A Mixed-Integer Programming Model for Assessing Energy-Saving Investments in Domestic Buildings under Uncertainty

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Abstract: A decision support tool has been developed to evaluate energy-saving intervention investments for domestic buildings. Various potential interventions are considered, each affecting energy consumption and savings, as well as the total financial cost of the investment. The decision problem is formulated as a mixed-integer programming problem. The implemented methodologies increase the efficiency and efficacy of the solution algorithms and can be applied to most realistic cases. The tool allows users to customize the problem based on their own preferences and find the optimal combination of investments. Uncertainty complicating the decision process is addressed by using interval analysis; therefore, the robustness of the optimal decision can be evaluated to facilitate the decision-making process. A domestic building in the Mediterranean area is used as a case study to demonstrate the functionality of this tool and to evaluate the impact of the decision-maker's uncertainty on the optimal decision.

Keywords: energy efficiency; interval analysis; mixed-integer programming; decision support systems

1. Introduction

Buildings account for more than 40% of total energy consumption, and the majority of this energy is for residential use [1,2]. Therefore, building energy efficiency has become a worldwide priority for environmental reasons, including emissions and sustainability [3]. To improve the energy performance of buildings, the E.U. has established the Energy Performance in Buildings Directive (EPBD, 2010/31/EU) and the Energy Efficiency Directive (EED, 2012/27/EU) has set specific goals and requirements for reducing building energy consumption. New constructions are now designed according to these standards; however, the majority of buildings in Europe and the United States were built before 2000 [4,5], meaning interventions and refurbishments could drastically increase their energy efficiency. Residential buildings built between 2000 and 2005 are 14% more efficient than those built in the 1980s and 40% more efficient than those built before 1950 [5]. Therefore, a strong interdependence exists between energy consumption and building age, explaining why some governments provide incentives for retrofitting old buildings.

The measures that improve a building's energy efficiency range from equipping with more efficient devices to complete renovation [6]. The most important measures are major interventions that significantly increase the building's energy savings; however, these are usually very expensive. Potential energy-saving interventions can be classified into categories to facilitate evaluation based on various criteria such as final layout of the building, financial cost, energy savings achieved, and/or environmental performance [7,8]. Practically, the aim of evaluating available retrofit solutions for a building is to improve its energy efficiency and increase its energy label, with the goal of being

a nearly Zero Energy Building [9]. In this study, domestic buildings are considered, since they are the major and simultaneously the most aged building category. The energy needs of a domestic building are usually broken down into two parts: heating and cooling needs that depend on thermal comfort and can be simulated using computational fluid dynamics (CFD) analysis [10], and electromechanical equipment consumption that depends on residents' schedule and habits. These two categories are different, but they interact if electricity is used for the heating and cooling of the house.

Therefore, a large set of interventions is available that needs to be evaluated based on various criteria, and the decision-maker needs to manage many conflicting objectives. Such multi-criteria problems are common in the energy sector [11]. In this study, to address the retrofit problem, weights are assigned to the evaluation criteria that are then incorporated into a unique function representing the total gain of the decision-maker from all points of view according to his preferences. The weighting method is a widely used generation method, since a set of Pareto optimal solutions can be generated from which the decision-maker can select one [12]. The objective function is then optimized using integer and mixed-integer programming techniques, and by changing the weights, different optimal solutions are obtained. The problem is formulated using binary variables that are convenient, and the algorithms converge to the optimal solution after proper reformulation of nonlinear criteria.

Weight assignment has been used in several recent energy efficiency studies, mostly originating from multi-criteria analysis and multi-objective optimization, to address the problem of selecting the most efficient and suitable measures [13]. These studies propose models that either mainly focus on complex computer-aided simulation [14], in which uncertainty can be addressed by testing scenarios, or offer a holistic approach to the problem by considering the retrofit cost and the respective energy savings [15]. In the latter case, software tools have also been developed to facilitate the decision process, without, however, addressing uncertainty [16]. Some studies include a third objective, such as environmental impact [17] or resident comfort [18], and the problem is solved using meta-heuristic algorithms and penalty functions that may lead to sub-optimal or even unrealistic solutions. These methods independently evaluate each intervention without addressing probable synergies, and most do not consider that, except for the criteria used to evaluate an energy intervention, other preferences and limitations could also exist, which act as constraints to the formulated problem. For example, there may be a budget limit, a time horizon limit depending on the rental or ownership of the residence, personal preferences regarding aesthetics and hassle resulting from the refurbishment, or even special requirements for some building types, such as historical buildings [19]. This means that there could be quantitative and qualitative criteria, and the respective thresholds should be considered during the decision process. Moreover, uncertainties faced during this evaluation and their influence on the optimal decision are not examined thoroughly in recent studies, since uncertainty is usually incorporated either by performing a sensitivity analysis of the optimal solution [20] or using probability distributions [21].

Uncertainty needs to be taken into consideration to study the robustness of solutions. Robustness and reliability of optimal solutions is an important issue for several studies using multi-criteria decision-making framework [22,23]. In this problem, uncertainty results from properly selecting the weights that correspond to the decision maker's preferences, which is one of the most critical problems in decision-making, as well as from the problem's constraints that might change unexpectedly, such as the available budget. Uncertainty can also exist in the investment's cost [24] or expected savings, such as if funding mechanisms are available or if the energy rates are not fixed. Although these uncertainties are not considered in this study, they can be addressed using the proposed methodology.

Several methodologies address parameter uncertainty in integer programming, such as Fuzzy Sets [25] and Stochasticity [26]. In this study, Interval Analysis is used, as it is considered more suitable for engineering problems [27], since the distribution information and the membership functions for the uncertain parameters are not usually known or required. In Interval Analysis, an unknown parameter x is substituted with an interval, meaning that the parameter's value lies between the limits of this interval. Any function applied to x produces another interval that contains all the possible values of

that function for all possible values of the uncertain parameter. Therefore, the decision maker can effectively obtain the range of the optimal solution and how the decision variables affect it.

A decision support model is thus used to incorporate nonlinear criteria and various constraints that depend on the decision-maker in order to address the retrofit problem. It is also discussed how the proposed model, extending recent studies, easily addresses synergies among the various interventions and qualitative criteria to solve even the most complicated realistic case studies. The uncertainty in the decision-maker's preferences is addressed to study the robustness of the decision variables and to estimate which decisions are most affected, resulting in a more efficient decision support tool. A case study is presented of a residence in the Mediterranean area that requires upgrading to improve its energy efficiency, and the available energy-saving interventions are evaluated. The developed decision support system is used, and the influence of the preferences, constraints, and uncertainty is studied using some examples. Except for energy efficiency investments, the proposed model can also be used to address other selection problems in various sectors in which investment decisions need to be assessed under uncertainty.

In summary, the main contributions of this paper are:

1. The development of a decision-support tool to evaluate energy-saving interventions, which can incorporate any number of quantitative and/or qualitative criteria for assessing the available interventions even when synergies exist.
2. The ability of the tool to address uncertainty, taking into account the lack of distribution information and robustness of the solutions.

This paper is organized as follows: the problem is formulated in Section 2, and Section 3 describes the interval analysis framework and the methodology used to solve the problem under uncertainty. In Section 4, a case study is presented, and some numerical examples are solved and discussed in Section 5 to evaluate the proposed methodology. Finally, in Section 6, the conclusions are summarized, and some future extensions are suggested.

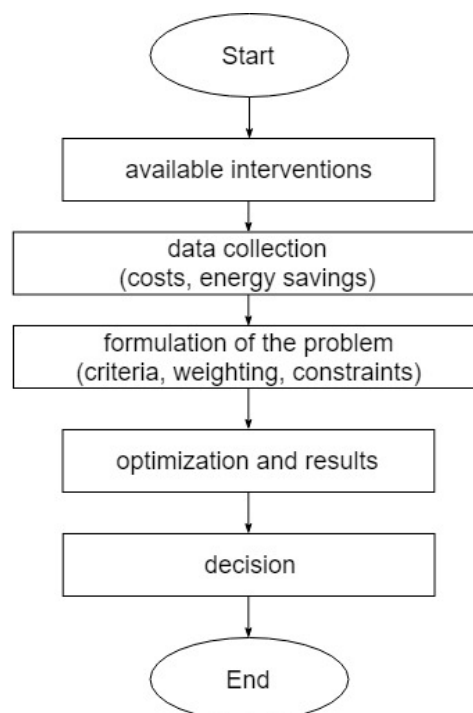The steps of the proposed methodology are summarized in the high-level flowchart in Figure 1.

Figure 1. High-level flowchart of the proposed methodology.

2. Formulation of the Problem

After understanding the current energy profile of a building and the potential interventions, the decision-maker needs to evaluate the options to decide how many and which interventions to select for financing. The criteria used should be necessary and sufficient for the stakeholder to decide. Various criteria can be used, but based on the opinions of real-estate market participants, the most important energy efficiency criteria for residential decision-makers are (1) the capital cost of each intervention; (2) the projected annual savings, which are the reduced costs due to improved energy efficiency; and (3) the payback time of the total investment based on its costs and gains. Capital cost is the most important deterrent to energy efficiency projects in the residential sector, since individuals lack funding options, technical knowledge, and long-term investment approach when compared to companies. Energy savings are used to evaluate the annual profits that result from an intervention. Savings are calculated using the estimated reduction in energy consumption due to an intervention and the respective energy cost. Finally, payback time is an important criterion in any investment decision, as it measures the relationship between the savings and the cost of capital. The payback time for a decision in the proposed model results from the costs and savings of all the interventions selected and not from their own payback time, because a decision-maker is mainly interested in the return on the total investment. All three criteria are quantitative, and their values can easily be calculated, even though payback time is not expressed in monetary terms.

Some decision-makers may wish to add more criteria to the decision process that they consider important, such as Net Present Value instead of payback time (the latter was selected because it is easier to understand by a residential decision-maker without a financial background) or carbon dioxide (CO_2) emissions to evaluate the environmental effect of an intervention if there is an ecological approach. Some criteria may even be qualitative, e.g., the extent of the renovation needed that could act as a deterrent. Therefore, various criteria could be of interest, since the prioritization and relevance can vary. However, a typical residential decision-maker's approach is simple, usually leading to the three above-mentioned criteria. Nevertheless, the methodology proposed in this study can address any number of criteria, and qualitative criteria can be incorporated into the model as well, as long as the decision-maker creates a representative numeric evaluation scale for each criterion.

Continuing with the mathematical formulation, the selection of an intervention I_i is modeled as a binary variable x_i. This means that the final decision is represented by a vector $\mathbf{x}$ of binary variables, in which zero means that the respective interventions are not suggested and one means that the respective interventions should be preferred. The criteria C_j ($j \in C$) that are considered in the decision process are the respective costs J_j. J_1 represents the total capital cost of an investment decision, J_2 is the total annual cost savings from a decision, and J_3 is the payback time of the decision. Costs J_j are given by the following equations, respectively:

$$J_1(\mathbf{x}) = \sum_{i=1}^{5} a_{1,i} x_i \tag{1}$$

$$J_2(\mathbf{x}) = \sum_{i=1}^{5} a_{2,i} x_i \tag{2}$$

$$J_3(\mathbf{x}) = \frac{J_1(\mathbf{x})}{J_2(\mathbf{x})} = \frac{\sum_{i=1}^{5} a_{1,i} x_i}{\sum_{i=1}^{5} a_{2,i} x_i} \tag{3}$$

in which J_1 is the aggregation of capital costs $a_{1,i}$ of the selected interventions, and it is assumed that no synergies exist between the interventions, so that J_2 results similarly from annual savings $a_{2,i}$. However, in many cases, there are synergies between two or more interventions that result in higher or lower savings than the independent aggregation of these savings. These synergies can be easily formulated by using products of binary variables, so that the terms that correspond to specific combinations of interventions are inserted into J_2 and J_3. For example, if the combination of

external insulation and frame replacement leads to 10% lower savings than the sum of their annual savings because these two interventions' savings overlap, J_1 remains the same, but J_2 becomes equal to $\sum_{i=1}^{5} a_{2,i} x_i - 0.1(a_{2,1} + a_{2,2}) x_1 x_2$, and the denominator of J_3 is modified, respectively. The products of the binary variables can then be substituted with simple binary variables by adding more constraints to the problem. Assuming there are n binary variables y_i, $i = 1, \ldots, n$, their product can be substituted by a binary variable y_p by imposing the additional constraints $y_s \le y_p + (n-1)$ and $y_s \ge n y_p$, in which y_s is the sum of the n binary variables. Through this method, the linear and nonlinear formulation of J_2 and J_3 are not affected, and the proposed methodology can be used to address even more complex real-world applications. After proper reformulation, any synergies lead again to the original cost formulations. Therefore, for simplicity, it is assumed that the annual savings resulting from an intervention are independent of the simultaneous selection of any other intervention.

Furthermore, the decision maker could set some limits for the criteria costs to meet. For example, there may be a budget limit or a certain time horizon. This means that upper and lower bounds $J_{j,min}$ and $J_{j,max}$ can be assigned to costs J_j and be used as constraints of the decision problem. These constraints can be either optional, or the decision-maker should appropriately select the values of the related parameters. They are formulated as:

$$J_{j,min} \le J_j \le J_{j,max},\ j = 1,\ 2,\ 3 \tag{4}$$

All the different costs that affect the final decision lead to the multi-objective mathematical programming (MMP) framework, which is an extension of traditional mathematical programming theory. MMP is part of the multi-criteria decision-making framework that includes various methodologies to facilitate the decision process [28,29]. To combine these objectives and obtain a single cost that acts as an objective function, the weighting method is used. Therefore, the decision maker assigns weights to these criteria according to his preferences, thus creating his own objective function that needs to be optimized. This method is simple and converges quickly to the optimal solution. In the end, the formulated problem is a binary integer programming problem [30]:

$$\min_{\mathbf{x}} (w_1 J_1 - w_2 J_2 + w_3 J_3) \tag{5}$$

Subject to

$$J_{j,min} \le J_j \le J_{j,max},\ j = 1,\ 2,\ 3 \tag{6}$$

A negative or positive sign is assigned to each J_j depending on its notion. Total capital cost and payback time should be as low as possible. Therefore, costs J_1 and J_3 are inserted into the objective function of the minimization problem with positive signs, whereas cost savings imply gain that should be as high as possible, so J_2 is inserted with a negative sign. Opposite signs would be used to maximize the objective function.

For every weight vector $\mathbf{w} = (w_1, w_2, w_3)$, an optimal solution exists that corresponds to the decision maker's preferences. The weights represent value trade-offs among criteria [31]. These weights can be considered as a measure of the importance of each criterion by assigning to them any positive value, but they can also be reformulated so as to add up to one, thus implying percentages. Moreover, the weights can also be used for the transformation of units so that the costs are comparable. If some weights are equal to zero, the respective costs are not considered in the objective function.

3. Optimization Procedure and Methods

When uncertainty is inserted into the model, the value of some parameters cannot be exactly estimated; therefore, typical integer programming algorithms may not be effective. In these cases, the methods used to address uncertainty are usually Fuzzy Sets, Stochasticity, or Interval Analysis. However, the first two methodologies require additional data and lead to complex sub-models that limit their applicability in practical problems. In stochastic and fuzzy programming, the probabilistic

distribution information and the membership function of each uncertain parameter are required, respectively. In practical engineering problems, a decision-maker can usually estimate the interval within which the uncertain parameter lies, but meaningful possibilistic or probabilistic information may be difficult to obtain. Conversely, Interval Analysis can be directly applied to the optimization problem and its uncertainty requirements are less demanding. This is why the interval analysis framework is proposed, and the uncertain parameters are expressed as intervals that represent the range of these parameters' values [27]. In this framework, any unknown parameter $x^\pm$ is expressed as an interval with an upper bound x^+ and a lower bound x^-, meaning its values range within $[x^-, \ x^+]$. The values allowed within this interval depend on whether the unknown parameter is continuous or discrete. All basic numerical operations can also be performed with intervals. Firstly, the interval linear problem is described, and then it is extended to address the nonlinear problem formulated in Section 2.

An interval linear programming problem can be formulated as

$$min f^\pm = C^\pm X^\pm \tag{7}$$

Subject to

$$A^\pm X^\pm \le B^\pm \tag{8}$$

$$X^\pm \ge 0 \tag{9}$$

in which $A^\pm, B^\pm, C^\pm, X^\pm$ are matrices, with elements belonging to a set of interval numbers $R^\pm$.

This problem can be solved by being split into two sub-models [32]. For a minimization problem, a sub-model corresponding to f^- is formulated, and, using its solutions, another sub-model corresponding to f^+ is then formulated and solved. Otherwise, the model corresponding to f^+ is formulated and solved first. Assuming that $b^\pm > 0$, the first submodel is

$$min f^- = \sum_{j=1}^{k_1} c_j^- x_j^- + \sum_{j=k_1+1}^{n} c_j^- x_j^+ \tag{10}$$

Subject to

$$\sum_{j=1}^{k_1} |a_{ij}|^+ Sign(a_{ij}^+) x_j^- / b_i^- + \sum_{j=k_1+1}^{n} |a_{ij}|^- Sign(a_{ij}^-) x_j^+ / b_i^+ \le 1, \ \forall i \tag{11}$$

$$x_j^\pm \ge 0, \ \forall j \tag{12}$$

in which $x_j^\pm$ for $j = 1, \ldots, k_1$ are continuous or discrete variables with positive cost coefficients $c_j^\pm$, and for $j = k_1 + 1, \ldots, n$ the cost coefficients are negative.

After solving the first sub-model, optimal values $x_{j,opt}^-$ for $= 1, \ldots, k_1, x_{j,opt}^+$ for $j = k_1 + 1, \ldots, n$ and f_{opt}^- are obtained. The second sub-model is similarly formulated by inverting the signs of Equations (10) and (11), and adding the constraints $x_j^+ \ge x_{j,opt}^-, \ j = 1, 2, \ldots, k_1$ and $x_j^- \le x_{j,opt}^+, \ j = k_1 + 1, k_1 + 2, \ldots, n$. After solving the two sub-models, the final solutions f_{opt}^-, f_{opt}^+ and $x_{j,opt}^-, x_{j,opt}^+$ for $j = 1, \ldots, n$ are obtained.

Using this methodology, a set of optimal and robust solutions for the decision variables becomes available, which results in a range of feasible optimal solutions for the objective function. This is superior to a best/worst case analysis that mainly focuses on the objective function without studying the robustness and feasibility of the decision variables.

In this study, the problem differs from the classic interval linear programming problem, because J_3 is nonlinear. However, this nonlinear cost can be reformulated using fractional programming techniques, since it is a quotient of linear functions [33].

Constraints of the form $\frac{\sum_i a_i x_i}{\sum_i d_i x_i} \le c$, which are not linear, can be reformulated as $\sum_i (a_i - c d_i) x_i \le 0$, since the denominators of the problem are always positive. Similarly, constraints of the form $\frac{\sum_i a_i x_i}{\sum_i d_i x_i} \ge c$ can also be transformed. If linear quotients appear in the objective function, the linear fractional programming problem can be reformulated into a mixed-integer programming (MIP) problem using binary and positive continuous variables.

The linear fractional programming problem with binary variables is

$$min \frac{a_1 x_1 + \cdots + a_k x_k + a_{k+1}}{c_1 x_1 + \cdots + c_k x_k + c_{k+1}} \tag{13}$$

Subject to

$$A_1 x_1 + \cdots + A_k x_k \le b \tag{14}$$

in which A_i, $i = 1 \ldots k$ and b are m-dimensional constant column vectors, x_i are binary variables, and a_i, c_i with $i = 1, \ldots, k+1$ are the respective constant coefficients.

By setting $y = 1/c_1 x_1 + \cdots + c_k x_k + c_{k+1}$, the problem is reduced to

$$min \quad a_1 x_1 y + \cdots + a_k x_k y + a_{k+1} y \tag{15}$$

Subject to

$$c_1 x_1 y + \cdots + c_k x_k y + c_{k+1} y = 1 \tag{16}$$

$$A_1 x_1 + \cdots + A_k x_k \le b \tag{17}$$

in which x_i are binary variables and $y \ge 0$.

Then, the transformation $z = xy$ can be used by adding four linear inequalities: $y - z \le K - Kx$, $z \le y$, $z \le Kx$, and $z \ge 0$, in which $K > y$ and x is a binary variable.

Following similar transformations, the general extended form of a linear fractional programming problem, with interval coefficients in the objective function, can be transformed [34]. In that case, the constraint in Equation (16) with interval coefficients $c_i^\pm$ is replaced by two inequalities:

$$c_1^+ z_1 + \cdots + c_k^+ z_k + c_{k+1}^+ y \ge 1 \tag{18}$$

$$c_1^- z_1 + \cdots + c_k^- z_k + c_{k+1}^- y \le 1 \tag{19}$$

Implementing the methodology described in this section, the original nonlinear interval problem can be written as

$$\min_x \left(\sum_{i=1}^{5} (w_1^\pm a_{1,i}^\pm - w_2^\pm a_{2,i}^\pm) x_i^\pm + \sum_{i=1}^{5} w_3^\pm a_{1,i}^\pm z_i \right) \tag{20}$$

Subject to

$$a_{2,1}^\pm z_1 + \cdots + a_{2,5}^\pm z_5 = 1 \tag{21}$$

$$y - z_i \le K - K x_i^\pm, \; z_i \le y, \; z_i \le K x_i^\pm, \; z_i \ge 0 \; \forall i \tag{22}$$

$$J_{1,min}^\pm \le \sum_{i=1}^{5} a_{1,i}^\pm x_i^\pm \le J_{1,max}^\pm \tag{23}$$

$$J_{2,min}^\pm \le \sum_{i=1}^{5} a_{2,i}^\pm x_i^\pm \le J_{2,max}^\pm \tag{24}$$

$$J_{3,min}^\pm \le \frac{\sum_{i=1}^{5} a_{1,i}^\pm x_i^\pm}{\sum_{i=1}^{5} a_{2,i}^\pm x_i^\pm} \le J_{3,max}^\pm \tag{25}$$

in which $y \ge 0$, $K > y$ and $x_i^\pm$ binary variables $\forall i$, $i = 1 \ldots 5$.

To solve the problem, the signs of the decision variables' coefficients are needed to separate those that contribute either to f^- or f^+. This is not always a simple task, since the objective function is nonlinear with respect to x_i. Therefore, these signs depend on the specific problem. Attention should be paid to the fact that coefficients $c_i^{\pm}$ of the linear interval programming problem correspond to $x_i^{\pm}$, whereas coefficients $w_i^{\pm}$ correspond to J_j; therefore, they differ. The first derivative and the monotony of the objective function were used to find certain conditions that determine these signs. Another method involves testing all scenarios for each decision variable. This procedure is considerably simplified, because decision variables are binary. Notably, in case synergies exist, additional constraints should be added to express the product and sum of the reformulated binary variables using the technique presented in Section 2.

4. Case Study

In this section, the residence that needs to be upgraded and the potential interventions are described. The house examined is situated in a city in which the climate is Mediterranean, meaning the weather is usually sunny and dry during summer, with temperatures up to 40 °C, and mild during winter without significant snowfall, with temperatures as low as 3 °C. This residence is an apartment on the top floor with an area of 75 m^2 in the city suburbs, so it is also protected from strong winds to a certain extent. It was built in the late 1980s, and its specifications are outdated; however, the building receives minimum solar exposure due to its orientation, and the other apartments of the building reinforce the floor insulation. Based on this data, the heat transfer coefficient and the annual energy needs for heating and cooling can be calculated. The annual needs for electricity were obtained from historical billing data.

The most widely suggested interventions for a residence to improve heating and cooling include external wall insulation, replacement of external window frames with newer and more efficient frames, and the installation of a heat pump for heating and cooling. Other interventions could include the use of geothermal energy if the house was detached, and the use of natural gas instead of heating oil. The residence examined is separate from the natural gas distribution network and uses heating oil for space heating and a solar thermal collector for water heating. The interventions proposed for reducing the electricity costs are mainly the installation of photovoltaic cells (PVs), since the Mediterranean climate is sunny and a feed-in tariff mechanism exists, and the replacement of incandescent and fluorescent lamps with light-emitting diodes (LEDs) that are easy to install and much more efficient. LED lights are one of the most widely adopted energy efficiency interventions in residences worldwide. Even though LEDs are more expensive than other lights, they offer the same light output (luminous flux), using much less power and increasing luminous efficiency up to 10-fold, and their lifespan is typically about 15 times longer. These advantages also hold compared with fluorescent lamps, although to a lesser degree. The energy consumption of a residence can be also reduced by replacing old devices and equipment with new technologically improved and more efficient devices, but these individual replacements are not considered interventions, with the exception of lamps that account for a large percentage of energy consumption.

The proposed interventions for the residence examined, along with their cost of capital and annual cost savings, are presented in Table 1. The capital costs used were based on market prices, after consulting energy services companies that undertake such projects in the area. Depending on the retrofitting period, the capital cost can be reduced by taking advantage of probable funding mechanisms that were not available during this study. In that case, the results may differ in accordance with each funding option's terms. The energy and cost savings stem from the annual energy needs and costs, along with the reduction expected from each intervention. The payback time is the ratio of capital cost to annual gain. For example, replacing 1000 W incandescent light fixtures with 120 W LED fixtures costs about €65 depending on the brand selected, which results in the same light output and 880 W less power installed. Given that the variable component of the electricity cost for this residence is circa 0.14 €/kWh including the supply cost, regulated costs, and all taxes and levies, and assuming

lights will stay on for about 6–6.5 h per day, the resulting annual savings were estimated to be about €277. Similarly, the costs and savings of the other available interventions were obtained.

Table 1. Available interventions. LEDs: light-emitting diodes; PVs: photovoltaic cells.

Interventions / Data	External Insulation	Frames	LEDs	PVs	Heat Pump
Capital cost (€)	$a_{1,1} = 6000$	$a_{1,2} = 8000$	$a_{1,3} = 65$	$a_{1,4} = 6900$	$a_{1,5} = 1100$
Annual savings (€)	$a_{2,1} = 1208$	$a_{2,2} = 1072$	$a_{2,3} = 277$	$a_{2,4} = 988$	$a_{2,5} = 325$

In case a decision-maker wants to assess less common or more innovative interventions that are not widely available for residences, they can be easily incorporated into the model as long as their capital cost and savings can be calculated.

5. Results

To solve the problem presented in the previous section, a suitable GAMS solver is used. GAMS is a high-level modeling system tailored for complex and large-scale optimization problems [35] and the LP/MIP solver used is CPLEX [36]. GAMS is widely used worldwide for modeling applications. In this section, only a few examples are presented, but using the same algorithms and methodology, any other numerical examples or case studies can be completed, since the scalability is obvious. The algorithm requires less than one second to solve the following examples in which five interventions are evaluated based on three criteria. Therefore, the methodology is also applicable and efficient for large-scale problems with many available interventions and various criteria.

In the following examples, the capital cost and the annual savings are expressed in thousands of Euros so that the three costs are comparable; otherwise, J_1 would dominate in the objective function. As already mentioned, another method to ensure comparability involves considering the scale of each cost during weight selection. Moreover, in the following examples and generally, in most cases x_2 and x_3 have a positive and a negative coefficient, respectively, as can be easily calculated. The rest of the coefficients' signs highly depend on the weight selection. It is highlighted that the algorithms converge to the optimal value of the objective function, but it is the decision variables for the selection of interventions that are actually of practical use to the decision-maker.

In the first examples (Table 2), the problem is solved without uncertainty; all parameters are deterministic, and the optimal solution is obtained in each scenario. This means that all data are available and the decision-maker is certain regarding his preferences and limitations.

As expected, as the thresholds in the constraints are tightened, the more the feasible solutions are restricted. Thus, the optimal solution includes fewer interventions, and the value of the objective function deteriorates. The opposite results hold true if the constraints are relaxed. For example, the relaxation of the constraints between the first and second example affects the selection of the PVs, even though the value of the objective function is only slightly affected. This is due to the fact that the PV cost of capital is high, thus increasing the total investment's payback time. These are the two constraints that differ in the two examples. Moreover, the importance assigned to each cost in the objective function can affect the optimal solution even if the constraints remain unchanged. For example, the difference between the first and last example in Table 2 is that the importance of payback time increases as opposed to the importance of savings. Therefore, the selection of external insulation, which provides high energy savings at a high capital cost, is affected.

Table 2. Examples without uncertainty.

		Inputs		Optimal Decision					
w_1	w_2	w_3	Constraints	x_1	x_2	x_3	x_4	x_5	J_{opt}
0.1	0.7	0.2	$J_{1,max} = 10,\ J_{2,min} = 0,\ J_{3,max} = 5$	1	0	1	0	1	-0.504
0.1	0.7	0.2	$J_{1,max} = 15,\ J_{2,min} = 0,\ J_{3,max} = 7$	1	0	1	1	1	-0.505
0.1	0.7	0.2	$J_{1,max} = 8,\ J_{2,min} = 0,\ J_{3,max} = 5$	0	0	1	0	1	-0.258
0.1	0.4	0.5	$J_{1,max} = 10,\ J_{2,min} = 0,\ J_{3,max} = 5$	0	0	1	0	1	-0.007

In the next set of examples (Table 3), it is assumed that the decision-maker can specify the constraints, but he cannot explicitly decide the weights that correspond to his preferences. Therefore, the objective function includes the interval values $w_j^{\pm}$ for $j = 1, 2, 3$, and the remainder of the parameters are assumed to be deterministic.

Table 3. Examples with interval weights.

		Inputs		Optimal Decision					
$w_1^{\pm}$	$w_2^{\pm}$	$w_3^{\pm}$	Constraints	$x_1^{\pm}$	$x_2^{\pm}$	$x_3^{\pm}$	$x_4^{\pm}$	$x_5^{\pm}$	$J_{opt}^{\pm}$
[0.3, 0.5]	[0.4, 0.6]	[0.1, 0.2]	$J_{1,max} = 10$ $J_{2,min} = 1.5$ $J_{3,max} = 6$	[1, 1]	[0, 0]	[1, 1]	[0, 0]	[1, 1]	[1.087, 2.905]
[0.1, 0.3]	[0.4, 0.7]	[0.2, 0.3]	$J_{1,max} = 15$ $J_{2,min} = 1$	[1, 1]	[0, 0]	[1, 1]	[0, 1]	[0, 1]	[-0.505, 1.296]

In the first example, it is observed that the optimal decision may not be affected by the uncertainty, in contrast with the second example, in which the robustness decreases as the uncertainty increases and/or different constraints are used. In the first example, three interventions, namely external insulation, LEDs, and heat pump, are recommended even though the decision-maker is not absolutely certain about his preferences. In the second example, the tool informs him that his uncertainty affects the optimal decision as far as the PV and heat pump installation are concerned. In both examples, the objective function's optimal value is within a certain range that depends on the weighting factors, meaning J_{opt} may vary independently of the choice of optimal interventions. In that case, the solution can be robust, since the decision-maker is more interested in the selection of optimal interventions rather than the range of J_{opt}.

Finally, in Table 4, the weights are assumed to be deterministic, but uncertainty exists in the constraints and more specifically in the limits imposed by the decision-maker. Therefore, intervals are used to express $J_{j,max}^{\pm}$ and $J_{j,min}^{\pm}$ for $j = 1, 2, 3$.

Table 4. Examples with uncertainty in the constraints.

		Inputs		Optimal Decision					
w_1	w_2	w_3	Constraints	$x_1^{\pm}$	$x_2^{\pm}$	$x_3^{\pm}$	$x_4^{\pm}$	$x_5^{\pm}$	$J_{opt}^{\pm}$
0.5	0.45	0.05	$J_{1,max} = 15$ $J_{2,min}^{\pm} = [1, 1.5]$	[1, 1]	[0, 0]	[1, 1]	[0, 0]	[0, 1]	[2.376, 2.780]
0.5	0.45	0.05	$J_{1,max}^{\pm} = [15, 20]$ $J_{2,min}^{\pm} = [1, 2.5]$	[1, 1]	[0, 0]	[1, 1]	[0, 1]	[0, 1]	[3.303, 7.775]

It is observed that the optimal value of the objective function may not be considerably affected, but the uncertainty influences the optimal solution. In the case of low demand for energy savings in the first example of Table 4, only interventions I_1 and I_3 are selected, i.e., external insulation and LEDs.

Conversely, if more savings are required, potentially due to a legislative provision, the fifth intervention (heat pump installation) should also be selected. In the second example, greater uncertainty leads to less robust optimal solutions, as expected. The range of the objective function's optimal value increases, but the most important outcome is that the number of interventions whose selection is uncertain also increases. Another method to assess the robustness of the optimal solution could include studying various examples with deterministic parameters that cover the entire range of the uncertainty. However, these examples should be appropriately selected to avoid incoherent decisions.

Judging from the examples tested, the uncertainty in the constraints seems to affect the robustness of the optimal solution more than the possible uncertainty in the weight parameters as far as this particular building is concerned. However, more examples need to be studied to verify if this observation also applies to other cases and buildings. For the residence examined, the replacement of external frames is not generally suggested for improving energy efficiency (this intervention could, however, be selected due to soundproofing if a decision-maker includes this qualitative criterion into the model), whereas the installation of LEDs is considered a cheap intervention with high energy savings and its selection is generally robust.

The uncertainty of both the weight parameters and the constraint thresholds is expected to have an aggregate effect on the robustness of the optimal solution and the objective function, rendering the optimal selection more difficult. The proposed framework helps the decision-maker to evaluate how different circumstances affect the optimal decisions. If the problem was solved based on the most probable scenario, all parameters would be deterministic, and the decision-maker would not be able to determine which decision variables are more sensitive to slight variations in the parameters.

6. Conclusions

Retrofitting domestic buildings has become a worldwide priority given the unexploited energy efficiency potential in this sector. For this reason, all available building interventions and energy efficiency measures need to be evaluated to optimize the improvement of a building's energy performance.

To facilitate the decision process, a decision support tool has been developed that incorporates the stakeholder preferences using the multi-criteria decision support framework and mixed-integer programming. The proposed methodology offers a holistic approach to the retrofit problem and also incorporates the effect of uncertainty on the optimal decision in a consistent and understandable manner. The uncertainty faced when both the importance and the limits of each criterion need to be decided is addressed using interval analysis, which is suitable for engineering problems, since the distribution information of uncertain parameters is usually unknown. Moreover, this approach is superior to only studying the best- and worst-case scenarios, since the decision-maker is provided with a set of feasible and robust solutions. The proposed methodology can also address both nonlinear fractional criteria, which are common in engineering problems, and synergies that arise among the interventions, thus increasing the practicality of the tool.

Future research could include large-scale simulations in building blocks using extensions of the proposed model so that more interventions and all their synergies are incorporated. Furthermore, the robustness of the solutions could be studied given uncertain capital costs and/or uncertain energy savings. The proposed model could also be applied to other multi-criteria selection problems in which quantitative and qualitative criteria need to be considered.

Nomenclature

Acronyms

EPBD	Energy Performance in Buildings Directive
EED	Energy Efficiency Directive
CFD	Computational Fluid Dynamics
MMP	Multi-objective Mathematical Programming
LP	Linear Programming
MIP	Mixed-Integer Programming
PV	Photovoltaic
LED	Light-emitting diodes
GAMS	General Algebraic Modeling System

Problem's functions

J_1	capital cost of the investment
J_2	annual energy savings of the investment
J_3	payback time of the investment

Problem's variables

x_i	intervention selection (binary)
y	variable used for reformulation
z	variable used for reformulation

Problem's parameters

i	potential interventions
j	evaluation criteria
$a_{1,i}$	capital cost of intervention i
$a_{2,i}$	annual savings due to intervention i
$J_{j,min}$	lower bound of cost J_j
$J_{j,max}$	upper bound of cost J_j
w_j	weight of cost J_j
K	sufficiently large positive number

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