1. Review of the Failure Criteria

In the online supplementary file, the failure criteria used in the present study are reviewed: the Morh-Coulomb (Section 1.1), Mogi-Coulomb (Section 1.2), modified Lade (Section 1.3) and Drucker-Prager (Section 1.4) criteria. The four criteria are widely used in wellbore stability analysis because they can be fully established by the Mohr-Coulomb failure parameters, such as a cohesion and an internal friction angle.

1.1. Mohr-Coulomb

Among the various rock failure criteria, the most frequently used during wellbore stability analysis is the Mohr-Coulomb failure criterion [13]. According to Labuz and Zang [42], the criterion works well with rocks when all principal stresses are compressive. Coulomb proposed a relationship between the shear strength of a rock and the normal stress at failure given by,

$$\sigma_s = C + \sigma_n \tan\phi \tag{1}$$

where, σ_s is the shear strength on the failure plane, C is the pure shear strength also known as cohesion, σ_n is the normal stress on the failure plane and ϕ is the internal friction angle. The relationship between the shear and normal stress can also be established by the Mohr's failure criterion given by,

$$\sigma_{s,max} = f(\sigma_{m,2}) \tag{2a}$$

where, $\sigma_{s,max}$ and $\sigma_{m,2}$ are respectively the maximum shear stress and the mean normal stress in 2-dimension given by,

$$\sigma_{s,max} = \frac{(\sigma_1 - \sigma_3)}{2} \tag{2b}$$

$$\sigma_{m,2} = \frac{(\sigma_1 + \sigma_3)}{2} \tag{3}$$

where, σ_1 and σ_3 are the maximum and minimum principal stresses, respectively. When a rock is at the moment of failure, the Mohr circle with the radius of $\sigma_{s,max}$ and the center of $(\sigma_{m,2}, 0)$ is tangent to the failure envelope. Therefore, the Coulomb criterion given by Equation (1) is a linear type of the Mohr criterion given by Equation (2a). Combining the criteria, we can finally reach the final expression of the Mohr-Coulomb failure criterion in $\sigma_1 - \sigma_3$ domain.

$$\sigma_1 - P_p = UCS + q(\sigma_3 - P_p) \tag{4}$$

where, UCS is the unconfined compressive strength of the rock and q is given by,

$$q = \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \tag{5}$$

1.2. Mogi-Coulomb

The Mogi-Coulomb criterion was proposed by Mogi [43], after a set of triaxial compression tests. The author suggested a relationship between the principal stresses at failure given by,

$$\sigma_{s,oct} = f(\sigma_{m,2}) \tag{6}$$

Al-Ajmi and Zimmerman [44] proposed that the function f in Equation (6) should be a linear function for simplification. Consequently, the simplified Mogi-Coulomb failure criterion is defined by,

$$\sqrt{\frac{1}{2}((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)} = 2C\cos\phi + (\sigma_1 + \sigma_3 - 2P_p)\sin\phi \tag{7}$$

1.3. Modified Lade

Ewy [14] proposed the modified Lade criterion for wellbore stability analysis. The original Lade criterion was proposed by Lade [45] and is given by,

$$\left(\frac{I_1{}^3}{I_3} - 27\right)\left(\frac{I_1}{p_a}\right)^n = \eta_1 \tag{8}$$

where, I_1 and I_3 are given by

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 \tag{9a}$$

$$I_3 = \sigma_1\,\sigma_2\,\sigma_3 \tag{9b}$$

And where, p_a is the atmospheric pressure, n and η_1 are material constants. From the original criterion, Ewy [14] modified Equations with consideration of the cohesion and the pore pressure.

$$\frac{(I_1{}'')^3}{I_3{}''} = 27 + \eta \tag{10}$$

where, $I_1{}''$ and $I_3{}''$are given by,

$$I_1{}'' = (\sigma_1 + S_1 - P_p) + (\sigma_2 + S_1 - P_p) + (\sigma_3 + S_1 - P_p) \tag{11a}$$

$$I_3{}'' = (\sigma_1 + S_1 - P_p)(\sigma_2 + S_1 - P_p)(\sigma_3 + S_1 - P_p) \tag{11b}$$

where, S_1 and η are the material constants, which can be directly derived from the cohesion and internal friction angle in the following equations,

$$\mathrm{S}_1 = \frac{C}{\tan\phi} \tag{12}$$

$$\eta = 4\tan^2\phi\,\frac{9 - 7\sin\phi}{1 - \sin\phi} \tag{13}$$

1.4. Drucker-Prager

Another failure criterion widely used for wellbore stability analysis is the Drucker-Prager criterion, which was proposed by Drucker and Prager [46]. The authors assumed that the octahedral shear stress and normal stress are linearly related as following,

$$\sigma_{s,oct} = k + m\,\sigma_{n,oct} \tag{14}$$

where, $\sigma_{s,oct}$ and $\sigma_{n,oct}$ are respectively the octahedral shear and normal stresses given by,

$$\sigma_{s,oct} = \frac{1}{3}\sqrt{((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)} \tag{15a}$$

$$\sigma_{n,oct} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \tag{15b}$$

where, k and m are material constants determined from the internal friction angle and cohesion. There are three choices in the Drucker-Prager constants calculation [4]. Assuming that the Drucker-Prager failure surface circumscribes the Mohr-Coulomb failure surface, then the Drucker-Prager criterion can be rearranged as,

$$\sqrt{\frac{1}{6}((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)} = a\left(\sigma_1 + \sigma_2 + \sigma_3 - 3P_p\right) + b \tag{16}$$

where, coefficients a and b are given by,

$$a = \frac{2\sin\phi}{\sqrt{3}(3 - \sin\phi)} \tag{17a}$$

$$b = \frac{6c\cos\phi}{\sqrt{3}(3 - \sin\phi)} \tag{17b}$$

If it is assumed that the Drucker-Prager failure surface middle-circumscribes, the constants a and b are differently calculated as,

$$a = \frac{2\sin\phi}{\sqrt{3}(3 + \sin\phi)} \tag{18a}$$

$$b = \frac{6c\cos\phi}{\sqrt{3}(3 + \sin\phi)} \tag{18b}$$

If the Drucker-Prager yield surface inscribes the Mohr-Coulomb yield surface,

$$a = \frac{\sin\phi}{\sqrt{9 + 3(\sin\phi)^2}} \tag{19a}$$

$$b = \frac{3c\cos\phi}{\sqrt{9 + 3(\sin\phi)^2}} \quad (19b)$$

2. Determination of the Critical Wellbore Pressure at Shear and Tensile Failure

In order to determine the critical wellbore pressure, the approach proposed by Al-Ajmi and Zimmerman [13] was used. The radial, tangential and vertical principal stresses at the wellbore wall can be calculated from the Kirsch equation.

$$\sigma_r = P_w \quad (20a)$$

$$\sigma_\theta = \sigma_H + \sigma_h - 2(\sigma_H - \sigma_h)\cos 2\theta - P_w \quad (20b)$$

$$\sigma_z = \sigma_v - 2\nu(\sigma_H - \sigma_h)\cos 2\theta \quad (20c)$$

Since the lower-bound shear failure occurs at $\theta = \pi/2$ or $3\pi/2$ the radial, tangential and axial stresses can be calculated by substituting θ in (20).

$$\sigma_{rl} = P_w \quad (21a)$$

$$\sigma_{\theta l} = 3\sigma_H - \sigma_h - P_w = A_l - P_w \quad (21b)$$

$$\sigma_{zl} = \sigma_v + 2\nu(\sigma_H - \sigma_h) \quad (21c)$$

Similarly, the upper-shear or tensile failures occur at $\theta = 0 \; or \; \pi$ and the stress condition at the wellbore wall also can be calculated with (20).

$$\sigma_{ru} = P_w \quad (22a)$$

$$\sigma_{\theta u} = 3\sigma_h - \sigma_H - P_w = A_u - P_w \quad (22b)$$

$$\sigma_{zu} = \sigma_v - 2\nu(\sigma_H - \sigma_h) \quad (22c)$$

$$\sigma_{\theta u} - P_p = 3\sigma_h - \sigma_H - P_w - P_p = -T_0 \quad (22d)$$

where, A_l and A_u are given by,

$$A_l = 3\sigma_H - \sigma_h \quad (23a)$$

$$A_u = 3\sigma_h - \sigma_H \quad (23b)$$

Constraints are required to determine the critical wellbore pressure by the Mohr-Coulomb, because it takes into account the order of the principal stresses. For lower-bound shear failure, the tangential stress at the wellbore wall is higher than the radial stress ($\sigma_\theta > \sigma_r$). Therefore, there are 3 cases available 1) $\sigma_1 = \sigma_z$ and $\sigma_3 = \sigma_r$, 2) $\sigma_1 = \sigma_\theta$ and $\sigma_3 = \sigma_r$, and 3) $\sigma_1 = \sigma_\theta$ and $\sigma_3 = \sigma_z$. For each case, the lower limit of the wellbore pressure can be calculated by substituting σ_1 and σ_3 in Equation (4). Similarly, the critical wellbore pressure inducing the upper-bound shear failure can be calculated. Table 1 shows the equations calculating the critical wellbore pressure at upper- and lower-bound shear failure.

In order to derive the equations for the critical wellbore pressure calculation by the Mogi-Coulomb criterion, the intermediate principal stress needs to be constrained. By substituting the principal stresses in Equation (7), the lower and upper critical wellbore pressures can be calculated as shown in Table 2.

For the modified Lade and Drucker-Prager criteria, no constraint is necessary for the critical wellbore determination. Instead, either the lower-bound shear failure constants in (21) or the upper-bound shear failure constants in (22) need to be substituted in Equations (10) and (16). Solving the combined equations, the smaller root with the former constants and the larger root with the latter constants are the final equations to calculate the critical wellbore pressures by the modified Lade and Drucker-Prager criteria as shown in Table 3 and Table 4, respectively.

The critical wellbore pressure at tensile failure can be simply determined by letting the effective tangential stress equal the rock tensile strength.

Therefore, the critical wellbore pressure and critical net pressure at tensile failure can be calculated by (22a) and (22b).

$$P_w = 3\sigma_h - \sigma_H - P_p + T_0 \quad (24a)$$

$$P_{NET} = 3\sigma_h - \sigma_H - 2P_p + T_0 \quad (24b)$$

where, P_{NET} is the difference between the internal wellbore pressure and the formation pore pressure, i.e. $P_w - P_p$.

Table 1. Equations for the critical wellbore pressure for upper- and lower-bound shear failure by the Mohr-Coulomb criterion.

Failure Type	Condition	Equations
Lower-Bound Shear Failure ($\sigma_\theta > \sigma_r$)	$\sigma_1 = \sigma_z$ and $\sigma_3 = \sigma_r$	$P_{wl} = \dfrac{\sigma_{zl} - UCS + (q-1)P_p}{q}$
	$\sigma_1 = \sigma_\theta$ and $\sigma_3 = \sigma_r$	$P_{wl} = \dfrac{A_l - UCS + (q-1)P_p}{1+q}$
	$\sigma_1 = \sigma_\theta$ and $\sigma_3 = \sigma_z$	$P_{wl} = A_l - q\,\sigma_{zl} - UCS + (q-1)P_p$
Upper-Bound Shear Failure ($\sigma_r > \sigma_\theta$)	$\sigma_1 = \sigma_r$ and $\sigma_3 = \sigma_z$	$P_{wu} = q\,\sigma_{zu} + UCS + (1-q)P_p$
	$\sigma_1 = \sigma_r$ and $\sigma_3 = \sigma_\theta$	$P_{wu} = \dfrac{A_u q + UCS + (1-q)P_p}{1+q}$
	$\sigma_1 = \sigma_z$ and $\sigma_3 = \sigma_\theta$	$P_{wu} = A_u + \dfrac{-\sigma_{zu} + UCS + (1-q)P_p}{q}$
where, $q = tan^2\left(\dfrac{\pi}{4} + \dfrac{\phi}{2}\right)$		

Table 2. Equations for the critical wellbore pressure for upper- and lower-bound shear failure by the Mogi-Coulomb criterion.

Condition	Failure Type	Equations
$\sigma_2 = \sigma_\theta$	Lower-Bound	$P_{wl} = \dfrac{-2GE_{1l} - 3A_l + \sqrt{-12D_l + 4D_lG^2 + 12{E_{1l}}^2 + 12GE_{1l}A_l + 9A_l^2}}{2G^2 - 6}$

<table>
<tr><td rowspan="2"></td><td>Shear Failure</td><td></td></tr>
<tr><td>Upper-Bound Shear Failure</td><td>$P_{wu} = \frac{-2GE_{1u} - 3A_u - \sqrt{-12D_u + 4D_uG^2 + 12{E_{1u}}^2 + 12GE_{1u}A_u + 9A_u^2}}{2G^2 - 6}$</td></tr>
<tr><td rowspan="2">$\sigma_2 = \sigma_z$</td><td>Lower-Bound Shear Failure</td><td>$P_{wl} = \frac{1}{6}(3A_l - \sqrt{3}\sqrt{-4D_l + 4{E_{2l}}^2 + 3A_l^2})$</td></tr>
<tr><td>Upper-Bound Shear Failure</td><td>$P_{wu} = \frac{1}{6}(3A_u + \sqrt{3}\sqrt{-4D_u + 4{E_{2u}}^2 + 3A_u^2})$</td></tr>
<tr><td rowspan="2">$\sigma_2 = \sigma_r$</td><td>Lower-Bound Shear Failure</td><td>$P_{wl} = \frac{2GE_{3l} - 3A_l + \sqrt{-12D_l + 4D_lG^2 + 12{E_{3l}}^2 - 12GE_{3l}A_l + 9A_l^2}}{2G^2 - 6}$</td></tr>
<tr><td>Upper-Bound Shear Failure</td><td>$P_{wu} = \frac{2GE_{3u} - 3A_u - \sqrt{-12D_u + 4D_uG^2 + 12{E_{3u}}^2 - 12GE_{3u}A_u + 9A_u^2}}{2G^2 - 6}$</td></tr>
<tr><td colspan="3">where,
$D_i = {A_i}^2 - A_i\,\sigma_{zi} + {\sigma_{zi}}^2$
$E_{1i} = 2C\cos\phi + G(\sigma_{\mathrm{zi}} - 2P_p)$
$E_{2i} = 2C\cos\phi + G(A_i - 2P_p)$
$E_{3i} = 2C\cos\phi + G(A_i + \sigma_{\mathrm{zi}} - 2P_p)$
$i = l\ or\ u$
$G = \sin\phi$</td></tr>
</table>

Table 3. Equations for the critical wellbore pressure for upper- and lower-bound shear failure by the modified Lade criterion.

Failure Type	Equations
Lower-Bound Shear Failure	$P_{wl} = \frac{1}{2}\left(A_l - \frac{\sqrt{K_l(27+\eta)(-(A_l - 2\sigma_{zl})^2(6J + 4A_l + \sigma_{zl}) + K_l(2J + A_l)^2\eta)}}{K_l(27+\eta)}\right)$

Upper-Bound Shear Failure	$P_{wu} = \frac{1}{2}\left(A_u + \frac{\sqrt{K_u(27+\eta)(-(A_u-2\sigma_{zu})^2(6J+4A_u+\sigma_{zu})+K_u(2J+A_u)^2\eta)}}{K_u(27+\eta)}\right)$
where, $\eta = 4\tan^2\phi \frac{9-7\sin\phi}{1-\sin\phi}$; $J = S1 - P_p$; $K_i = J + \sigma_{zi}$, $i = l, u$	

Table 4. Equations for the critical wellbore pressure for upper- and lower-bound shear failure by the Drucker-Prager criterion that circumscribes the Mohr-Coulomb failure surface.

Failure Type	Equations
Lower-Bound Shear Failure	$P_{wl} = \frac{1}{6}\left(3A_l - \sqrt{36\left[b + a\left(\sigma_{zl} + A_l - 3P_p\right)\right]^2 - 3(A_l - 2\sigma_{zl})^2}\right)$
Upper-Bound Shear Failure	$P_{wu} = \frac{1}{6}\left(3A_u + \sqrt{36\left[b + a\left(\sigma_{zu} + A_u - 3P_p\right)\right]^2 - 3(A_u - 2\sigma_{zu})^2}\right)$
where, $a = \frac{2\sin\phi}{\sqrt{3}(3-\sin\phi)}$; $b = \frac{6c\cos\phi}{\sqrt{3}(3-\sin\phi)}$	