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Mechanism Analysis of Sympathetic Inrush in Traction Network Cascaded Transformers Based on Flux-Current Circuit Model⁺

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Abstract: When electric multiple units (EMU) pass the neutral zone, the traction transformer may generate sympathetic inrush, which will cause a malfunction in the transformer differential protection. In order to study the mechanism of the sympathetic inrush of the cascaded traction transformer, the flux–current model of the transformer, line impedance, power system voltage source, and other loads was established. On the basis of the flux–current circuit model, the influence of different factors on the sympathetic inrush of the traction transformer was analyzed. The analysis results were verified by simulation. Research results show that the remanence, closing angle, line impedance, and load will affect the duration and amplitude of the sympathetic inrush.

Keywords: traction network; sympathetic inrush; cascaded transformer; transformer saturation; flux–current circuit model

1. Introduction

In order to ensure the reliability and flexibility of power supply, the electrified railway traction power supply system generally adopts partition power supply [1,2]. There is no power zone between different power supply arms. When the EMU passes the electric phase separation, the locomotive transformer will perform the opening and closing operations in sequence. Moreover, when the locomotive transformer is restored to power, inrush current may occur [3,4]. The inrush current generated by the closing transformer will lead to the sympathetic inrush generated by the traction transformer in the same power supply area, which is the main cause of differential protection errors in traction transformers [5–8].

Many scholars at home and abroad have done a lot of research on the mechanism analysis and influencing factors of sympathetic inrush and inrush current. These studies were carried out on the basis of the T-type equivalent model [9–13]. First, the equivalent model of the transformer was established, and then the generation of the inrush current was analyzed. It is believed that the sympathetic inrush is mainly caused by the non-periodic fluctuation of the voltage on the common node caused by the non-periodic component in the inrush current of the adjacent transformer flowing through the system resistance. The authors in [14,15] compared and analyzed the structure of parallel transformers and cascade transformers. It is considered that parallel transformers are more likely to generate and inrush currents because of its smaller parameters. It must be pointed out that the T-equivalent circuit is based on Kirchhoff's law, and the voltage and current on the secondary side of the transformer are converted to the primary side [16,17]. Therefore, it does not reflect the internal magnetic variables of the transformer. The model does not contain information such as transformer



geometry parameters and topology structure, and does not reflect information such as magnetic flux and flux density at various locations within the core. The T type equivalent model is unable to directly reflect the impact of remanence, line impedance, over-phase closing phase angle, transformer capacity, and load conditions [18,19]. Therefore, it is necessary to build a new model that can research the mechanism of sympathetic inrush and its impacts.

Some scholars have focused on the sympathetic inrush of two or more EMU transformers, which are connected in parallel. Studies in [20,21] analyzed the parallel connection of multiple electric locomotive transformers, which work simultaneously under the same power supply arm, while [22,23] analyzed the effects of locomotives at different locations and directions of travel on the sympathetic inrush. During the analysis, they supposed that the voltage of the catenary was a fixed value [24,25]. That is to say, they ignored the fluctuation of the traction network voltage. The analysis of the sympathetic inrush in [26,27] was based on the case where the capacity of the EMU transformer and the traction transformer was not very different. When the closing of the EMU transformer generates 6–8 times of inrush current, its influence on the traction transformer is not negligible [28]. Therefore, it is necessary to research the inrush current of the cascaded traction transformer and analyze the influence of various factors on the duration and amplitude of the sympathetic inrush.

This paper presents a method for analyzing the sympathetic inrush of a traction network cascade transformer based on the flux–current circuit model. First, the flux–current model of the transformer, line impedance, power system voltage source, and other loads was established. Then, the flux–current circuit model of the traction network was formed. On the basis of the flux–current circuit model, we analyzed the causes of the sympathetic inrush as well as the effects of remanence, over-phase closing phase angle, line impedance, and load conditions on the magnitude and duration of the sympathetic inrush. The analysis results were verified by simulation. The rest of this paper is organized as follows. Section 2 presents the modeling of the flux–current mode, Section 3 presents the analysis of the mechanism and the impacts. Section 4 details the simulation and analysis, and Section 5 presents our conclusions.

2. Modeling of the Flux-Current Model

The traction supply system shown in Figure 1 includes the public power grid, traction transformer, traction network, partitions, EMU1 and EMUs, where EMU1 passes through neutral devices and EMUs travel at a constant speed.



Figure 1. Supply system.

When the EMU1 passes no power zone, the transformer of EMU1 will perform the opening and closing operations in sequence, which will last several seconds. Then, we can use the timed logic switch to simulate the process. Furthermore, when we investigated the EMU1 passing through neutral devices, the EMUs can be regarded as a constant power load. Therefore, the simplified circuit of the traction supply system is shown in Figure 2:



Figure 2. Simplified circuit of the traction supply system.

Next, the flux–current model of the main elements will be built in this part, which include transformers (containing traction transformer and EMU transformer), line impedance (containing power system line impedance, and traction electric network line impedance), power system voltage source, and other loads.

2.1. Transformers

Both the EMU transformer and the traction transformer are magneto electric conversion elements. Take the traction transformer as an example of establishing its flux–current circuit model. As shown in Figure 3a below, the relationship curve between the flux and exciting current of the transformer had hysteretic and saturation characteristics. Figure 3a can be linearized, as shown in Figure 3b. The curve is divided into three sections. When the range of the exciting current I_{ST} is in the range of $-I_{kST}$ and I_{kST} , the transformer works in the linear area. Meanwhile, when the exciting current I_{ST} is less than $-I_{kST}$ or greater than I_{kST} , the transformer works in the saturation area.



Figure 3. Relationship of the transformer flux-exciting current. (**a**) Transformer flux-exciting current curve; (**b**) Transformer flux-exciting current curve after piece-wise linearization.

Furthermore, Figure 3b can also be described as follows.

$$\Phi_{ST} = \begin{cases} L_{kST} \left(I_{ST} + \frac{\Phi_{kST}}{L_{mST}} \right) - \Phi_{kST} & (\Phi_{ST} \le -\Phi_{kST}) \\ L_{mST} I_{ST} & (-\Phi_{kST} < \Phi_{ST} < \Phi_{kST}) \\ L_{kST} \left(I_{ST} - \frac{\Phi_{kST}}{L_{mST}} \right) + \Phi_{kST} & (\Phi_{ST} \ge \Phi_{kST}) \end{cases}$$
(1)

where Φ_{ST} is the flux of the traction transformer; Φ_{kST} is the forward flux saturation threshold, and $-\Phi_{kST}$ is the negative flux saturation threshold; I_{ST} is the critical saturation current; L_{mST} and L_{kST} are the equivalent inductors in the linear and nonlinear regions, respectively.

Equation (1) can also be described as an equivalent circuit, as shown is Figure 4, which points out the relationship between the transformer flux and exciting current. The equivalent circuit is the flux–current circuit model of the transformer.



Figure 4. Flux-current circuit model of the transformer.

As EMU1 passes neutral devices, the EMU1 transformer carries no load, so its exciting current is only provided by the primary winding. Its primary winding current is its exciting current. However, the traction transformer usually carries loads, whose exciting current is provided by the primary winding current and the secondary winding current. Therefore, it is necessary to compute the exciting current, as shown below.

$$N_{ST1}i_s + N_{ST2}i_{oh} = N_{ST1}i_{aST} \tag{2}$$

where N_{ST1} is the turns of primary side and N_{ST2} is the turns of the secondary side winding of the traction transformer; i_s is the winding current of primary side; i_{oh} is the winding current of secondary side; and i_{aST} is the traction transformer excitation current.

The voltage and current of the secondary side of the traction transformer can be converted to the primary side of the traction transformer, as shown in the following formula.

$$\begin{cases}
 i_{oh}^{*} = -i_{oh} \frac{N_{ST1}}{N_{ST2}} \\
 i_{oh} = \frac{u_{ST2}}{Z_{ST2}} \\
 i_{oh}^{*} = \frac{u_{ST1}}{Z_{ST2}^{*}} \\
 \frac{u_{ST1}}{u_{ST2}} = \frac{N_{ST1}}{N_{ST2}}
\end{cases}$$
(3)

where i_{oh} is the secondary side current of traction transformer; i_{oh}^* is the equivalent current, which is converted from the secondary side of the traction transformer to the primary side; Z_{ST2} is the secondary side impedance of the traction transformer including traction network impedance Z_{oh} and electric locomotive equivalent effect impedance; U_{ST1} and U_{ST2} are the primary side and secondary side voltages of the traction transformer, respectively; and Z_{ST2}^* is the equivalent impedance after converting all the secondary side impedance to the primary side.

In order to facilitate writing, the following secondary side resistance was switched to the primary side without adding *.

2.2. Line Impedance Components

This part builds the flux–current model of line impedance. The line impedance contains line resistance and line inductance.

According to Ohm's law, the relationship between the voltage at both ends of the resistance and the current passing current is shown in the following formula.

$$u_R = Ri_R \tag{4}$$

If the initial state of the resistance is not taken into account, then the Equation (4) integral can be obtained

$$\Phi_R(t) = R \int_0^t i_R(t) dt$$
(5)

where Φ_R is the equivalent flux of *R*.

From Equation (5), we can see that the resistance has the characteristics of a "magnetic trap" in the flux–current circuit. That is to say, the resistance captures the DC magnetic bias and converts it

into equivalent flux through energy consumption. Equation (5) is formally similar to the relationship between the voltage and current at both ends of capacitance. We might as well use the capacitance symbol in the circuit to represent the resistance in the "flux–current" circuit, which is shown in Table 1.

Table 1. Symbolic representation of elements in the "voltage-circuit" circuit in the "flux-current" circuit.

Components in the "Voltage-current" Circuit	In the "Flux-Current" Circuit Representation
Voltage source	AC Quasi-power symbol
Resistance	Quasi-Capacity symbol
Inductance	Quasi-Resistance symbol

Note: In order to distinguish the symbol in the voltage-current circuit, all flux-current circuit elements are framed, and named as "quasi".

For inductance, the relationship between the voltage and the current is shown in the following formula.

$$u_L = L \frac{du_L}{dt} \tag{6}$$

If we do not take account of the initial state of the inductor, the Equation (6) integral is obtained.

$$\Phi_L(t) = \int_0^t u_L dt = Li_L \tag{7}$$

where Φ_L is the equivalent flux of *L*.

By observing Equation (7), it can be seen that the flux of the inductance is proportional to the current flowing through the inductance. It can be seen that the inductance has the characteristics of "damping" in the "flux–current" circuit. We might as well use the resistance symbol in the circuit to represent the inductance in the "flux–current" circuit, as shown in Table 1.

2.3. Power System Voltage Source

The voltage source includes an AC source and DC source, which can be expressed as Equation (8). When $\omega = 100\pi$, the following formula represents the 50 HZ AC source.

$$u_s = A\cos(\omega t) \tag{8}$$

If the initial state of the power supply is not taken into account, the upper formula integral is obtained.

$$\Phi_s = \int_0^t A\cos(\omega t)dt = \frac{A}{\omega}\sin(\omega t)$$
(9)

where Φ_S is the equivalent flux of the voltage source.

According to Equation (9), the flux–current model of the voltage is still a sine function. We can use the power symbol in the voltage–current loop to represent the power supply in the "flux–current" circuit model, as depicted in Table 1.

2.4. Transformer with Load

When analyzing the process of EMU1 passing through neutral devices, other EMUs can be represented by controlling the current source. The flux–current circuit model of other loads is described built in this section.

When EMUs ride in uniform speed, they can be looked on as constant power load (CPL). The modeling process of constant power load is as follows.

$$P = P_0 \cdot \left(\frac{U}{U_0}\right)^{MP} \tag{10}$$

where MP = 0. *P* is the real time power of the load; *P*₀ is the initial power of the load; *U* is the real time voltage of the load; and *U*₀ is the initial voltage of the load.

Through the Taylor expansion, the equation can be written as follows.

$$\Delta I \approx P_0 \cdot (MP - 1) \cdot \Delta U \tag{11}$$

When MP = 0, the load is the constant power load (CPL), and the formula can be written as follows.

$$\Delta I \approx -P_0 \cdot \Delta U \tag{12}$$

where ΔI has a reverse positive proportional relationship with ΔU .

The load can be described as the formula below.

$$I = \frac{P_0}{\Phi'_{E0}} - \left(\Phi'_E - \Phi'_{E0}\right) \cdot P_0 \tag{13}$$

Thus, the flux-current circuit model of other EMUs is as shown in Figure 5.



After building the flux–current circuit model of each element of the traction supply system, the flux–current circuit model of traction supply system is as follows.

In the Figure 6, Φ_s is the flux–current model of the public power grid; Z_s is the flux–current model of the impedance between the public power grid and the traction transformer, Φ_{ST} is the flux–current model of the traction transformer; Z_{oh1}^* is the flux–current model of the impedance from the transformer of the over-phase locomotive to the traction transformer; Φ_T is the transient magnetic flux; Φ_E is the flux–current model of the over-phase transformer; Z_{oh2}^* is the flux–current model of the impedance between other locomotive transformers under the same power supply arm and the traction transformer; and CPL is the flux–current model of other locomotive transformers under the same power supply arm.



Figure 6. The flux-current circuit model of the traction network-EMU coupling system.

3. Analysis of the Mechanism and the Impacts

The characteristics of sympathetic inrush are mainly reflected in amplitude and duration. This section will analyze the influence of remanence and the over-phase switching angle, line impedance components, transformer capacity, and transformer with loads on sympathetic inrush from the perspective of magnetic flux. According to Figure 6, the magnetic flux of the traction transformer is

superimposed by the steady-state magnetic flux corresponding to the power supply and the transient magnetic flux corresponding to the closing moment, and its value satisfies the following equation:

$$\Phi_{ST} = \Phi_S + \Phi_T \tag{14}$$

where Φ_S is the steady-state magnetic flux and Φ_T is the transient magnetic flux.

The flux–current circuit model of the traction transformer consists of three branches. When the traction transformer works normally, only linear branch #1 is turned on; in the case of forward saturation or negative saturation, #2 or #3 is turned on while keeping #1 turned on. Due to the conduction of #2 or #3, the equivalent impedance of the traction transformer is reduced, so the traction transformer generates sympathetic inrush with a maximum amplitude. The influences of remanence, over-phase switching angle, line impedance components, transformer capacity, and transformer with load condition on sympathetic inrush will be analyzed one by one.

3.1. Remanence and Over-Phase Switching Angle

The relationship between the remanence, over-phase switching angle, and the transient flux is given in Equation (15) and the detailed derivation process can be found in [15]. On the basis of this, we drew a three-dimensional graph, which is shown in Figure 7. It can be seen from the figure that if the remanence remains unchanged, the transient magnetic flux exhibits a cosine law change with the switching angle, the transient flux increases and then decreases in the range of 0 to 2π , and reaches the maximum value when the switching angle is 0. If the switching angle remains unchanged, the transient flux and remanence show a linear relationship, that is, in the interval of -1 to 1, the transient magnetic flux increases with the increase of remanence; the transient flux varies from -2 to 2 under the combined action of remanence and over phase switching angle.

$$\Phi_{T0} = \Phi_{ET}^{0-} - \Phi_{ET}^{0+} = \Phi_{ET} \cos \theta_{V0} - \Phi_R \tag{15}$$

where Φ_{T0} is the transient flux at closing moment; Φ_{ET} is the amplitude of the steady-state flux; Φ_R is remanence; and θ_{V0} is the switching angle.



Figure 7. Transient flux of the traction transformer affected by remanence and switching angle.

As a DC excitation source, the transient magnetic flux transfers the magnetic flux to Z_S and Z_{oh1} continuously. After the balance, the DC flux on both sides of Z_S and Z_{oh1} satisfies the following relationship.

$$\Phi_{Zs} + \Phi_{Zoh1} = \Phi_{T0} \tag{16}$$

It is necessary to point out that the allocation of Φ_{Zs} and Φ_{Zoh1} is only related to the line and transformer parameters. That is to say, the value of $\frac{\Phi_{Zs}}{\Phi_{Zoh1}}$ will not change when the parameters of the line and transformer remain unchanged. Therefore, the transient magnetic flux on both sides of the traction transformer is shown in the following formula.

$$\Phi_T = k \left(\theta_{V0}, t \right) \Phi_{T0} \tag{17}$$

Based on Equations (15) and (17), the relationship between transient magnetic flux, closing angle, and remanence of the traction transformer can be obtained as shown in Figure 7.

According to Figure 7, the transient flux Φ_{T0} of the traction transformer is related to phase angle θ_{V0} and remanence Φ_R . If we keep θ_{V0} constant, then $|\Phi_{T0}|$ is proportional to $|\Phi_R|$. The bigger the $|\Phi_R|$, the bigger the $|\Phi_{T0}|$; if Φ_R remains unchanged, Φ_{T0} and θ_{V0} are cosine functions. The magnetic flux of the traction transformer is superimposed by the steady-state magnetic flux and the transient magnetic flux, which are independent of each other after over-phase closing. For the transformer, the saturation threshold generally has a margin of about 15%. Therefore, if the saturation threshold is exceeded under the combined action of the remanence and the over-phase switching angle, the traction transformer will generate sympathetic inrush. If the leakage inductance and winding impedance of the transformer are neglected, the relationship between sympathetic inrush and magnetic flux is shown as in Equation (18).

$$i_{aST} = \frac{\Phi_{kST}}{L_{mST}} + \frac{\Phi_{ST} - \Phi_{kST}}{L_{kST}}$$
(18)

When the traction transformer is saturated, its equivalent inductance value is sharply reduced to less than half of the normal value, and its magnetic flux is about three times of the normal value. Under the combined action of these two factors, the inrush current should reach 6–8 times that of the normal current.

3.2. Line Impedance Components

According to the flux–current circuit model of the traction supply system shown in Figure 6, the line impedance has a "magnetic trap" characteristic, which is similar to the "electricity-storage" characteristic of capacitance in the circuit. This section will analyze the influence of line impedance on sympathetic inrush by analogy.

The line impedance of the traction network shown above is similar to the RC circuit and its time constant is $\tau = \frac{L_{olt}}{R_{oh}}$.

The saturation flux of the traction transformer is set as Φ_{kST} . The magnetic flux of the traction transformer at the moment of passing through phase is Φ_{ST0} . The attenuation time of the sympathetic inrush is *t*, so we can obtain the following equation:

$$\Phi_{ST0} - \Phi_{kST} = \Phi_{ST0} \left(1 - e^{-\frac{t}{\tau}} \right)$$
(19)

which is solved by Equation (20), then we can get

$$t = \tau \ln \left(\frac{\Phi_{ST0}}{\Phi_{ST0} - \Phi_{kST}} \right) \tag{20}$$

where $\Phi_{kST} \leq \Phi_{ST0}$.

If the saturation threshold of the traction transformer and the flux of the EMU transformer remain unchanged at the moment of passing phase, the decay time of the sympathetic inrush is proportional to τ . Thus, the line impedance angle of the traction network line can be expressed as:

$$\theta_{Zoh} = \arctan\left(\frac{L_{oh}}{R_{oh}}\right) = \arctan\tau$$
(21)

Therefore, the decay time of sympathetic inrush increases monotonically with the increase in the impedance angle. Since the impedance of the traction transformer is inductive, when the electric locomotive gradually moves from the far end of the traction network to the traction substation, the equivalent impedance angle of the contact line will increase, and the decay time of the sympathetic inrush will change, as shown in Figure 8.



Figure 8. Effect of the line impedance angle on the duration of the sympathetic inrush.

3.3. Traction Transformer Capacitance

Electric locomotives will go through different traction substations during operation. The transformer capacity of different traction substations is not the same, so the amplitude of the sympathetic inrush is not the same. For the same type of transformer, the capacity of a transformer is proportional to the magnetic flux, that is, when the transformer capacity is large, the critical saturation flux is also large. The relationship between the critical saturation flux and the current is as follows.

$$i_{aST} = \frac{\Phi_{kST}}{L_{mST}} + \frac{\Phi_{\max ST} - \Phi_{kST}}{L_{kST}}$$
(22)

where Φ_{maxST} is the maximum flux in over-phase process.

Generally speaking, $L_{mST} \gg L_{kST}$. According to Equation (22), if Φ_{maxST} remains unchanged, the smaller the critical flux, the smaller the transformer capacity, and the greater the sympathetic inrush amplitude. As shown in Figure 9.



Figure 9. The relationship between the sympathetic inrush amplitude and the capacity of the traction transformer.

The "magnetization" time required from the moment of the excessive phase to the maximum of the sympathetic inrush is shown in Equation (23). It can be concluded that the smaller the critical saturation flux, the smaller the transformer capacity, and the shorter the time required for the sympathetic inrush to reach the maximum amplitude.

$$t = \tau \ln \left(\frac{\Phi_{STmax}}{\Phi_{STmax} - \Phi_{STk}} \right)$$
(23)

3.4. Transformer with Load

For the traction transformer on the same supply arm, the power supply is not only for EMU1 in the over-phase, but also for EMUs in running. According to Figure 6, the traction transformer flux satisfies the following equation.

$$\Phi_{ST} = L_{ST}(i_{EMU1} + ni_{EMUS}) \tag{24}$$

Therefore, we can obtain the relationship between the magnetic flux of the traction transformer and the current of EMU1 and EMUs, which is shown in Figure 10.



Figure 10. The relationship between the magnetic flux of the traction transformer and the current of EMU1 and EMUs.

According to Figure 10, the magnetic flux of the traction transformer is related to the current of the EMU1 and EMUs. If the current of EMU1 remains unchanged, as the current of the EMUs increases in the positive or negative direction, the traction transformer will exceed the saturation threshold and enter the saturation zone, thus leading to the generation of the sympathetic inrush. Moreover, the larger the current amplitude of the EMUs, the easier it is for the traction transformer to become saturated.

4. Simulation and Analysis

The flux–current circuit model of the traction supply system is shown in Figure 6. The simulation model was built by MATLAB, then we analyzed the mechanism of sympathetic inrush and the influence of different factors. The simulation parameters are as follows. The Line parameters shown in Table 2 were from the measured data of a high-speed railway, the traction transformer data in Table 3 were from [14], and the EMU data in Table 4 were from the measured data of the traction substations of a high-speed railway.

Parameters	Value
The equivalent impedance of power grid to traction network	$0.47 + j 0.85 \Omega / \mathrm{km}$
Contact line length	25 km
Contact line impedances	$0.15 + j 0.85 \Omega/\mathrm{km}$

Table 2. L	ine parameters.

Parameters	Value
Primary side voltage	220 kV
Secondary side voltage	25 kV
Slope of the saturation area	0.35
Frequency	50 Hz
Capacity	31.5 MW
Mutual impedance	$0.08 + j 2.1 \Omega$
Self-impedance	$0.02 + j 0.8 \; \Omega$

Table 3. Traction transformer parameters.

Table 4. EMU transformer parameters.

Parameters	Value
Primary side voltage	25 kV
Secondary side voltage	15 kV
Slope of the saturation area	0.8
Frequency	50 Hz
Capacity	22 MW
Mutual impedance	$0.04 + j 4.1 \Omega$
Self-impedance	$0.02+j1.5~\Omega$

This section analyzes the mechanism of sympathetic inrush and its impacts, which is as follows. During the simulation, the current is uniformly converted to the primary side of the traction transformer.

4.1. Impact of Remanence and Switching Angle

The relationship between the remanence, switching angle, and transient flux is as shown in Equation (16). The change of remanence and switching angle will cause the change in the transient magnetic flux, and finally cause the traction transformer to be saturated, and then generate sympathetic inrush.

This section will analyze the characteristics of sympathetic inrush under different transient magnetic flux. The transient flux Φ_T was set as 0, 0.3, or 0.7 (p.u.), respectively. The simulation waveform is shown below.

According to Figure 11, with the increase of transient flux, the amplitude of sympathetic inrush increases and the generation time of sympathetic inrush is advanced, which is consistent with the conclusion in the previous section.



Figure 11. Sympathetic inrush under different transient flux conditions.

4.2. Impact of Line Impedance

The effect of line impedance on traction transformer sympathetic inrush was deduced in the previous section. The impedance angle was different, and the decay time of the sympathetic inrush was also different, and the smaller the impedance angle, the faster the attenuation will be. This section describes the simulation verification of this conclusion. The impedance of the traction network was set as $0.1 + j 0.35 \Omega$, $0.12 + j 0.55 \Omega$, $0.15 + j 0.85 \Omega$, respectively. The simulation waveform is shown below in Figure 12.

According to Figure 12, as the impedance angle increases, the generation time of the sympathetic inrush will gradually be pushed back, but the amplitude of the sympathetic inrush will always remain around 400 A.



Figure 12. Sympathetic inrush under different line impedance conditions.

4.3. Impact of Traction Transformer Capacity

In the previous section, the influence of the capacity of the traction transformer on the traction transformer sympathetic inrush was deduced, and a conclusion was drawn that the smaller the capacity of traction transformer, the greater the sympathetic inrush amplitude, and the earlier the time that the sympathetic inrush is generated. The following will be verified separately. We set the transformer capacity as 0.9 p.u., 1.0 p.u., and 1.1 p.u., respectively. The simulation waveform is shown below in Figure 13.



Figure 13. Influence of traction transformer capacity on sympathetic inrush.

According to the figure above, the smaller the capacity of the traction transformer, the greater the amplitude of sympathetic inrush, and the earlier the generation time of the sympathetic inrush.

4.4. Impact of Load Conditions

The previous section has already deduced the influence of traction transformer loading on the traction transformer and the sympathetic inrush, and concluded that the larger the load current, the larger the amplitude of the sympathetic inrush. The following will be verified separately. The load current was set as 150 A, 180 A, and 350 A, respectively. The simulation waveform is shown below in Figure 14.



Figure 14. Sympathetic inrush of the traction transformer under different load conditions.

According to Figure 14, the larger the load carried by the traction transformer, the easier it is to generate the sympathetic inrush, and the greater the amplitude of the sympathetic inrush, the earlier the generation time.

5. Conclusions

This paper presented a method for analyzing the sympathetic inrush of a traction network cascade transformer on the basis of the flux–current circuit model. First, the flux–current model of the transformer, line impedance, power system voltage source, and other loads was established. Then, the flux–current circuit model of traction network was formed. On the basis of the flux–current circuit model, we analyzed the causes of the sympathetic inrush of the traction transformer as well as the effects of remanence, over-phase closing phase angle, line impedance, and load conditions on the magnitude and duration of the sympathetic inrush and obtained the following conclusions:

- (1) When the sum of the steady-state flux and the transient flux of the traction transformer exceeds the saturation flux, the traction transformer will generate the sympathetic inrush.
- (2) Remanence and the over-phase closing angle affect the transient magnetic flux. With the increase in transient flux, the traction transformer will enter the saturation region, the amplitude of sympathetic inrush increases, and the generation time of the sympathetic inrush is advanced.
- (3) The greater the impedance angle of the line, and the later the generation time of the sympathetic inrush, but the line impedance angle has no effect on the amplitude of the sympathetic inrush.

- (4) The smaller the capacity of the traction transformer, the greater the amplitude of the sympathetic inrush, and the earlier the generation time of the sympathetic inrush
- (5) The larger the load carried by the traction transformer, the easier it is to generate sympathetic inrush, and the greater the amplitude of the sympathetic inrush, the earlier the generation time.

This paper only analyzed the generation mechanism of the sympathetic inrush and its influencing factors, and did not analyze how to suppress sympathetic inrush. In the next step, the suppression measures of the sympathetic inrush will be studied.

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