_b + \rho(w_t - w_b) = 0$$
(8)



Figure 9. An arbitrary lubricating gap.

The subscripts *t* stands for top surface, and b stands for bottom surface. Each dimensional variable can be described using an original dimension (subscript 0) and a linear scaling factor, as demonstrated in Equation (9). The linear factor is consistent for each variable.

$$x = \lambda x_0 \quad y = \lambda y_0 \quad z = \lambda z_0 \quad \overrightarrow{v} = \overrightarrow{v}_0 \quad \overrightarrow{u} = \overrightarrow{u}_0$$
$$\nabla \cdot \overrightarrow{\mathbf{A}} = \frac{1}{\lambda} \cdot \left(\nabla \cdot \overrightarrow{\mathbf{A}} \right)_0 \quad \nabla a = \frac{1}{\lambda} \cdot (\nabla a)_0 \tag{9}$$

By applying Equation (9) to Equation (8), the Reynolds equation becomes:

$$\nabla \cdot \left(-\frac{\rho \lambda h_0^3}{12\mu} \nabla p(\lambda) \right) + \frac{(\vec{v}_{t0} + \vec{v}_{b0})}{2} \cdot \nabla (\rho h_0) - \rho \vec{v}_t \cdot \nabla h_{t0} + \rho \vec{v}_b \cdot \nabla h_{b0} + \rho (w_t - w_b) = 0$$
(10)

Comparing Equation (10) to Equation (8), the remaining scaling factor makes the Reynolds equation size-dependent. Therefore, the pressure distribution in a linearly scaled lubricating gap is not able to retain the behavior of the gap in the pre-scaled unit.

4.2.2. Solid Body Elastic Deformation

The solid body elastic deformation due to the pressure and thermal loads also plays an essential role in the performance of the lubricating interface. The nodal displacement under the pressure and thermal load is determined using the principle of minimal potential energy. The total potential energy Π is the sum of the strain energy U and the applied load potential energy V:

$$\Pi = U + V \tag{11}$$

where the applied load potential energy *V* is a function of the nodal force \mathbf{f}_E and the nodal displacement vector \mathbf{u} :

$$V = -\mathbf{u}^T \mathbf{f}_E \tag{12}$$

and the strain energy *U* is a function of the elastic strain ε_F , and the stress σ :

$$U = \frac{1}{2} \int_{\delta V} \boldsymbol{\varepsilon}_F^T \boldsymbol{\sigma} dV \tag{13}$$

The relationship between the stress and the elastic strain can be expressed by the constitutive matrix **C**:

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon}_F = \mathbf{C}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_T) \tag{14}$$

where the constitutive matrix **C** contains the isothermal elastic modulus *E* and the Poisson's ratio *v*:

$$\mathbf{C} = \frac{E}{(1-\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0\\ \nu & 1-\nu & \nu & 0 & 0 & 0\\ \nu & \nu & 1-\nu & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$
(15)

The matrix **B** expresses the strain as a function of the nodal displacement:

$$\varepsilon = \mathbf{B}\mathbf{u} \tag{16}$$

Combining Equations (13), (14), and (16):

$$U = \frac{1}{2} \int_{\delta V} \left(\mathbf{B} \mathbf{u} - \boldsymbol{\varepsilon}_T \right)^T \mathbf{C} (\mathbf{B} \mathbf{u} - \boldsymbol{\varepsilon}_T) dV$$
(17)

The total potential energy then becomes:

$$\Pi = \frac{1}{2} \int_{\delta V} \left(\mathbf{B} \mathbf{u} - \boldsymbol{\varepsilon}_T \right)^T \mathbf{C} (\mathbf{B} \mathbf{u} - \boldsymbol{\varepsilon}_T) dV - \mathbf{u}^T \mathbf{f}_E$$
(18)

According to the minimal potential energy principle, by setting the differential of the total potential energy with respect to the nodal displacement to zero, the elasticity equation is obtained:

$$\int_{\delta V} \mathbf{B}^{\mathrm{T}} \mathbf{C} \mathbf{B} dV \mathbf{u} = \int_{\delta V} \mathbf{B}^{\mathrm{T}} \mathbf{C} \boldsymbol{\varepsilon}_{T} dV + \mathbf{f}_{E}$$
(19)

Equation (19) can be written more compactly as:

$$\mathbf{k}\mathbf{u} = \mathbf{f}_T + \mathbf{f}_E \tag{20}$$

In order to examine the size-dependence of the solid body deformation under both pressure and thermal loading, the loading pressure distribution and temperature distribution is proportionally scaled in order to allow for a direct comparison of the effects of scaling across different unit sizes. Therefore, the nodal force vector due to the external load scales with area, which scales according to the second order of the linear scaling factor:

$$\mathbf{f}_E(\lambda) = \lambda^2 \mathbf{f}_{E0} \tag{21}$$

When using scalable dimensions, Equation (13) therefore becomes:

$$\boldsymbol{\varepsilon}(\lambda) = \mathbf{B}(\lambda)\mathbf{u}(\lambda) \tag{22}$$

Again, scaling the pressure distribution assures that the stress remains the same, while the nodal displacement scales according to the linear scaling factor. Accordingly, Equation (22) becomes:

$$\boldsymbol{\varepsilon}_0 = \lambda \mathbf{B}(\lambda) \mathbf{u}_0 \tag{23}$$

It can be seen from Equation (20) that the matrix **B** scales according to the reciprocal of the linear scaling factor:

$$\mathbf{B}(\lambda) = \lambda^{-1} \mathbf{B}_0 \tag{24}$$

The nodal forces vector due to thermally-induced stress then scales as:

$$\mathbf{f}_{T}(\lambda) = \int_{\delta V} \mathbf{B}(\lambda)^{T} \mathbf{C} \boldsymbol{\varepsilon}_{T} dV(\lambda) = \lambda^{2} \int_{\delta V} \mathbf{B}_{0}^{T} \mathbf{C} \boldsymbol{\varepsilon}_{T} dV_{0} = \lambda^{2} \mathbf{f}_{T0}$$
(25)

The element stiffness matrix **k** scales as:

$$\mathbf{k}(\lambda) = \int_{\delta V} \mathbf{B}(\lambda)^{T} \mathbf{C} \mathbf{B}(\lambda) dV(\lambda) = \lambda \int_{\delta V} \mathbf{B}_{0}^{T} \mathbf{C} \mathbf{B}_{0} dV_{0} = \lambda \mathbf{k}_{0}$$
(26)

With scaling, the elasticity equation becomes:

$$\lambda \mathbf{k}_0 \mathbf{u} = \lambda^2 \mathbf{f}_{T0} + \lambda^2 \mathbf{f}_{E0} \tag{27}$$

Therefore, the resulting nodal displacement due to pressure and thermal loading is scaled based on the first order of the scaling factor:

$$\mathbf{u}(\lambda) = \lambda \mathbf{u}_0 \tag{28}$$

In conclusion, the elastic deformation due to both the pressure and thermal loading is linearly scalable. The solid parts of the lubricating interfaces deform linearly with respect to the first-order linear scaling factor when the pressure and temperature distribution is kept consistent. Therefore, the elastic deformation principle itself does not cause any performance difference when scaling an axial piston machine.

To verify this conclusion, simulation cases A, B, and C are reconfigured in order to ensure that they remain comparable in terms of both the pressure and thermal loading. The temperature distribution is made identical for all three cases by turning off the solid body heat transfer model. Instead of solving the heat transfer problem, the solid body temperatures remain at their initial values. The heat transfer phenomenon is further discussed in future sections. However, from the conclusion of the Reynolds equation size dependence study associated with Equation (10), the pressure distribution is not able to

$$\mu = \lambda \cdot \mu_0 \tag{29}$$

With the viscosity as described by Equation (26), Equation (7) becomes:

$$\nabla \cdot \left(-\frac{\rho h_0^3}{12\mu_0} \nabla p(\lambda) \right) + \frac{(\vec{v}_{t0} + \vec{v}_{b0})}{2} \cdot \nabla(\rho h_0) - \rho \vec{v}_t \cdot \nabla h_{t0} + \rho \vec{v}_b \cdot \nabla h_{b0} + \rho(w_t - w_b) = 0$$
(30)

In this way, the pressure distribution is maintained consistent.

The inputs for simulation cases A, B, and C are then characterized as shown in Table 2. Figure 10 shows a comparison of the normalized energy dissipation, the normalized leakage flow, and the normalized torque loss of a single piston/cylinder interface for the three simulation cases, A, B, and C. The fluid viscosity is artificially scaled with the linear scaling factor, and the fluid and the solid body heat transfer models are turned off to maintain a consistent temperature distribution between cases. The figure shows that the normalized energy dissipation, the normalized leakage, and the normalized torque loss are very similar throughout the three simulation cases. The performance differences are less than 4%.

Table 2. Summary of operating conditions with scaled fluid viscosity.

	Α	В	С
Linear factor λ	0.5	1	2
Unit size [cc]	9.375	75	600
Pressure [bar]	400	400	400
Displacement [%]	100	100	100
Speed [rpm]	7200	3600	1800
Fluid viscosity	$0.5 \cdot \mu_0$	μ_0	$2 \cdot \mu_0$



Figure 10. Normalized piston/cylinder interface performance comparison for different sizes.

Figure 11 shows a comparison of the normalized energy dissipation, the normalized leakage, and the normalized torque loss of the cylinder block/valve plate interface simulation for cases A, B, and C. Again, the fluid viscosity is artificially scaled with the linear scaling factor, and the fluid and the solid body heat transfer models are turned off to maintain a proportionally identical temperature distribution. Similar to the piston/cylinder interface comparison results, the cylinder block/valve plate interface results show that the interface performs very similarly for the three simulated unit sizes.



Figure 11. Normalized cylinder block/valve plate interface performance comparison for different sizes.

Figure 12 shows a comparison of the normalized energy dissipation, the normalized leakage, and the normalized torque loss of the slipper swashplate interface simulation for cases A, B, and C. As was done for the piston/cylinder interface and the cylinder block/valve plate interface, the fluid viscosity is artificially scaled with the linear scaling factor, and the fluid and solid body heat transfer models are turned off to maintain the temperature distribution consistent. The normalized energy dissipation and normalized torque loss are almost identical for the different sizes that are simulated. The normalized leakages from the three cases are also very close to each other.



Figure 12. Normalized slipper/swashplate interface energy dissipation comparison for different sizes.

The simulation results in Figures 10-12 validate all of the conclusions in the Reynolds equation size-dependence study and the elastic deformation size-dependence study. In summary, without the consideration of heat transfer, the scaled axial piston machine is able to achieve the same performance as the pre-scaled unit with a different fluid viscosity.

4.2.3. Multi-Domain Heat Transfer

To investigate the size-dependence of the multi-domain heat transfer, the simulation cases A, B, and C, as shown in Table 2, are rerun with both the fluid domain heat transfer model and the solid domain heat transfer model turned on.

Figure 13 shows a comparison of the normalized energy dissipation, the normalized leakage, and the normalized torque loss of a single piston/cylinder interface for the three simulation cases A, B, and C, again with the fluid viscosity artificially scaled with the linear scaling factor. Also considered in these simulations are the elastic solid body deformations due to the pressure and thermal loads. The fluid temperature distribution is calculated from a three-dimensional fluid heat transfer model, and the solid body temperature distribution is calculated from a three-dimensional solid body heat

transfer model. In comparison to Figure 10, the piston/cylinder interface shows much more variation across the different sizes in Figure 13.



Figure 13. Normalized piston/cylinder interface performance comparison for different sizes.

Figure 14 compares the normalized energy dissipation, the normalized leakage, and the normalized torque loss of the cylinder block/valve plate interface for cases A, B, and C, again with the fluid viscosity artificially scaled, and with both the fluid and the solid heat transfer models turned on. The fluid temperature distribution is calculated from a three-dimensional fluid heat transfer model, and the solid temperature distribution is calculated from a three-dimensional solid body heat transfer model. The figure shows that the normalized energy dissipation remains at roughly the same level for all three cases, but the normalized leakage and the normalized torque loss have major differences across the different unit sizes.



Figure 14. Normalized cylinder block/valve plate interface performance comparison for different sizes.

Figure 15 compares the normalized energy dissipation, the normalized leakage, and the normalized torque loss of the slipper/swashplate interface for the three simulated cases, A, B, and C, with the fluid viscosity artificially scaled, and with both the fluid and the solid heat transfer models turned on. Just as for the piston/cylinder interface and the cylinder block/valve plate interface, the fluid temperature distribution is calculated from a three-dimensional fluid heat transfer model, and the solid temperature distribution is calculated from a three-dimensional solid body heat transfer model. The different size slipper/swashplate interfaces behave very similarly, even with the heat transfer model turned on.

To further investigate the size-dependence of the thermal behavior of the three lubricating interfaces, the resulting solid body temperature distributions of the piston, cylinder block, and the slipper are studied. In order to compare the solid body temperature distributions between the different sizes, the solid parts are stretched to the same size, as illustrated in Figure 16.



Figure 15. Normalized slipper/swashplate interface energy dissipation comparison for different sizes.



Figure 16. Demonstration of solid body temperature comparison for different sizes.

Figure 17 shows the piston temperature distributions of cases A, B, and C. Clearly, the piston solid body temperature distributes differently over the three sizes. The temperature distribution of the larger unit shows more variation than its smaller counterpart.



Figure 17. Piston temperature distribution comparison for different sizes.

Figure 18 shows the cylinder block temperature distribution of cases A, B, and C. The cylinder block solid body temperature distribution also shows more variation for the larger size.

Figure 19 shows the slipper temperature distribution of cases A, B, and C. Similar to the piston and the cylinder block solid body temperatures, the slipper solid body temperature also shows more variation for the larger size. However, unlike the piston and the cylinder block, the slipper solid body thermal deformation under the temperature distributions that are shown in Figure 19 enlarges the solid body proportionally. The surface deflection due to thermal load has rather a limited impact on the fluid film behavior in the slipper/swashplate interface. Therefore, comparing Figure 8, Figure 12, and Figure 15, even though the thermal behavior is size-dependent, the slipper/swashplate interface size-dependence is mainly determined by the pressure distribution.



Figure 18. Cylinder block temperature distribution comparison for different sizes.



Figure 19. Slipper temperature distribution comparison for different sizes.

In order to understand the solid body temperature distribution, the fluid temperature distribution must be solved first. The energy equation that describes the convective–diffusive effects in the fluid domain is used to solve for the fluid temperature distribution:

$$\nabla \cdot (\rho \mathbf{V}T - \Gamma \nabla T) = S \tag{31}$$

where the diffusion coefficient is:

$$\Gamma = \frac{\kappa_{fluid}}{c_p} \tag{32}$$

and the source term *S* is:

$$S = \frac{\mu}{c_p} \left(\frac{\partial v}{\partial h}\right)^2 \tag{33}$$

Integrating the energy equation over a volume and applying the divergence theorem:

$$\int_{A} (\rho \mathbf{V}T - \Gamma \nabla T) \cdot dA\mathbf{n} = \int_{V} SdV$$
(34)

With scalable dimensions, the source term *S* becomes:

$$S(\lambda) = \frac{\mu(\lambda)}{c_p} \Phi_d(\lambda) = \frac{\mu_0}{c_p} \lambda^{-2} \Phi_{d0} = \lambda^{-2} S_0$$
(35)

Then, the energy equation for the scaled fluid domain yields:

$$\int_{A} (\rho \mathbf{V} T(\lambda) - \Gamma \nabla T(\lambda)) \cdot \lambda dA_0 \mathbf{n} = \int_{V} S_0 dV_0$$
(36)

By comparing Equations (34) and (36), it can be concluded that the temperature distribution in the fluid domain is not linearly scalable.

The governing energy equation for the three-dimensional heat transfer problem is commonly written as the sum of the convective part and the conductive part:

$$\nabla \cdot (\mathbf{q}_{cv} + \mathbf{q}_{cd}) = 0 \tag{37}$$

For a control volume, the energy equation yields:

$$\int_{A} \mathbf{q}_{cv} \cdot \mathbf{n} dA + \int_{V} \nabla \cdot \mathbf{q}_{cd} dV = 0$$
(38)

There are two types of convection boundaries commonly applied to the solid body surfaces:

-

• Neumann condition:

$$\mathbf{q}_{cv} \cdot \mathbf{n} dA = \frac{1}{2} \left[\int_{V} \mu \Phi_d dV - \int_{S} c_p \rho \mathbf{V} T \cdot \mathbf{n} dS \right]$$
(39)

• Mixed condition:

$$\mathbf{q}_{cv} \cdot \mathbf{n} dA = h(T_S - T_\infty) dA \tag{40}$$

The Neumann condition is applied to the running surfaces of the lubricating interface, where the heat flux into each of the solid parts adjacent to the fluid can be calculated as the energy dissipation, minus the energy taken away with the passing flow. The mixed condition is applied on the surfaces where the temperature of the surrounding environment is constant. The heat flux of the mixed condition can be calculated using the solid body surface temperature, the temperature of the surrounding environment, and a heat transfer coefficient.

The conductive part of the energy equation yields:

$$\int_{V} \nabla \cdot \mathbf{q}_{cd} dV = \int_{V} \nabla \cdot (-\kappa_{solid} \nabla T) dV$$
(41)

When use scalable dimensions, the Neumann-type convection boundary becomes:

$$\mathbf{q}_{cv}(\lambda) \cdot \mathbf{n} dA(\lambda) = \frac{1}{2} \left[\int_{V} \mu(\lambda) \lambda^{-2} \Phi_{d0} \lambda^{3} dV_{0} - \int_{S} c_{p} \rho \mathbf{V} T \cdot \mathbf{n} \lambda^{2} dS_{0} \right]$$
(42)

The mixed-type convection boundary becomes:

$$\mathbf{q}_{cv}(\lambda) \cdot \mathbf{n} dA(\lambda) = h(T_S - T_\infty) \lambda^2 dA_0 = \lambda^2 \mathbf{q}_{cv0} \cdot \mathbf{n} dA_0$$
(43)

The conductive term of the energy equation scales as:

$$\int_{V} \nabla \cdot (-\kappa_{solid} \nabla T(\lambda)) dV(\lambda) = \lambda^{2} \int_{V} \nabla \cdot (-\kappa_{solid} \nabla T(\lambda)) dV_{0}$$
(44)

Equations (42)–(44) show that the scaled solid body parts will result in a different dimensional heat transfer performance than when using linear scaling.

So far, the analytical investigation of the temperature distribution in both the fluid domain and the solid domain proved that the thermal characteristic of the lubricating interface is not scalable. For the purposes of scientific research, the artificial size-dependent viscosity in Equation (29) was used in Equation (45):

$$S(\lambda) = \lambda^{-1} S_0 \tag{45}$$

Then, the energy equation for the scaled fluid domain yields:

$$\int_{A} (\rho \mathbf{V} T(\lambda) - \Gamma \nabla T(\lambda)) \cdot dA_0 \mathbf{n} = \int_{V} S_0 dV_0$$
(46)

To keep the temperature distribution consistent in the scaled fluid domain, the temperature and the temperature gradient scale as:

$$T(\lambda) = T_0$$

$$\nabla T(\lambda) = \lambda^{-1} \nabla T_0$$
(47)

Equation (46) then becomes:

$$\int_{A} \left(\rho \mathbf{V} T_0 - \Gamma \lambda^{-1} \nabla T_0 \right) \cdot dA_0 \mathbf{n} = \int_{V} S_0 dV_0 \tag{48}$$

Equation (48) shows that consistency in the temperature distribution can be achieved if the fluid is not only given the viscosity of Equation (29), but also the following fluid diffusion coefficient:

$$\Gamma(\lambda) = \lambda \Gamma_0 \tag{49}$$

A similar approach can be used for the three-dimensional solid body heat transfer analysis. Applying Equation (29) to Equation (42):

$$\mathbf{q}_{cv}(\lambda) \cdot \mathbf{n} dA(\lambda) = \frac{1}{2} \left[\int_{V} \lambda \mu_0 \lambda^{-2} \Phi_{d0} \lambda^3 dV_0 - \int_{S} c_p \rho \mathbf{V} T \cdot \mathbf{n} \lambda^2 dS_0 \right]$$

$$= \lambda^2 \mathbf{q}_{cv0} \cdot \mathbf{n} dA_0$$
(50)

Then, together with Equation (43), the convective term of the energy equation scales as:

$$\mathbf{q}_{cv}(\lambda) \cdot \mathbf{n} dA(\lambda) = \lambda^2 \mathbf{q}_{cv0} \cdot \mathbf{n} dA_0 \tag{51}$$

Substituting Equations (41) and (51) into Equation (38):

$$\lambda^{2} \int_{A} \mathbf{q}_{cv0} \cdot \mathbf{n} dA_{0} + \lambda^{2} \int_{V} \nabla \cdot (-\kappa_{solid} \nabla T(\lambda)) dV_{0} = 0$$
(52)

The consistency of the temperature distribution in the scaled solid domain can be achieved if the solid part's conductivity scales with the first-order linear scaling factor:

$$\boldsymbol{\kappa}_{solid}(\lambda) = \lambda \boldsymbol{\kappa}_{solid_o} \tag{53}$$

In order to verify the analysis for the temperature distribution in the fluid and solid domains, the simulation cases A, B, and C are reconfigured to reflect Equations (29), (49), and (53). Since the fluid diffusion coefficient is a function of fluid conductivity κ_{fluid} and the fluid heat capacity c_p , in this study, the fluid conductivity was changed with the linear scaling factor, while the heat capacity remained the same. The simulation inputs for cases A, B, and C become those shown in Table 3.

Figures 19–21 show the simulated temperature distribution of the piston, the cylinder block, and the slipper using the artificially scaled fluid viscosity, fluid conductivity, and solid body conductivity, as specified in Table 3. The simulations are conducted with all of the fluid and structure physical phenomena, including heat transfer in the fluid and solid domains. Comparing Figures 19–21 to Figures 16–18, it can be observed that the solid body heat transfer is no longer size-dependent when the fluid viscosity, and the fluid and solid conductivities, are artificially scaled with the unit size.

	Α	В	С
Linear factor λ	0.5	1	2
Unit size [cc]	9.375	75	600
Pressure [bar]	400	400	400
Displacement [%]	100	100	100
Speed [rpm]	7200	3600	1800
Fluid viscosity	$0.5 \cdot \mu_0$	μ_0	$2 \cdot \mu_0$
Fluid conductivity	$0.5 \cdot \kappa_{fluid 0}$	$\kappa_{fluid 0}$	$2 \cdot \kappa_{fluid 0}$
Solid conductivity	$0.5 \cdot \kappa_{\text{solid}}$	κ_{solid} 0	$2 \cdot \kappa_{\text{solid}}$

Table 3. Summary of operating conditions with scaled fluid viscosity.



Figure 20. Piston temperature distribution comparison for different sizes.



Figure 21. Cylinder block temperature distribution comparison for different sizes.

Figures 22–24 show the normalized energy dissipation, the normalized leakage, and the normalized torque loss of the piston/cylinder interface, the cylinder block/valve plate interface, and the slipper/swashplate interface from the simulations A, B, and C, as shown in Table 3.

According to Figure 23, the normalized energy dissipation and normalized torque loss of the piston/cylinder interface from cases A, B, and C are almost identical for the unit sizes, which differ in size by a factor of 64 from the smallest to the largest. The normalized leakages, even though they are not identical, are very close to each other.

Figure 24 shows that the cylinder block/valve plate interfaces of cases A, B, and C behave very similarly. Both the magnitude and the shape of the normalized energy dissipation, the normalized leakage, and the normalized torque loss are very close to each other across the different sizes.

Similar to the piston/cylinder interface, Figure 25 shows that the normalized energy dissipation and normalized torque loss of slipper/swashplate interfaces from cases A, B, and C are almost identical. The normalized leakages, even though are not identical, are very close to each other.



Figure 22. Slipper temperature distribution comparison for different sizes.





Figure 25. Normalized slipper/swashplate interface performance for different size units.

Figures 19–24 show that the size-dependence of the performance of the axial piston machine can be eliminated by artificially scaling the fluid viscosity, the fluid conductivity, and the solid conductivity. The reason that the common practice, which is to proportionally scale an axial piston machine design to larger or smaller sizes, cannot achieve the pre-scaled performance can be found in the presented analysis and simulation studies, which show the size-dependent pressure distribution and the size-dependent multi-domain heat transfer.

5. Findings and Scaling Guides

According to Figures 5–7, by proportionally scaling the axial piston machine, the performance of the three lubricating interfaces does not match the pre-scaled one, which agrees with the result published before [27,28]. The reasons that were found through the analysis and demonstrated through the simulation are that:

- Pressure distribution and heat transfer in both the fluid domain and the solid domain are the only contributions to the size-dependence of the axial piston machine's lubricating interface performance.
- As long as the pressure distribution and temperature distribution are proportionally scaled, the deformation due to the pressure and thermal load stays constant, and therefore does not contribute to the size-dependence of the lubricating interface performance.

From the analysis conducted, it is also can be found that:

- A size-independent fluid pressure distribution can be achieved by scaling the viscosity with the linear scaling factor. The simulation results in Figures 10–12 demonstrated this by showing an identical normalized performance for all three lubricating interfaces when using a scaled viscosity and no heat transfer.
- The size-independent temperature distribution in both the fluid and solid domains can be achieved by scaling the fluid and solid conductivity with the linear scaling factor. The simulation results in Figures 20–25 demonstrated this by showing an identical normalized performance for all three lubricating interfaces with a scaled viscosity and conductivity.

From the findings of the analysis and simulation studies, the following scaling guide for all three lubricating interfaces emerges:

- When scaling a swashplate-type axial piston machine to a different size, the size-dependent
 pressure distribution-induced performance bias can be eliminated by using hydraulic fluid at a
 different viscosity grade.
 - Use higher viscosity for up-scaling.
 - Use lower viscosity for down-scaling.
 - Choose the viscosity based on the linear scaling factor as in Equation (1).
- The viscosity of the fluid can also be controlled by:
 - Increasing operating temperature for down-scaling
 - O Decreasing operating temperature for up-scaling.
- When using a fluid of different viscosity is not feasible, design modifications should compensate for the size-dependent sealing function of the scaled lubricating interfaces:
 - For the piston/cylinder interface, use a lower normalized clearance for up-scaling, and use a higher normalized clearance for down-scaling [28].
 - For the cylinder block/valve plate interface, increase the sealing land area for up-scaling, and decrease the sealing land area for down-scaling [27].

- For the slipper/swashplate interface, increase the sealing land area for up-scaling, and decrease the sealing land area for down-scaling.
- To compensate for the size-dependent heat transfer, the design of the lubricating interfaces needs to be modified:
 - When up-scaling, the lubricating interface design should be modified in order to increase the cooling performance of the lubricating gap, e.g., adding a flow channel beneath the valve plate to smooth the temperature distribution.
 - When down-scaling, the lubricating interface design should be modified to create more thermal deformation, e.g., by using a bi-material solid body [30].

Even though the simulation studies that were shown pertain to the fluid–structure and thermal interaction model for all three lubricating interfaces in a swashplate-type axial piston machine, the analytical study is not limited to a single type of hydraulic pump. The findings will hold true for not only the tribological interfaces in axial piston machines, radial piston machines, internal and external gear pumps and motors, gerotors, and vane pumps, but also for all the thermal elastohydrodynamic tribological interfaces in bearings and seals.

6. Conclusions

This paper presented an analytical study of the size-dependence of thermal elastohydrodynamic tribological interfaces. The physical phenomena that were studied include the hydrostatic and hydrodynamic pressure distributions, the heat transfer and heat generation in the fluid film, the heat transfer in the solid domain, and the solid body deformation due to both the pressure loading and the thermal loading.

The analysis finds that the performance change due to size variation is unavoidable. The findings indicates that the pressure distribution in the lubricating gap, and the heat transfer in the fluid and in the solid bodies are the only size-dependent physical phenomena; therefore, they are the only contributors to the performance change of the thermal elastohydrodynamic tribological interfaces in response to scaling.

A state-of-the-art fluid-structure-thermal interaction model for the three lubricating interfaces in swashplate-type axial piston machines was used to demonstrate the consequences of the analytical study. Three different pump sizes were modeled in a series of simulation studies, the largest of which was 64 times bigger than the smallest. These simulation studies verified the findings of the analysis, and allowed for the creation of a general guide to scaling aimed at maintaining the efficiency of the original unit. This scaling guide enables pump manufacturers to apply the developed scaling laws, and efficiently generate scaled pump designs of much higher efficiency than that allowed for by the traditional approach of linear scaling.

Author Contributions: L.S. and M.I. conceptualized the scaling problem, performed analysis and conducted the simulation study. This paper is wrote by L.S.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

Symbols	Denotation	Unit
В	Strain-displacement matrix	[1/m
С	Constitutive matrix	[Pa]
d	Diameter	[m]
f	Vector of nodal force	[N]
h	Gap height	[m]

Н	Maximum stroke	[m]
k	Element stiffness matrix	[N/m]
1	Length	[m]
Р	Power	[W]
р	Pressure	[Pa]
q	Rate of heat flux	$[W/m^2]$
S	Stroke	[m]
Т	Temperature	[K]
U	Strain energy	[J]
u	Vector of nodal displacement	[m]
V	Applied load potential energy	[J]
υ	Speed	[m/s]
w	Normal squeezing velocity	[m/s]
β	Swashplate angle	[rad]
Ε	Elastic modulus	[Pa]
υ	Poisson's ratio	[-]
Г	Diffusion coefficient	$[kg/m \cdot s]$
$\boldsymbol{\epsilon}_F$	Vector of elastic strain	[-]
κ	Conductivity	$[W/m{\cdot}K]$
λ	Linear scaling factor	[-]
μ	Fluid dynamic viscosity	[Pa·s]
П	Total potential energy	[J]
ρ	Density	$[kg/m^3]$
σ	Vector of elastic stress	[Pa]
Φ	Energy dissipation rate	[W]
φ	Shaft angle	[rad]
ω	Shaft speed	[rad/s]
Subscripts	Denotation	
В	Cylinder block	
Ь	Bottom surface	
cd	Conductive	
сυ	Convective	
Ε	Pressure load	
fluid	Fluid domain	
k	Piston	
opt	Optimal	
solid	Solid domain	
SQ	Loss due to leakage	
ST	Loss due to friction	
t	Top surface	
Т	Thermal load	
0	Pre-scaled	

References

- 1. Ivantysyn, J.; Ivantysynova, M. *Hydrostatic Pumps and Motors: Principles, Design, Performance, Modelling, Analysis, Control and Testing*; Tech Books International: New Delhi, India, 2003; ISBN 81-88305-08-1.
- 2. Merritt, H. *Hydraulic Control Systems*; John Wiley & Sons: Hoboken, NJ, USA, 1967; ISBN 0-471-59617-5.
- 3. Manring, N.D. Friction forces within the cylinder bores of swash-plate type axial-piston pumps and motors. *J. Dyn. Syst. Meas. Control* **1999**, *121*, 531–537. [CrossRef]
- Manring, N.D. Valve-plate design for an axial piston pump operating at low displacements. *J. Mech. Des.* 2003, 125, 200–205. [CrossRef]
- 5. van der Kolk, H.-J. Beitrag zur Bestimmung der Tragfähigkeit des Stark Verkanteten Gleitlagers Kolben/Zylinder an Axialkolbenpumpen der Schrägscheibenbauart, Verlag Nicht Ermittelbar; Rodenbusch: 1972.

- Ivantysynova, M. An Investigation of Viscous Flow in Lubricating Gaps; SVST Bratislava: Bratislava, Slovakia, 1983.
- 7. Wieczorek, U.; Ivantysynova, M. Computer aided optimization of bearing and sealing gaps in hydrostatic machines—The simulation tool CASPAR. *Int. J. Fluid Power* **2002**, *3*, 7–20. [CrossRef]
- 8. Huang, C. An advanced gap flow model considering piston micro motion and elastohydrodynamic effect. In Proceedings of the 4th FPNI PhD symposium, Sarasota, FL, USA, 13–17 June 2006; pp. 181–196.
- 9. Pelosi, M.; Ivantysynova, M. The impact of axial piston machines mechanical parts constraint conditions on the thermo-elastohydrodynamic lubrication analysis of the fluid film interfaces. *Int. J. Fluid Power* **2013**, *14*, 35–51. [CrossRef]
- 10. Pelosi, M.; Ivantysynova, M. A geometric multigrid solver for the piston–cylinder interface of axial piston machines. *Tribol. Trans.* **2012**, *55*, 163–174. [CrossRef]
- Shang, L.; Ivantysynova, M. Advanced heat transfer model for piston/cylinder interface. In Proceedings of the 11th IFK International Fluid Power Conference, Aachen, Germany, 19–21 March 2018; Volume 1, pp. 587–595.
- Shang, L.; Ivantysynova, M. Thermodynamic Analysis on Compressible Viscous Flow and Numerical Modeling Study on Piston/Cylinder Interface in Axial Piston Machine. In Proceedings of the 10th JFPS International Symposium on Fluid Power, Fukuoka, Japan, 24–27 October 2017; pp. 475–581.
- 13. Sartchenko, J.S. *Force Balance Condition of the Valve Plate and Rotor of an Axial Piston Pump*; British Hydromechanics Research Association: Bingley, UK, 1950; p. 10.
- 14. Franco, N. Pump design by force balance. Hydraul. Pneum. 1961, 14, 101–107.
- Zecchi, M.; Ivantysynova, M. A novel fluid structure interaction model for the cylinder block/valve plate interface of axial piston machines. In Proceedings of the 52nd National Conference on Fluid Power, Las Vegas, NV, USA, 23–25 March 2011.
- Zecchi, M.; Ivantysynova, M. A novel approach to predict the cylinder block/valve plate interface performance in swash plate type axial piston machines. In Proceedings of the ASME/Bath Symposium on Fluid Power and Motion Control, Valencia, Spain, 8–12 July 2012; pp. 13–28.
- 17. Zecchi, M. A Novel Fluid Structure Interaction and Thermal Model to Predict the Cylinder Block/Valve Plate Interface Performance in Swash Plate Type Axial Piston Machines; Purdue University: West Lafayette, IN, USA, 2013.
- 18. Shute, N.A.; Turnbull, D.E. The Minimum Power Loss of Hydrostatic Slipper Berrings; BHRA: Bingley, UK, 1962.
- 19. Shute, N.A.; Turnbull, D.E. *The Losses of Hydrostatic Slipper Bearings under Various Operating Conditions*; British Hydromechanics Research Association: Bingley, UK, 1962.
- 20. Hooke, C.J.; Li, K.Y. The lubrication of overclamped slippers in axial piston pumps—Centrally loaded behaviour. *Proc. Inst. Mech. Eng. Part C* **1988**, *202*, 287–293. [CrossRef]
- 21. Hooke, C.J.; Li, K.Y. The lubrication of slippers in axial piston pumps and motors—The effect of tilting couples. *Proc. Inst. Mech. Eng. Part C* **1989**, 203, 343–350. [CrossRef]
- 22. Schenk, A.; Ivantysynova, M. The influence of swashplate elastohydrodynamic deformation. In Proceedings of the 7th FPNI PhD Symposium, Reggio Emilia, Italy, 27–30 June 2012.
- 23. Schenk, A.; Ivantysynova, M. A transient thermoelastohydrodynamic lubrication model for the slipper/swashplate in axial piston machines. *J. Tribol.* **2015**, *137*, 031701. [CrossRef]
- 24. Schenk, A.; Ivantysynova, M. An investigation of the impact of elastohydrodynamic deformation on power loss in the slipper swashplate interface. In Proceedings of the 8th JFPS International Symposium on Fluid Power, Okinawa, Japan, 25–28 October 2011.
- 25. Schenk, A.T. *Predicting Lubrication Performance between the Slipper and Swashplate in Axial Piston Hydraulic Machines;* Purdue University: West Lafayette, IN, USA, 2014.
- 26. Shang, L.; Ivantysynova, M. Port and case flow temperature prediction for axial piston machines. *Int. J. Fluid Power* **2015**, *16*, 35–51.
- Shang, L.; Ivantysynova, M. An Investigation of Design Parameters Influencing the Fluid Film Behavior in Scaled Cylinder Block/Valve Plate Interface. In Proceedings of the 9th FPNI PhD Symposium on Fluid Power; American Society of Mechanical Engineers, Florianópolis, SC, Brazil, 26–28 October 2016; p. V001T01A027.
- Shang, L.; Ivantysynova, M. A Path toward Effective Piston/Cylinder Interface Scaling Approach. In Proceedings of the ASME/BATH 2017 Symposium on Fluid Power and Motion Control, Sarasota, FL, USA, 16–19 October 2017; p. V001T01A072.

- 29. Reynolds, O. IV. On the theory of lubrication and its application to Mr. Beauchamp tower's experiments, including an experimental determination of the viscosity of olive oil. *Philos. Trans. R. Soc. Lond.* **1886**, 177, 157–234. [CrossRef]
- 30. Shang, L.; Ivantysynova, M. A temperature adaptive piston design for swash plate type axial piston machines. *Int. J. Fluid Power* **2017**, *18*, 38–48. [CrossRef]



© 2018 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).