



Article Dual-Zone Gas Flow Characteristics for Gas Drainage Considering Anomalous Diffusion

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Abstract: Gas drainage in deep coal seam is a critical issue ensuring the safety of mining and an important measure to obtain gas as a kind of clean available energy. In order to get a better understanding of gas flow and diffusion for gas drainage in deep coal seams, a dual-zone gas flow model, including the drainage damage zone (DDZ) and the non-damaged zone (NDZ), are characterized by different permeability models and anomalous diffusion models to analyze the influence of damage induced by drilling boreholes on gas flow. The permeability model and anomalous diffusion model are verified with experiment and field data. A series of finite-element numerical simulations based on developed models are carried out, indicating that, compared with normal diffusion model, the anomalous diffusion is more accurate and appropriate to field test data. The coal fracture permeability increases rapidly with the distance decreasing from the borehole, and the area of DDZ is increasing significantly with the extraction time. Moreover, with the increasing of fractional derivative order, the diffusion model transforms the anomalous diffusion to the normal gradually, and the decay of gas pressure is aggravated. The higher value of non-uniform coefficient results in the larger increment of fracture permeability. The permeability–damage coefficient increase makes the increment of fracture permeability bigger.

Keywords: gas drainage; anomalous diffusion; coal permeability; drainage damage zone; nondamage zone

1. Introduction

There is a large quantity of free and adsorbed gas stored in coal as a porous medium comprising organic and mineral substances [1]. Gas is a major cause of gas explosions and coal or gas outburst disasters [2,3], which are more prone to happen in deep coal seams; meanwhile, it is an efficient clean energy. Unconventional reservoirs, including shale oil and gas, tight sand oil and gas, and coalbed methane, will change energy development patterns of the world [4]. Gas drainage through drilling boreholes, as an available and common measure to extract the gas in coal seams, has been the focus of relevant researches. In order to reduce gas accidents and expand gas collection and utilization, mastering the characteristic of gas flow during gas drainage is critical. Furthermore, it is supposed that the gas flow in the coal seam follows Darcy's law in this study; that is, in the Darcy's law equation, when the gas pressure gradient is constant, the greater the permeability, the greater the gas drainage flow rate. As the most crucial parameter for the gas production that determines the law of gas flow, the permeability of coal is incredibly essential and meaningful in studying gas drainage.



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There are a number of theoretical models of permeability evolution law. Seidle [5] put forward the matchstick model and applied it to the stress-dependent permeability of coals and deduced that the permeability changed exponentially with effective stress. After that, many scholars have researched and developed the permeability models on the basis of this geometric model. Shi [6] established a model for pore pressure-dependent cleat permeability under uniaxial strain conditions. Cui [7] thought we should take strain induced by adsorption, pressure and mechanical properties into account. Wu [8] put forward that the gas flow and transport happened in matrix and fracture of coal, which was summarized "dual porosity-dual permeability model". Pan [9] demonstrated a lot of permeability models considering effective stress and gas adsorption. This research is about permeability evolution of compression stage in coalbed methane mining based on uniaxial strain assumption, and there are few models describing the whole evolution process including permeability evolution in dilation stage. Zhou [10] suggested an improved permeability model which first decreases, then maintains an unchanged level, and then increases during creep tests in deep coal. On the one hand, these above models are proposed based on uniaxial strain hypothesis, while the coal is under triaxial compression circumstance during gas drainage. On the other hand, in the theoretical research of gas drainage, the evolution of permeability is monotone with effective stress or strain in previous models. In the actual gas extraction process, however, the permeability in some zones increases due to the development of fractures and aggravation of damage. In order to describe all the process of gas migration when coal and gas are mined together under the condition of three-dimensional compression, a more comprehensive model in line with mining is needed for study.

At present, most scholars believe that the CBM reservoir is a dual-porosity system composed by microporous matrix and natural random fractures. The matrix has the great ability to store gas adsorbed, while the fracture networks play a quite significant role in transporting free gas [11]. Song [12] considered simultaneously the diffusion and gas flow for nonlinear gas transport in multiscale porous media. Gao [13] developed a coupling model of gas diffusion and seepage based on Fick's law. Ye [14] proposed a new multi-field coupling model, which considered fracture and pore structure in coal seam. Liu [15] developed a multi-field coupling seepage model by introducing a dynamic diffusion coefficient taking the timeliness of diffusion into account. However, the classical Fick's law was not suitable for describing the tracer transport in porous media [16,17]. In this paper, the anomalous diffusion [18] is introduced to describe the diffusion behavior in deep coal.

Actually, the gas migration in coal seams is a series and parallel connection process of diffusion and seepage. However, the amount of gas directly entering the borehole by diffusion is quite small. As a result, the gas migration can be simplified as a series process between diffusion and seepage. The first step is the gas diffusion from matrix to fracture in the form of anomalous diffusion, and gas flow is the second step from fractures to borehole in Darcy's law.

In this work, the permeability model and anomalous diffusion model based on triaxial strain hypothesis are applied to simulate gas drainage in coal seams. Considering damage induced by drilling boreholes, a novel dual-zone model is proposed to character drainage damage zone (DDZ) and non-damage zone (NDZ). Permeability and gas pressure evolution with time and distance is investigated by a series of numerical simulations. The effects of anomalous derivative order, non-uniform coefficient and permeability-damage coefficient on gas flow are analyzed for serious scenarios under various conditions.

2. Conceptual Model and Governing Equations

Underground gas drainage is one of main methods for relieving gas pressure and reducing gas concentration in coal seams. To ensure the safety and efficiency of gas drainage, it is necessary to analyze the diffusion and seepage of gas in matrix and fracture, respectively. This paper takes the deep coal mines in the Pingdingshan area as the engineering background, with the stress state of the coal seam set under the in situ conditions [19]. As shown in Figure 1a, boreholes are made for extracting gas from coal seams. Figure 1b illustrates the process of gas drainage. On the one hand, the generation of buried tubes makes deviatory stress of coal increase induced by pressure relief around boreholes, which leads to severe deformation and damage; on the other hand, the size of boreholes is small relative to the coal seams, which causes stress concentration. Furthermore, more fractures emerge around the drilling boreholes (Figure 1c) and a damage zone is formed accordingly. In order to illustrate the issue vividly, a physical conceptual model is proposed. The drainage damage zone (DDZ) and the non-damage zone (NDZ) are defined.



Figure 1. Schematic diagrams ((**a**) gas extraction in coal seam; (**b**) the process of gas extraction; (**c**) DDZ and NDZ).

Aiming at simplify issues during the process of model derivation based on representative elementary volume (REV), some desirable assumptions are stated [20–22].

- (1) Coal is a dual-porosity, elastic material, consisting of fractures and a matrix with pores.
- (2) Only fracture permeability is considered.
- (3) Coal behaves to be isotropic and gas is deemed to be an ideal gas.
- (4) The migration of gas in coal seam is treated as an isothermal process.
- (5) Both pore system and fracture system are continuous media systems.
- (6) The fissures of coal are filled with free gas, and the gas in coal matrix exists in two forms: adsorption and free gas.

2.1. Coal Deformation

Gas flow in coal seam is affected by permeability evolution, which is the performance of gas–solid coupling. In the case of external stress changes, the deformation caused by loading is the main factor affecting the permeability evolution of coal. Stress equilibrium relations can be presented as:

$$\sigma_{ij,j} + f_i = 0 \tag{1}$$

where f_i is the body force.

The geometric equation describing the relation between strain and displacement is:

$$\varepsilon_{ij} = \frac{1}{2} \left(u_{i,j} + u_{j,i} \right) \tag{2}$$

where ε_{ij} means the component of the strain tensor. u_i is the displacement in direction of *i*, and *i*, *j* = *x*, *y*, *z*.

On the basis of the generalized Hooke's law, the governing equation of coal deformation can be written as [23,24]:

$$Gu_{i,jj} + \frac{G}{1-2\nu}u_{j,ji} - \beta p_i - K\varepsilon_{s,i} + f_i = 0$$
(3)

where *G* is the shear modulus, GPa and $G = \frac{E}{2(1+\nu)}$; *E* is elastic modulus, GPa, and ν is the Poisson's ratio. u_i is the displacement of the direction *i* (*i* = *x*, *y*, *z*); β is the Biot's coefficient; *p* is gas pressure, MPa. *K* is the bulk modulus, GPa, and $K = \frac{E}{3(1-2\nu)}$. ε_s is the adsorption swelling or desorption shrinkage strain. f_i is the is the body force of coal.

2.2. Gas Flow Model in Coal Fractures

It is assumed that the fracture deformation affected by external factors only has an impact on the fracture size but has no effect on the matrix deformation. Then when the change of volume strain of the fracture occurs, nothing but the fracture aperture varies. Fracture porosity change can be calculated by fracture strain [15]:

$$\frac{\phi_f}{\phi_{f0}} = \frac{L_f + \Delta L_f}{L_f} = 1 + \Delta \varepsilon_f \tag{4}$$

where ϕ_f is fracture porosity; ϕ_{f0} is the initial value; L_f is the width of fracture; ΔL_f is the increment of fracture width; $\Delta \varepsilon_f$ is the fracture strain, which is caused by effective stress and adsorption in this paper.

Considering fracture as an elastic medium, the fracture strain induced by effective stress $\Delta \varepsilon_f^E$ can be obtained by generalized Hooke law under triaxial conditions, i.e., [25]

$$\Delta \varepsilon_f^E = \frac{1}{E_f} \left[\Delta \sigma_y^e - \nu_f (\Delta \sigma_x^e + \Delta \sigma_z^e) \right]$$
(5)

where E_f is the elastic modulus of fracture, GPa. $\Delta \sigma_x^e$, $\Delta \sigma_y^e$ and $\Delta \sigma_z^e$ are increment of effective stress in x, y, and z directions, respectively. v_f means Poisson's ratio of fracture, which is the same value to that of coal with fractures.

To simplify the problem in process of model derivation, we assume that the coal is composed by coal matrix and fracture, thus the whole aperture of coal mass L equals the sum of width of matrix L_m and that of fracture L_f in geometry,

$$L = L_m + L_f \tag{6}$$

Referring to the literature from Liu [26], the fracture porosity ϕ_f can be expressed as:

$$\phi_f = \frac{L^3 - L_m^3}{L^3} \cong \frac{3L_f}{L_m} \tag{7}$$

As shown in Figure 2, the matrix swelling deformation ΔL_m^S induced by gas adsorption result in the coal mass deformation ΔL^S and fracture deformation ΔL_f^S , the relation can be given by:

$$\Delta L_m^S = \Delta L^S + \Delta L_f^S \tag{8}$$



Figure 2. Schematic diagram of matrix swelling deformation.

The mineral fillers between adjacent fracture surfaces limit matrix deformation into fracture to some extent, and the matrix will not expand evenly and equally on both sides [27]. Therefore the non-uniform deformation coefficient μ is introduced to describe the contribution of the matrix deformation to fracture ($0 < \mu \le 0.5$), then the deformation change of coal mass behaves $(1 - \mu)\Delta L_m^s$ given by:

$$\Delta L_f^s = \mu \Delta L_m^s, \ \Delta L^s = (1 - \mu) \Delta L_m^s \tag{9}$$

In extreme circumstances, when there are no mineral fillers or the surfaces of the fracture are parallel each other, the coal matrix expands uniformly on both sides and the contribution of matrix deformation on fracture and coal mass are both 0.5, i.e., $\mu = 0.5$. On the contrary, if there were no fractures, all the matrix deformation acts on coal mass, and the parameter μ , would be equal to 0.

Combining Equations (7)–(9), one obtains:

$$\Delta \varepsilon_f^s = \frac{\Delta L_f^s}{L_f} = \frac{\mu \Delta L_m^s}{L_f} = \frac{3\mu \Delta \varepsilon_m^s}{\phi_f} \tag{10}$$

where ϕ_f is fracture porosity of coal. $\Delta \varepsilon_m^S$ is the matrix strain resulting from gas adsorption. Assuming that expansion of matrix caused by gas adsorption is uniform in *x*, *y* and *z* directions, i.e.,

$$\Delta \varepsilon_m^S = \frac{1}{3} \Delta \varepsilon_{mV}^S \tag{11}$$

where $\Delta \varepsilon_{mV}^{s}$ is volumetric strain owing to gas adsorption and it can be expressed by Langmuir Equation [28]:

$$\Delta \varepsilon_{mV}^{S} = \varepsilon_L \left(\frac{p}{p + p_L} - \frac{p_0}{p_0 + p_L} \right) \tag{12}$$

where ε_L is the Langmuir volumetric strain. p_L is the Langmuir pressure constant, MPa. p and p_0 are the pore pressure and initial values, respectively, MPa.

Combining Equations (10)–(12), the fracture strain induced by adsorption can be written as:

$$\Delta \varepsilon_f^S = \frac{\mu \varepsilon_L}{\phi_f} \left(\frac{p}{p + p_L} - \frac{p_0}{p_0 + p_L} \right) \tag{13}$$

Taking into account the effects of gas adsorption and effective stress, the whole fracture strain of coal can be given by:

$$\Delta \varepsilon_f = \Delta \varepsilon_f^S + \Delta \varepsilon_f^E \tag{14}$$

Substituting Equations (5) and (13) into Equation (14) yields:

$$\frac{\phi_f}{\phi_{f0}} = 1 + \frac{\mu\varepsilon_L}{\phi_{f0}} \left(\frac{p}{p+p_L} - \frac{p_0}{p_0+p_L} \right) + \frac{1}{E_f} [\Delta\sigma_y^e - \nu_f (\Delta\sigma_x^e + \Delta\sigma_z^e)]$$
(15)

The permeability model considering effective stress and adsorption swelling can be expressed according to the cubic law,

$$\frac{k_f}{k_{f0}} = \left\{ 1 + \frac{\mu\varepsilon_L}{\phi_{f0}} \left(\frac{p}{p+p_L} - \frac{p_0}{p_0+p_L} \right) + \frac{1}{E_f} \left[\Delta \sigma_y^e - \nu_f (\Delta \sigma_x^e + \Delta \sigma_z^e) \right] \right\}^3 \tag{16}$$

During the process of gas drainage, the development of borehole induces stress redistribution, stress concentration and damage in coal seams. In order to characterize the extent of damage, the calculation formula of damage [29] is introduced from the perspective of strain in this paper, which can be described as:

$$D = \begin{cases} 0 & 0 < \varepsilon \le \varepsilon_t \\ \frac{\varepsilon_u(\varepsilon - \varepsilon_t)}{\varepsilon(\varepsilon_u - \varepsilon_t)} & \varepsilon_t < \varepsilon < \varepsilon_u \end{cases}$$
(17)

where ε_t is the strain threshold value of damage evolution; ε_u is the ultimate strain. In other words, there is no damage when strain does not reach the threshold.

The variation tendency of the damage variable obtained by this model is drawn in Figure 3. Meanwhile, the aggravation of damage degree is illustrated combined with the rupture diagram of coal sample obtained from laboratory experiment.



Figure 3. Diagram of damage variable model.

Considering to the effect of damage on permeability evolution and referring to Zhu [30] and Ren [31], the exponential item of permeability change induced by damage is incorporated into original permeability expression, Equation (16). The permeability evolution model is derived under the comprehensive effects of effective stress and damage, which is shown as Equation (18).

$$\frac{k_f}{k_{f0}} = \begin{cases} \left\{ 1 + \frac{\mu\varepsilon_L}{\phi_{f0}} \left(\frac{p}{p+p_L} - \frac{p_0}{p_0+p_L} \right) + \varepsilon_f \right\}^3 & 0 < \varepsilon_f \le \varepsilon_t \\ \left\{ 1 + \frac{\mu\varepsilon_L}{\phi_{f0}} \left(\frac{p}{p+p_L} - \frac{p_0}{p_0+p_L} \right) + \varepsilon_f \right\}^3 \cdot \exp\left[\lambda \cdot \frac{\varepsilon_u(\varepsilon_f - \varepsilon_t)}{\varepsilon_f(\varepsilon_u - \varepsilon_t)} \right] & \varepsilon_t < \varepsilon_f < \varepsilon_u \end{cases}$$
(18)

where λ is permeability–damage coefficient presenting the influence of damage on permeability.

Assuming that fracture is the only channel of permeability, then the mass balance equation of gas in coal fracture is the following equation:

$$\frac{\partial m_f}{\partial t} + \nabla \cdot \left(\rho_g q_g\right) = Q_m \tag{19}$$

where m_f means the free-phase gas content of coal fracture, *t* is the gas flow time, ρ_g is the gas density, and q_g is the Darcy velocity vector, which can be obtained by Equation (20), and Q_m is gas mass exchange between pores and fractures, kg/(m³·s).

$$q_g = -\frac{k_f}{\omega} \nabla p \tag{20}$$

where ω is dynamic viscosity. Substituting Equation (20) into (19), we can obtain the mass balance equation of gas in the coal fracture.

$$\frac{\partial m_f}{\partial t} - \nabla \cdot \left(\rho_g \nabla p \frac{k_f}{\mu} \right) = Q_m \tag{21}$$

Combining the novel fracture permeability model containing damage (Equation (18)) with Equation (21), the gas flow model in coal fracture can be used in simulations work.

2.3. Gas Diffusion Model in Coal Matrix

(1) The governing equation of gas mass exchange [32] is given by:

$$Q_m = F\chi \frac{M}{RT} (p_m - p_f)$$
⁽²²⁾

where *F* is the gas diffusion coefficient, m^2/s , χ is the shape factor of matrix, and $\chi = \frac{3\pi^2}{L^2}$ (*L* is the width of fractures). *M* is the molar mass of CH₄, kg/mol, and *R* is the gas constant (8.3143 J/(mol·K)). p_m and p_f are gas pressure in pores and fractures, respectively, MPa, and they equal under initial conditions.

(2) The matrix gas content for unit volume m_m [33] is obtained by Langmuir equation:

$$m_m = \frac{abp_m \rho_c M}{(1+bp_m)V_m} + \varphi_m \frac{Mp_m}{RT}$$
(23)

where *a*,*b* are Langmuir constants, $V_m = 22.4 \text{ L/mol}$, ρ_c is the apparent density of coal, kg/m³, and φ_m is the matrix porosity, %.

(3) According to the law of mass conservation, the governing equation of matrix gas diffusion is described by:

$$\frac{\partial m_m}{\partial t} + Q_m = 0 \tag{24}$$

Substituting Equations (22) and (23) into Equation (24), the governing equation of gas pressure in pores can be pointed out as follows [34]:

$$\frac{\partial p_m}{\partial t} = -\frac{3\pi^2 F V_m (1+bp_m)^2 (p_m - p_f)}{L^2 [ab\rho_c RT + \varphi_m V_m (1+bp_m)^2]}$$
(25)

Considering to a space and time memory effects of gas in porous media, anomalous diffusion is introduced in this paper. According a new approach, Zhou [35] described the anomalous diffusion based on conformable derivative, which can be presented by the fractional derivative:

$$\frac{\partial^{\alpha} p_m}{\partial t^{\alpha}} = -\frac{3\pi^2 F V_m (1+bp_m)^2 (p_m-p_f)}{L^2 [ab\rho_c RT + \varphi_m V_m (1+bp_m)^2]}$$
(26)

Take the function f(t) for example, the definition of conformable derivative is [36]:

$$T_{\alpha}f(t) = \lim_{\varepsilon \to \infty} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}$$
(27)

where α is the conformable derivative order and $\alpha \in (0, 1]$.

Furthermore, the relationship of conformable fractional derivative and the first derivative can be represented as [36]:

$$T_{\alpha}f(t) = t^{1-\alpha}\frac{df(t)}{dt}$$
(28)

Substituting Equation (28) into Equation (26), the anomalous diffusion model describing gas transport in pores of coal can be achieved:

$$t^{1-\alpha}\frac{\partial p_m}{\partial t} = -\frac{3\pi^2 F V_m (1+bp_m)^2 (p_m - p_f)}{L^2 [ab\rho_c RT + \varphi_m V_m (1+bp_m)^2]}$$
(29)

The anomalous diffusion model (Equation (29)) and the gas flow model (Equation (21)) are combined to simulate gas flow and diffusion behaviors.

3. Geometric Models and Parameters

A 2-D rectangular numerical model with length of 10 m and width of 5 m is established (Figure 4). The borehole represented by a circle of radius 0.3 m is created in the center of the numerical model. The lateral and bottom boundaries are fixed; at the same time, the top boundary is loaded by stress of 25 MPa. The initial gas pressure of 6 MPa is applied. For drainage borehole, the pressure is atmospheric, i.e., 101.325 kPa. Except for drainage borehole boundary, all the boundaries are in a no flow condition. We take the red point away from the center of the borehole as a measure point.



Figure 4. Numerical model.

The numerical simulation work is completed by using COMSOL Multiphysics software and employing solid mechanics module and Darcy flow in fluid flow module. The discretization method of the numerical study is finite element method (FEM), which is applicable and stable solving gas drainage problems. In addition, parameters utilized in this numerical simulation and their sources are listed in Table 1.

Table 1. Basic parameters for numerical simulation.

Parameter	Value	Data Sources
Elastic modulus <i>E</i> , GPa	4.5	Zhou [37]
Poisson's ratio ν	0.38	Experiments
Shear modulus G, GPa	1.63	Experiments
Bulk modulus <i>K</i> , GPa	9.38	Experiments
Initial fracture permeability k_{f0} , m ²	$5 imes 10^{-18}$	Experiments
Non-uniform deformation coefficient μ	0.4	Fitting data
Langmuir volumetric strain constant ϵ_L	0.01266	Zhao [38]
Langmuir volumetric constant a , m ³ /kg	0.048	Zhao [38]
Langmuir pressure constant p_L , MPa	2	Zhao [38]
Initial fracture porosity φ_{f0}	0.1	Experiments
Critical strain ε_t	0.01	Experiments
Residual stain ε_u	0.045	Experiments
Permeability-damage coefficient λ	$7.2 imes 10^{-7}$	Fitting data

4. Simulation Results and Model Verification for Gas Drainage

4.1. The Fracture Permeability Evolution around the Gas Drainage Center

By simulating gas flow during drainage process, permeability evolution curves and nephograms are exhibited in Figure 5 with the distance from the center of the research object at diverse time points 1 day, 10 days and 30 days, respectively.





Figure 5. Fracture permeability evolution ((**a**) the permeability curves with distance and (**b**) the permeability nephograms at different times).

The evolution of fracture permeability with distance (Figure 5a) presents a symmetrical picture centered on the borehole. The permeability in the zone close to borehole (i.e., DDZ) increases rapidly from the initial value of 5×10^{-18} m² with distance decreasing. On the contrary the permeability of the zone away from borehole (i.e., NDZ) declines firstly and levels out eventually from the initial fracture permeability as the distance from center is increasing. As seen in Figure 5a, take the dotted line of $k_{f0} = 5 \times 10^{-18}$ m² as the demarcation line, the zone above the line is drainage damage zone (DDZ) of permeability increasing and below is the non-damage zone (NDZ).

As an additional illustration, the permeability nephogram (Figure 5b) is drawn. With gas drainage time going on, the area of drainage damage zone (the dotted circle) is increasing, with the reason that, during gas extraction, pore pressure decreases while the effective stress rises. The deformation and damage around the borehole increase. Therefore, the fracture permeability increases. At the same time permeability, the DDZ gradually increases and is significantly greater than that of the NDZ at any time, which is consistent with the permeability evolution during the process of gas drainage in a high gassy mine.

4.2. The Effect of Fractional Derivative Order on Gas Diffusion

In Equation (29), when $\alpha = 1$, the model represents the normal diffusion model, while $\alpha \neq 1$, it means the abnormal diffusion model. To illustrate the accuracy of anomalous diffusion on field, the relevant data about gas flow is adopted from Zhao [38]. By fitting the field data, the comparison result of anomalous diffusion ($\alpha = 0.87$) and normal diffusion ($\alpha = 1$) is manifested in Figure 6, where gas flow rate ratio refers to the ratio of the gas flow rate to the initial value. The anomalous diffusion is better to describe the field data while the gas flow rate declines more quickly during gas drainage with the normal diffusion model. In view of this feature, the anomalous diffusion model is used in this paper to analyze gas diffusion characteristic in the coal matrix.



Figure 6. Matching result comparison of anomalous and normal diffusion with field test data.

It can be seen from the anomalous diffusion formula that the change of fractional derivative order affects the gas pressure. Therefore, the influence of conformable derivative order on gas pressure evolution with gas drainage time is discussed by curves and nephograms (Figure 7).





Taking the measure point as an example, there is a huge damping of gas pressure at the beginning of gas drainage followed by a long-time equilibrium state. As the fractional derivative order increases, the decline of gas pressure is faster. From another perspective when α rises from 0.6 to 1, the conversion process happens from anomalous diffusion to normal diffusion. In other words, the normal diffusion model takes a longer time to reach a state of equilibrium and the damping decrement of gas pressure is larger. The nephogram describes the gas pressure changing in the whole study zone at 30 days. Similar result that the gas pressure drops off obviously with conformable derivative orders increasing is

found. It demonstrates that the gas pressure with anomalous diffusion model is greater than that with normal diffusion law.

4.3. The Effect of Non-Uniform Coefficient on Fracture Permeability

In order to depict the attribution of matrix deformation to fracture, the non-uniform coefficient is introduced in the permeability model. Numerical simulations for various scenarios are conducted to illustrate the effect of non-uniform coefficient on fracture permeability. The fracture permeability trend versus gas drainage time is given in Figure 8.



Figure 8. The effect of non-uniform coefficient on fracture permeability evolution.

When the drainage proceeds for 30 days, compared with the initial value, the fracture permeability increases 5%, 7% and 10% for μ = 0.2, 0.3 and 0.4, respectively. The cause of the change can be traced that pore pressure decreases during gas extraction, which results in shrinkage of coal matrix. Accordingly, the width of the fracture increases, and while μ rises, the increasing attribution of the matrix deformation on the fracture results in the fracture deformation growing, as illustrated vividly in Figure 9. Consequently, the increment of permeability is larger. Meanwhile, a larger non-uniform coefficient demonstrates fewer mineral fillers between fracture faces or the surfaces of fracture are closer to parallel, furthermore the rise tendency of permeability with time is more evident.



Figure 9. Schematic diagram of fracture change induced by non-uniform coefficient (the colorful circles represent mineral filler and other inclusion in fracture of coal).

4.4. The Effect of Permeability-Damage Coefficient on Fracture Permeability

According to Equation (16), when the strain is more than the threshold value, there will be an additional factor affecting permeability, i.e., mechanical damage (Equation (30)). To illustrate the influence of the damage item on fracture permeability, the permeability evolution is studied against pore pressure for the various permeability–damage coefficient λ , depicted in Figure 10.

$$\frac{k_f}{k_{f0}} = \left\{ 1 + \frac{\mu\varepsilon_L}{\phi_{f0}} \left(\frac{p}{p + p_L} - \frac{p_0}{p_0 + p_L} \right) + \varepsilon_f \right\}^3 \cdot \exp\left[\lambda \cdot \frac{\varepsilon_u(\varepsilon_f - \varepsilon_t)}{\varepsilon_f(\varepsilon_u - \varepsilon_t)} \right]$$
(30)



Figure 10. The effect of permeability-damage coefficient on fracture permeability.

As shown in Figure 10, fracture permeability increases when gas pressure reduces from 6 MPa to atmospheric pressure. The reason is that gas pressure in coal fracture decreases gradually as gas drainage is carried out, which leads to gas desorption. Fracture permeability increases as a result of matrix shrinkage. On the other side, the reduction of gas pressure results in effective stress rising and the fracture aperture dilates for drainage damage zone that causes increasing permeability.

Furthermore, that the increase of fracture permeability with gas pressure decreasing is various for three scenarios, i.e., $\lambda = 0$, 0.5, 1, and can be observed in Figure 10. Under the same gas pressure condition, the permeability is comparatively small when the permeability-damage coefficient is equal to 0 (no damage). With λ increasing, the increment of fracture permeability increases. To be specific, fracture permeability behaves at 1.97, 2.05 and 2.13 times increase compared with the initial values for different permeability–damage coefficients. The result illustrates that fracture permeability of coal may be underestimate if the damage is ignored.

5. Conclusions

Based on the engineering background of gas drainage in deep coal seams, a series of numerical simulations were carried out to analyze gas flow. The drainage damage zone (DDZ) caused by drilling borehole and the non-damaged zone (NDZ) were characterized respectively by a damage–induced permeability evolution model. The anomalous diffusion model expressed by fractional derivative were validated by field data. The sensitivity of conformable derivative order on gas pressure and the influence of non-uniform and permeability-damage coefficients on fracture permeability evolution were analyzed and possible causes are stated in this paper. The crucial conclusions can be concluded as follows:

- (1) The permeability of DDZ increases from the initial value with distance from center decreasing, while it rapidly declines firstly and tends to be stable in NDZ with distance increasing away from the borehole center. Furthermore, as gas drainage time goes on, the area of DDZ increases evidently.
- (2) The conversion process happens from normal diffusion to anomalous diffusion as the fractional derivative order goes down from 1. Compared with normal diffusion model, the gas flow rate declines more slowly using the anomalous diffusion model, which is more consistent with the field data.
- (3) When the non-uniform deformation coefficient is larger, the increment of permeability curves is more pronounced. Fracture permeability is large considering damage compares with the condition of no damage during gas drainage, and it may be underestimate if the damage is ignored.

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Nomenclature

- *α* Conformable derivative order
- *a,b* Langmuir constant
- β Biot's coefficient
- D Damage variable
- ε_{ij} Strain tensor
- ε_s Adsorption strain
- $\varepsilon_t, \varepsilon_u$ Threshold and ultimate strain
- ε_L Langmuir volumetric strain
- $\Delta \varepsilon_f$ Fracture strain
- $\Delta \varepsilon_f^E$ Fracture strain induced by effective stress
- $\Delta \varepsilon_f^s$ Fracture strain induced by gas adsorption
- $\Delta \varepsilon_{mV}^{s}$ Volumetric strain induced by gas adsorption
- $\Delta \varepsilon_m^s$ Matrix strain induced by gas adsorption
- *E* Elastic modulus
- E_f Elastic modulus of fracture
- f_i Body force
- *F* Gas diffusion coefficient
- G Shear modulus
- *K* Bulk modulus
- *k*_f Fracture permeability
- k_{f0} Initial fracture permeability
- *L* Aperture of coal mass
- *L_m* Matrix width
- L_f Fracture width
- ΔL^S Coal mass deformation induced by gas adsorption
- ΔL_f Increment of fracture width

- ΔL_m^S Matrix swelling deformation induced by gas adsorption
- ΔL_f^S Fracture deformation induced by gas adsorption
- m_f Free-phase gas content in fracture
- m_m Gas content for unit volume
- M Molar mass of CH₄
- μ Non-uniform coefficient
- *p* Gas pressure
- p_m , p_f Gas pressure in pores and fractures
- p_L Langmuir pressure constant
- ∇p Increment of gas pressure
- q_g Darcy velocity vector
- Q_m Gas mass
- R Gas constant
- T Temperature
- u_i Displacement ν Poisson's ratio
- v_f Poisson's ratio of fracture
- V_m Gas molar volume
- ω Dynamic viscosity
- χ Shape factor of matrix
- λ Permeability-damage coefficient
- ρ_{g},ρ_c Gas density and coal density
- φ_m Matrix porosity
- ϕ_f Fracture porosity
- ϕ_{f0} Initial fracture porosity
- σ_{ij} Stress tensor
- $\Delta \sigma_i^e$ Increment of effective stress

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