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Safe Operation Conditions of Electrical Power System Considering Power Balanceability among Power Generators, Loads, and Storage Devices

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Abstract: The introduction of an energy storage system plays a vital role in the integration of renewable energy by keeping a stable operation and enhancing the flexibility of the power flow system, especially for an islanding microgrid which is not tied to a grid and for a self-contained microgrid which tries to stay independent from a grid as much as possible. To accommodate the effects of power fluctuations of distributed energy resources and power loads on power systems, a power flow assignment under power balance constraint is essential. However, due to power limitations of power devices, the capacity of storage devices, and power flow connections, the power balance may not be achieved. In this paper, we proposed a system characterization which describes the relation among power generators, power loads, power storage devices, and connections that must be satisfied for a system to operate by keeping *SOC* limitations of power storage devices. When we consider one power generator, one power load, and one power storage device connected at a single node, the generated energy by the generator minus the consumed energy by the load from some start time will increase/decrease the state of charge (*SOC*) for the storage device; hence, keeping *SOC* max/min limitations relies on whether the difference between the generated energy and the consumed energy stays within a certain range or not, which can be computed from the capacity *Ess* and other parameters. Our contribution in this paper is an extension and generalization of this observation to a system that consists of multiple fluctuating power generators, multiple fluctuating power loads, multiple storage devices, and connections that may not be a full connection between all devices. By carefully enumerating the connection-dependent flow paths of generated energy along the flow direction from generators to storages and loads, and enumerating the connection-dependent flow paths of consumed energy along the counter-flow direction from loads to storages and generators, we have formulated the increase/decrease of *SOCs* of storage devices caused by the imbalance between generated energy and consumed energy. Finally, considering the max/min limitations of *SOCs* and fluctuations of power generators and power loads, the conditions that the power generators and the power loads must have for *SOCs* of storage devices to maintain individual max/min limitations have been derived. The system characterization provides guidelines for a power flow system that can continue safe operation in the presence of power fluctuations. That is, in order for a system to have a feasible power flow assignment, there are the issues of how large the capacity of a power storage device should be, how large/small the maximum/minimum power/demand levels of the power generators and the power loads should be, and how the connection should be configured. Several examples using our system characterization are demonstrated to show the possible applications of our results.

Keywords: renewable resources; energy storage devices; power fluctuations; power balanceability

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1. Introduction

To face the dynamic rise in power demand, the decrease of conventional power resources, and the necessity of reducing gas emissions, renewable energy resources have been promoted throughout the world [1,2]. Renewable energy resources (RESs), such as wind and solar devices, are different from conventional power generation sources, such as coal, gas, nuclear, etc. They are environmentally friendly alternatives; however, the power generated from these power sources is uncontrollable, uncertain, and exhibits large power fluctuations due to their intermittent nature [3,4]. These characteristics of renewable power generation devices are critical obstacles in integrating renewable energy sources into the existing power grid system. On the other hand, the power load side also shows power fluctuations, which together with fluctuated power generation brings challenges to the stable operation of the power grid [5–7].

Numerous solutions have been presented to improve the incorporation of the generated power of renewable resource penetration to the existing power grid. One of the major challenges of the integration of renewable generation into the power grid remains in the matching of the fluctuated power supply with the dynamic power demand. One potential approach is the introduction of energy storage devices which can play a vital role in the integration of renewable energy sources by maintaining the continuous operation of a power system and enhancing the flexibility of the usage of power devices [8–10]. That is, from the perspective of increased absorption of the excess of power and supply in a shortage of power, energy storage devices are an essential part of any power system.

For example, the use of power storage devices can help in supplying power to various power loads to fulfill their requested power demand and also in consuming power when the power generation from renewable generators is higher than demand.

The power grid should have enough capacity to meet the power demands of consumers. However, the power demand varies dynamically due to daily and seasonal schedules, and matching generation with demand is a critical problem to solve. The power system must have a battery storage system to provide continuous capacity to meet real-time power demands. There are many existing models which consider renewable generation together with storage systems under different applications and objectives. Morais et al. [11] proposed a non-robust optimization using a mixed-integer linear program (MILP) for power control. The author considered a power system consisting of wind turbines, PV panels, storage batteries, and power loads of controllable type. Handschin et al. [12] showed a stochastic optimization-based MILP to dispatch power in the microgrid. Their proposed robustness approach, however, is used at the cost of greatly increasing the size of the MILP optimization problem. There are also online model predictive control (MPC) [13,14] approaches that have been proposed for power management for a micro-grid with storage systems and renewable generation. In these approaches, an optimization problem is designed and solved over a rolling window for each time step. These optimization approaches focus on economic costs and balancing energy flows. The ESS capacity determination is another problem in terms of ensuring power system reliability. To solve this issue, capacity optimization approaches are discussed [15,16] to show minimum capacity based on the state of charge (SOC) limitation. These approaches also consider the situation when ESS and controllable power loads coordinate with each other to reduce the burden of ESS capacity. Therefore, these approaches are essential to ensure the minimum ESS capacity required for a given power system.

To accommodate power fluctuations triggered by fluctuating power generators and loads, a power flow assignment is essential. The power flow assignment finds power levels for controllable power devices and connections between power devices while keeping the power limitation constraints. The most fundamental objectives of the power flow assignment include: (i) the generated power of fluctuating power generators is fully consumed by loads or stored in power storage devices, (ii) the power demand of power loads is fully satisfied as requested, and (iii) the state of charge (SOC) of the power storage device always stays within the maximum and minimum energy limits. In this paper, we discuss a system characterization which describes the relation among power generators, power loads, power storage devices, and connections that must be satisfied for a system to operate by keeping the SOC limitations of power storage devices. The purpose of our system characterization is to provide guidelines for a power flow system that can continue safe operation in the presence of power fluctuations. That is, in terms of a system having a feasible power flow assignment, the issues of how large the capacity of the power storage device should be, how large/small the maximum/minimum power/demand levels of the power generators and the power loads should be, and how the connection should be configured are the main contributions of this paper.

In particular, as a first attempt on the system characterization, power systems with fluctuating power generators and loads are discussed in this paper together with energy storage devices. In the future, we will extend our system characterization of power systems with controllable power devices.

The rest of the paper is organized as follows. Section 2 presents related works. Section 3 shows the system characterization problem. This section describes the power devices, such as the power generators, loads, power storage devices, and the connections between them. It also explains the modeling of the system components, types of power generator/load devices, and power flow connections among power devices. An energy conservation law is presented for a simple power flow system in this section. The proposed system characterization problem with given power levels for fluctuating power devices is explained in Section 4. To show the validation of the proposed system characterization problem, simulation results are presented in Section 5. Finally, the concluding remarks are discussed in Section 6.

2. Related Works

Since the power demands of electric consumers must be satisfied, the electric grid needs to have enough generation capacity. However, due to daily and seasonal power demand variations, the operating power generators cannot meet power demand [17]. Furthermore, advanced information and communication technology equipped with sensing and controlling capabilities, e.g., smart sensors and actuators, have been attached to power devices to measure real-time power sensing, transmission, and control activities [18–20]. The high accuracy and controllability of smart sensing and controlling devices along with the transfer of power data can assist in controlling power levels accurately on each power flow connection.

One possible strategy to mitigate the variability of renewable generated power is the use of an energy storage system (ESS) that combines the renewable generation system with other types of equipment. Energy storage is a key element of any power flow system in diversifying power generators and adding renewable sources into the power system. The integration of an energy storage system can compensate for the power generation variation situations along with power demand dynamic behavior. These ESSs can be used for any power flow system in a variety of ways, such as providing services for renewable power smoothing, peak demand shaving, frequency regulation, etc. Energy storage systems are utilized for managing the peak power demand of the consumer load profile by discharging power during peak hours. As a result, customer bills can be greatly reduced by reducing the power supplied by the electric grid [21–23].

There have been many previous studies on energy consumption reduction [24], renewable energy integration [3,4], and usage of energy storage system schemes with different objectives and methods [8–10]. However, the design guidelines of a power flow system in the presence of power fluctuations caused by fluctuating power generators and loads are not discussed in existing work.

The reduction in power demand from the grid helps in reducing demand charges, which is considered in many papers for different consumers, such as residential consumers, commercial consumers, industrial consumers, etc. One potential solution is the use of an ESS to increase benefits related to the reduction in demand charges as discussed in [25–27]. Another paper shows the demand reduction using ESS in the U.S., which is discussed in [25]. The economic feasibility of implementing ESS for the reduction in demand changes is considered in [26]. In [27], a deployment algorithm for optimization is proposed which tries to optimize a commercial customer who owns an ESS for decreasing demand charges. However, the authors did not consider the relation between power generators, loads, and ESS, which is the main goal of our paper, considered as the *"System Characterization problem"*. Moreover, some algorithms in the literature consider PV generation with ESS for minimizing energy bills as shown in [28–32]. In the given algorithms in [28–30], the reduction in demand charges is considered for large-scale commercial consumers, while [31,32] focus on residential consumers. All these works focused on the reduction in demand charges with or without photovoltaic installation, considering ESS for different consumers based on the type of the application.

In [33], the authors present a DFT approach as a tool to identify the optimal size of storage that is required for a power flow system combined with a renewable energy system. In [34], the author shows another approach to mitigating the unpredictability of renewable energy in demand-side management through flexible incentives for consumers by shifting peak power demand periods. However, both works lack the definition of a power flow system that has a feasible power flow assignment, the consideration of storage design for a given system, the physical constraints of power generators and loads, and the arrangements of connections. These are the main contributions of this paper.

The goal of the proposed system characterization is to provide system guidelines for a power system to design a safe and reliable power system in the presence of power fluctuations. This particular paper can solve the issues of how large the capacity of the power storage device should be, how large/small the maximum/minimum power/demand levels of the power generators and the power loads should be, and how the connection should be configured for a power flow system, which are not addressed in existing studies.

3. System Characterization Problem

First, a system model representation for a power flow system which consists of power generators, power loads, power storage devices, and connections between them is introduced. After that, the power flow assignment to maintain power balance and energy preservation and the system characterization are formulated.

3.1. System Model

The system architecture model comprises three sets of power devices, such as power generators $(\mathcal{PG})$, power loads $(\mathcal{PL})$, power storage devices $(\mathcal{PS})$, and a set $\mathcal{X}$ of connections between power devices: $\mathcal{X} \subseteq (\mathcal{PG} \times \mathcal{PL}) \cup (\mathcal{PG} \times \mathcal{PS}) \cup (\mathcal{PS} \times \mathcal{PL})$.

A power generator (PG) is an electric device that can provide electric power to power loads and power storage devices. A power load (PL) can be described as an electric device which consumes electric power transferred from power generators and power storage devices. A power storage (PS) is an electrical device which can charge the electric energy received from power generators and discharge it for supplying power to various power loads. Figure 1 illustrates our system model schematically. With respect to power generators and power loads, two distinct types are considered: *controllable* and *uncontrollable*. A controllable power device PG^c/PL^c can control its power generation/consumption accurately. These power devices are responsible for supplying power or absorbing power to manage power fluctuations caused by fluctuating power devices. On the other hand, an uncontrollable power device cannot control its generating or consuming power. In this paper, the word *fluctuating* is used to refer *uncontrollable* power generators and loads. All fluctuating power devices can be represented as PG^f, PL^f.

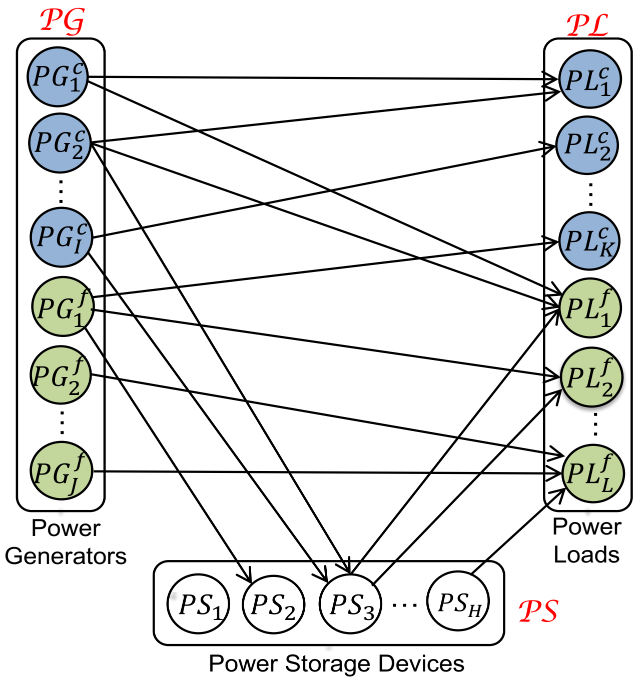

Figure 1. Presentation of power generators, power loads, power storage devices, and connections between power devices.

All power generators are indexed separately for controllable ones and for fluctuating ones as $PG_i^c, 1 \leq i \leq I$, $PG_j^f, 1 \leq j \leq J$, or generically as $PG_m, 1 \leq m \leq I+J$, where I and J show the numbers of controllable power generators and fluctuating power generators, respectively. Similarly, power loads are indexed separately for controllable ones and for fluctuating ones as $PL_k^c, 1 \leq k \leq K, PL_\ell^f, 1 \leq \ell \leq L$, or generically as $PL_n^f, 1 \leq n \leq K+L$, where K and L show the numbers of controllable and fluctuating power loads. All power storage devices are indicated as $PS_h, 1 \leq h \leq H$, where H shows the total number of power storage devices.

The time-varying actual power levels for power generators will be shown as $pg_i^c(t)$ and $pg_j^f(t)$ for PG_i^c and PG_j^f, respectively. Similarly, the actual power levels for power loads are denoted as $p\ell_k^c(t)$ and $p\ell_\ell^f(t)$ for controllable power load PL_k^c and fluctuating power load PL_ℓ^f, respectively. The actual input and output power levels for a power storage device PS_h are represented as $ps_h^{in}(t)$ and $ps_h^{out}(t)$, respectively.

A connection is presented as a pair of devices. For example, a connection can be established between a PG and a PL as (PG_m, PL_n), between a power generator and power storage as (PG_m, PS_h), and between a power storage and a power load as (PS_h, PL_n). Note that a connection between same type of power devices is not considered in this paper. Each connection is associated with a time-varying power level in Watts as $x(PG_m, PL_n, t)$, $x(PG_m, PS_h, t)$ and $x(PS_h, PL_n, t)$ that show the amount of power transferred from a particular power device represented with the first argument to another power device represented with the second argument via this connection at time t.

Each power generator and power load have power limitations to show the minimum and maximum power levels.

$$pg_i^{c.min} \leq pg_i^c(t) \leq pg_i^{c.max} \quad (1)$$

$$pg_j^{f.min} \leq pg_j^f(t) \leq pg_j^{f.max} \quad (2)$$

$$p\ell_k^{c.min} \leq p\ell_k^c(t) \leq p\ell_k^{c.max} \quad (3)$$

$$p\ell_{\ell}^{f.min} \leq p\ell_{\ell}^{f}(t) \leq p\ell_{\ell}^{f.max} \tag{4}$$

where, $pg_i^{c.min}$, $pg_j^{f.min}$, $p\ell_k^{c.min}$, and $p\ell_\ell^{f.min}$ are given lower power bounds, and $pg_i^{c.max}$, $pg_j^{f.max}$, $p\ell_k^{c.max}$, and $p\ell_\ell^{f.max}$ are given upper power bounds.

The energy levels of a power generator and a power load generated and consumed during the times from 0 to t can be shown as $Epg_m(t)$ and $Ep\ell_n(t)$, respectively.

$$Epg_m(t) = \int_0^t pg_m(\tau)d\tau$$

$$Ep\ell_n(t) = \int_0^t p\ell_n(\tau)d\tau$$

The parameter SOC is the state of charge of a power storage device which is calculated by (5), where η_c and η_d are the charging and discharging efficiency. In addition, $E_{ss(h)}$ is the energy storage capacity and $SOC(0)$ is the initial state of charge of the power storage device.

$$SOC_h(t) = SOC_h(0) + \frac{\eta_c}{E_{ss(h)}} \cdot \int_0^t ps_h^{in}(t)dt - \frac{\eta_d}{E_{ss(h)}} \cdot \int_0^t ps_h^{out}(t)dt \tag{5}$$

In order to prevent the forced shutdown of the power storage device due to overcharge or over-discharge of the power storage device, SOC needs to stay within a certain range shown by (6):

$$SOC_h^{min} \leq SOC_h(t) \leq SOC_h^{max} \tag{6}$$

In addition, $ps_h^{in}(t)$ and $ps_h^{out}(t)$ are also assumed to be bounded as

$$ps_h^{in.min} \leq ps_h^{in}(t) \leq ps_h^{in.max} \tag{7}$$

$$ps_h^{out.min} \leq ps_h^{out}(t) \leq ps_h^{out.max} \tag{8}$$

3.2. Power Flow Assignment and System Characterization Problem

To accommodate power fluctuations caused by fluctuating power sources and loads, a power flow assignment is required. The power flow assignment finds power levels for controllable power devices and connections between power devices while keeping (1), (3), and (6)–(8). The most fundamental objectives of the power flow assignment include: (i) the generated power of fluctuating power generators is fully consumed by loads or stored in power storage devices, (ii) the power demand of power loads is fully satisfied as requested, and (iii) the SOC of the power storage device always stays within the maximum and minimum power limits.

Our concern in this paper is not the issue of how to solve this power flow assignment problem but the issue of the structural property (system characterization) for a power flow system to have a feasible power flow assignment. That is, for a power flow system to have a feasible power flow assignment, the issues of how large the capacity of power storage device should be, how large/small the maximum/minimum power/demand levels of the power generators and the power loads should be, and how the connection should be configured are our main concern. Of course, these issues are related to each other, and hence the characterization is given as the relations between the capacities and the minimum and maximum levels of the states of charge for storage devices, power level limitations of power generators and power loads (i.e., minimum and maximum power limits), and the limitation on connectivity between devices.

3.3. Energy Conservation for Simple Power System: A Case Study

At first, in order to illustrate the basic idea of energy balancing between different types of devices, a simple power flow system which consists of a fluctuating power generator, PG_1^f, a fluctuating power load, PL_1^f, a power storage device, PS_1, and connections between them is considered (see Figure 2). The power flow assignment for this system is to determine $x_1(t)$, $x_2(t)$ and $x_3(t)$, $0 \leq t \leq T$, with given power levels $pg_1^f(t)$ and $p\ell_1^f(t)$, $0 \leq t \leq T$.

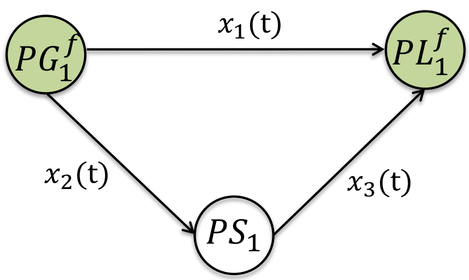

Figure 2. An overview of a simple power flow system.

The power levels $x_1(t)$ and $x_2(t)$ together can make the total instantaneous power generation $pg_1^f(t)$ at any time t of power generator PG_1^f, which can be written as

$$x_1(t) + x_2(t) = pg_1^f(t), \qquad 0 \leq t \leq T$$

In the energy levels, this means

$$Epg_1^f(t) = \int_0^t x_1(\tau)d\tau + \int_0^t x_2(\tau)d\tau, \qquad 0 \leq t \leq T$$

Similarly, the power levels $x_1(t)$ and $x_3(t)$ can make the total instantaneous power consumption $p\ell_1^f(t)$ at time step t of power load PL_1^f, which can be represented as

$$x_1(t) + x_3(t) = p\ell_1^f(t), \qquad 0 \leq t \leq T$$

and

$$Ep\ell_1^f(t) = \int_0^t x_1(\tau)d\tau + \int_0^t x_3(\tau)d\tau, \qquad 0 \leq t \leq T$$

The SOC_1 of the storage device in an ideal lossless case where $\eta_c = \eta_d = \eta$ can be written as

$$\begin{aligned} SOC_1(t) &= SOC_1(0) + \frac{\eta}{E_{ss1}}\left(\int_0^t x_2(\tau)d\tau - \int_0^t x_3(\tau)d\tau\right) \\ &= SOC_1(0) + \frac{\eta}{E_{ss1}}\left(Epg_1^f(t) - Ep\ell_1^f(t)\right) \end{aligned}$$

This can be rewritten in energy-based form as

$$Epg_1^f(t) - Ep\ell_1^f(t) = \frac{E_{ss1}}{\eta} \cdot (SOC_1(t) - SOC_1(0)) \tag{9}$$

Due to the constraint (6) of power storage, we have

$$(SOC_1^{min} - SOC_1(0)) \cdot \frac{E_{ss1}}{\eta} \leq (SOC_1(t) - SOC_1(0)) \cdot \frac{E_{ss1}}{\eta} \leq (SOC_1^{max} - SOC_1(0)) \cdot \frac{E_{ss1}}{\eta} \tag{10}$$

Applying this to (9), we have

$$(SOC_1^{min} - SOC_1(0)) \cdot \frac{E_{ss1}}{\eta} \leq Eps_1^f(t) - Ep\ell_1^f(t) \leq (SOC_1^{max} - SOC_1(0)) \cdot \frac{E_{ss1}}{\eta}$$

Finally, energy supply can be bounded as

$$Epg_1^f(t) \leq Ep\ell_1^f(t) + (SOC_1^{max} - SOC_1(0)) \cdot \frac{E_{ss1}}{\eta} \tag{11}$$

$$Epg_1^f(t) \geq Ep\ell_1^f(t) + (SOC_1^{min} - SOC_1(0)) \cdot \frac{E_{ss1}}{\eta} \tag{12}$$

In order to keep the SOC bounds of a power storage device, $Epg_1^f(t)$ must lie between the power limitations $Ep\ell_1^f(t) + (SOC_h^{min} - SOC_h(0)) \cdot \frac{E_{ss1}}{\eta}$ and $Ep\ell_1^f(t) + (SOC_h^{max} - SOC_h(0)) \cdot \frac{E_{ss1}}{\eta}$ during the time integral $[0, T]$ (see Figure 3).

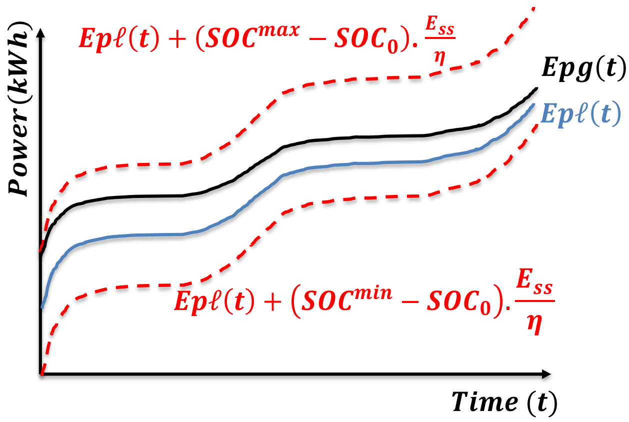

Figure 3. In order to keep SOC limitations, Epg must stay between two dotted lines determined by $Ep\ell + SOC_{min}$ and $Ep\ell + SOC_{max}$.

The above result seems simple and trivial. However, when a system contains multiple power generators, power loads, and power storage devices, and the connections between devices are limited, the characterization for a system to operate while maintaining the individual minimum and maximum power limitations for generators, loads, and storage devices is neither simple nor straightforward.

4. System Characterization with Given Power Levels for Fluctuating Power Devices

In the following part of this paper, a power system consisting of fluctuating power generators, fluctuating power loads, power storage devices, and connections between them is studied. An extended discussion for a system that contains both controllable and fluctuating power generators and loads is reserved as future work.

4.1. Main Theorem

In a tripartite graph model representation as shown in Figure 1, if a connection (PX, PY) exists, PY is called a neighbor of PX, and vice versa. The notation $N(PX)$ is used for representing the set of neighbors of PX. If PX is a set $\{PX_1, PX_2, ..., PX_p\}$, then $N(PX)$ is the union of $N(PX_p)$, $PX_p \in PX$. When it is needed, the set of type-specified neighbors is used, such as $NFL(PX)$, $NFG(PX)$ and $NS(PX)$ for the sets of neighboring fluctuating loads, neighboring fluctuating generators, and neighboring storage devices, respectively, (see Figures 4–6 for a representation of neighbors).

For notational simplicity, $f(A, B, t)$, as the total power sent from a set of devices (or a single device) A to another set of devices (or a single device) B, is introduced, which can be expressed as follows:

$$f(A, B, t) = \sum_{a \in A, b \in B, (a,b) \in \mathcal{X}} x(a, b, t)$$

As the first result for characterizing a system with storage devices to have a feasible power flow assignment and as the main contribution of this paper, the following Theorem 1 is submitted.

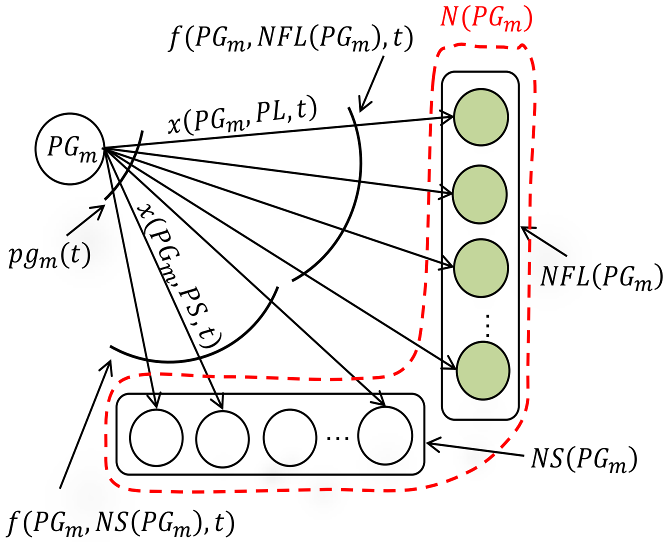

Figure 4. Connections between power generators and neighboring power devices.

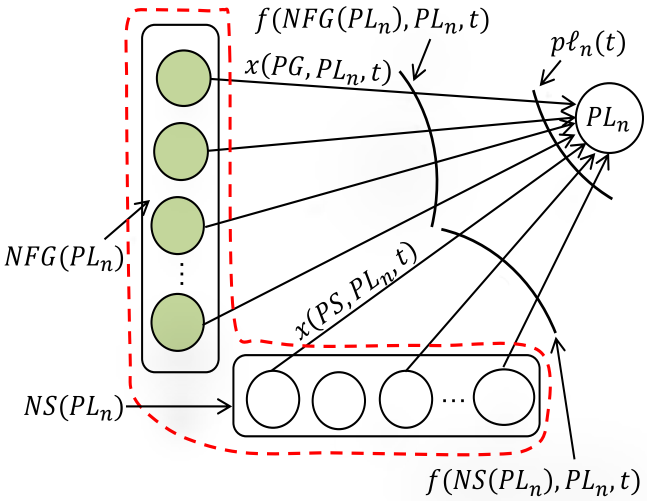

Figure 5. Connections between power loads and neighboring power devices.

Theorem 1. *If the power flow assignment problem has a feasible solution, then the following two system characterization conditions are satisfied.*
Condition 1-1: $\forall S \subseteq \mathcal{PG}$

$$\sum_{PG_j^f \in S} Epg_j^f(t) \leq \sum_{PL_\ell^f \in NFL(S) \cup NFL(NS(S))} Ep\ell_\ell^f(t) + \sum_{PS_h \in NS(S)} (SOC_h^{max} - SOC_h(0)) \cdot \frac{E_{ssh}}{\eta}$$

Condition 1-2: $\forall T \subseteq \mathcal{PL}$

$$\sum_{PG_j^f \in NFG(T) \cup NFG(NS(T))} Epg_j^f(t) \geq \sum_{PL_\ell^f \in T} Ep\ell_\ell^f(t) + \sum_{PS_h \in NS(T)} \left(SOC_h^{min} - SOC_h(0)\right) \cdot \frac{E_{ssh}}{\eta}$$

When we consider any subset S of $\mathcal{PG}$, the total energy generated by power generators in S is partly consumed by power loads in $NFL(S) \cup NFL(NS(S))$ (partly sent directly to power loads and partly sent via power storage devices in $NS(S)$), and the other is stored in storage devices in $NS(S)$. Note that the latter results in the increase of the state of charge (SOC) from an initial SOC. Since storage devices in $NS(S)$ may possibly receive energy from other generators outside S, and power loads in $NFL(S) \cup NFL(NS(S))$ may receive energy from other power generators outside S and from other storage devices outside $NS(S)$, the sum of the energy consumption by power loads in $NFL(S) \cup NFL(NS(S))$ and the increase of energy in storage devices in $NS(S)$ is no smaller than the total generated energy by power generators in S. From this observation, we have

$$\sum_{PG_j^f \in S} Epg_j^f(t) \leq \sum_{PL_\ell^f \in NFL(S) \cup NFL(NS(S))} Ep\ell_\ell^f(t) + \sum_{PS_h \in NS(S)} (SOC_h(t) - SOC_h(0)) \cdot \frac{E_{ssh}}{\eta} \tag{13}$$

To keep $SOC_h(t)$ no larger than SOC_h^{max}, Condition 1-1 is necessary from (13) and $SOC_h(t) \leq SOC_h^{max}$.

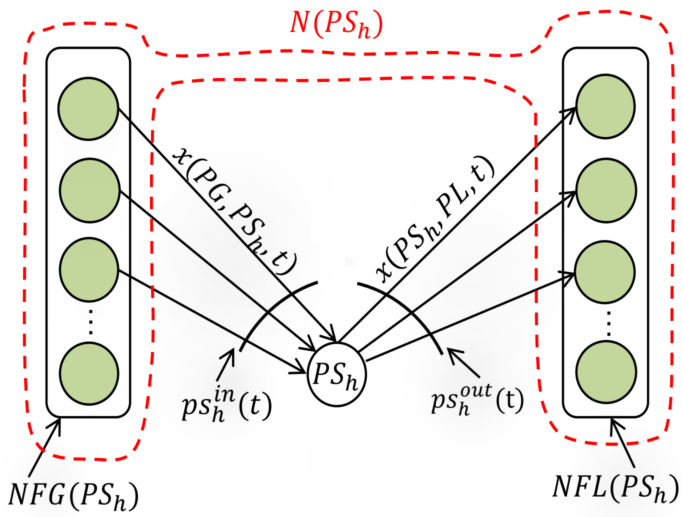

Figure 6. Connections between power storage and neighboring power devices.

On the other hand, when we consider any subset T of $\mathcal{PL}$, the total energy consumed by power loads in T is provided only from power generators in $NFG(T)$ (the energy is directly sent to T), power generators in $NFG(NS(T))$ (the energy is sent once to storage devices in $NS(T)$ and then conveyed to T with/without time delay), and storage devices in $NS(T)$. Note that the storage originated energy is only the initially stored energy, and the part of storage originated energy which is sent to T is measured as the decrease of stored energy from the initially stored energy. Since power generators in $NFG(T) \cup NFG(NS(T))$ may provide energy to other storage devices outside $NS(T)$ and other power loads outside T, and storage devices in $NS(T)$ may provide energy to other power loads outside T, the total consumed energy by power loads in T is no larger than the sum of the energy generated by power generators in $NFG(T) \cup NFG(NS(T))$ and the decreases of stored energy from the initial states of storage devices in $NS(T)$. From the above observation, we have

$$\sum_{PL_\ell^f \in T} Ep\ell_\ell^f(t) \leq \sum_{PG_j^f \in NFG(T) \cup NFG(NS(T))} Epg_j^f(t) + \sum_{PS_h \in NS(T)} (SOC_h(0) - SOC_h(t)) \cdot \frac{E_{ssh}}{\eta} \tag{14}$$

To keep $SOC_h(t)$ no smaller than SOC_h^{min}, Condition 1-2 is necessary from (14) and $SOC_h(t) \geq SOC_h^{min}$. A formal proof of the necessity of Condition 1-1 and Condition 1-2 is shown in the following subsection.

4.2. Proof of the Theorem

Proof of Theorem 1. Let S be a subset of fluctuating power generators. The neighboring power devices of power generators in S are fluctuating power loads and power storage devices which can be represented as $NFL(S)$ and $NS(S)$, respectively. Furthermore, $NFL(NS(S))$ are fluctuating power load neighbors of $NS(S)$ (see Figure 7).

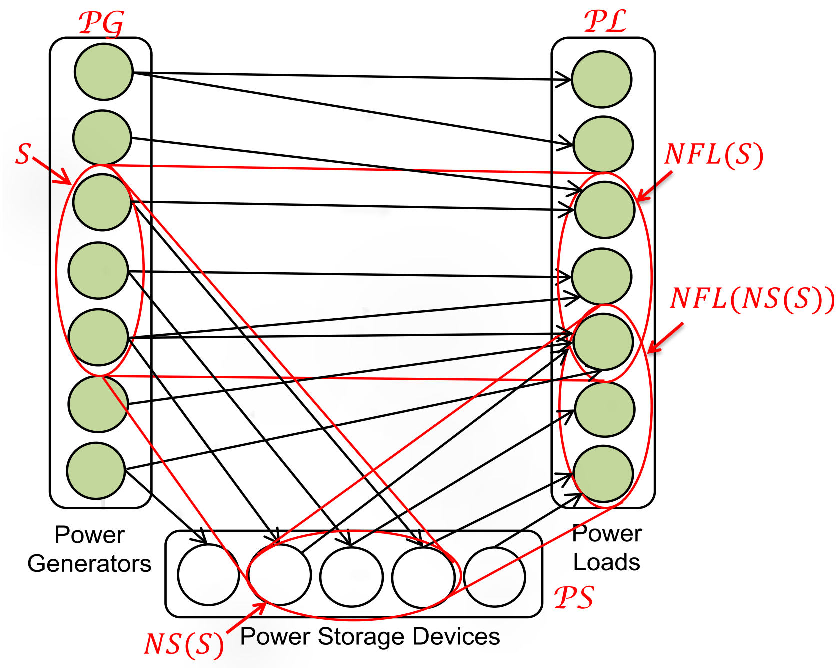

Figure 7. Subset S of power generators and the neighbor connected set of power devices.

The power sent from S to $NFL(S)$ is represented as

$$\sum_{PG_j^f \in S, PL_\ell^f \in NFL(S), (PG_j^f, PL_\ell^f) \in \mathcal{X}} x(PG_j^f, PL_\ell^f, t) = f(S, NFL(S), t)$$

On the other hand, the power sent from S to $NS(S)$ is given as

$$\sum_{PG_j^f \in S, PS_h \in NS(S), (PG_j^f, PS_h) \in \mathcal{X}} x(PG_j^f, PS_h, t) = f(S, NS(S), t)$$

The total of the above two powers must be the same as the total generated power of power generators in S, i.e.,

$$f(S, NFL(S), t) + f(S, NS(S), t) = \sum_{PG_j^f \in S} pg_j^f(t)$$

Power storage devices in $NS(S)$ may receive power not only from S. The total incoming power to the storage devices in $NS(S)$ is no less than the power sent from S to $NS(S)$.

$$\sum_{PG_j^f \in S} pg_j^f(t) = f(S, NFL(S), t) + f(S, NS(S), t) \le f(S, NFL(S), t) + \sum_{PS_h \in NS(S)} ps_h^{in}(t) \tag{15}$$

Next, we consider the power consumed by power loads in $NFL(S) \cup NFL(NS(S))$, which includes (i) the power sent from S, (ii) the power sent from $NS(S)$, and (iii) the other power sent from power generators outside S and storage devices outside $NS(S)$.

Hence,

$$\sum_{PL_\ell^f \in NFL(S) \cup NFL(NS(S))} p\ell_\ell^f(t) \geq \sum_{PG_j^f \in S, PL_\ell^f \in NFL(S), (PG_j^f, PL_\ell^f) \in X} x(PG_j^f, PL_\ell^f, t)$$
$$+ \sum_{PS_h \in NS(S), PL_\ell^f \in NFL(NS(S)), (PS_h, PL_\ell^f) \in X} x(PS_h, PL_\ell^f, t) = f(S, NFL(S), t) + f(NS(S), NFL(NS(S)), t)$$

Note that the power sent from $NS(S)$ to $NFL(NS(S))$ is identical to the total outgoing power from $NS(S)$:

$$f(NS(S), NFL(NS(S)), t) = \sum_{PS_h \in NS(S)} ps_h^{out}(t)$$

As a result, we have

$$\sum_{PL_\ell^f \in NFL(S) \cup NFL(NS(S))} p\ell_\ell^f(t) \geq f(S, NFL(S), t) + \sum_{PS_h \in NS(S)} ps_h^{out}(t) \tag{16}$$

By summing two inequalities (15) and (16), we have

$$\sum_{PL_\ell^f \in NFL(S) \cup NFL(NS(S))} p\ell_\ell^f(t) + \sum_{PS_h \in NS(S)} ps_h^{in}(t) \geq \sum_{PG_j^f \in S} pg_j^f(t) + \sum_{PS_h \in NS(S)} ps_h^{out}(t) \tag{17}$$

By integrating the above with respect to t from 0 to t,

$$\sum_{PG_j^f \in S} Epg_j^f(t) \leq \sum_{PL_\ell^f \in NFL(S) \cup NFL(NS(S))} Ep\ell_\ell^f(t) + \sum_{PS_h \in NS(S)} \int_0^t ps_h^{in}(t) - ps_h^{out}(t) dt$$

$$= \sum_{PL_\ell^f \in NFL(S) \cup NFL(NS(S))} Ep\ell_\ell^f(t) + \sum_{PS_h \in NS(S)} \frac{Ess_h}{\eta}(SOC_h(t) - SOC_h(0))$$
$$\leq \sum_{PL_\ell^f \in NFL(S) \cup NFL(NS(S))} Ep\ell_\ell^f(t) + \sum_{PS_h \in NS(S)} \frac{Ess_h}{\eta}(SOC_h^{max}(t) - SOC_h(0))$$

Similarly, let T be a subset of fluctuating power loads. The neighboring power devices of power loads in T are fluctuating power generators and power storage devices which can be represented as $NFG(T)$ and $NS(T)$, respectively. Furthermore, $NFG(NS(T))$ are fluctuating power generating neighbors of the subset $NS(T)$ of power storage devices (see Figure 8). The power consumed by power loads in T is sent from $NFG(T)$ and from $NS(T)$.

$$\sum_{PL_\ell^f \in T} p\ell_\ell^f(t) = f(NFG(T), T, t) + f(NS(T), T, t)$$

Since the power storage devices in $NS(T)$ may supply power to other power loads outside T, the power sent from $NS(T)$ to T is no larger than the total outgoing power from $NS(T)$.

$$f(NS(T), T, t) \leq \sum_{PS_h \in NS(T)} ps_h^{out}(t)$$

As a result, we have

$$\sum_{PL_\ell^f \in T} p\ell_\ell^f(t) \le f(NFG(T), T, t) + \sum_{PS_h \in NS(T)} ps_h^{out}(t) \tag{18}$$

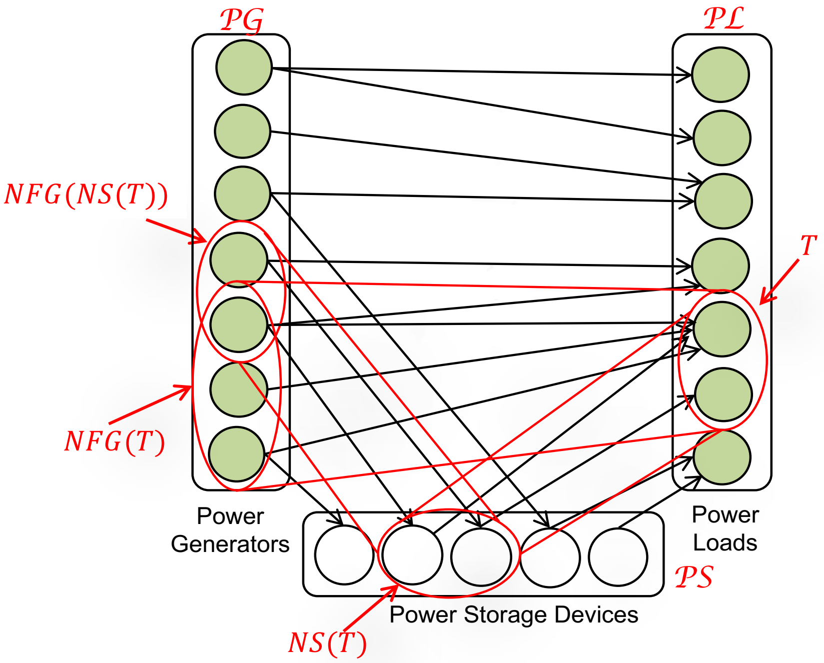

Figure 8. Subset T of power loads and the neighbor connected set of power devices.

On the other hand, the power generated by power generators in $NFG(T) \cup NFG(NS(T))$ is sent out to (i) the power loads in T, (ii) the storage devices in $NS(T)$, and (iii) other power loads outside T and other storage devices outside $NS(T)$. Hence the total generated power by power generators in $NFG(T) \cup NFG(NS(T))$ is no smaller than the sum of the above factor (i) and factor (ii):

$$\sum_{PG_j^f \in NFG(T) \cup NFG(NS(T))} pg_j^f(t) \ge f(NFG(T), T, t) + f(NFG(NS(T)), NS(T), t)$$

Since the power sent from $NFG(NS(T))$ to $NS(T)$ is all the power that $NS(T)$ receives,

$$f(NFG(NS(T)), NS(T), t) = \sum_{PS_h \in NS(T)} ps_h^{in}(t)$$

As a result, we have

$$\sum_{PG_j^f \in NFG(T) \cup NFG(NS(T))} pg_j^f(t) \ge f(NFG(T), T, t) + \sum_{PS_h \in NS(T)} ps_h^{in}(t) \tag{19}$$

From two inequalities (18) and (19), we have

$$\sum_{PL_\ell^f \in T} p\ell_\ell^f(t) + \sum_{PS_h \in NS(T)} ps_h^{in}(t) \le \sum_{PG_j^f \in NFL(T) \cup NFL(NS(T))} pg_j^f(t) + \sum_{PS_h \in NS(T)} ps_h^{out}(t)$$

By integrating above with respect to t from 0 to t,

$$\sum_{PG_j^f \in NFL(T) \cup NFL(NS(T))} Epg_j^f(t) \geq \sum_{PL_\ell^f \in T} Ep\ell_\ell^f(t) + \sum_{PS_h \in NS(T)} \int_0^t ps_h^{in}(t) - ps_h^{out}(t) dt$$

$$= \sum_{PL_\ell^f \in T} Ep\ell_\ell^f(t) + \sum_{PS_h \in NS(T)} \frac{Ess_h}{\eta}(SOC_h(t) - SOC_h(0)) \geq \sum_{PL_\ell^f \in T} Ep\ell_\ell^f(t) + \sum_{PS_h \in NS(T)} \frac{Ess_h}{\eta}(SOC_h^{min}(t) - SOC_h(0))$$

□

4.3. Note on Treatment of Power Level Constraints

In this paper, the system characterization with respect to the constraint (6) has been discussed. On the other hand, with respect to the constraints (1), (3), (7), and (8), we can apply the result discussed in [5,6] by treating input power levels of power storage devices as controllable power loads and output power levels of power storage devices as controllable power generators, as they are shown in Figures 9 and 10.

These power constraints are related to instantaneous power levels, and the system condition for such a system to have a feasible power flow assignment which satisfies these constraints is explained in these papers.

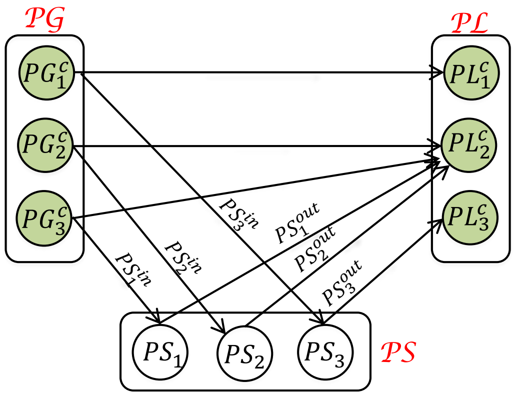

Figure 9. Power system considered in this paper.

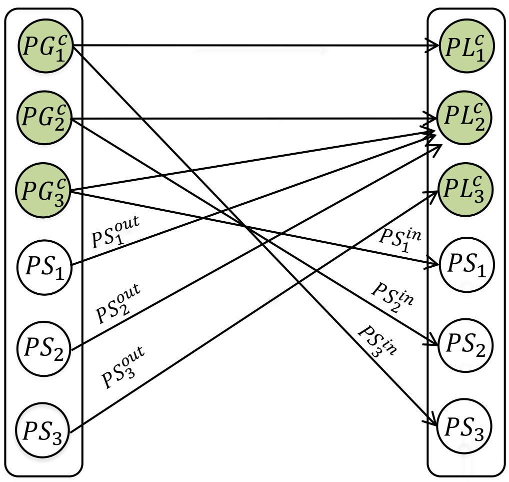

Figure 10. An equivalent imaginary system.

5. Demonstrations

This section shows the application and validation of our proposed system characterization conditions to have the existence of a feasible solution of a given power flow system. To show different applications of the proposed system in a quasi real-world environment, subsections are provided as (i) Minimum *Ess* and worst-case behavior, (ii) Estimation of upper and lower power bounds and minimum *Ess*, (iii) Minimum *Ess* with relation to connections, and (iii) Placement and sizing of energy storage device. All the demonstrations have been done in a discrete-time domain from time 0 up to time 10 with the interval time 1.

Our experiments are based on a graph model representation of power generators, loads, storage devices, and connections between them. In the demonstration section, we tried to evaluate different scenarios to validate the effectiveness of our proposed conditions.

In the first demonstration scenario, the worst case is considered to show that the conditions are valid even if the power generators are operating at their maximum power and power loads are operating on the minimum power level, and vice versa. In addition, it is also verified that the specified *Ess* is the minimal value that satisfies both Condition 1-1 and Condition 1-2 in their individual worst cases.

In the second demonstration scenario, the main objective is to show the effect of minimum and maximum power bounds on the minimum *Ess* of the power storage devices. In this demonstration, an estimation of power generation and consumption is achieved from several patterns of historical data. The estimated data are then analyzed to obtain the minimum capacity of power storage devices.

In the third demonstration scenario, the set of four power generators and the set of five power loads are given, where each of them has individual power generation and consumption profiles. This given power system uses four storage devices, and each storage device is requested to stay within 0.2 and 0.9, and we try to reduce the total *Ess* as much as possible.

In the fourth demonstration scenario, the location and size of the storage devices are analyzed to improve the efficiency of the storage device along with social and economic benefits. For this purpose, the locations of power generators, power loads, and connections between them are fixed. Three storage devices are installed, and the effects of the location and arrangement of connections have been analyzed to achieve a minimum *Ess*.

5.1. Minimum Ess and Worst-Case Behavior

The proposed theorem in this paper is tested for the given power flow system and verifies that the system satisfies Condition 1-1 and Condition 1-2 even for their individual worst cases, i.e., using the upper limits for generating power and the lower limits for power consumption when Condition 1-1 is tested, while using lower limits for generating power and upper limits for power consumption when Condition 1-2 is tested. In addition, it is also verified that the specified *Ess* is the minimal value that satisfies both Condition 1-1 and Condition 1-2 in their individual worst cases. This shows that, if a smaller *Ess* value is used, *SOC* limitation is not kept when the worst-case scenario is produced.

Here, a power flow system is considered with four power generators (PG_1, PG_2, PG_3, PG_4), five power loads (PL_1, PL_2, PL_3, PL_4, PL_5), and four power storage devices (PS_1, PS_2, PS_3, PS_4) with power flow connections which are given (see Figure 11 for the representation of power devices and connections between them). All power generators and loads are fluctuating power devices, represented as PG_j^f and PL_ℓ^f. The generated power of each power generator is bounded between the minimum and maximum power limitations, shown as $pg_j^{f.min}$ and $pg_j^{f.max}$. Similarly, the power demand of all five fluctuating power loads is also restricted between power limitations as $p\ell_\ell^{f.min}$ and $p\ell_\ell^{f.max}$ (see Figure 11 for power limitations of each power device).

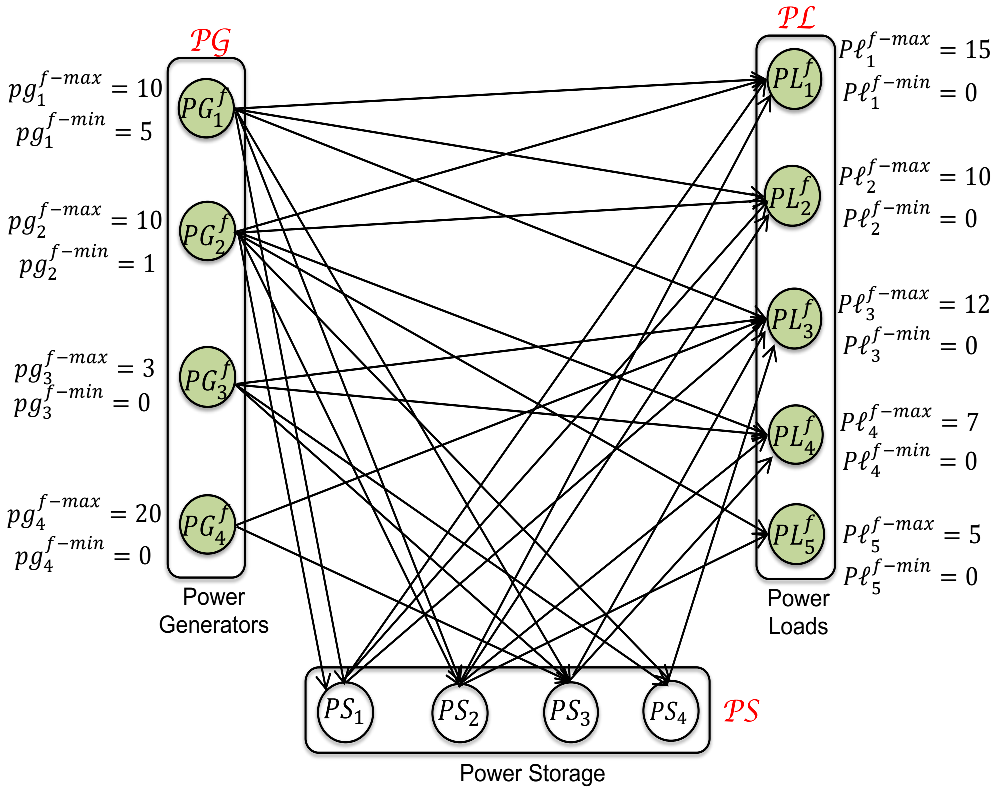

Figure 11. Simulation scenario consisting of power generators, power loads, power storage devices, and connections between them.

The power limitations of four power storage devices, such as input power ps_h^{in} and output power ps_h^{out} to/from the storage battery, and state of charging SOC bounds are given in Table 1.

Furthermore, the initial state of charge $SOC(0)$ for all power storage devices is considered same as 50%.

To represent a real-world scenario with the generated power of a photo-voltaic generator, precise power generation data should be determined first based on historical power generation patterns. From these generation patterns, power bounds, i.e., the upper limit of generating power and lower limit of generating power, are assumed to be given as functions of time t as illustrated in Figure 12. The time domain actual upper and lower limits of generating power for each power generator are set as shown in Figure 13. Similarly, the time domain upper and lower limits of power consumption for each power load are given as shown in Figure 14.

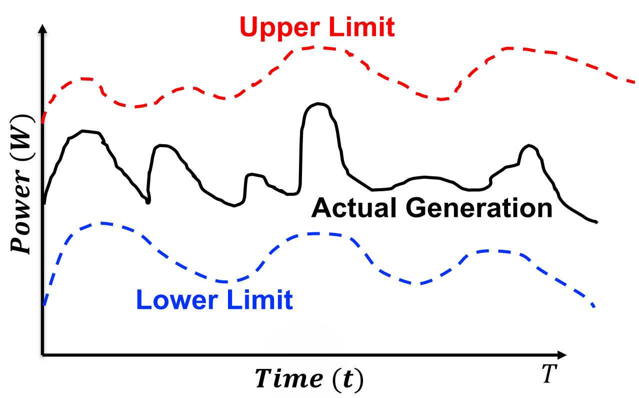

Figure 12. Power supply limitation based on historic data.

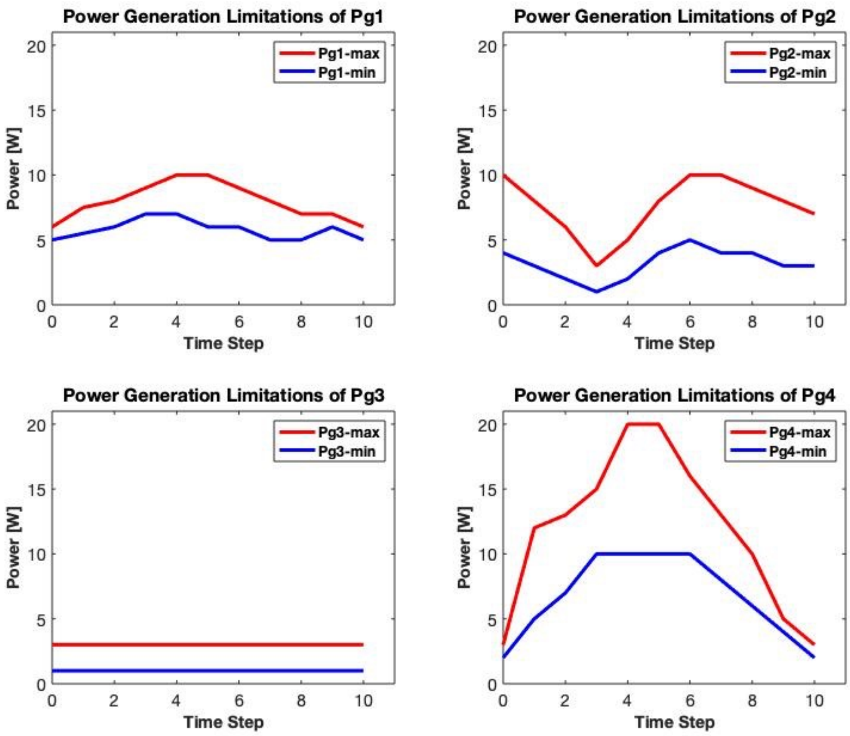

Figure 13. Power generation limitation.

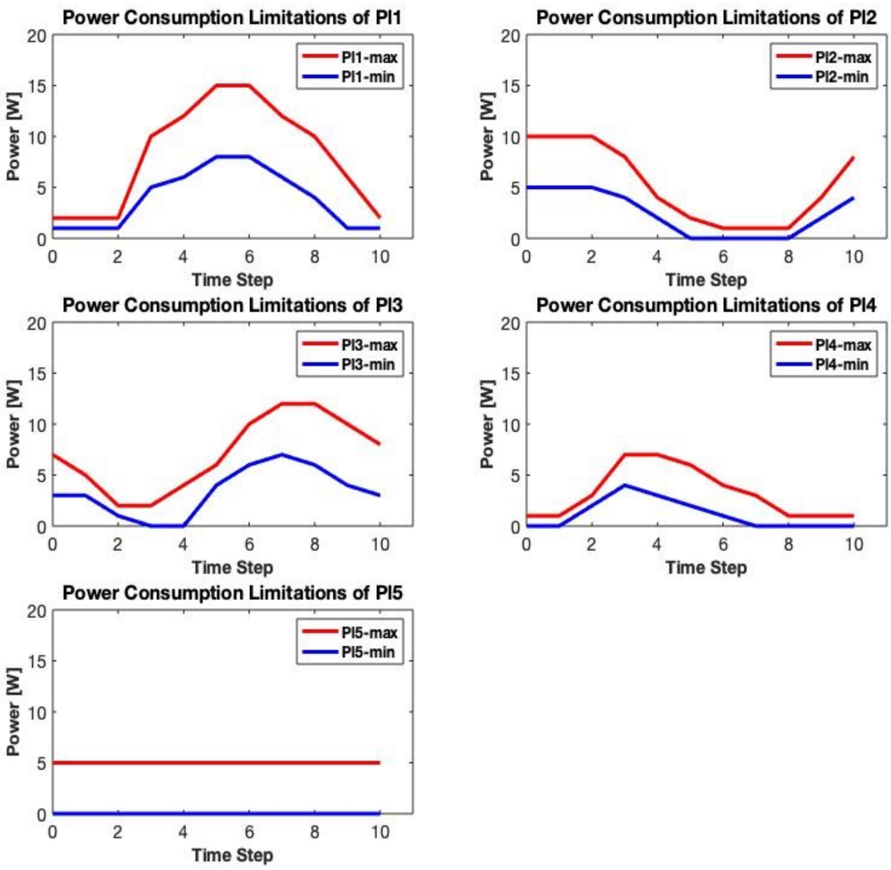

Figure 14. Power consumption limitation.

Table 1. List of power storage devices with power limitation.

ps_h	ps_h^{in} (W)		ps_h^{out} (W)		SOC		Ess_h (kWh)
	min	max	min	max	min	max	
ps_1	0	10	0	13	0.2	0.9	100
ps_2	0	10	0	15	0.2	0.9	220
ps_3	0	20	0	10	0.2	0.9	203
ps_4	0	5	0	5	0.2	0.9	75

While the proposed theorem provides only the necessary condition for a system to operate while keeping SOC limitations, the presence of the power flow assignment solution $x(PX, PY, t), 0 \leq t \leq 10$ is verified, which satisfies SOC limitations even in the worst-case scenario. Figure 15 shows the change of SOC in time for each power storage device when all power generators keep generating their individual lower limits of generating power and all power loads keep consuming their individual upper limits of power consumption. Similarly, Figure 16 shows the change of SOC in time for each power storage device when all power generators generate their upper limits of generating power and power loads consume their lower limits of power consumption.

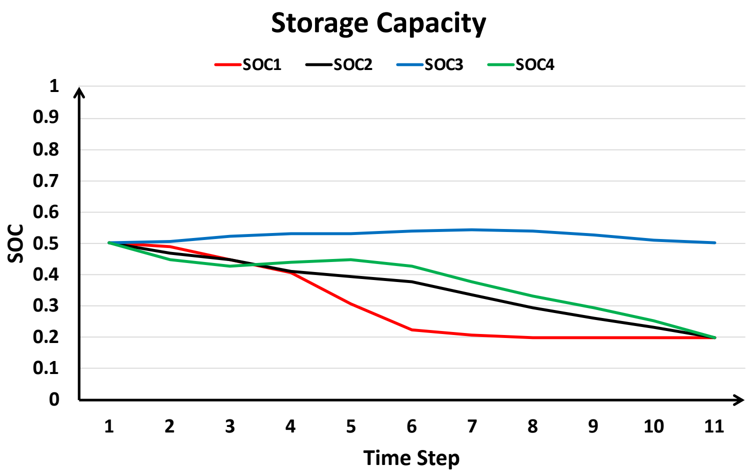

Figure 15. Change of SOC in time when power generators generate lower limits and power loads consume upper limits.

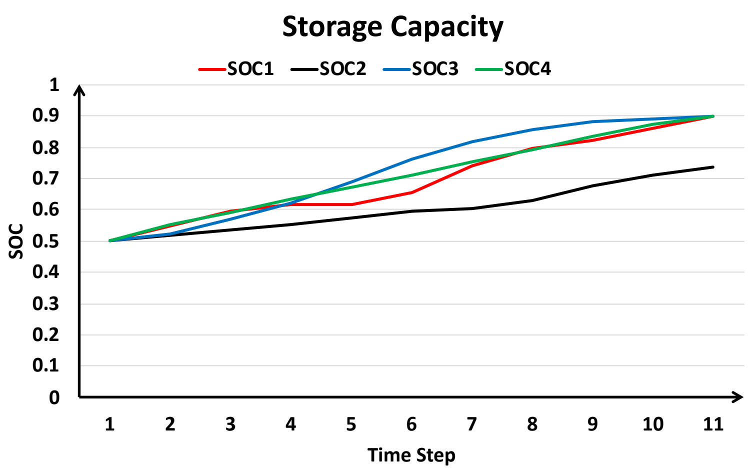

Figure 16. Change of SOC in time when power generators generate upper limits and power loads consume lower limits.

5.2. Estimation of Upper and Lower Power Bounds and Minimum Ess

The main objective of this demonstration is to show the effect of the minimum and maximum power bounds on minimum *Ess* of the power storage devices. In this demonstration,an estimation of power generation and consumption is achieved from several patterns of historical data. Based on the estimated data, a minimum *Ess* is analyzed. For this purpose, the power system given in Figure 17 is considered.

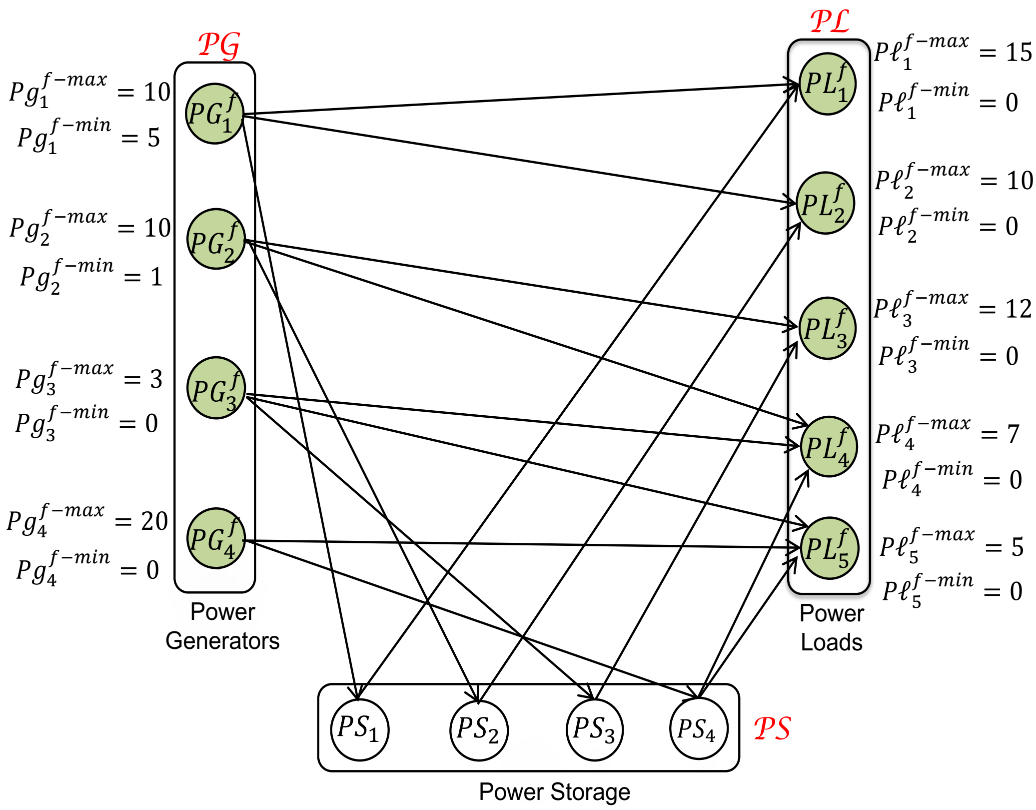

Figure 17. Demonstration scenario considered for power bounds and minimum *Ess*.

Here, three patterns of minimum and maximum power generation are used for one power generator. In Case-1 of power bounds, no historical data are used for power generation and consumption. In this case, the physical power bounds of the power device itself are used which are given by the device manufacturer. In Case-2, upper and lower bounds are estimated from historical data. In this case, the estimated power bounds are loose and include a relatively large margin because the historical data are not enough. In Case-3, enough historical data are available to estimate power bounds more tightly. The power bounds for all three cases for one power generator are shown in Figure 18; the power bounds for the rest of the power devices can be estimated similarly.

Considering power bounds for each case, minimum *Ess* is achieved as shown in Figure 19. In Case-1 and Case-2, the worst case energy gap between generated energy and consumed energy becomes larger, and the energy to be charged or discharged in the storage device tends to increase. As a result, the requirement of a larger *Ess* is expected. On the contrary, if tighter bounds are available as in Case-3, the worst case energy gap becomes smaller, and a smaller *Ess* is expected.

Our experimental results clearly reflect our expectation concerning the relation between the accuracy of the estimated bounds and the minimum *Ess*.

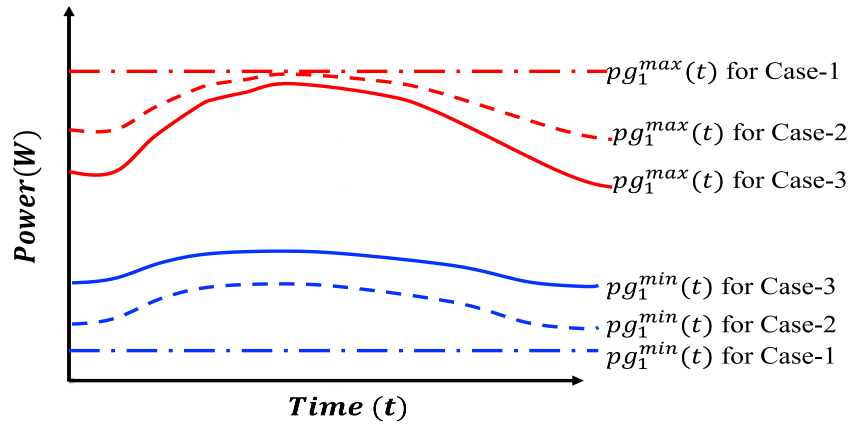

Figure 18. Maximum and minimum power bounds for three cases.

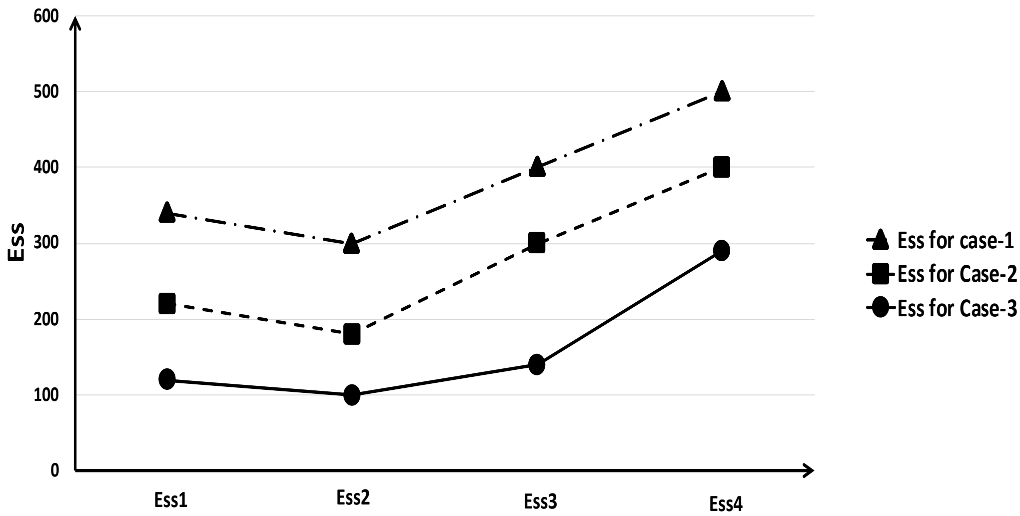

Figure 19. Minimum *Ess* based on power bounds for three cases.

5.3. Minimum Ess with Relation to Connections

In this subsection, one simple system design is simulated, and how our proposed conditions are used during the system design is demonstrated. In this simulation, the set of four power generators and the set of five power loads are given, where each of them has the same device characteristics used in the previous subsection. That is, each generator/load has the individual minimum and maximum instantaneous power limits shown in Figure 11 and the maximum and minimum power level profiles in time domain shown in Figures 13 and 14. We design the power flow system by installing four power storage devices and connections. We suppose that the *SOC* of each storage device is requested to stay within 0.2 and 0.9, and we try to reduce the total *Ess* as much as possible.

As the first step of the simulated system design, we have computed the minimum possible total *Ess* by checking our conditions for an imaginary system having complete connections, and finally, we have the minimum total *Ess* = 520 kWh. As the second step of the simulated system design, we have one trial set of connections shown in Figure 20. For this system configuration, we have applied our conditions and found the minimum *Ess* for each power storage. The result is shown in Table 2, and the total capacity becomes *Ess* = 770 kWh.

In the following step in the simulated system design, we try to reduce the *Ess* by adding a connection. Before adding a connection, as for the reference information concerning the above system configuration, we have checked the critical portion which prevents a further reduction of Ess. That is, we found that Condition 1-2 with the set $T = T_1 = \{PL_1, PL_2, PL_3\}$ fails when we try to reduce the Ess_1 from 120 kWh. The violation of Condition 1-2 arises when the generated energy is smaller than the power demand. Thus, in order to mitigate the limitation of Ess_1, we need to arrange the system configuration so that $NFG(T_1) \cup NFG(NS(T_1))$ contains more power generators.

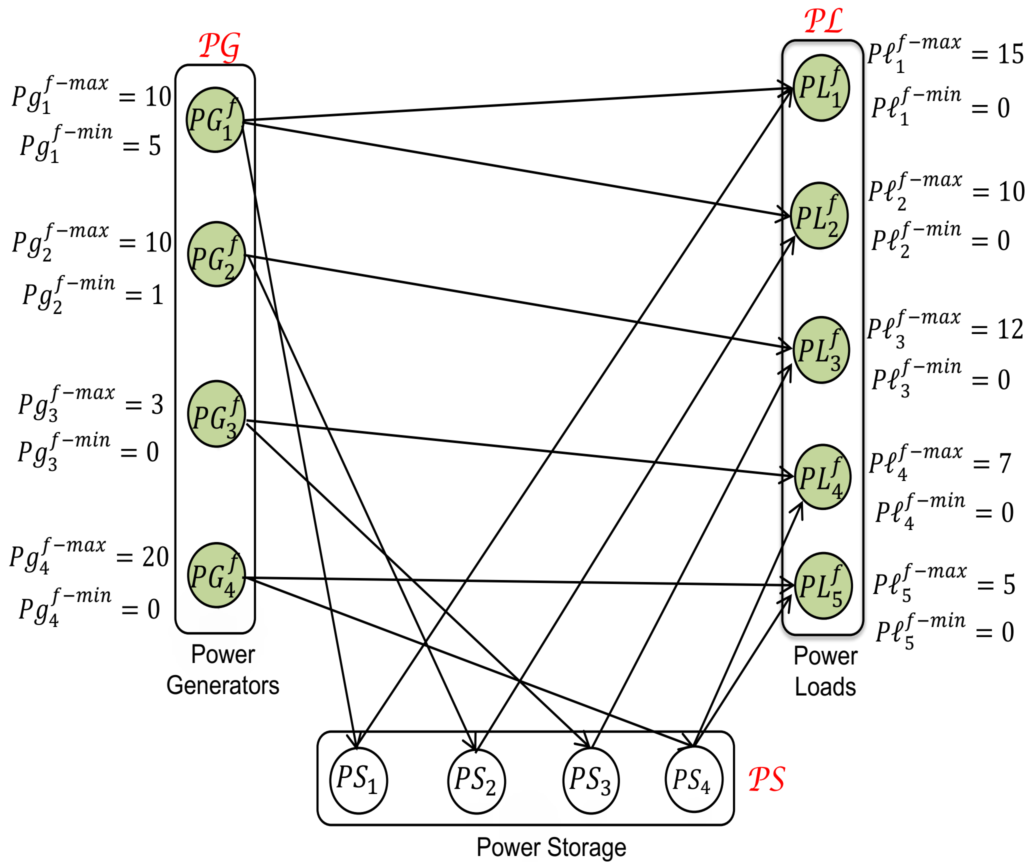

Figure 20. Simulation scenario consisting of power devices and least connections among them.

Table 2. List of power storage devices with power limitation.

ps_h	ps_h^{in} (W)		ps_h^{out} (W)		SOC		Ess_h (kWh)
	min	max	min	max	min	max	
ps_1	0	10	0	13	0.2	0.9	120
ps_2	0	10	0	15	0.2	0.9	140
ps_3	0	20	0	10	0.2	0.9	100
ps_4	0	5	0	5	0.2	0.9	290

Table 3 summarizes the critical condition and a possible counteraction for each storage device, and finally, we decide to add a new connection between PG_4^f and PL_3^f. Figure 21 shows the set T_1 and its neighbors as well as a new power generator PG_4^f which is included in the neighbors by adding a new connection (PG_4^f, PL_3^f). Figure 22 shows the set S_1 and its neighbors as well as a new power load PL_3^f which is included in the neighbors by adding a new connection (PG_4^f, PL_3^f). Finally, the entire system with the new connection can reduce the Ess as shown in Table 4, and the total capacity becomes 520 kWh.

Table 3. Critical condition and counteraction.

Limitation on Storage	Critical Condition	Counteraction
ESS_1	Condition 1-2 with $T = T_1 = \{PL_1, PL_2, PL_3\}$	Include new power generator in $NFG(T_1) \cup NFG(NS(T_1))$
ESS_2	Condition 1-2 with $T = T_1 = \{PL_1, PL_2, PL_3\}$	Include new power generator in $NFG(T_1) \cup NFG(NS(T_1))$
ESS_3	Condition 1-2 with $T = T_1 = \{PL_1, PL_2, PL_3\}$ Condition 1-2 with $T = T_2 = \{PL_3\}$	Include new power generator in $NFG(T_1) \cup NFG(NS(T_1))$ Include new power generator in $NFG(T_2) \cup NFG(NS(T_2))$
ESS_4	Condition 1-1 with $S = S_1 = \{PG_4\}$	Include new power load in $NFL(S_1) \cup NFL(NS(S_1))$

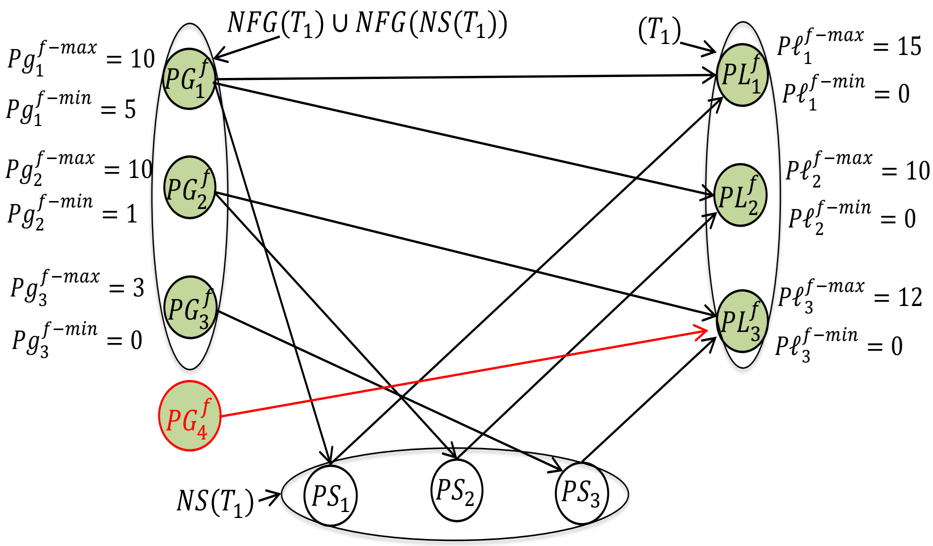

Figure 21. One possible new connection (red line) for making new generator a member of $NFG(T_1) \cup NFG(NS(T_1))$. This connection also contributes to adding a new generator to $NFG(T_2) \cup NFG(NS(T_2))$.

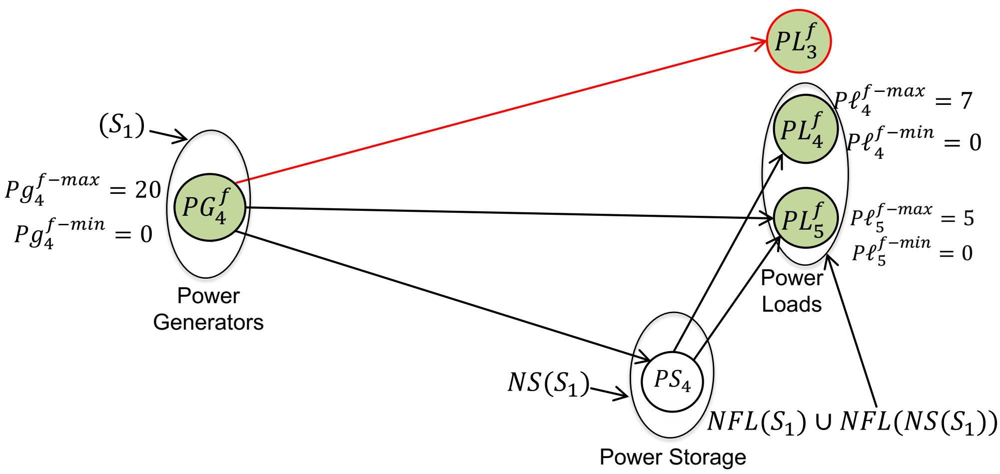

Figure 22. One possible new connection (red line) for including new power load into $NFL(S_1) \cup NFL(NS(S_1))$.

Table 4. List of power storage devices with power limitation.

ps_h	ps_h^{in} (W)		ps_h^{out} (W)		SOC		Ess_h (kWh)
	min	max	min	max	min	max	
ps_1	0	10	0	13	0.2	0.9	110
ps_2	0	10	0	15	0.2	0.9	130
ps_3	0	20	0	10	0.2	0.9	70
ps_4	0	5	0	5	0.2	0.9	210

5.4. Placement and Sizing of Energy Storage Device

To improve the efficiency of energy storage along with social and economic benefits, it is important to analyze the location and size of the storage.

In the simulation scenario, the locations of power generators, power loads, and connections between them are fixed. The power generation and consumption patterns used for simulation are the same as given in Figures 13 and 14. We are going to install three new storage devices. The location of the power storage affects the arrangement of connections and thus the minimum *Ess* of each storage device which can be analyzed in the following configurations. Here, we have tested four configurations: one is preliminary for checking the lower bound of minimum total *Ess*, and the other three are candidate configurations obtained from different localization patterns.

In the first configuration given in Figure 23, all three power storage devices can be seen as one shared power storage device with complete connections (i.e., connected to all power sources and loads). The total capacity of power storage devices needed for a given power system that satisfies both Condition 1-1 and Condition 1-2 is *Ess* = 520 kWh. This shows that the power devices absorb electric power and supply electric power at the same level when connections are available between power devices.

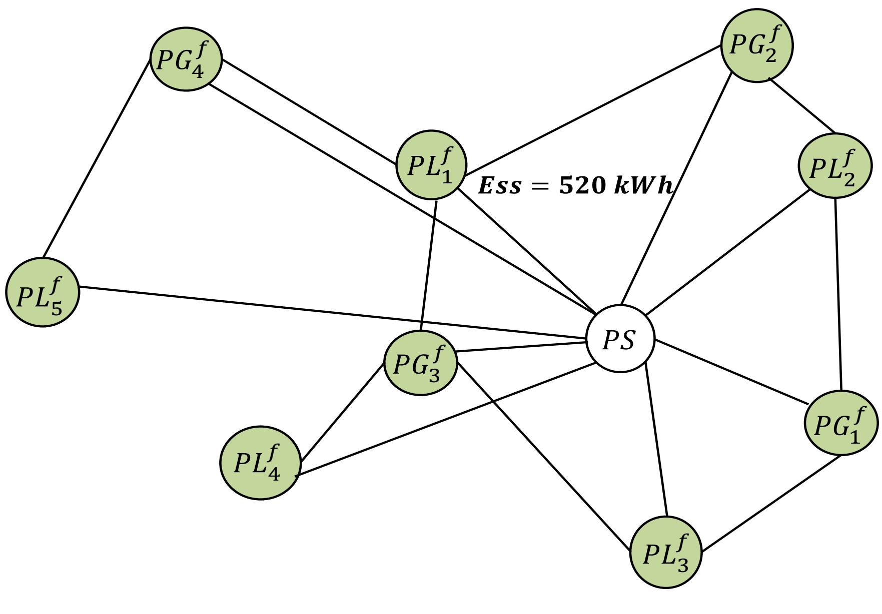

Figure 23. Imaginary system configuration for checking the lower bound for total *Ess*.

In second configuration which is given in Figure 24, the location of three power storage devices is shown, which also affects the arrangement of connections. The minimum capacity levels of three power storage devices are obtained as Ess_1 = 220 kWh, Ess_2 = 120 kWh, and Ess_3 = 250 kWh, respectively. The total capacity by summing up the individual capacity of each power device is computed as *Ess* = 590 kWh.

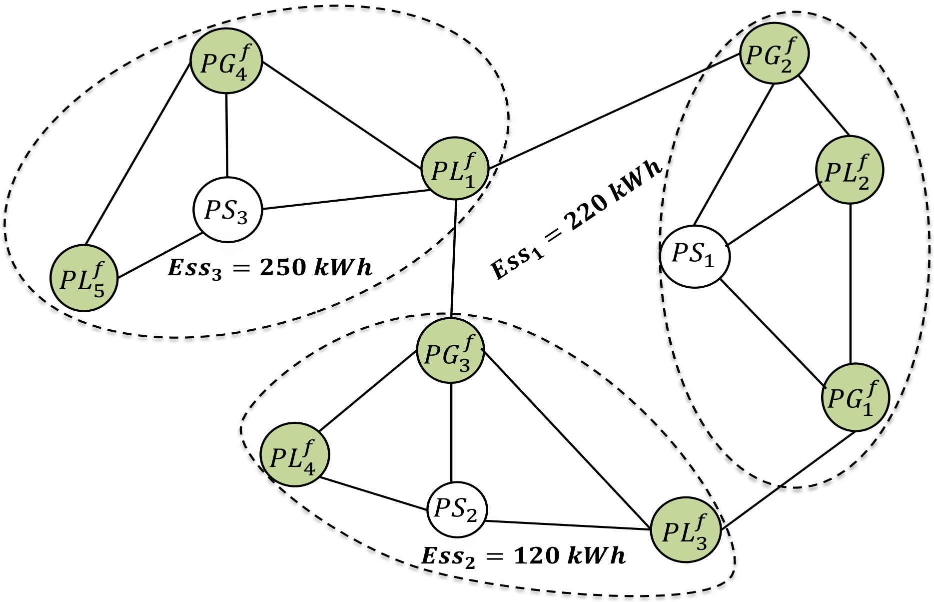

Figure 24. One possible localization for installing storage devices.

In the third configuration shown in Figure 25, the location of three power storage devices is altered, and the minimum capacity level of all power storage devices is obtained as Ess_1 = 190 kWh, Ess_2 = 350 kWh, and Ess_3 = 80 kWh. The total capacity is computed as Ess = 620 kWh.

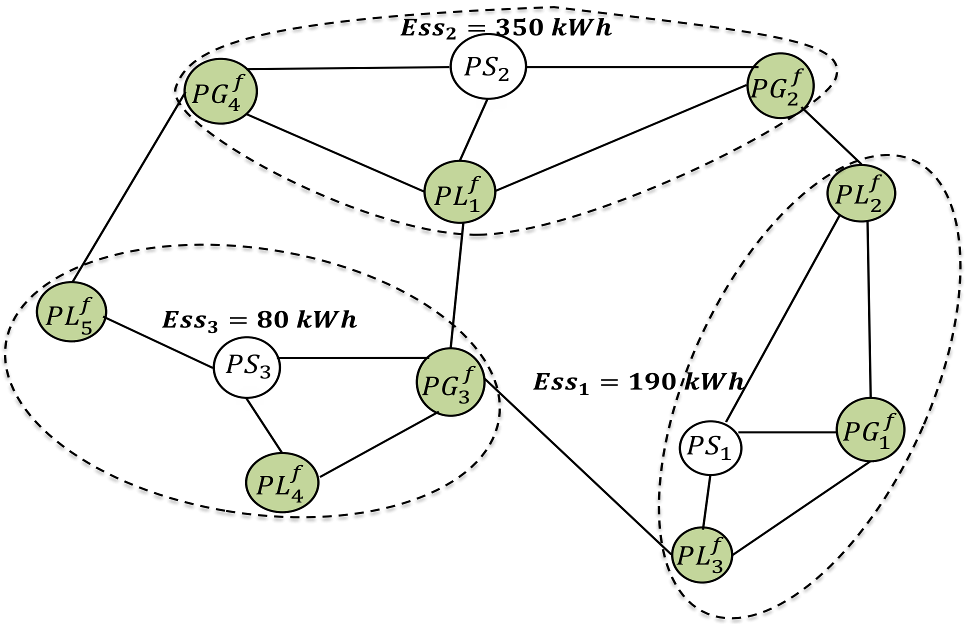

Figure 25. Second possible localization.

In the last configuration shown in Figure 26, the location of three power storage devices is changed, and the minimum capacity level of all power storage devices is noticed as Ess_1 = 200 kWh, Ess_2 = 70 kWh, and Ess_3 = 250 kWh. The total capacity is computed as Ess = 520 kWh.

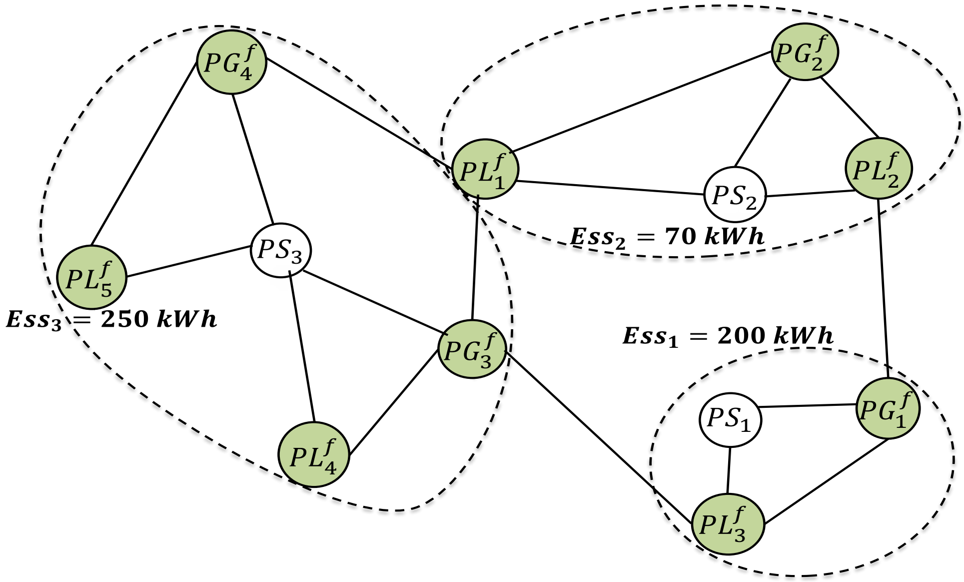

Figure 26. Third possible localization.

In summary, it is observed that the different connection arrangements and different localizations of power storage devices will result in different *Ess*. The above demonstration shows that our system characterization provides good insight into the localization and installation of power storage devices for each local storage device.

6. Concluding Remarks

The variability and intermittent nature of renewable power resources such as photovoltaic and wind power generation bring numerous challenges to the steady operation of power systems. To fully integrate with the power fluctuation caused by these power sources, the energy storage system plays an essential role in renewable energy integration due to its control flexibility and fast response capacity. Moreover, from the viewpoint of increasing the absorption of generated power and supply in power shortages, energy storage systems should be considered.

In this paper, power storage devices are used to compensate for the situations of excessive power and shortage of power caused by fluctuating power generators and fluctuating power loads in an islanding microgrid that is not tied to a grid and hence cannot gain any support from a grid. To accommodate power fluctuations of fluctuating power devices, a power flow assignment is essential to keep the power balance between supply and demand. However, due to the power limitations of power devices, the capacity of storage devices, and power flow connections, the power balance may not be achieved.

This paper proposes a system characterization which describes the relation among power generators, power loads, power storage devices, and the connections that must be satisfied for a system to operate by keeping the *SOC* limitations of power storage devices. The simulation results show that the application of the characterization conditions validates the existence of a feasible solution of a given power flow system. Since this paper provides a necessary condition to find a feasible solution, discussions concerning sufficiency and its application remain as important future works. Furthermore, we will extend our system characterization of power systems with controllable power devices.

Our method can be used for both optimal storage allocation and the planning phase. The proposed system in its current form cannot be used in a real/physical power network. The implementation in the real physical world requires additional knowledge about types of devices, their power limitations, and the scale of the power network. However, our discussions can be used as a prerequisite for detailed real physical design.

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