



Optimization of Induction Motor Equivalent Circuit Parameter Estimation Based on Manufacturer's Data

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Abstract: This paper presents a two-stage optimization of the parameters of a seven-parameter equivalent circuit of three-phase induction motor. The initial parameters of this equivalent circuit are estimated by a method called the Engineering Method using the data given in the manufacturer's data sheet. The two-stage parameter optimization procedure was developed to minimize the errors between the estimated and the actual values in motor torque and current. In the first stage, the method is targeted to optimize the parameters of the stator only. The second stage, if necessary, aims at optimizing the rotor-circuit parameters. Normalized least squares method is used to formulate the optimization problem. An objective function is established to minimize the errors between the calculated starting torque and current and the pullout torque and the given values in the manufacturer's data sheet. The model parameters of ten industrial induction motors are estimated without and with optimization. The obtained results are compared with the Engineering Method and the actual manufacturer's data to verify the effectiveness of the proposed method.

Keywords: equivalent circuit; induction motor; manufacturer's data; optimization; parameters estimation; squirrel cage

1. Introduction

Due to its reliability and low cost, induction motors are the most widely used motors in most industrial applications. Performance analysis of an induction motor and its behavior prediction during faults and different normal operating regimes require the representation of the motor by an adequate mathematical model [1–4]. The accuracy of this analysis depends on the adopted equivalent circuit in the mathematical model of the induction motor.

According to the IEEE standard 112TM-2004 [5], which is a revision of the IEEE standard 112-1996, the test procedure for poly-phase induction motors and generators include (1) a no-load test to determine the core, windage and friction losses; (2) a load test to determine the efficiency, power factor, speed, current, and temperature rise; (3) a dc test to measure the stator resistance; and (4) a locked rotor test to determine the total leakage reactance. However, at the design level of a power supply system with induction motors or when there is no means (or it is too expensive) to perform these tests on existing motors, the parameters of the equivalent circuit of the motor can be estimated from the manufacturer's data sheet [6–17].

In [6], the authors proposed an iterative method for equivalent circuit parameter identification using the nameplate data, which includes the rated power, voltage, efficiency, power factor, speed and number of poles, NEMA design type, and code letter. The authors in [7] used speed, rated torque, starting torque, efficiency, and power factor values at 100%, 75% and 50% of the rated load to estimate



the parameters of the equivalent circuit of the induction motor. In [8], a method was proposed to estimate the induction motor model parameters using the nameplate data, the ratio of starting torque to the full load torque, and the power factor and efficiency at 100% and 50% of the rated full load. In [9] and [10], a detailed two-part study was performed on induction motor modeling and model parameters estimation based on the electrical and geometrical data generally given after the electromagnetic design. However, the equivalent circuit of the rotor of squirrel-cage motors in [6,10] is modeled by one resistive-inductive branch that does not accurately reflect the non-uniformity distribution of the current in the rotor bars of squirrel-cage motors. In addition, some of these methods require data at operating conditions other than 100% of the rated load. In [11], the researchers presented a methodology for determining the equivalent circuit parameters for an induction machine by estimating the rotor parameters as a function of slip. In [12] the authors suggested a method for obtaining the circuit parameters of National Electrical Manufacturers Association (NEMA) design A and B induction motors. Their method was based on the formulation of a set of nonlinear equations derived from the induction machine equivalent circuit with the manufacturers' data. Another parameter identification method was proposed in [13] using the results obtained through varying frequency tests.

Enhancement of the starting characteristics in squirrel-cage induction motors is achieved by the double-cage or deep-bar design of the rotor bars. In these bars, the current density is not uniform due to the skin effect. The skin effect causes variations in the resistance and reactance of the equivalent circuit of the rotor bars [10,14]. An eight-parameter equivalent circuit was proposed in [15] to represent the rotor by series inductance connected in parallel with two parallel-connected resistive-inductive branches. A seven-parameter equivalent circuit was presented in [16] to represent the rotor by two parallel-connected resistive-inductive branches. It was reported in the paper that the resistance of the stator is the least significant parameter in the parameters estimation. A method called the "Engineering Method" was proposed in [17] to determine the parameters of a seven-parameter equivalent circuit model of the induction motor. According to this engineering method, the error when comparing the data provided by the manufacturer is about 1–3%.

This paper aims at optimizing the initial model parameters obtained via the Engineering Method to minimize the errors between the model predictions and the manufacturer's data. The optimized parameters obtained through the proposed two-stage optimization method better match the manufacturer's data than those developed in the previous works. The rest of this paper is structured as follows: An introduction to different equivalent circuits of the induction motor and methods of model parameters identification is given in Section 1; the Engineering Method developed in [17] is introduced and summarized in Section 2; the proposed two-stage parameters optimization procedure is described in Section 3; the results are given and discussed in Section 4, followed by the concluding remarks in Section 5.

2. Engineering Method for Parameter Estimation

The Engineering Method is a method proposed in [17] to estimate the per-unit parameters of a seven-parameter equivalent circuit model of the induction motor, from the data given in the manufacturer's data sheet. The seven parameters of this model are shown in Figure 1 and explained below. The equivalent circuit of the rotor of the squirrel-cage motor is modeled by two parallel connected resistive-inductive branches 1 and 2. The superscripts (1) and (2) are used to denote branches 1 and 2, respectively. This method requires the following data: the rated slip or speed, power factor, efficiency, the ratios of the starting and pullout torques to the full load torque, and the ratio of the starting current to the rated current. To evaluate the parameters in actual values, if needed, the rated voltage and power of the motor will be required as well.



Figure 1. Equivalent circuit for the three-phase induction motor.

According to [17], the Engineering Method for parameter identification of the per unit equivalent circuit of the induction motor is given as follows:

1. The resistance (R_s) and leakage inductance ($x_{\sigma s}$) of the stator are estimated as:

$$R_s \approx S_{rated},$$
 (1)

$$x_{\sigma s} = \frac{1}{(2 \sim 3)K_{i_{start}}},\tag{2}$$

where (S_{rated}) is the rated slip of the machine and $(K_{i_{start}})$ is the ratio of the starting current over the rated current. In [17], the authors calculated the leakage reactance of the stator as one third of the input reactance of the motor, while in [18], the authors stated that the leakage reactance of the stator can be estimated as half of the input reactance.

2. Since the core loss is not represented in the model under investigation [17], the values of the given efficiency and power factor are corrected as follows:

$$\eta' = 1 - R_s - \frac{\eta \times \cos \phi \times S_{rated}}{1 - S_{rated}},\tag{3}$$

$$\cos\phi' = \frac{\eta \times \cos\phi}{\eta'},\tag{4}$$

where η and $\cos \phi$ are the motor efficiency and power factor at full load, respectively. In [18] and [19], the authors proposed a nine-parameter equivalent circuit that does not ignore the core loss in the machine. In that case, this correction step is not needed.

3. The reactance of the magnetization branch (x_{μ}) is calculated as follows:

$$x_{\mu} = \frac{1}{i_{\mu}} - x_{\sigma s},\tag{5}$$

where the magnetization current (i_{μ}) is calculated by:

$$i_{\mu} = \sqrt{1 - \cos \phi'^2} - (K_{\tau_{max}} - \sqrt{K_{\tau_{max}}^2 - 1}) \times \cos \phi', \qquad (6)$$

 $K_{\tau_{max}}$ in (6) is the ratio of the pullout torque to the full load torque given in the manufacturer's data sheet.

4. The input resistance and reactance, at the rated slip (S_{rated}) and the starting slip (S = 1), are estimated as follows:

$$R_{input}^{S_{rated}} = \cos \phi' \,, \tag{7}$$

$$x_{input}^{S_{rated}} = \sin \phi' = \sqrt{1 - \cos \phi'^2},\tag{8}$$

$$R_{input}^{S=1} = R_s + \frac{K_{\tau_{start}}^{corr.} \eta' \times \cos \phi'}{(K_{i_{start}}^{corr.})^2 \times (1 - S_{rated})}$$
(9)

$$x_{input}^{S=1} = \sqrt{\frac{1}{(K_{i_{start}}^{corr.})^2 - (R_{input}^{S=1})^2}}$$
(10)

To increase the accuracy, the ratio of the starting torque to the full load torque ($K_{\tau_{start}}$) and the ratio of the starting current to the rated current ($K_{i_{start}}$) are corrected as:

$$K_{i_{start}}^{corr.} = 0.99 K_{i_{start}},$$
(11)

$$K_{\tau_{start}}^{corr.} = 1.01 K_{\tau_{start}},\tag{12}$$

5. The conductance and susceptance of the rotor, at the rated slip (S_{rated}) and the starting slip (S = 1), are determined as follows:

$$G_r^{S_{rated}} = \frac{R_{input}^{S_{rated}} - R_s}{\left(R_{input}^{S_{rated}} - R_s\right)^2 + \left(x_{input}^{S_{rated}} - x_{\sigma s}\right)^2},\tag{13}$$

$$B_{r}^{S_{rated}} = \frac{x_{input}^{S_{rated}} - x_{\sigma s}}{\left(R_{input}^{S_{rated}} - R_{s}\right)^{2} + \left(x_{input}^{S_{rated}} - x_{\sigma s}\right)^{2}} - \frac{1}{x_{\mu}},$$
(14)

$$G_r^{S=1} = \frac{R_{input}^{S=1} - R_s}{\left(R_{input}^{S=1} - R_s\right)^2 + \left(x_{input}^{S=1} - x_{\sigma s}\right)^2}$$
(15)

$$B_r^{S=1} = \frac{x_{input}^{S=1} - x_{\sigma s}}{\left(R_{input}^{S=1} - R_s\right)^2 + \left(x_{input}^{S=1} - x_{\sigma s}\right)^2} - \frac{1}{x_{\mu}}$$
(16)

6. The resistance and the leakage reactance of the first rotor-circuit branch are calculated as:

$$R_{r}^{(1)} = \frac{G_{r}^{S_{rated}}}{\left(G_{r}^{S_{rated}}\right)^{2} + \left(B_{r}^{S_{rated}}\right)^{2}} S_{rated},$$
(17)

$$x_{\sigma r}^{(1)} = \frac{B_r^{S_{rated}}}{\left(G_r^{S_{rated}}\right)^2 + \left(B_r^{S_{rated}}\right)^2} S_{rated},\tag{18}$$

7. The resistance and leakage reactance of the second rotor-circuit are calculated as:

$$R_r^{(2)} = \frac{G_r^{(2)}}{\left(G_r^{(2)}\right)^2 + \left(B_r^{(2)}\right)^2},\tag{19}$$

$$x_{\sigma r}^{(2)} = \frac{B_r^{(2)}}{\left(G_r^{(2)}\right)^2 + \left(B_r^{(2)}\right)^2},\tag{20}$$

where the conductance $(G_r^{(2)})$ and susceptance $(B_r^{(2)})$ of the second rotor circuit, at starting slip (S = 1), are determined as follows:

$$G_r^{(2)} = G_r^{S=1} - \frac{R_r^{(1)}}{\left(R_r^{(1)}\right)^2 + \left(x_{\sigma r}^{(1)}\right)^2} , \qquad (21)$$

$$B_r^{(2)} = B_r^{S=1} - \frac{x_{\sigma r}^{(1)}}{\left(R_r^{(1)}\right)^2 + \left(x_{\sigma r}^{(1)}\right)^2} , \qquad (22)$$

To validate the above summarized algorithm, the authors in [17] calculated the stator current and torque at different values of slip (rated, critical and starting) and compared them to those values given in the data sheet as follows:

$$K_{i}^{calc.}(S) = \frac{1}{\sqrt{\left(R_{input}^{(S)}\right)^{2} + \left(x_{input}^{(S)}\right)^{2}}},$$
(23)

$$K^{calc.}(S) = i_s^2(S) \ (R_{input}^{(s)} - R_s) \frac{1 - S_{rated}}{\eta' \cdot \cos \phi'} ,$$
⁽²⁴⁾

The critical slip (S_{cr}) is calculated by:

$$S_{cr} = \frac{K_{\tau_{max}} \pm \sqrt{K_{\tau_{max}}^2 - 1 - k_2(1 - K_{\tau_{max}})}}{1 + k_2(1 - K_{\tau_{max}})} S_{rated},$$
(25)

where the coefficients of k_1 and k_2 can be calculated as:

$$k_1 = 1 + \frac{x_{\sigma s}}{x_{\mu}} ,$$

$$k_2 = \frac{2R_s S_{rated}}{k_1 R_s^{(1)}}$$

Since there are two values for the critical slip calculated by (25), the extraneous value is excluded.

In this paper, the algorithm of the Engineering Method has been realized using MATLAB. A comparison of the manufacturer's ratios of the starting and pullout torques, and the starting current with those calculated by the Engineering Method for different motors are shown in Table 1. The first record for DAZO-1569-8/10 is given as an example in [17], where the percentage errors are in the range of 1–3%. The other nine motors were selected randomly from different tables with different technical specifications, such as rated speed, rated power, power factor and efficiency. The data sheets of these induction motors, which are listed in Table 1, are available in [20] and [21]; Table A1 in Appendix A summarizes the technical specifications of these motors.

As shown in Table 1, the errors between the calculated values and the real manufacturer's values for the motors with numbers 2, 4, 5 and 10 are about 1%; thus, for these motors, the parameters of the equivalent circuit determined by the Engineering Method may be sufficient for engineering calculations.

However, when applying this method to estimate the parameters of the equivalent circuits for other motors (with numbers 3, 6, 7, 8 and 9), it is found that the error might be more than the acceptable value for engineering calculations. For instance, the error of ($K_{\tau_{max}}$) for 1LA8 455-8AD is over 9% and over 7% for 1PQ8 458-8PD, as shown in Table 1. Thus, it is necessary to tune the model parameters to reduce the errors. To achieve this goal, a parameter optimization procedure for the Engineering Method is proposed in Section 3.

No	Motor Tuno	Courses of Data	V	V	V	Error, %				
INO.	wotor type	Source of Data	K _{istart}	Λ ø _{start}	κ _{ø_{max}}	K _{istart}	K _{Østart}	K _{Ømax}		
4	D 170 15(0 0/10	Data sheet	5.500	0.800	2.700	1 000	1 000	Komax 2.991 1.007 3.346 0.703 0.790 9.113 6.463 5.779 7.710 0.274		
1.	DAZO-1569-8/10	Engineering Method	5.445	0.808	2.619	1.000	1.000			
2		Data sheet	7.000	1.800	2.800	1.000	1 000	1.007		
2.	1LA8 317-2AC	Engineering Method	6.930	1.818	2.828	1.000	1.000	1.007		
2	11 AO 215 (AD	Data sheet	6.500	2.000	2.500	1.000	1 000	2.246		
з.	1LA8 313-0AD	Engineering Method	6.435	2.020	2.584	1.000	1.000	3.340		
4 1DO8 257 2DC	Data sheet	6.500	1.800	2.600	1 000	1 000	0.702			
4.	4. IFQ8 557-2FC	Engineering Method	6.435	1.818	2.618	1.000	1.000	0.703		
F	1DO8 407 4DB	Data sheet	6.800	1.900	2.700	1 000	1.000 0.7	0.700		
5.	11 Q0 407-41 D	Engineering Method	6.732	1.919	2.721	1.000		0.790		
6		Data sheet	7.000	1.200	2.700	1 000	1.000	0.112		
0.	1LA8 455-8AD	Engineering Method	6.930	1.212	2.946	1.000		9.115		
7	11 48 458 440	Data sheet	6.800	0.900	2.500	1 000	1 000	6 162		
7.	1LA0 430-4AD	Engineering Method	6.732	0.909	2.662	1.000	1.000	0.405		
Q	1PO8 453-6PD	Data sheet	6.500	1.200	2.500	1 000	1 000	5 770		
о.	11 Q0 455-61 D	Engineering Method	6.435	1.212	2.644	1.000	1.000	3.779		
0	1008 458 800	Data sheet	6.500	1.000	2.600	1 000	1 000	7 710		
9.	11 Q0 430-01 D	Engineering Method	6.435	1.010	2.800	1.000	1.000	7.710		
10	2 A 2NA 2500 / 6000VVI A	Data sheet	5.300	0.900	2.300	1 000	1 000	0.274		
10.	2A31v1-2300/60001AL4	Engineering Method	5.247	0.909	2.294	1.000	1.000	0.274		

Table 1. Comparison of the manufacturers' ratios of the starting and pullout torques, and the starting current with those calculated by the engineering method.

3. Proposed Optimization Procedure

This paper aims at optimizing the Engineering Method summarized in Section 2 to minimize the errors shown in Table 1. The optimization procedure is proposed to be performed in two stages. In the first stage, the optimization is carried out for the stator parameters only; rewriting (1), (2), (11) and (12) yields:

$$R_S = C_1 S_{rated} , \qquad (26)$$

$$x_{\sigma S} = \frac{1}{C_2 K_{i_{start}}} , \qquad (27)$$

$$K_{i_{start}}^{corr.} = C_3 K_{i_{start}} , \qquad (28)$$

$$K_{\tau_{start}}^{corr.} = C_4 K_{\tau_{start}} , \qquad (29)$$

The coefficients (C_1 , C_2 , C_3 and C_4) are bounded as follows:

$$\begin{cases}
0.9 \le C_1 \le 1.1 \\
2 \le C_2 \le 3 \\
0.99 \le C_3 \le 1.01 \\
0.99 \le C_4 \le 1.01
\end{cases}$$
(30)

The reasons for selecting those boundaries are as follows:

 C_1 : the Engineering Method claims that R_S is approximately equal to the rated slip. For practical calculations, it is assumed to be equal. We represent this approximation by +-1%.

 C_2 : many researchers [17,18] claim that the leakage reactance of the stator is one third to half of the input impedance at starting.

 C_3 and C_4 : the Engineering Method claims that C_3 is 0.99 and C_4 is 1.01 to correct the starting current and torque ratios at starting. In this paper, we assumed them to range from 0.99 to 1.01. Results, in this paper, show that this correction is not necessary in most cases.

The goal of the model parameter optimization is to minimize the errors between the calculated values and the real manufacturer values. In this study, the focus is given to the starting current ratio $(K_{i_{start}})$, starting torque ratio $(K_{\tau_{start}})$, and the pull-out torque ratio $(K_{\tau_{max}})$. Define

$$f_1(x) = \left(\frac{K_{i_{start}} - K_{i_{start}}^{calc.}}{K_{i_{start}}}\right)^2,$$
(31)

$$f_2(x) = \left(\frac{K_{\tau_{start}} - K_{\tau_{start}}^{calc.}}{K_{\tau_{start}}}\right)^2,$$
(32)

$$f_3(x) = \left(\frac{K_{\tau_{max}} - K_{\tau_{max}}^{calc.}}{K_{\tau_{max}}}\right)^2,\tag{33}$$

where $x = \begin{bmatrix} C_1 & C_2 & C_3 & C_4 \end{bmatrix}'$. The starting ratios are calculated by (23) and (24) at S = 1. In addition, the pullout torque ratio is calculated by (24) at the critical slip.

Taking into account (30), the parameter optimization problem can be formulated as follows:

$$\min \sum_{i=1}^{3} \lambda_i f_i, \tag{34}$$

Subject to constraints in (30).

where λ_i , i = 1, 2, 3 are the weight factors. For the optimization process, the weight factors in an objective function are of a great importance and should be chosen carefully. For the case in this paper, it is desirable to give each of the torque-slip and current-slip characteristics equal weights [15]. In other words, $\lambda_i = 1$, i = 1, 2, 3 for the study in this paper.

In this paper, MATLAB, based on the default interior-point algorithm, is used to solve the nonlinear optimization problem with bounded constraints. The procedure used in the first stage of optimization is summarized in Algorithm 1.

Algorithm 1: First Stage of Optimization.

Step 1: Import induction machine parameters from the manufacturer's data sheet **Step 2:** Obtain the parameters of the equivalent circuit of the machine using the Engineering Method **Step 3:** Minimize the error functions f_1 , f_2 and f_3 , and iterate until the optimization criterion is met **Step 4:** Obtain the optimized values for the independent variables C_1 , C_2 , C_3 and C_4 **Step 5:** Recalculate the parameters of the equivalent circuit using the optimized values of C_1 , C_2 , C_3 and C_4

Applying the above optimization procedure to the motors listed in Table 1, the errors are significantly reduced, as shown in Table 2.

Table 2. Comparison between the Engineering Method and the 1st optimization stage.

Na	Motor Tuno	Mathad	C	C	C	C.		Error, %	
INO.	wotor type	Method	\mathbf{c}_1	C_2	C3	c_4	K _{istart}	$K_{o_{start}}$	K _{ømax}
	DA 70 15(0.0/10	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	2.991
1.	DAZO-1569-8/10	1st optimization stage	0.900	2.002	1.005	1.000	0.510	0.030	1.020
2	11 49 217 240	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	1.007
Ζ.	1LA8 517-2AC	1st optimization stage	1.100	2.995	0.998	1.000	0.240	0.020	rror, % K _{0,tart} K _{0max} 1.000 2.991 0.030 1.020 1.000 3.046 0.130 2.760 1.000 0.703 0.010 0.540 1.000 0.790 0.010 0.540 1.000 9.113 0.990 8.560 1.000 5.779 0.500 5.340 1.000 7.710 0.990 7.200 1.000 0.274
2	11 A 9 21E (A P	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	3.346
5.	1LA8 315-6AB	1st optimization stage	1.100	2.999	0.990	1.001	0.990	0.130	2.760
4.	1009 257 200	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	0.703
	11 Q8 337-21 C	1st optimization stage	1.100	3.000	0.999	1.000	0.120	0.010	0.540
-	1DO9 407 4DB	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	1.000 0.790 0.010 0.640
5.	11 Q8 407-41 B	1st optimization stage	1.099	2.993	0.999	1.000	0.130	0.010	
		Engineering	1.000	3.000	0.990	1.010	1.000	1.000	9.113
6.	1LA8 455-8AD	1st optimization stage	1.100	3.000	0.990	1.010	1.000	0.990	$\begin{array}{c} 0.540 \\ 0.790 \\ 0.640 \\ 9.113 \\ 8.560 \\ 6.463 \\ (170) \end{array}$
-	11 49 459 445	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	6.463
7.	ILA8 458-4AD	1st optimization stage	1.100	3.000	0.990	1.006	1.000	0.570	Kemax 00 2.991 30 1.020 00 1.007 20 0.800 00 3.346 30 2.760 00 0.703 10 0.540 00 0.790 10 0.640 00 9.113 90 8.560 00 6.463 70 6.170 00 5.340 00 7.200 00 0.274 00 0.274
0	1PO9 452 4PD	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	5.779
8.	11 Q8 455-61 D	1st optimization stage	1.100	3.000	0.990	1.005	1.000	0.500	5.340
0	1009 459 900	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	7.710
9.	11 Q8 458-81 D	1st optimization stage	1.100	3.000	1.009	0.990	1.000	0.900	7.200
10	2 A 2 M 2 E 00 / 6000 V VI 4	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	0.274
10.	2A3WI-2300/6000YAL4	1st optimization stage	1.048	2.692	1.000	1.000	0.000	0.000	0.010

Based on the results in Table 2, C_1 is the most critical coefficient compared to C_2-C_4 , which is due to the direct relation between cupper losses in the motor and C_1 . Since the resistance of the stator in each unit is initially equal to the rated slip of the machine, correction is needed to satisfy the given efficiency of a machine. The results given in Table 2 show that the errors are reduced to almost zero for some machines after the first optimization stage (motors with numbers 1, 2, 4, 5 and 10) and decreased for other motors (numbers 3, 6, 7, 8 and 9); however, for these motors, the errors are still large. This leads to the second stage of optimization, as shown in Table 3.

In the second stage, the focus is to optimize the rotor parameters; rewriting (17)–(20) yields:

$$R_{r}^{(1)} = C_{5} \frac{G_{r}^{S_{rated}}}{\left(G_{r}^{S_{rated}}\right)^{2} + \left(B_{r}^{S_{rated}}\right)^{2}} S_{rated},$$
(35)

$$x_{\sigma r}^{(1)} = C_6 \frac{B_r^{S_{rated}}}{\left(G_r^{S_{rated}}\right)^2 + \left(B_r^{S_{rated}}\right)^2} S_{rated},\tag{36}$$

$$R_r^{(2)} = C_7 \frac{G_r^{(2)}}{\left(G_r^{(2)}\right)^2 + \left(B_r^{(2)}\right)^2},\tag{37}$$

$$x_{\sigma r}^{(2)} = C_8 \frac{B_r^{(2)}}{\left(G_r^{(2)}\right)^2 + \left(B_r^{(2)}\right)^2},\tag{38}$$

In this stage, the vector of the independent variables is:

$$x = \begin{bmatrix} C_5 & C_6 & C_7 & C_8 \end{bmatrix}'$$

All the other parameters including C_1 , C_2 , C_3 and C_4 are kept constant after the first stage of optimization.

Table 3. Comparison between the results obtained by the engineering method and the 2nd optimization stage.

No	Motor Type	Method	C.	C	C	C.	<i>C</i> -	C.	<i>C</i> -	Ca	Error, %		
INO.	wotor type	Wiethou	c_1	C ₂	03	04	05	C6	C7	C8	K _{istart}	$K_{\theta_{start}}$	$K_{\theta_{max}}$
	DATO 15(0.0/10	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	1.000	1.000	1.000	1.000	2.991
1.	DAZO-1569-8/10	2nd optimization stage	0.900	2.002	1.005	1.000	0.895	0.946	1.055	1.014	0.000	0.000	0 2.991 0 0.000 0 1.007 0 0.000 0 3.346 0 0.000 0 0.703 0 0.000 0 0.790 0 0.000
2	11 49 217 240	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	1.000	1.000	1.000	1.000	1.007
۷.	ILA8 517-2AC	2nd optimization stage	1.100	2.995	0.998	1.000	0.895	1.001	0.988	0.998	0.000	0.000	0.000
2	11 AO 21E (AD	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	1.000	1.000	1.000	1.000	3.346
з.	1LA6 515-6AD	2nd optimization stage	1.100	2.999	0.990	1.001	0.884	1.033	0.959	0.990	0.000	0.000	6 2.991 0.000 1.007 0.000 3.346 0.000 0.793 0.000 0.790 0.000 0.790 0.000 9.113 0.000 6.463 0.000 5.779 0.000 7.710 0.000 0.274 0.000
4.	1009 257 200	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	1.000	1.000	1.000	1.000	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
	IF Q8 357-2FC	2nd optimization stage	1.100	3.000	0.999	1.000	0.899	0.998	0.995	1.000	0.000	0.000	
-	1DO8 407 4DB	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	1.000	1.000	1.000 1.00	1.000	0.790
5.	11 Q8 407-41 B	2nd optimization stage	1.099	2.993	0.999	1.000	0.899	0.999	0.994	1.000	0.000	0.000	0.000
		Engineering	1.000	3.000	0.990	1.010	1.000	1.000	1.000	1.000	1.000	1.000	9.113
6.	ILA6 455-6AD	2nd optimization stage	1.100	3.000	0.990	1.010	0.833	1.133	0.947	0.977	0.000	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.000
	11 49 459 440	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	1.000	1.000	1.000	1.000	6.463
7.	1LA0 438-4AD	2nd optimization stage	1.100	3.000	0.990	1.006	0.868	1.090	0.946	0.973	0.000	0.000	bb; / o Gattert Komax 000 2.991 000 0.000 000 1.007 000 0.000 000 3.346 000 0.000 000 0.000 000 0.703 000 0.000 000 0.000 000 0.000 000 0.000 000 6.463 000 0.000 000 5.779 000 0.000 000 0.274 000 0.000
0	1PO8 453-6PD	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	1.000	1.000	1.000	.000 1.000 !	5.779
ð.	11 Q0 455-01 D	2nd optimization stage	1.100	3.000	0.990	1.005	0.864	1.076	0.948	0.976	0.000	0.000	0.000
0	1PO8 458 8PD	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	1.000	1.000	1.000	1.000	7.710
9.	11 Q0 450-01 D	2nd optimization stage	1.100	3.000	1.009	0.990	0.842	1.112	0.943	0.974	0.000	0.000	0.000
10	2 A 2NA 2500 /6000XXI 4	Engineering	1.000	3.000	0.990	1.010	1.000	1.000	1.000	1.000	1.000	1.000	0.274
10.	2/13101-2300/ 00001 AL4	2nd optimization stage	1.048	2.692	1.000	1.000	0.894	0.987	1.000	1.000	0.000	0.000	0.000

An unconstrained nonlinear programming optimization method based on the quasi-newton algorithm is used to solve the optimization problem with the same objective function as in (34), but without constraints imposed on the new x vector. The procedure of the second stage of optimization is summarized in Algorithm 2.

Algorithm 2: Second Stage of Optimization.

Step 1: Import induction machine parameters from the manufacturer's data sheet **Step 2:** Obtain the parameters of the equivalent circuit and the results of the first stage of optimization **Step 3:** Keep C_1 , C_2 , C_3 and C_4 constants in the second stage of optimization **Step 4:** Minimize the functions f_1 , f_2 and f_3 with respect to the new variables: C_5 , C_6 , C_7 and C_8 ; and find their optimal values

Step 5: Recalculate the parameters of the equivalent circuit using the values of C_5 , C_6 , C_7 and C_8

4. Results and Discussion

The Engineering Method [17] and the two-stage optimization procedure proposed in Section 3 are programmed using MATLAB according to Algorithm 1 and 2. Table 4 shows the seven parameters of the equivalent circuit models determined by the Engineering Method and the two-stage optimization method for ten different induction motors. The results show that after the two-stage optimization, the errors in starting current, starting torque and pull-out torque are eliminated. The first stage optimization is targeted for the stator parameters, while the second stage, if necessary, aims at the optimization of the rotor parameters.

Figure 2 shows the torque- and current-slip characteristics of the induction motor number 10 in Table 4 as an example. It is clearly seen from Figure 2 that the characteristic curves are close to each other since the errors without optimization are within 1%. Figure 3 shows the torque- and current-slip characteristic curves of the induction motor number 1 in Table 4. It can be seen from Table 4 and Figure 3 that the errors without optimization are much greater than 1%, but within 3%. For motor number 1, one stage of optimization is acceptable since the error is decreased to 1.02% after this optimization stage.

No	Motor Type	Mathad	R.	Xmc	<i>x</i> -	$\mathbf{p}^{(1)}$	(1)	$R_r^{(2)} = R_r^{(2)}$	R ⁽²⁾	(2)		Error, %	
110.	inotor type	Wiethou	nş	- ues		\mathbf{K}_r	<i>x</i> _{œr}		x _{œr}	K _{istart}	$K_{\theta_{start}}$	$K_{\theta_{max}}$	
		Engineering method	0.0093	0.061	2.429	0.0103	0.163	0.318	0.430	1.000	1.000	2.991	
1.	DAZO-1569-8/10	1st opt. stage	0.0084	0.091	2.391	0.0100	0.128	0.187	0.238	0.510	0.030	1.020	
		2nd opt. stage	0.0084	0.091	2.391	0.0090	0.121	0.245	0.249	0.000	0.000	0.000	
		Engineering method	0.0070	0.048	3.173	0.0074	0.155	0.169	0.154	1.000	1.000	1.007	
		1st opt. stage	0.0077	0.048	3.186	0.0074	0.155	0.161	0.154	0.240	0.020	0.800	
		2nd opt. stage	0.0077	0.048	3.186	0.0066	0.155	0.159	0.152	0.000	0.000	0.000	
		Engineering method	0.0110	0.051	2.671	0.0120	0.189	0.155	0.145	1.000	1.000	3.346	
3.	1LA8 315-6AB	1st opt. stage	0.0121	0.051	2.683	0.0119	0.189	0.152	0.145	0.990	0.130	2.760	
		2nd opt. stage	0.0121	0.051	2.683	0.0106	0.195	0.139	0.141	0.000	0.000	0.000	
		Engineering method	0.0060	0.051	3.363	0.0063	0.165	0.193	0.148	1.000	1.000	0.703	
		1st opt. stage	0.0066	0.051	3.376	0.0063	0.165	0.183	0.149	0.120	0.010	0.540	
		2nd opt. stage	0.0066	0.051	3.376	0.0057	0.165	0.183	0.148	0.000	0.000	0.000	
		Engineering method	0.0053	0.049	3.056	0.0057	0.161	0.177	0.142	1.000	1.000	0.790	
5.	1PQ8 407-4PB	1st opt. stage	0.0059	0.049	3.064	0.0057	0.162	0.169	0.143	0.130	0.010	0.640	
		2nd opt. stage	0.0059	0.049	3.064	0.0051	0.161	0.168	0.142	0.000	0.000	0.000	
		Engineering method	0.0080	0.048	2.129	0.0093	0.187	0.083	0.193	1.000	1.000	9.113	
		1st opt.	0.0088	0.048	2.134	0.0093	0.188	0.082	0.191	1.000	0.990	8.560	
		2nd opt. stage	0.0088	0.048	2.134	0.0077	0.213	0.064	0.168	0.000	0.000	0.000	
		Engineering method	0.0053	0.049	3.012	0.0057	0.180	0.083	0.215	1.000	1.000	6.463	
7.	1LA8 458-4AD	1st opt. stage	0.0059	0.049	3.020	0.0057	0.181	0.083	0.214	1.000	0.570	6.170	
		2nd opt. stage	0.0059	0.049	3.020	0.0050	0.197	0.066	0.191	0.000	0.000	0.000	
		Engineering method	0.0080	0.051	2.477	0.0089	0.191	0.110	0.208	1.000	1.000	5.779	
		1st opt. stage	0.0088	0.051	2.484	0.0089	0.191	0.109	0.207	1.000	0.500	5.340	
		2nd opt. stage	0.0088	0.051	2.484	0.0077	0.206	0.091	0.189	0.000	0.000	0.000	
		Engineering method	0.0080	0.051	2.168	0.0092	0.191	0.092	0.226	1.000	1.000	7.710	
9.	1PQ8 458-8PD	1st opt. stage	0.0088	0.051	2.173	0.0092	0.191	0.091	0.225	1.000	0.900	7.200	
		2nd opt. stage	0.0088	0.051	2.173	0.0077	0.213	0.071	0.197	0.000	0.000	0.000	
		Engineering method	0.0083	0.063	4.557	0.0084	0.175	0.288	0.320	1.000	1.000	0.274	
		1st opt. stage	0.0087	0.070	4.569	0.0084	0.168	0.244	0.282	0.000	0.000	0.010	
		2nd opt. stage	0.0087	0.070	4.569	0.0075	0.165	0.257	0.280	0.000	0.000	0.000	

Table 4. Equivalent circuit model parameters determined by the engineering method and the two-stage optimization.



Figure 2. Characteristics of the induction motor number 10 (2A3M-2500/6000YXL4): (**a**) Torque-slip, (**b**) Current-slip.



Figure 3. Characteristics of the induction motor number 1 (DAZO-1569-8/10): (a) Torque-slip, (b) Current-slip.

Figure 4 also shows the torque- and current-slip characteristics of the induction motor number 6 in Table 4. It is clear from Figure 4 and Table 4 that the errors are large without optimization and the two stages of optimization are then necessary. The results verified that the proposed two-stage optimization can effectively be used in tuning induction model parameters and eliminating the model output errors.



Figure 4. Characteristics of the induction motor number 6 (1LA8 455-8AD): (a) Torque-slip, (b) Current-slip.

The actual results obtained for ten induction motors were given and compared with the Engineering Method. The results show that the optimized parameters obtained via the two-stage optimization method well match the manufacturer's data, which successfully verified the effectiveness of the proposed parameter optimization method for induction motors.

5. Conclusions

A two-stage optimization method was proposed for parameter estimation of the seven-parameter equivalent circuit model of three-phase induction motors. The optimization problem was formulated to minimize the objective function that is the sum of the errors between the calculated ratios of starting torque and current and pullout torque and the given values in the manufacturer's data sheet. The proposed method was compared with the Engineering Method used in industry. Although the Engineering Method is simple, the errors introduced by the method can be large for some induction motors.

The proposed two-stage optimization method has successfully addressed this issue via the two stages of parameter optimization: the first stage optimization on the stator parameters and the second stage optimization on the rotor parameters.

The Engineering Method can be applied for any of the three-phase squirrel-cage induction motors provided in the manufacturers' data. The nameplate data of the motor only is insufficient to apply this method and the proposed two-stage optimization.

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Nomenclature

R_s	Resistance of the stator, in per-unit (pu)
$x_{\sigma s}$	Leakage inductance of the stator, pu
S _{rated}	Rated slip of the machine
K _{istart}	Ratio of the starting current over the rated current
η	Motor efficiency at full load
$\cos\phi$	Power factor at full load
η'	Corrected value of efficiency
$\cos \phi'$	Corrected value of power factor
x_{μ}	Reactance of the magnetization branch, pu
i_{μ}	Magnetization current, pu
$K_{\tau_{max}}$	Ratio of the pullout torque to the full load torque
$K_{\tau_{start}}$	Ratio of the starting torque to the full load torque
$G_r^{S_{rated}}$	Conductance of the rotor, at the rated slip (S_{rated}), pu
$B_r^{S_{rated}}$	Susceptance of the rotor, at the rated slip (S_{rated}) , pu
$R_{r}^{(1)}$	Resistance of the first rotor-circuit branch, pu
$x_{\sigma r}^{(1)}$	Leakage reactance of the first rotor-circuit branch, pu
$R_{r}^{(2)}$	Resistance of the second rotor-circuit, pu
$x_{\sigma r}^{(2)}$	Leakage reactance of the second rotor-circuit, pu
$G_{r}^{(2)}$	Conductance of the second rotor circuit, at starting slip (S = 1), pu
$B_{r}^{(2)}$	Susceptance of the second rotor circuit, at starting slip (S = 1), pu
S _{cr}	Critical slip, slip at maximum torque for a given induction machine
$K_{\tau_{start}}^{corr}$	Corrected starting torque ratio
K ^{corr} _{istart}	Corrected starting current ratio
$\lambda_i, i = 1, 2, 3$	Weight factors for the error functions, for the optimization process
$C_1, C_2, \ldots C_8$	Correction factors of the equivalent circuit parameters
$f_i, i = 1, 2, 3$	Error functions between the calculated values and the real manufacturer values

Appendi	ix
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No.	Motor Type	Output Power, kW	Rated Voltage, V	Rated Current, A	Rated Speed, rpm	Efficiency %
1	DAZO-1569-8/10	800	6000	94	991	92.5
2	1LA8 317-2AC	315	415	504	2979	96.6
3	1LA8 315-6AB	200	415	338	989	95.7
4	1PQ8 357-2PC	500	415	797	2982	97.0
5	1PQ8 407-4PB	675	690	654	1492	690
6	1LA8 455-8AD	560	415	999	744	96.3
7	1LA8 458-4AD	1125	690	1103	1492	97.0
8	1PQ8 453-6PD	630	415	1080	992	96.6
9	1PQ8 458-8PD	670	690	717	744	96.5
10	2A3M-2500/6000YXL4	2500	6000	260	2975	96.8

Table A1. Technical specifications of the induction motors used in the analysis [17], [20] and [21].

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