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An Approach for the Analysis of Energy Resource Selection Based on Attributes by Using Dombi T-Norm Based Aggregation Operators

Mujab Waqar [1], Kifayat Ullah [1,*], Dragan Pamucar [2,*], Goran Jovanov [3] and Đordje Vranješ [4]

1 Department of Mathematics, Lahore Campus, Riphah International University, Lahore 54000, Pakistan; 26956@students.riphah.edu.pk or mujabwaqar13@gmail.com
2 Department of Logistics, Military Academy, The University of Defense in Belgrade, 11000 Belgrade, Serbia
3 Department of Forensics, University of Criminal Investigation and Police Studies, 11080 Belgrade, Serbia; drgoranjovanov@hotmail.com
4 Environ Doo Beograd-Čukarica, 11030 Belgrade, Serbia; djordjevranjes@yahoo.com
* Correspondence: kifayat.khan.dr@gmail.com or kifayat.ullah@riphah.edu.pk (K.U.); dragan.pamucar@va.mod.gov.rs (D.P.)

Abstract: Dombi t-norm (DTN) and t-conorm (TCN) are among the most effective triangular norms in fuzzy systems for aggregation purposes. The environment of interval-valued intuitionistic fuzzy (IVIF) set gives us precision in expressing uncertain information by using a membership grade (MG) and non-membership grade (NMG) in the form of closed subintervals of $[0, 1]$. The goal of this paper is to introduce DTN-based aggregation operators (AOs) for IVIF numbers (IVIFNs) and study their performance in the evaluation of the worth of energy recourses to be opted in Pakistan to deal with the energy crises situation. We first introduced some DTN and TCN-based operations for IVIFNs and developed two new AOs known as IVIF Dombi weighted averaging (IVIFDWA) and IVIF Dombi weighted geometric (IVIFDWG) operators. The validity and fitness of the proposed operators are tested. A case study is presented where the energy resources of Pakistan are discussed and the problem of the selection of sustainable energy resources in the context of Pakistan is investigated. The sensitivity analysis of the proposed IVIFDWA and IVIFDWG operators is studied and a comparative analysis of the current work with previous studies is established.

Keywords: Dombi aggregation operators; interval-valued intuitionistic fuzzy sets; multi-attribute decision-making

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Citation: Waqar, M.; Ullah, K.; Pamucar, D.; Jovanov, G.; Vranješ, Đ. An Approach for the Analysis of Energy Resource Selection Based on Attributes by Using Dombi T-Norm Based Aggregation Operators. *Energies* **2022**, *15*, 3939. https://doi.org/10.3390/en15113939

Academic Editor: Mojtaba Ahmadieh Khanesar

Received: 4 April 2022
Accepted: 23 May 2022
Published: 26 May 2022

1. Introduction

Several phenomena are unable to be described by using classical set theory, such age, height, intelligence, etc. Classical theory narrates that an element either belongs to or it entirely does not belong to a set and there is no other way around it; hence, the MG of a certain element of this type of framework can assign only two numeric values, i.e., 0 or 1. The above-mentioned limitations of classical theory led Zadeh [1] to evolve the idea of fuzzy set (FS) theory. The concept of MG of an element was put in place by Zadeh and is elaborated by a membership function on a unit interval $[0, 1]$. The unpredictability of human decisions can be described by the idea of Zadeh's membership function.

Only the MG of an undetermined event can be explained by the FS framework and is unable to provide any information about the NMG of the event. Human judgment for a certain occurrence is not always unidirectional, which led Atanassov [2] to put forward the idea of the intuitionistic fuzzy set (IFS). IFS constructed an MG and an NMG denoted by μ and v, respectively, with a constraint that their sum must not exceed 1, i.e., $sum\ (\mu, v) \in [0, 1]$. Further, the term $1 - sum\ (\mu, v)$ indicates a hesitancy degree (HD). The frame of IFS has been utilized in many practical problems and several results have been developed for IFSs. Kahraman et al. [3] utilized the conception of IVIFSs in a MADM problem involving radiators of automobiles. Romaniuk and Hryniewicz [4] utilized the conception of interval-valued FSs for resampling methods of statistics. Tyagi [5] investigated the reliability analysis of the power loom plant by using the frame of IVIFSs. Song et al. [6] developed some divergence and uncertainty measures for IFS and applied them in decision-making. Wei et al. [7] developed an information-based score function(SF) of the IVIF set (IVIFS) and discussed its applications in multi-attribute decision-making (MADM). Xue and Deng [8] proposed the idea of decision-making under measure-based granular uncertainty with IFSs. Joshi and Kumar [9] introduced an enhanced score formula for the processing of intuitionistic fuzzy information by making a comparison with other score computing techniques.

IFS illustrated an uncertain phenomenon by using an MG and NMG only where the values of the MG and NMG were assigned from $[0, 1]$ interval. Human opinion, when expressed in the form of a crisp number may lead to information loss. Consider a scenario where an opinion is given using a crisp number say a from $[0, 1]$ where in the same scenario, the same opinion can be of the closed subinterval form of $[0, 1]$, i.e., $[a, b]$. In the former one, there is a chance of inaccuracy in the opinion, while in the latter one, the uncertainty is described with more accuracy. Due to this fact, the theory of IVIFS is developed by Atanassov and Gargove [10] where the MG and NMG were expressed by using closed subintervals of $[0, 1]$. A significant amount of work is achieved so far by using the frame of IVIFSs. Liu and Jiang [11] developed a new distance measure of IVIFSs and utilized them in decision-making. Joshi and Kumar [9] proposed an improved accuracy function for IVIFSs by observing the limitations of the previously defined SFs. Kumar and Chen [12] analyzed the MADM approaches based on IVIF numbers, SF of connection numbers, and the set pair analysis theory. Liu et al. [13] proposed a variable weight-based hybrid approach for MADM under IVIFSs. Haiyun et al. [14] analyzed the innovation strategies of the energy sector for green supply chain management using decision-making based on IVIF information. Liu et al. [15] discussed a three-way decision support model based on IVIF information and Xie et al. [16] studied the sustainability of the supply chain by prioritizing the risk factors using IVIF information. Hu et al. [17] introduced some generalized Heronian means based on IVIF information for MADM problems. Jan et al. [18] investigated some very basic theories of IVIF graphs and revised several definitions keeping in view their limitations. Li et al. [19] used IVIF information to deal with decision-making with graph patterns. A two-sided matching model was presented by Liang et al. [20] using intuitionistic fuzzy information with MG and NMG as $[0, 1]$ sub-intervals that are closed.

MADM is one of the widely discussed topics as an application of FS theory and a tremendous amount of work is recorded on the topic. MADM is also applied to solve several problems in the energy sector and a significant amount of work is on the record. Baumann et al. [21] carried out a comprehensive review of MADM approaches for evaluating energy storage systems for grids. Ijadi Maghsoodi et al. [22] developed the use of integrated H-SWARA-MULTIMOORA to solve the renewable energy technology selection problem approach. Souzangarzadeh et al. [23] used the approach of MADM based on MULTIMOORA for the selection of the most efficient design of conical segmented aluminum tubes that can be used as an absorber of energy. Ilbahar et al. [24] studied the implementations of MADM faced by the energy renewable sector. Taylan et al. [25] proposed using fuzzy set theory, the activity of energy efficiency in the petrochemical industry, the essential model of MADM. Xu et al. [26] developed a phase change material selection for thermal energy storage in solar air conditioning systems. Wang et al. [27] presented a CoCoSo-D method using Pythagorean fuzzy information based on sine trigonometric interaction operational laws. Akram et al. [28] made the interval-valued T-spherical fuzzy Bonferroni mean operators used to evolve the performance development of solar energy cells. Pamucar et al. [29] proposed for the selection of power generation technology, a linguistic neutrosophic CODAS method was used. Riaz et al. [30] developed by using Einstein operations a robust q-rung orthopair fuzzy information aggregation system was developed for sustainable energy planning decision management. Riaz et al. [31] proposed an unfamiliar q-rung orthopair prioritized AOs dependent on priority sustainable energy planning.

In MADM problems, the main tool that is used is AO and most of the AOs are based on T-norms (TNs) and TCNs. There are several TNs and TCNs including algebraic [32], Hamacher [33], Einstein [34], Frank [35], and Aczel–Alsina t-norms [36]. DTN and TCN [37] are also utilized extensively to deal with MADM by using Dombi AOs. The theory of Dombi AOs is rich and some recent work on Dombi AOs can be found in [38–43].

The theory of Dombi AOs proposed by Seikh and Mandal [38] has applications in MADM problems, but it faces issues and information loss is likely to occur. The reason for information loss is the expression of the MG and NMG by using single values from $[0,\ 1]$ interval. To overcome this issue, we aim to use the frame of IVIFS and proposed Dombi AOs based on IVIFNs to deal with MADM problems. Another goal is to apply the theory of Dombi AOs to the energy sector by discussing the energy crises of Pakistan. A case study is discussed and the selection of the most profitable energy resource in Pakistan is carried out by using the MADM approach based on IVIF information and the theory of DTNs.

This article is divided into seven different sections. In Section 1, we provide a brief overview of the research background as well as the goals and objectives of the proposed work. In Section 2, some very basic definitions are studied. In Section 3, Dombi operational laws for IVIFNs are defined. In Section 4, Dombi AOs are discussed and the validity of the proposed Dombi AOs is also studied. In Section 5, we developed a method of MADM by using IVIF information where we discussed the case study, the proposed algorithm, and a comprehensive algorithm. In Section 6, we established a comparative study of current work with that of previous work. In Section 7, we conclude this paper by drawing some remarks and discussing some future work.

2. Preliminaries

In this section, we will go through some fundamental definitions and preliminaries. We discuss the research difference for the proposed task. Throughout the paper, X stands for a non-empty set, and μ and v are the MG and NMG, respectively.

Definition 1 ([2])**.** *Let X be the set. An IFS B is:*

$$B = \{(\varkappa,\ (\mu_B(\varkappa),\ v_B(\varkappa)))\ :\ 0 \le\ \mu_B(\varkappa) + v_B(\varkappa) \le 1 \text{ and } \varkappa \in X\}$$

where we represent the MG and NMG of the element $\varkappa \in X$ *to the set B by* $\mu_B(\varkappa)\ :\ X \to [0,\ 1]$ *and* $v_B(\varkappa)\ :\ X \to [0,\ 1]$, *respectively. The HD is denoted by* $\pi_B(\varkappa) = 1 - \mu_B(\varkappa) - v_B(\varkappa)$. *For simplicity,* $B = (\mu_B,\ v_B)$ *is called an intuitionistic fuzzy number (IFN).*

Definition 2 ([12])**.** *Let X be the set. An IVIFS B is:*

$$B = \left\{\left(\varkappa,\ \left(\left[\mu_B^l(\varkappa),\ \mu_B^u(\varkappa)\right],\ \left[v_B^l(\varkappa),\ v_B^u(\varkappa)\right]\right)\right)\ : \varkappa \in X\right\}$$

where $\left[\mu_B^l(\varkappa),\ \mu_B^u(\varkappa)\right]$ *and* $\left[v_B^l(\varkappa),\ v_B^u(\varkappa)\right]$ *are closed subintervals of* $[0,\ 1]$ *representing the MG and NMG of the element* $\varkappa \in X$ *to the set B, respectively, thereby satisfying* $0 \le\ \mu_B^u(\varkappa) +\ v_B^u(\varkappa)\ \le 1$. *The HD is defined by* $\left[\pi_B^l(\varkappa),\ \pi_B^u(\varkappa)\right] = \left[1 - \mu_B^u(\varkappa) - v_B^u(\varkappa),\ 1 - \mu_B^l(\varkappa) - v_B^l(\varkappa)\right]$. *For simplicity,* $B = \left(\left[\mu_B^l(\varkappa),\ \mu_B^u(\varkappa)\right], \left[v_B^l(\varkappa),\ v_B^u(\varkappa)\right]\right)$ *is called IVIFN.*

Definition 3 ([10])**.** *Let* $M = \left(\left[\mu_M^l(\varkappa), \mu_M^u(\varkappa)\right],\ \left[v_M^l(\varkappa),\ v_M^u(\varkappa)\right]\right)$ *and* $N = \left(\left[\mu_N^l(\varkappa), \mu_N^u(\varkappa)\right],\ \left[v_N^l(\varkappa),\ v_N^u(\varkappa)\right]\right)$ *be two IVIFNs and* $\lambda > 0$ *be any arbitrary real number. Then:*

1. $M \subseteq N$ iff $\mu_M^l(\varkappa) \le \mu_N^l(\varkappa),\ \mu_M^u(\varkappa) \le \mu_N^u(\varkappa)$ and $v_M^l(\varkappa) \ge v_N^l(\varkappa), v_M^u(\varkappa) \ge v_N^u(\varkappa)$.
2. $M = N$ iff $\mu_M^l(\varkappa) = \mu_N^l(\varkappa),\ \mu_M^u(\varkappa) = \mu_N^u(\varkappa)$ and $v_M^l(\varkappa) = v_N^l(\varkappa) = v_M^u(\varkappa) = v_N^u(\varkappa)$

3. $M\varkappa N = \left(\left[\mu_M^l(\varkappa) \vee \mu_N^l(\varkappa),\ \mu_M^u(\varkappa) \vee \mu_N^u(\varkappa)\right],\ \left[v_M^l(\varkappa) \wedge v_N^l(\varkappa),\ v_M^u(\varkappa) \wedge v_N^u(\varkappa)\right]\right)$
4. $M \cap N = \left(\left[\mu_M^l(\varkappa) \wedge \mu_N^l(\varkappa),\ \mu_M^u(\varkappa) \wedge \mu_N^u(\varkappa)\right],\ \left[v_M^l(\varkappa) \vee v_N^l(\varkappa),\ v_M^u(\varkappa) \vee v_N^u(\varkappa)\right]\right.$.
5. $M^c = \left(\left[v_M^l(\varkappa),\ v_M^u(\varkappa)\right],\ \left[\mu_M^l(\varkappa), \mu_M^u(\varkappa)\right]\right)$
6. $M \oplus N = \begin{pmatrix} \left[\mu_M^l(\varkappa) + \mu_N^l(\varkappa) - \mu_M^l(\varkappa)\mu_N^l(\varkappa)\ ,\ \mu_M^u(\varkappa) + \mu_N^u(\varkappa) - \mu_M^u(\varkappa)\mu_N^u(\varkappa)\right], \\ \left[v_M^l(\varkappa)v_M^l(\varkappa),\ v_M^u(\varkappa)v_N^u(\varkappa)\right] \end{pmatrix}$.
7. $M \otimes N = \begin{pmatrix} \left[\mu_M^l(\varkappa)\mu_M^l(\varkappa),\ \mu_M^u(\varkappa)\mu_N^u(\varkappa)\right], \\ \left[v_M^l(\varkappa) + v_N^l(\varkappa) - v_M^l(\varkappa)v_N^l(\varkappa),\ v_M^u(\varkappa) + v_N^u(\varkappa) - v_M^u(\varkappa)v_N^u(\varkappa)\right] \end{pmatrix}$.
8. $\lambda M = \left[1 - \left(1 - \mu_M^l(\varkappa)\right)^\lambda 1 - \left(1 - \mu_M^u(\varkappa)\right)^\lambda\right], \left[\left(v_M^l(\varkappa)\right)^\lambda \left(v_M^u(\varkappa)\right)^\lambda\right]$.
9. $M^\lambda = \left[\left(\mu_M^l(\varkappa)\right)^\lambda \left(\mu_M^u(\varkappa)\right)^\lambda\right], \left[1 - \left(1 - v_M^l(\varkappa)\right)^\lambda 1 - \left(1 - v_M^u(\varkappa)\right)^\lambda\right]$.

Definition 4 ([7]). *Let $\alpha = \left(\left[\mu_\alpha^l,\ \mu_\alpha^u\right],\ \left[v_\alpha^l,\ v_\alpha^u\right]\right)$ be any IVIFN. Then, the SF is:*

$$\Phi(\alpha) = \frac{(\mu_\alpha^l)\left(1 - v_\alpha^l\right) + (\mu_\alpha^u)(1 - v_\alpha^u)}{2} \quad (1)$$

where $\Phi(\alpha) \in [0, 1]$.

Definition 5 ([12]). *Let $\alpha = \left(\left[\mu_\alpha^l,\ \mu_\alpha^u\right],\ \left[v_\alpha^l,\ v_\alpha^u\right]\right)$ and $\beta = \left(\left[\mu_\beta^l,\ \mu_\beta^u\right],\ \left[v_\beta^l,\ v_\beta^u\right]\right)$ be two IVIFNs. Then:*

1. If $\Phi(\alpha) > \Phi(\beta)$, then $\alpha > \beta$.
2. If $\Phi(\alpha) < \Phi(\beta)$, then $\alpha < \beta$.

Definition 6 ([44]). *Let the real numbers be a and b . Then, DTN is:*

$$Dom(\mathrm{a},\ \mathrm{b}) = \frac{1}{1 + \left\{\left(\frac{1-\mathrm{a}}{\mathrm{a}}\right)^\phi + \left(\frac{1-\mathrm{b}}{\mathrm{b}}\right)^\phi\right\}^{\frac{1}{\phi}}} \quad (2)$$

where $\phi \geq 1$ and $(a,\ b) \in [0,\ 1] \times [0,\ 1]$.

Definition 7. *Let the real numbers be a and b. Then, Dombi TCN is:*

$$Dom'(\mathrm{a},\ \mathrm{b}) = 1 - \frac{1}{1 + \left\{\left(\frac{\mathrm{a}}{1-\mathrm{a}}\right)^\phi + \left(\frac{\mathrm{b}}{1-\mathrm{b}}\right)^\phi\right\}^{\frac{1}{\phi}}} \quad (3)$$

Based on DTN and TCN, Seikh and Mandal [38] introduced some Dombi AOs for IFSs and studied their applicability in MADM problems. The Dombi AOs proposed by Seikh and Mandal [38] are given by:

Definition 8 ([38]). *Let $\alpha_i = (\mu_i,\ v_i)(i = 1,\ 2,\ \ldots,\ n)$ be a number of IFNs. Then, intuitionistic fuzzy Dombi weighted averaging (geometric) operators are defined by:*

$$IFDWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \overset{n}{\underset{i=1}{\oplus}} \xi_i \alpha_i = \left(\left(1 - \frac{1}{1 + \left\{ \sum_{i=1}^{n} \xi_i \left(\frac{\mu_i}{1-\mu_i} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right), \left(\frac{1}{1 + \left\{ \sum_{i=1}^{n} \xi_i \left(\frac{1-v_i}{v_i} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right) \right) \tag{4}$$

$$IFDWG(\alpha_1, \alpha_2, \ldots, \alpha_n) = \overset{n}{\underset{i=1}{\oplus}} \xi_i \alpha_i = \left(\left(1 - \frac{1}{1 + \left\{ \sum_{i=1}^{n} \xi_i \left(\frac{1-\mu_i}{\mu_i} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right), \left(\frac{1}{1 + \left\{ \sum_{i=1}^{n} \xi_i \left(\frac{v_i}{1-v_i} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right) \right) \tag{5}$$

It is observed that representation of uncertain information by using MG and NMG in the form of a single digit from $[0,\ 1]$ interval leads to information loss. Due to this reason, the notion of IVIFS was introduced by Atanassov and Gargove [10]. An IFS becomes a special case of IVIFS and has less applicability than IVIFSs. The goal of this paper is to establish some Dombi AOs for IVIFNs and to investigate their applications.

3. IVIF Dombi Operational Laws

In this section, we will go over Dombi operations in terms of IVIFNs with the view of DTN and TCN. Some properties of the proposed Dombi operations are also discussed.

Definition 9 ([43])**.** *Let* $\alpha = \left(\left[\mu_\alpha^l,\ \mu_\alpha^u \right],\ \left[v_\alpha^l,\ v_\alpha^u \right] \right)$, $\beta = \left(\left[\mu_\beta^l,\ \mu_\beta^u \right],\ \left[v_\beta^l,\ v_\beta^u \right] \right)$ *be two IVIFNs, and* $\phi \geq 1$ *and* $\lambda > 0$ *be any real number. Then, DTN and TCN operations of IVIFNs are:*

1. $$\alpha \oplus \beta = \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{ \left(\frac{\mu_\alpha^l}{1-\mu_\alpha^l} \right)^{\phi} + \left(\frac{\mu_\beta^l}{1-\mu_\beta^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}},\ 1 - \frac{1}{1+\left\{ \left(\frac{\mu_\alpha^u}{1-\mu_\alpha^u} \right)^{\phi} + \left(\frac{\mu_\beta^u}{1-\mu_\beta^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{ \left(\frac{1-v_\alpha^l}{v_\alpha^l} \right)^{\phi} + \left(\frac{1-v_\beta^l}{v_\beta^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}},\ \frac{1}{1+\left\{ \left(\frac{1-v_\alpha^u}{v_\alpha^u} \right)^{\phi} + \left(\frac{1-v_\beta^u}{v_\beta^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right] \end{array} \right)$$

2. $$\alpha \otimes \beta = \left(\begin{array}{c} \left[\frac{1}{1+\left\{ \left(\frac{1-\mu_\alpha^l}{\mu_\alpha^l} \right)^{\phi} + \left(\frac{1-\mu_\beta^l}{\mu_\beta^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}},\ \frac{1}{1+\left\{ \left(\frac{1-\mu_\alpha^u}{\mu_\alpha^u} \right)^{\phi} + \left(\frac{1-v_\beta^u}{\mu_\beta^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right], \\ \left[1 - \frac{1}{1+\left\{ \left(\frac{\mu_\alpha^l}{1-v_\alpha^l} \right)^{\phi} + \left(\frac{v_\beta^l}{1-v_\beta^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}},\ 1 - \frac{1}{1+\left\{ \left(\frac{v_\alpha^u}{1-v_\alpha^u} \right)^{\phi} + \left(\frac{v_\beta^u}{1-v_\beta^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right] \end{array} \right)$$

3. $$\lambda\alpha=\left(\begin{array}{c}\left[1-\frac{1}{1+\left\{\lambda\left(\frac{\mu_{\alpha}^{l}}{1-\mu_{\alpha}^{l}}\right)^{\phi}\right\}^{\frac{1}{\phi}}},\ 1-\frac{1}{1+\left\{\lambda\left(\frac{\mu_{\alpha}^{u}}{1-\mu_{\alpha}^{u}}\right)^{\phi}\right\}^{\frac{1}{\phi}}}\right],\\ \left[\frac{1}{1+\left\{\lambda\left(\frac{1-v_{\alpha}^{l}}{v_{\alpha}^{l}}\right)^{\phi}\right\}^{\frac{1}{\phi}}},\ \frac{1}{1+\left\{\lambda\left(\frac{1-v_{\alpha}^{u}}{v_{\alpha}^{u}}\right)^{\phi}\right\}^{\frac{1}{\phi}}}\right]\end{array}\right)$$

4. $$\alpha^{\lambda}=\left(\begin{array}{c}\left[\frac{1}{1+\left\{\lambda\left(\frac{1-\mu_{\alpha}^{l}}{\mu_{\alpha}^{l}}\right)^{\phi}\right\}^{\frac{1}{\phi}}},\ \frac{1}{1+\left\{\lambda\left(\frac{1-\mu_{\alpha}^{u}}{\mu_{\alpha}^{u}}\right)^{\phi}\right\}^{\frac{1}{\phi}}}\right],\\ \left[1-\frac{1}{1+\left\{\lambda\left(\frac{v_{\alpha}^{l}}{1-v_{\alpha}^{l}}\right)^{\phi}\right\}^{\frac{1}{\phi}}},\ 1-\frac{1}{1+\left\{\lambda\left(\frac{v_{\alpha}^{u}}{1-v_{\alpha}^{u}}\right)^{\phi}\right\}^{\frac{1}{\phi}}}\right]\end{array}\right)$$

Theorem 1. *If* $\alpha=\left(\left[\mu_{\alpha}^{l},\ \mu_{\alpha}^{u}\right],\ \left[v_{\alpha}^{l},\ v_{\alpha}^{u}\right]\right)$ *, and* $\beta=\left(\left[\mu_{\beta}^{l},\ \mu_{\beta}^{u}\right],\ \left[v_{\beta}^{l},\ v_{\beta}^{u}\right]\right)$ *be two IVIFNs and* $\lambda\ ,\ \lambda_1\ ,\lambda_2\in\mathbb{R}^{+}$ *; then, we have:*

1. $\alpha\oplus\beta=\beta\oplus\alpha$
2. $\alpha\otimes\beta=\beta\otimes\alpha$
3. $\lambda(\alpha\oplus\beta)=\lambda\alpha\oplus\lambda\beta$
4. $(\lambda_1+\lambda_2)\alpha=\lambda_1\alpha\oplus\lambda_2\alpha$
5. $(\alpha\otimes\beta)^{\lambda}=\alpha^{\lambda}\otimes\beta^{\lambda}$
6. $\alpha_1^{\lambda}\otimes\alpha_2^{\lambda}=\alpha^{\lambda_1+\lambda_2}$

The proofs can be followed from [38].

4. Dombi AOs

In this section, we apply the Dombi operational laws on averaging and geometric AOs to develop some Dombi AOs for IVIFSs. We shall purpose the IVIFDWA, IVIFDOWA, IVIFDHA, IVIFDWG, IVIFDOWG, and IVIFDHG AOs. Throughout this paper, $\xi=(\xi_1,\xi_2,\ldots,\xi_n)^{t}$ will denote the weight vector of IVIFNs $\alpha_i(i=1,2,\ldots,n)$, $\xi_i>0$, and $\sum_{i=1}^{n}\xi_i=1$ and $(i=1,2,\ldots,n)$ shall be used for indexing purposes.

Definition 10 ([38])**.** *The IVIFDWA operator for some IVIFNs* $\alpha_i=\left(\left[\mu_i^{l},\ \mu_i^{u}\right],\ \left[v_i^{l},\ v_i^{u}\right]\right)$ *is a function* $\alpha^{n}\rightarrow\alpha$, *such that*

$$IVIFDWA(\alpha_1,\alpha_2,\ldots,\alpha_n)=\overset{n}{\underset{i=1}{\oplus}}\xi_i\alpha_i$$

Theorem 2. *The aggregated result of IVIFNs* $\alpha_i = \left(\left[\mu_i^l,\ \mu_i^u\right],\ \left[v_i^l,\ v_i^u\right]\right)$ *using an IVIFDWA operator is also an IVIFN and*

$$IFDWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{\sum_{i=1}^{n} \xi_i \left(\frac{\mu_i}{1-\mu_i}\right)^{\phi}\right\}^{\frac{1}{\phi}}}, \frac{1}{1+\left\{\sum_{i=1}^{n} \xi_i \left(\frac{1-v_i}{v_i}\right)^{\phi}\right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{\sum_{i=1}^{n} \xi_i \left(\frac{1-v_l^i}{v_l^i}\right)^{\phi}\right\}^{\frac{1}{\phi}}}, \frac{1}{1+\left\{\sum_{i=1}^{n} \xi_i \left(\frac{1-v_l^i}{v_l^i}\right)^{\phi}\right\}^{\frac{1}{\phi}}} \right] \end{array} \right)$$

Proof. Here, we apply the induction approach to this theorem.

When $n = 2$, we obtain the following result based on Dombi operations of IVIFNs as:

$$IVIFDWA(\alpha_1, \alpha_2) = \xi_1\alpha_1 \oplus \xi_2\alpha_2$$

$$= \left(\begin{array}{c} \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{\xi_1 \left(\frac{\mu_\alpha^l}{1-\mu_\alpha^l}\right)^{\phi}\right\}^{\frac{1}{\phi}}}, 1 - \frac{1}{1+\left\{\xi_1 \left(\frac{\mu_\alpha^u}{1-\mu_\alpha^u}\right)^{\phi}\right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{\xi_1 \left(\frac{1-v_\alpha^l}{v_\alpha^l}\right)^{\phi}\right\}^{\frac{1}{\phi}}}, \frac{1}{1+\left\{\xi_1 \left(\frac{1-v_\alpha^u}{v_\alpha^u}\right)^{\phi}\right\}^{\frac{1}{\phi}}} \right] \end{array} \right) \oplus \\ \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{\xi_2 \left(\frac{\mu_\alpha^l}{1-\mu_\alpha^l}\right)^{\phi}\right\}^{\frac{1}{\phi}}}, 1 - \frac{1}{1+\left\{\xi_2 \left(\frac{\mu_\alpha^u}{1-\mu_\alpha^u}\right)^{\phi}\right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{\xi_2 \left(\frac{1-v_\alpha^l}{v_\alpha^l}\right)^{\phi}\right\}^{\frac{1}{\phi}}}, \frac{1}{1+\left\{\xi_2 \left(\frac{1-v_\alpha^u}{v_\alpha^u}\right)^{\phi}\right\}^{\frac{1}{\phi}}} \right] \end{array} \right) \end{array} \right)$$

$$= \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{ \xi_1 \left(\frac{\mu_1^l}{1-\mu_1^l} \right)^{\phi} + \xi_2 \left(\frac{\mu_2^l}{1-\mu_2^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, 1 - \frac{1}{1+\left\{ \xi_1 \left(\frac{\mu_1^u}{1-\mu_1^u} \right)^{\phi} + \xi_2 \left(\frac{\mu_2^u}{1-\mu_2^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{ \xi_1 \left(\frac{1-v_1^l}{v_1^l} \right)^{\phi} + \xi_2 \left(\frac{1-v_2^l}{v_2^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, \frac{1}{1+\left\{ \xi_1 \left(\frac{1-v_2^u}{v_2^u} \right)^{\phi} + \xi_2 \left(\frac{1-v_2^u}{v_2^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right] \end{array} \right)$$

$$= \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{ \sum_{i=1}^{2} \xi_i \left(\frac{\mu_i^l}{1-\mu_i^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, 1 - \frac{1}{1+\left\{ \sum_{i=1}^{2} \xi_i \left(\frac{\mu_i^u}{1-\mu_i^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{ \sum_{i=1}^{2} \xi_i \left(\frac{1-v_i^l}{v_i^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, \frac{1}{1+\left\{ \sum_{i=1}^{2} \xi_i \left(\frac{1-v_i^u}{v_i^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right] \end{array} \right)$$

Hence the result is correct for $n = 2$.
Assume that the given result is correct for $n = s$. As a result, we have:

$$IVIFDWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \overset{s}{\underset{i=1}{\oplus}} \xi_i \alpha_i = \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{ \sum_{i=1}^{s} \xi_i \left(\frac{\mu_i^l}{1-\mu_i^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, 1 - \frac{1}{1+\left\{ \sum_{i=1}^{s} \xi_i \left(\frac{\mu_i^u}{1-\mu_i^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{ \sum_{i=1}^{s} \xi_i \left(\frac{1-v_i^l}{v_i^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, \frac{1}{1+\left\{ \sum_{i=1}^{s} \xi_i \left(\frac{1-v_i^u}{v_i^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right] \end{array} \right)$$

Now, for $n = s + 1$:

$$IVIFDWA(\alpha_1,\alpha_2,\ldots,\alpha_s,\alpha_{s+1}) = \overset{s+1}{\underset{i=1}{\oplus}} \xi_i\alpha_i = \left(\begin{array}{c} \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{ \sum_{i+1}^{s} \xi_1 \left(\frac{\mu_i^l}{1-\mu_i^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, 1 - \frac{1}{1+\left\{ \sum_{i+1}^{s} \xi_1, \left(\frac{\mu_i^u}{1-\mu_i^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{ \sum_{i+1}^{s} \xi_1, \left(\frac{1-v_\alpha^l}{v_\alpha^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, \frac{1}{1+\left\{ \xi_1 \left(\frac{1-v_\alpha^u}{v_\alpha^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right] \end{array} \right) \oplus \\ \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{ \xi_2 \left(\frac{\mu_\alpha^l}{1-\mu_\alpha^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, 1 - \frac{1}{1+\left\{ \xi_2 \left(\frac{\mu_\alpha^u}{1-\mu_\alpha^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{ \xi_2 \left(\frac{1-v_\alpha^l}{v_\alpha^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, \frac{1}{1+\left\{ \xi_2 \left(\frac{1-v_\alpha^u}{v_\alpha^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right] \end{array} \right) \end{array} \right)$$

$$= \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{ \xi_1 \left(\frac{\mu_1^l}{1-\mu_1^l} \right)^{\phi} + \xi_2 \left(\frac{\mu_2^l}{1-\mu_2^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, 1 - \frac{1}{1+\left\{ \xi_1 \left(\frac{\mu_1^u}{1-\mu_1^u} \right)^{\phi} + \xi_2 \left(\frac{\mu_2^u}{1-\mu_2^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{ \xi_1 \left(\frac{1-v_1^l}{v_1^l} \right)^{\phi} + \xi_2 \left(\frac{1-v_2^l}{v_2^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, \frac{1}{1+\left\{ \xi_1 \left(\frac{1-v_2^u}{v_2^u} \right)^{\phi} + \xi_2 \left(\frac{1-v_2^u}{v_2^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right] \end{array} \right)$$

Consequently, the result is true for $n = s + 1$ and it is also true for $n = s$. It is also valid for $n = 2$. Hence, the given result is true for all natural numbers n by using the induction method. □

Every aggregation function must satisfy the properties of idempotency, boundedness, and monotonicity. Below, we show that the Dombi AOs satisfy these properties.

Theorem 3. *If* $\alpha_i = \left(\left[\mu_i^l, \ \mu_i^u \right], \ \left[v_i^l, \ v_i^u \right] \right)$ *be a number of IVIFNs such that all are identical, i.e.,* $\alpha_i = \alpha = \left(\left[\mu^l, \mu^u \right], \ \left[v^l, v^u \right] \right)$ *for all i, then:*

$$IVIFDWA\ (\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha$$

Proof. We have

$$
\begin{aligned}
&IVIFDWA(\alpha_1, \alpha_2 \ldots, \alpha_n) \\
&= \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{\mu^l}{1-\mu^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}},\ 1 - \frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{\mu^u}{1-\mu^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{1-v^l}{v^l} \right)^{\phi} \right\}^{\frac{1}{\phi}}},\ \frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{1-v^u}{v^u} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right] \end{array} \right) \\
&= \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{ \left(\frac{\mu^l}{1-\mu^l} \right)^{\phi} \sum_{i=1}^{n} \xi_i \right\}^{\frac{1}{\phi}}},\ 1 - \frac{1}{1+\left\{ \left(\frac{\mu^u}{1-\mu^u} \right)^{\phi} \sum_{i=1}^{n} \xi_i \right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{ \left(\frac{1-v^l}{v^l} \right)^{\phi} \sum_{i=1}^{n} \xi_i \right\}^{\frac{1}{\phi}}},\ \frac{1}{1+\left\{ \left(\frac{1-v^u}{v^u} \right)^{\phi} \sum_{i=1}^{n} \xi_i \right\}^{\frac{1}{\phi}}} \right] \end{array} \right) \\
&= \left(\begin{array}{c} \left[1 - \frac{1}{1+\left(\frac{\mu^l}{1-\mu^l} \right)},\ 1 - \frac{1}{1+\left(\frac{\mu^u}{1-\mu^u} \right)} \right], \\ \left[\frac{1}{1+\left(\frac{1-v^l}{v^l} \right)},\ \frac{1}{1+\left(\frac{1-v^u}{v^u} \right)} \right] \end{array} \right) = \left(\left[\mu^l, \mu^u \right],\ \left[v^l, v^u \right] \right) = \alpha
\end{aligned}
$$

□

Theorem 4. *Let* $\alpha_i = \left(\left[\mu_i^l,\ \mu_i^u \right],\ \left[v_i^l,\ v_i^u \right] \right)$ *be a number of IVIFNs. Let* $\alpha^- = min\ \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ *and* $\alpha^+ = max\ \{\alpha_1, \alpha_2, \ldots,\ \alpha_n\}$*. Then,* $\alpha^- \leq IVIFDWA\ (\alpha_1, \alpha_2, \ldots, \alpha_n)\ \leq\ \alpha^+$*.*

Proof. Let $\alpha_i = \left(\left[\mu_i^l,\ \mu_i^u \right],\ \left[v_i^l,\ v_i^u \right] \right)$ be a number of IVIFNs and let $\alpha^- = min\ \{\alpha_1, \alpha_2, \ldots\ldots, \alpha_n\}$ $= (\mu^-,\ v^-)$ and $\alpha^+ = max\ \{\alpha_1, \alpha_2, \ldots\ldots, \alpha_n\} = (\mu^+, v^+)$. Then:

$$1-\frac{1}{1+\left\{\sum_{i=1}^{n}\xi_i\left(\frac{\mu^{l-}}{1-\mu^{l-}}\right)^{\phi}\right\}^{\frac{1}{\phi}}},1-\frac{1}{1+\left\{\sum_{i=1}^{n}\xi_i\left(\frac{\mu^{u-}}{1-\mu^{u-}}\right)^{\phi}\right\}^{\frac{1}{\phi}}}$$
$$\leq 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\xi_i\left(\frac{\mu_i^{l}}{1-\mu_i^{l}}\right)^{\phi}\right\}^{\frac{1}{\phi}}},\ 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\xi_i\left(\frac{\mu_i^{u}}{1-\mu_i^{u}}\right)^{\phi}\right\}^{\frac{1}{\phi}}}$$
$$\leq 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\xi_i\left(\frac{\mu^{l+}}{1-\mu^{l+}}\right)^{\phi}\right\}^{\frac{1}{\phi}}},1-\frac{1}{1+\left\{\sum_{i=1}^{n}\xi_i\left(\frac{\mu^{u+}}{1-\mu^{u+}}\right)^{\phi}\right\}^{\frac{1}{\phi}}}$$
$$\frac{1}{1+\left\{\sum_{i=1}^{n}\xi_i\left(\frac{1-v^{l+}}{v^{l+}}\right)^{\phi}\right\}^{\frac{1}{\phi}}},\frac{1}{1+\left\{\sum_{i=1}^{n}\xi_i\left(\frac{1-v^{u+}}{v^{u+}}\right)^{\phi}\right\}^{\frac{1}{\phi}}}$$
$$\geq \frac{1}{1+\left\{\sum_{i=1}^{n}\xi_i\left(\frac{1-v_i^{l}}{v_i^{l}}\right)^{\phi}\right\}^{\frac{1}{\phi}}},\frac{1}{1+\left\{\sum_{i=1}^{n}\xi_i\left(\frac{1-v_i^{u}}{v_i^{u}}\right)^{\phi}\right\}^{\frac{1}{\phi}}}$$
$$\geq \frac{1}{1+\left\{\sum_{i=1}^{n}\xi_i\left(\frac{1-v^{l-}}{v^{l-}}\right)^{\phi}\right\}^{\frac{1}{\phi}}},\frac{1}{1+\left\{\sum_{i=1}^{n}\xi_i\left(\frac{1-v^{u-}}{v^{u+}}\right)^{\phi}\right\}^{\frac{1}{\phi}}}$$

Therefore:

$$\alpha^{-} \leq IVIFDWA(\alpha_1,\alpha_2,\ldots,\alpha_n) \leq \alpha^{+}.$$

□

Theorem 5. *Letα_i and$\varkappa_i$ be two sets of IVIFNs, $\alpha_i \leq \varkappa_i$ for all i. Then:*

$$IVIFDWA(\alpha_1,\ \alpha_2,\ldots,\ \alpha_n) \leq IVIFDWA(\varkappa_1,\ \varkappa_2,\ \ldots,\ \varkappa_n)$$

Now, we would like to introduce the IVIFDOWA operator. The ordered weighted averaging operators are useful in cases where the sequence of the information is important.

Definition 11. *Let $\alpha_i = \left(\left[\mu_i^{l},\ \mu_i^{u}\right],\ \left[v_i^{l},\ v_i^{u}\right]\right)$ be a number of IVIFNs. Then, the IVIFDOWA operator of dimensionn is a function $\alpha^{n} \to \alpha$, such that:*

$$IVIFDOWA(\alpha_1,\alpha_2,\ldots,\alpha_n) = \mathop{\oplus}_{i=1}^{n}\xi_i\alpha_{p(i)}$$

where $(p(1),\ p(2),\ldots,\ p(n))$ are such that $\alpha_{p(i-1)} \geq \alpha_{p(i)}$ for all i.

Based on Definition 10 of Dombi operational laws, we propose the following result.

Theorem 6. *Let* $\alpha_i = \left(\left[\mu_i^l,\ \mu_i^u\right],\ \left[v_i^l,\ v_i^u\right]\right)$ *be a number of IVIFNs. Then IVIFDOWA operator is also an IIVFN expressed as:*

$$IVIFDOWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{\sum_{i=1}^{n} \xi_i \left(\frac{\mu^l_{p(i)}}{1-\mu^l_{p(i)}}\right)^{\phi}\right\}^{\frac{1}{\phi}}},\ 1 - \frac{1}{1+\left\{\sum_{i=1}^{n} \xi_i \left(\frac{\mu^u_{p(i)}}{1-\mu^u_{p(i)}}\right)^{\phi}\right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{\sum_{i=1}^{n} \xi_i \left(\frac{1-v^l_{p(i)}}{v^l_{p(i)}}\right)^{\phi}\right\}^{\frac{1}{\phi}}},\ \frac{1}{1+\left\{\sum_{i=1}^{n} \xi_i \left(\frac{1-v^u_{p(i)}}{v^u_{p(i)}}\right)^{\phi}\right\}^{\frac{1}{\phi}}} \right] \end{array} \right)$$

Remark 1. *The aggregation properties mentioned in Theorems 3, 4, and 5 are likely to be satisfied by the IVIFDOWA operator.*

It is noted that the IVIFDOWA operator gives importance to the ordered position of the uncertain information during the aggregation process, while IVIFDWA operators weigh only the uncertain information without considering their ordered position. An AO that has the feature of both IVIFDWA and IVIFDOWA operators would be of great importance and hence we propose the notion of an IVIFDHA operator.

Definition 12. *Let* α_i *and*$\varkappa_i$ *be two sets of IVIFNs. Then, the IVIF Dombi hybrid averaging (IVIFDHA) operator of dimensionn is a function* $\alpha^n \rightarrow \alpha$ *, such that:*

$$IVIFDHA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \underset{i=1}{\overset{n}{\oplus}} \xi_i \dot{\alpha}_{p(i)} = \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{\sum_{i=1}^{n} \xi_i \left(\frac{\dot{\mu}^l_{p(i)}}{1-\dot{\mu}^l_{p(i)}}\right)^{\phi}\right\}^{\frac{1}{\phi}}},\ 1 - \frac{1}{1+\left\{\sum_{i=1}^{n} \xi_i \left(\frac{\dot{\mu}^u_{p(i)}}{1-\dot{\mu}^u_{p(i)}}\right)^{\phi}\right\}^{\frac{1}{\phi}}} \right], \\ \left[1 - \frac{1}{1+\left\{\sum_{i=1}^{n} \xi_i \left(\frac{\dot{v}^l_{p(i)}}{1-\dot{v}^l_{p(i)}}\right)^{\phi}\right\}^{\frac{1}{\phi}}},\ 1 - \frac{1}{1+\left\{\sum_{i=1}^{n} \xi_i \left(\frac{\dot{v}^u_{p(i)}}{1-\dot{v}^u_{p(i)}}\right)^{\phi}\right\}^{\frac{1}{\phi}}} \right] \end{array} \right) \quad (6)$$

where $\dot{\alpha}_{p(i)}$ *is the* i^{th} *biggest weighted IVIF number of* $\alpha_i(\dot{\alpha}_i = n\xi_i\alpha_i$ *and n is the balancing coefficient.*

Definition 13 ([38]). *Let* $\alpha_i = \left(\left[\mu_i^l,\ \mu_i^u\right],\ \left[v_i^l,\ v_i^u\right]\right)$ *be a number of IVIFNs. Then, IVIF Dombi weighted geometric (IVIFDWG) operator is a function* $\alpha^n \rightarrow \alpha$ *such that:*

$$IVIFDWG(\alpha_1, \alpha_2 \ldots, \alpha_n) = \underset{i=1}{\overset{n}{\otimes}} (\alpha_i)^{\xi_i}$$

$$= \left(\begin{array}{c} \left[\frac{1}{1+\left\{\sum_{i=1}^n \xi_i \left(\frac{1-\mu_i^l}{\mu_i^l}\right)^{\phi}\right\}^{\frac{1}{\phi}}},\ \frac{1}{1+\left\{\sum_{i=1}^n \xi_i \left(\frac{1-\mu_i^u}{\mu_i^u}\right)^{\phi}\right\}^{\frac{1}{\phi}}} \right], \\ \left[1-\frac{1}{1+\left\{\sum_{i=1}^n \xi_i \left(\frac{v_i^l}{1-v_i^l}\right)^{\phi}\right\}^{\frac{1}{\phi}}},\ 1-\frac{1}{1+\left\{\sum_{i=1}^n \xi_i \left(\frac{v_i^u}{1-v_i^u}\right)^{\phi}\right\}^{\frac{1}{\phi}}} \right] \end{array} \right)$$

where $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^t$ *be the weight vector of* α_i , $\xi_i > 0$ *and* $\sum_{i=1}^n \xi_i = 1$

Proof. The proof follows the same pattern as Theorem 2. □

Theorem 10. *If* $\alpha_i = \left(\left[\mu_i^l,\ \mu_i^u\right],\ \left[v_i^l,\ v_i^u\right]\right)$ *be number of IVIFNs, all are identical, i.e.,* $\alpha_i = \alpha$ *for all* i, *where* $\alpha = \left(\left[\mu^l, \mu^u\right],\ \left[v^l, v^u\right]\right)$. *Then:*

$$IVIFDWG(\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha$$

Proof . The proof follows the same pattern as Theorem 3. □

Theorem 11. *Let* $\alpha_i = (\mu_i, v_i)$ $(i = 1, 2, \ \ldots, \ n)$ *be number of IVIFNs. Let* $\alpha^- = min\ \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ *and* $\alpha^+ = max\ \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$.*Then* $\alpha^- \ \leq\ IVIFDWG\ (\alpha_1, \alpha_2, \ldots, \alpha_n)\ \leq \alpha^+$.

Proof. The proof follows the same pattern as Theorem 4. □

Theorem 12. *Let* α_i *and* $\varkappa_i$ *be two sets of IVIFNs,* $\alpha_i\ \leq\ \varkappa_i$ *for all* i *: Then:*

$$IVIFDWG\ (\alpha_1,\ \alpha_2, \ldots,\ \alpha_n) \leq IVIFDWG\ (\varkappa_1,\ \varkappa_2,\ \ldots,\ \varkappa_n)$$

Now, the IVIF Dombi ordered weighted geometric (IVIFDOWG) operator is introduced.

Definition 14 ([38]). *Let* $\alpha_i = \left(\left[\mu_i^l,\ \mu_i^u\right],\ \left[v_i^l,\ v_i^u\right]\right)$ *be number of IVIFNs. Then, IVIF Dombi order weighted geometric (IVIFDOWG) operator of dimension n is a function* $\alpha^n \rightarrow \alpha$, *such that:*

$$IVIFDOWG\ (\alpha_1,\ \alpha_2,\ \ldots, \alpha_n) = \underset{i=1}{\overset{n}{\otimes}} (\alpha_{p(i)})^{\xi_i}$$

where $(p(1),\ p(2),\ \ldots,\ p(n))$ *are the permutation such that* $\alpha_{p(i-1)} \geq \alpha_{p(i)}$, *for all* i.

Based on Definition 10 of Dombi operational laws, we proposed the following result.

Theorem 13. *Let* $\alpha_i = \left(\left[\mu_i^l,\ \mu_i^u\right],\ \left[v_i^l,\ v_i^u\right]\right)(i = 1,\ 2,\ \ldots,\ n)$ *be number of IVIFNs, and then, the aggregated value of them using IVIFDOWG operation is also IVIFN and*

$$IVIFDOWG\ (\alpha_1,\ \alpha_2,\ \ldots,\alpha_n) = \overset{n}{\underset{i=1}{\otimes}} (\alpha_{p(i)})^{\xi_i}$$

$$= \left(\begin{array}{c} \left[\frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{1-\mu^l_{p(i)}}{\mu^l_{p(i)}} \right)^{\phi} \right\}^{\frac{1}{\phi}}},\ \frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{1-\mu^u_{p(i)}}{\mu^u_{p(i)}} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right], \\ \left[1 - \frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{v^l_{p(i)}}{1-v^l_{p(i)}} \right)^{\phi} \right\}^{\frac{1}{\phi}}},\ 1 - \frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{v^u_{p(i)}}{1-v^u_{p(i)}} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right] \end{array} \right)$$

where $(p(1),\ p(2),\ \ldots,\ p(n))$ *are the permutation such that* $\alpha_{p(i-1)} \geq \alpha_{p(i)}$, *for all* i.

Remark 2. *The IVIFDOWG operator is likely to satisfy the aggregation properties discussed in Theorem 3, Theorem 4, and Theorem 5, respectively.*

Theorem 14. *If* $\alpha_i = \left(\left[\mu_i^l,\ \mu_i^u\right],\ \left[v_i^l,\ v_i^u\right]\right)$ *be a number of IVIFNs, all are identical, i.e.,* $\alpha_i = \alpha$ *for all* i, *where* $\alpha == \left(\left[\mu^l, \mu^u\right],\ \left[v^l, v^u\right]\right)$. *Then:*

$$IVIFDOWG\ (\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha$$

Theorem 15. *Let* $\alpha_i = \left(\left[\mu_i^l,\ \mu_i^u\right],\ \left[v_i^l,\ v_i^u\right]\right)$ *be a number of IVIFNs. Let* $\alpha^- = \phi in(\alpha_1, \alpha_2, \ldots, \alpha_n)$ *and* $\alpha^- = \phi \alpha \varkappa(\alpha_1, \alpha_2, \ldots, \alpha_n)$. *Then:*

$$\alpha^- \leq\ IVIFDOWG\ (\alpha_1, \alpha_2, \ldots, \alpha_n)\ \leq\ \alpha^+.$$

Theorem 16. *Let* α_i *and* $\varkappa_i$ *be two sets of IFNs,* $\alpha_i\ \leq\ \varkappa_i$ *for all* i *: Then:*

$$IVIFDOWG(\alpha_1,\ \alpha_2,\ \phi, \alpha_n)\ \leq IVIFDOWG\left(\alpha_1',\ \alpha_2',\ \ldots,\ \alpha_n'\right)$$

It is found that the IVIFDOWG operator weighs the ordered position of the IVIF information in the aggregation phenomena, while the IVIFDWG operator lacks such characteristics and weighs the information only without considering the position of the information. Due to this reason, we present the notion of IVIFDHG operators that has the properties of both IVIFDWG and IVIFDOWG operators.

Definition 15 ([38]). *Let α_i and $\varkappa_i$ be two sets of IVIFNs. Then, IVIF Dombi hybrid geometric (IVIFDHG) operator of dimension n is a function $\alpha^n \to \alpha$, such that:*

$$IVIFDHG\ (\alpha_1,\ \alpha_2,\ \ldots,\ \alpha_n) = \overset{n}{\underset{i=1}{\oplus}} (\dot{\alpha}_{p(i)})^{\xi_i}$$
$$= \left(\begin{array}{c} \left[\frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{1-\dot{\mu}^l_{p(i)}}{\dot{\mu}^l_{p(i)}} \right)^{\phi} \right\}^{\frac{1}{\phi}}},\ \frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{1-\dot{\mu}^u_{p(i)}}{\dot{\mu}^u_{p(i)}} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right], \\ \left[1-\frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{\dot{v}^l_{p(i)}}{1-\dot{v}^l_{p(i)}} \right)^{\phi} \right\}^{\frac{1}{\phi}}},\ 1-\frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{\dot{v}^u_{p(i)}}{1-\dot{v}^u_{p(i)}} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right] \end{array} \right) \quad (7)$$

where ξ is the weight vector of α_i, $\xi_i \in [0,1]$, and $\sum_{i=1}^{n} \xi_i = 1$ $\alpha_{p(i)}$ is the i^{th} biggest weighted intuitionistic fuzzy value of $\varkappa_i(\varkappa_i = n\xi_i\varkappa_i,\ i = 1, 2,\ \ldots,\ n)$. n is the balancing coefficient.

5. Model for MADM Using IVIF Information

MADM is a vastly discussed technique used for decision-making, especially when it comes to human opinion under uncertain situations. The frame for IVIFS is capable of representing human opinion more precisely by discussing two aspects of opinion in the form of duplets. Our aim is to develop a MADM technique based on the proposed novel Dombi AOs of IVIFSs.

In the MADM problem, we have a finite set of choices $J_1,\ J_2,\ \ldots, J_m$ that are examined based on some finite attributes $Q_1,\ Q_2,\ \ldots, Q_n$ in order to select the most favorable choice. In the proposed algorithm, the ϕ choices are assessed under n attributes and the information is based on IVIFNs where two aspects of the information are covered. The aggregation process is carried out by using IVIFDWA and IVIFDWG operators where the weights of the attributes are denoted by ξ_k plays a key role. Suppose that $M = (\tau_{ij})_{\phi\times n} = (\mu_{ij}, v_{ij})_{\phi\times n}$ be the decision matrix containing information based on IVIFNs about the given choices based on finite attributes. The detailed steps of the MADM algorithm are given in Section 5.1.

5.1. Algorithm

The purpose of the below-given algorithm is to solve a MADM problem based on IVIFNs by using IVIFDWA and IVFDWG operators.

Step 1: Insert decision matrix $M = (\tau_{ij})_{\phi\times n} = (\mu_{ij}, v_{ij})_{\phi\times n}$ >based on IVIFNs.

Step 2: Normalize the decision matrix $M = (\tau_{ij})_{\phi\times n} = (\mu_{ij}, v_{ij})_{\phi\times n}$ into $M' = (\tau_{ij})'_{\phi\times n} = (\mu_{ij}', v_{ij}')_{\phi\times n}$ by the following equation:

$$\tau_{ij}' = \begin{cases} (\mu_{ij}, v_{ij}),\ if\ Q_j\ is\ benefit\ attribute; \\ (v_{ij}, \mu_{ij}),\ if\ Q_j\ is\ cost\ attribute; \end{cases}$$

Step 3: Calculate the alternative J_i collective information σ_k by the following equation:

$$\sigma_k = IVIFDWA((\tau_{k1}, \tau_{k2}, \ldots, \tau_{kn}) = \underset{i=1}{\overset{n}{\oplus}} \xi_i \tau_{ki}$$
$$= \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{\mu_{ki}^{l}}{1-\mu_{ki}^{l}} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, 1 - \frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{\mu_{ki}^{u}}{1-\mu_{ki}^{u}} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{1-v_{ki}^{l}}{v_{ki}^{l}} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, \frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{1-v_{ki}^{u}}{v_{ki}^{u}} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right] \end{array} \right)$$

and

$$\sigma_k = IVIFDWG(\tau_{k1}, \tau_{k2}, \ldots, \tau_{kn}) = \underset{i=1}{\overset{n}{\otimes}} \xi_i^{\tau_{ki}}$$
$$= \left(\begin{array}{c} \left[1 - \frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{\mu_{ki}^{l}}{1-\mu_{ki}^{l}} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, 1 - \frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{\mu_{ki}^{u}}{1-\mu_{ki}^{u}} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right], \\ \left[\frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{1-v_{ki}^{l}}{v_{ki}^{l}} \right)^{\phi} \right\}^{\frac{1}{\phi}}}, \frac{1}{1+\left\{ \sum_{i=1}^{n} \xi_i \left(\frac{1-v_{ki}^{u}}{v_{ki}^{u}} \right)^{\phi} \right\}^{\frac{1}{\phi}}} \right] \end{array} \right)$$

Step 4: Compute the SF $\Phi(\sigma_l\ )$ for each alternative J_l by the following equation.

$$\Phi(\alpha) = \frac{(\mu_{\alpha}^{l})\left(1 - v_{\alpha}^{l}\right) + (\mu_{\alpha}^{u})(1 - v_{\alpha}^{u})}{2}$$

Step 5: The optimal decision is to select J_k if $\Phi(\sigma_k\ ) = max_l\ \{\Phi(\sigma_l)\}$.

Now we give a numerical example to demonstrate the steps of the decision-making process given by taking the energy sector of Pakistan into consideration.

5.2. Case Study

Pakistan is facing various obstacles to its economic growth, and one of the main obstacles is Pakistan's energy sector. Since 2013, Pakistan has coped with and mitigated the major power blackout issue of the last decade, but still, this sector is in crisis and is underdeveloped due to certain reasons. Apart from the lack of long-term and sustainable energy planning, weak governance and poor policymaking by the government, aging, and insufficiency of transmission and distribution systems, shortages in electricity and chronic natural gas, dependency on imported energy products and expensive fuel resources are some of the main reasons that exist behind the calamity of this entire sector.

In 2013, the government introduced various support policies to nourish renewable energy development in the country, and due to these initial measures, if we look over the past five years, twenty wind power and six solar power projects are successfully providing a total of 1598 MW of electricity to the grid. The wind corridor of Sindh alone possesses an active potential to produce 50,000 MW of electricity to the grid.

Due to the initiative of the government, we also have some local industry resources that must be considered while looking for a solution for this particular sector of the economy. The Alternate Energy Development Board (AEDB) was established in May 2003 to promote and encourage the development of renewable energy. Pakistan Council for Renewable Energy Technologies and Pakistan Solar Association (PSA) are also working actively for the successful implementation of the aforementioned alternate energy production resources in the country.

To achieve the goal of the above-mentioned resources, one option is to utilize the US Department of Commerce's Export Assistance Centers (USEAC) in the United States and US Commercial Service Offices in the United States Embassy in Islamabad and the US Consulates General in Karachi and Lahore. Obtaining assistance from these departments should be considered mandatory before hunting for the best opportunities in the complex market of Pakistan.

Pakistan still requires more thermal and coal-based power generation plants to cope with the energy generation gap. Air/wind energy and solar energy production options are now mandatory for the survival of the energy sector of the country and due to this reason, there is huge competition in the international market for the provision of alternate energy production resources in the market. Here, we discussed the problem of selection of the energy resources to overcome the energy deficiency of Pakistan in the following example.

5.3. Example

There are many energy resources that can be used in Pakistan and the four most favorable among them include

J_1 as solar energy, J_2 as geothermal energy, J_3 as biomass energy, and J_4 as wind energy. Since all the mentioned alternatives have a different source that produces energy, the most suitable energy resource would be assessed based on some connecting attributes given as follows:

- Q_1*: Cost.*
- Q_2*: Quantity.*
- Q_3*: Reliability.*
- Q_4*: Sustainability.*

In this case, making the selection is tough because if we examine the data in Table 1, we can say that the cost of all energy resources is almost the same in MD, but in NMD solar energy resource reflects relatively higher figures than the other three sources. In the quantity attribute, geothermal and wind energy are elevating in MD, while in NMD, solar energy shows higher values than all other resources. Solar energy resources are much more reliable and durable than all other resources mentioned in Table 1. While, as per the information, biomass energy resources are most sustainable as compared to wind and solar energy. Geothermal/biomass energy resources are not dependent on sun/wind to produce energy, and they can run 24 h a day and 365 days a year consistently.

Table 1. Information for the assessment of energy resources based on 4 attributes.

	Q_1				Q_2				Q_3				Q_4			
	MD		**NMD**		**MD**		**NMD**		**MD**		**NMD**		**MD**		**NMD**	
	L	**U**	**L**	**U**	**L**	**U**	**L**	**U**	**L**	**U**	**L**	**U**	**L**	**U**	**L**	**U**
J_1	0.25	0.7	0.07	0.23	0.03	0.05	0.7	0.2	0.1	0.3	0.5	0.4	0.08	0.01	0.01	0.05
J_2	0.57	0.39	0.03	0.04	0.3	0.2	0.1	0.4	0.06	0.05	0.11	0.31	0.69	0.3	0.05	0.03
J_3	0.85	0.1	0.04	0.02	0.06	0.02	0.08	0.01	0.05	0.03	0.07	0.02	0.65	0.2	0.08	0.01
J_4	0.7	0.2	0.03	0.04	0.2	0.5	0.62	0.23	0.07	0.23	0.08	0.45	0.15	0.5	0.13	0.4

Assume that the alternative J_i is evaluated in the form matrix $M = (\tau_{ij})_{\phi \times n} = (\mu_{ij}, v_{ij})_{\phi \times n}$ based on IVIFNs for the attribute Q_j. The evaluation of the alternatives is shown in Table 1 where two aspects of decision-makers are expressed by MGs and NMGs, respectively.

To select the most preferable energy resources $J_k (k = 1, 2, 3, 4)$, we exploited the IVIFDWA and IVIFDWG operators to aggregate the uncertain information. The aggregated results are portrayed in Table 2 below.

Table 2. Aggregated data based on IVIFDWA and IVIFDWG operators.

	IVIFDWA Operator	**IVIFDWG Operator**
J_1	$([0.1212, 0.3768], [0.2500, 0.3969])$	$([0.1327, 0.2713], [0.3558, 0.2342])$
J_2	$([0.5490, 0.2538], [0.0926, 0.1511])$	$([0.1644, 0.1349], [0.0701, 0.1818])$
J_3	$([0.6550, 0.1178], [0.0914, 0.0282])$	$([0.1205, 0.0575], [0.0692, 0.0150])$
J_4	$([0.3690, 0.3901], [0.0911, 0.1538])$	$([0.1828, 0.3443], [0.2033, 0.3550])$

After the aggregation of information, ranking is an important step. For the ranking purpose, we use the SF as discussed in Equation (1) to compute the scores. All the results are listed in Table 3, below.

Table 3. Aggregated information w.r.t Scores.

Scores	**IVIFDWA Operator**	**IVIFDWG Operator**
J_1	0.1591	0.1466
J_2	0.3568	0.1316
J_3	0.3548	0.0844
J_4	0.3328	0.1838

The score values in Table 3 show a high ranking for J_2 in the case of IVIFDWA operators and J_4 in the case of the IVIFDWG operator. The detailed ranking analysis is given in Table 4, below.

Table 4. Analysis of ranking.

	Ranking Analysis
$IVIFDWA\ operator$	$J_2 > J_3 > J_4 > J_1$
$IVIFDWG\ operator$	$J_4 > J_1 > J_2 > J_3$

For better observation, we portrayed the ranking analysis of Table 4 in Figure 1 below where the blue colored data are based on IVIFDWA, and the brown color data are based on IVIFDWG operators.

The ranking results portrayed in Table 4 and Figure 1 clearly indicate that according to the IVIFDWA operator, J_2 is best and according to IVIFDWG operator, J_4 is the best energy resource option in Pakistan to overcome energy problems. The ranking results are not the same in the case of IVIFDWA and IVIFDWG operators. Both operators are good and give the result in their manner. However, the selection of the aggregation operator is dependent on the decision-maker.

The above-aggregated results are computed for some specific values of the variable parameters, and they can vary upon the variation in parameters. To analyze the impact of the parameter m on ranking results, and to establish stability in the results, a detailed survey is performed in the next section.

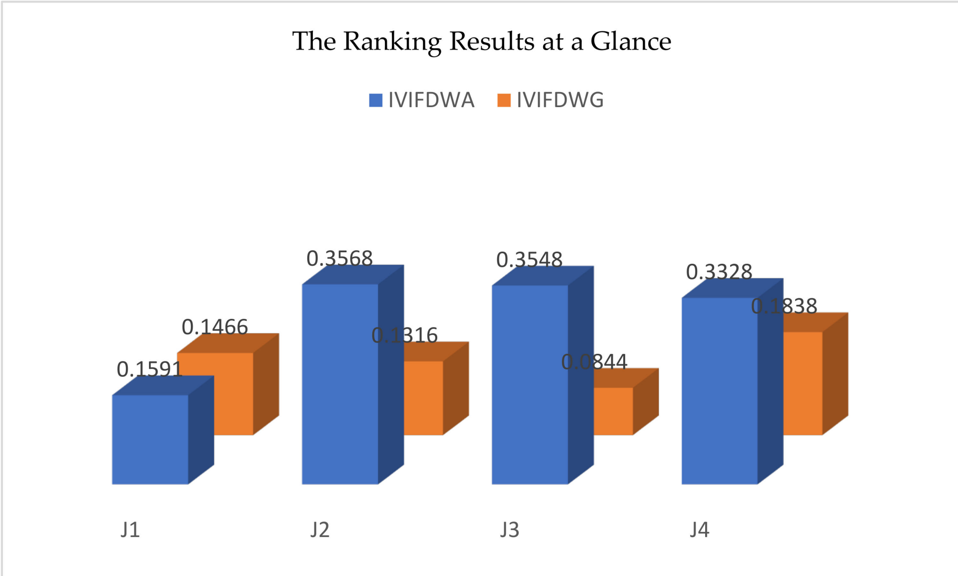

Figure 1. A graphical depiction of the ranking results is given in Table 4.

5.4. Impact of *m* on Ranking Results

In IVIF information, usually, the least value of m is taken in the aggregation process. However, it has been observed that for larger m the ranking results may vary. Stability in the results is always our need and to achieve that, a value of m is needed after which we do not observe any variation, and such value of m is considered as stability valued.

First, we observe the variation in m of both operators IVIFDWA and IVIFDWG. It is observed that the effect of m is on the ranking order. When $m = 1$, the best parameter is J_2, when $m = 2$ the best parameter is J_4, and when $m = 3, 4, 5, \ldots, 30$ the best parameter is still J_4. The variation in result can be seen in Tables 5 and 6 below.

Table 5. By using IVIFDWA, the impact of the parameter m on decision-making result.

M	Ranking Order	Optimal Alternative
1	$J_2 > J_3 > J_4 > J_1$	J_2
2	$J_4 > J_3 > J_2 > J_1$	J_4
3	$J_4 > J_2 > J_3 > J_1$	J_4
7	$J_4 > J_2 > J_3 > J_1$	J_4
10	$J_4 > J_2 > J_3 > J_1$	J_4
30	$J_4 > J_2 > J_3 > J_1$	J_4

Table 6. By using IVIFDWG, the impact of the parameter m on the decision-making result.

M	Ranking Order	Optimal Alternative
1	$J_4 > J_1 > J_2 > J_3$	J_4
4	$J_4 > J_1 > J_2 > J_3$	J_4
10	$J_4 > J_2 > J_3 > J_1$	J_4
20	$J_4 > J_2 > J_3 > J_1$	J_4

In the case of IVIFDWG, we check the effect of m on the ranking order. When $m = 1, 2, 3, \ldots, 20$ the best parameter is J_4.

6. Comparative Study

In this section, we will compare the aggregated results achieved by using IVIFDWA and IVIFDWG operators with various other IVIF information based on AOs. For this purpose, we applied averaging and geometric AOs [45] of the IVIFs framework, the

Hamy mean operators of IVIFSs [46], and IVIF Bonferroni mean operators [47] on the IVIF information given in Table 1, and the results are given in Table 7, below.

Table 7. Comparative study.

Operator	Results
IVIFDWA operator	$J_4 > J_3 > J_2 > J_1$
IVIFDWG operator	$J_4 > J_2 > J_3 > J_1$
IVIFWA operator [48]	$J_2 > J_3 > J_4 > J_1$
IVIFWG operator [49]	$J_2 > J_4 > J_3 > J_1$
IVIFHM operator [50]	$J_2 > J_4 > J_3 > J_1$
IVIFWGBM operator [51]	$J_2 > J_4 > J_3 > J_1$

IVIFDWA operator [52] shows a continuous increase in the results if we observe it from J_1 to J_4 showing J_4 as most suitable energy resouce. IVIFDWG operator [47] also declared J_4 as highly suitable. In IVIFWA operator J_2 is higher and J_1 shows the lowest levels as shown in Figure 2. IVIF Hamy mean operators [50] show a higher level of J_2. IVIF Bonferroni mean operators [51] indicate that J_2 is most optimum among the given options. The results obtained using the proposed approach have considerably higher accuracy due to the representation of the information in the form of closed sub-intervals of $[0, 1]$ instead of crisp numbers. This argument was justified by Ullah et al. [53] who prove the significance of using the information in the form of intervals numerically to see their significance. Such representation provides higher accuracy than the existing approaches of IFSs where information is not expressed in the form of intervals. Further, a recent study by [54] showed optimal results for Dombi t-norms in a classification problem; hence, showing the worth of using Dombi TN-based AOs. All this leads to the comprehensibility of the proposed theory. The pictorial view of Table 7 is given in Figure 2 below.

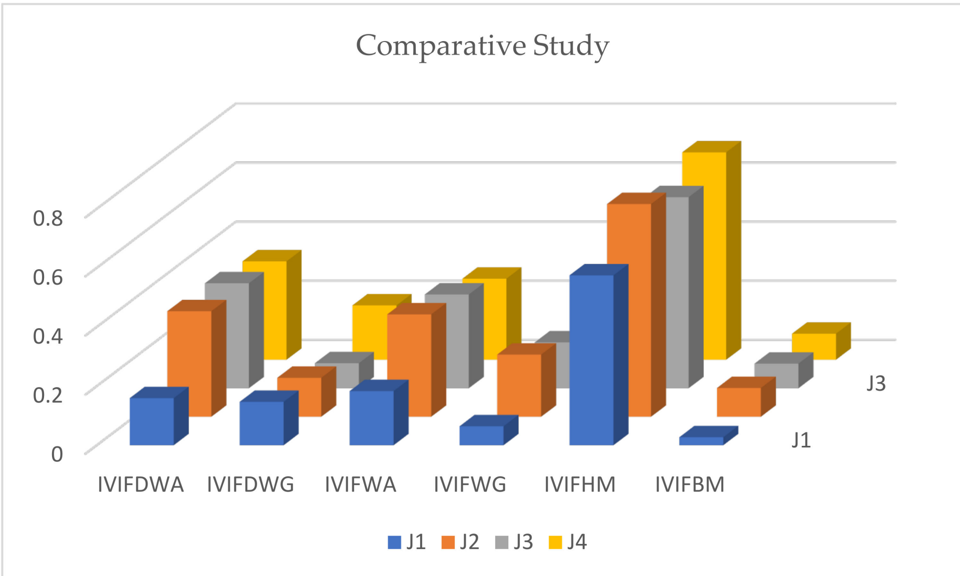

Figure 2. A graphical representation of the comparative study is given in Table 7.

7. Conclusions

In this article, we studied the Dombi AOs using IVIF information to cope with MADM problems. We obtained the IVIF Dombi operations based on DTN and TCN. The work is further used to develop two novel AOs known as IVIFDWA and IVIFDWG operators. The developed operators are tested for validity, and it is determined that the proposed Dombi operators satisfied the basic properties of aggregation including idempotency, monotonicity, and boundedness. We applied the work to a case study of an energy problem to discuss energy resource selection in the context of Pakistan by using the algorithm of MADM. The advantage of using IVIF information based Dombi operations is capturing uncertain information using two kinds of human opinion by expressing them using closed sub-intervals of $[0, 1]$. This helps in reducing information loss and the results obtained become reliable. In near future, we aim to associate Dombi operations with Maclaurin symmetric mean (MSM) operators [55], and Bonferroni mean operators [28]. We also aim to study the Dombi AOs in the context of bipolar fuzzy [43,56] structures. The performance of Dombi AOs when associated with Hamy [57] and Heronian [17] mean can also give us productive results in MADM problems. Further, such AOs-based theories can be utilized in Fintech risks [58], in the evaluation of noise reduction devices [59], and in the evaluation of green supply chains [59,60].

Author Contributions: Conceptualization, M.W. and K.U.; methodology, M.W., K.U. and D.P.; software, G.J., Đ.V. and K.U.; validation, D.P., M.W. and K.U.; investigation, M.W., K.U. and D.P.; writing—original draft preparation, M.W., K.U., D.P., G.J. and Đ.V.; writing—review and editing, M.W., K.U., D.P., G.J. and Đ.V.; supervision, M.W., K.U. and D.P. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: Not applicable.

Acknowledgments: The authors are thankful to the office of Research, Innovation, and Commercialization (ORIC) of Riphah International University Lahore for supporting this research under the project: Riphah-ORIC-21-22/FEAS-20.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

DTN	Dombi t-norm
TN	T-norm
TCN	T-conorm
MG	Membership grade
NMG	Non-MG
AO	Aggregation operator
IVIF	Interval-valued intuitionistic fuzzy
IVIFN	IVIF number
IVIFDWA	IVIF Dombi weighted averaging
IVIFDWG	IVIF Dombi weighted geometric
FS	Fuzzy set
IFS	Intuitionistic FS
HD	Hesitancy degree
MADM	Multi-attribute decision-making
IVIFDOWA	IVIF Dombi ordered weighted averaging
IVIFDHA	IVIF Dombi hybrid averaging
IVIFDWG	IVIF Dombi ordered weighted geometric
IVIFDHG	IVIF Dombi hybrid geometric

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