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Design of a Three-Phase Shell-Type Distribution Transformer Using Evolutionary Algorithms

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Abstract: In this paper, three metaheuristic optimization algorithms: genetic algorithm (GA), particle swarm optimization (PSO), and differential evolution (DE) are compared in terms of minimizing the total owning cost (TOC) of the active part of a three-phase shell-type distribution transformer. The three methods use six inputs: power rating, primary voltage, secondary voltage, primary and secondary winding connections, and frequency. The TOC of the transformer, which includes the cost of the basic materials of the transformer plus the cost of losses, is minimized under the imposed constraints (excitation current, impedance, no-load losses, load losses, and efficiency) usually specified in the standards. As a case study, the three algorithms are applied to optimize the design of a three-phase shell-type distribution transformer of 750 kVA. All applied metaheuristic algorithms provide good results, while DE avoids local optima leading to better TOC reduction. The results of the optimization algorithms used are superior to those of the manufacturer, showing a 6% TOC reduction. Optimization of the design of a power transformer may have important implications for reducing greenhouse gas emissions and extending the lifetime of the equipment.

Keywords: evolutionary algorithms; genetic algorithm; particle swarm optimization; differential evolution; total owning cost

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1. Introduction

To survive in a highly competitive world, transformer manufacturers need to run algorithms capable of producing optimal designs in a short duration. A transformer is an important element in the power supply system. It is a device in which a common magnetic circuit links two or more electrical circuits into one system. Since the invention of the first transformer in 1885 by the Hungarians K. Zipernowsky, O. Blaty, and M. Dery, the technology of transformer production has evolved significantly. The computer was first used in transformer design in 1955 [1]. Since then, computer technology has become an important element in the transformer design process. Several specialized computational techniques have been developed after 1970 [2] in this field. Then, the so-called metaheuristic optimization methods have become a suitable alternative in transformer design after 1999 [2] because they require less time and less computational resources (compared to the Finite Element Method, for instance) to obtain competitive solutions while minimizing the TOC of the transformer.

In [3], the so-called brute force methodology for the design of a single-phase transformer was presented. However, the disadvantage of the implemented method was that it analyzed all possible combinations of the input values to achieve a good solution. In [4], GA and Simulated Annealing techniques were used to minimize the mass of the windings and the core of a high-frequency transformer, resulting in a reduction in mass compared to the results of the geometric programming. A GA was also used to optimize the active part of single-phase and three-phase transformers in [5]. The results were compared with those of FEM and the specifications of the physical prototype, showing a significant reduction in total losses. However, the cost increased in the case of the three-phase transformer. Rectifier transformer optimization using GA and PSO metaheuristics was performed in [6]. The overall estimated cost as well as the core and load losses were minimized, resulting in a similar performance compared to traditional methods. In [7], a GA was implemented to design a three-phase core-type transformer. The results were also compared with those of the conventional design methods. Two programs have been employed, both considering four objective functions: total active part cost, total losses, percentage impedance, and transformer tank volume. Unlike the second, no additional constraints were imposed in the first program. In [8], the DE was used with several strategies to minimize the cost of the active part of a distribution transformer, obtaining a significant reduction of the TOC.

Other recent optimization methods have performed well in transformer design. In [9], the designs of transformers with cores made of grain-oriented steels and amorphous materials are compared, considering the temperature effects in the windings and dielectric oil in the design process. In [10], the covariance matrix adaptation evolution strategy (CMA-ES) and Self-adaptative differential evolution (SaDE) algorithms were used to optimize the transformer design. It was demonstrated that CMA-ES shows faster convergence than SaDE in the four analyzed objective functions (purchase cost design, mass design, total loss design, and total lifetime cost). In [11], metaheuristic algorithms such as Crow Search Algorithm (CSA), Moth Flame Optimization (MFO), Vortex Optimization Algorithm (VOA), Particle Swarm Optimization (PSO), and Social Learning-Particle Swarm Optimization (SL-PSO) were used to determine the main design parameters of a dry-type transformer. The specifications obtained by using the metaheuristic algorithms are validated with Finite Element Method (FEM) analysis. The problem of transformer designing as a mixed-integer non-linear programming problem with the branch-and-bound method was formulated in [12]. The results obtained are confirmed by FEM. The authors claim that the respective software created is more user-friendly than metaheuristic algorithms. In [13], the so-called Harmony Search Multi-Objective (MOHS) and Rick Harmony Search Map (MHSR) methods were used to solve the problem of optimizing the design of a transformer as a multi-objective problem. Both proved to be equally effective and resulted in similar design parameters. In [14], a tree-pruning method was implemented inspired by plant growth and fertility. This method reduces the cost of materials in a 200 MVA transformer compared to the results obtained with other optimization algorithms such as Genetic Algorithm, Unlimited Population Algorithm, Evolutionary Multi-objective Optimization Algorithm, and Heuristic Algorithm. In [15], geometric programming with the branch-and-bound method is used in designing autotransformers to deal with short-circuit impedance sensitivity.

The literature on minimizing the TOC of three-phase distribution transformers does not mention the use of the statistical verification method when applying metaheuristic optimization. However, this is an important issue due to the stochastic nature of these methods. The need for additional minimization of TOC of the transformers of this type is due to the fact that some manufacturers use the so-called brute force methodology, which is less efficient.

In this paper three evolutionary algorithms solve the distribution transformer optimization problem by minimizing TOC, including material cost and total loss. Over time, standards evolve calling for higher equipment efficiency, which is happening as governments worldwide introduce stricter energy efficiency policies to deal with greenhouse gas emissions. Optimizing the design of the transformer is the key to solving these problems. Although transformers are mostly made from the same materials and designed according to the laws of electromagnetism, each manufacturer has his own design programs that are difficult to apply directly to another manufacturer.

The proposed design methodology satisfies the customer specifications and all constraints imposed by national or international standards and manufacturing process constraints. The useful life of a transformer is strictly dependent on its design, so design optimization is critical to increasing the useful life of the transformer, especially since some companies do not use optimization techniques for design, and in some cases, brute force or spreadsheets.

The advantages and disadvantages of each of the three evolutionary algorithms are shown in one example, where cost and material savings and loss reduction are shown using a 3Φ, 750 kVA, 13.2/0.44 kV transformer at 60 Hz. We consider the comparison of three different algorithms to be more representative since most scientific articles provide information on only one or two algorithms.

A comparison of our results with the results of the manufacturer showed a decrease in TOC by 6% and a significant reduction in the calculation time. This test is especially interesting because it is rare to find such comparisons in the literature since manufacturers are extremely jealous of the information regarding the transformer design. Furthermore, this is a new approach to the problem, as only a few papers test the design optimization results on a manufactured prototype (see Table 1).

Metaheuristic optimization algorithms have become an alternative design technique due to their efficiency, low time, and computational resource consumption. Following are some applications where metaheuristic algorithms have proven to be successful: electrical network design [16], transformer equivalent circuit parameter estimation [17], induction motor equivalent circuit parameter estimation [18], speed and position estimation for AC motors [19], electricity cost forecasting [20], economic dispatching [21,22], reactive power flow dispatching [23], optimal power flows [24], control tuning [25,26], and automatic design analog integrated circuits [27].

Table 1. Recently applied transformer design methods.

References	Method Applied	Validation	Number of Design Equations	Number of Objective Functions	Transformer Type	Transformer Rating
[9]	No information	FEM	8	1	Shell-type, 3-phase.	800 kVA, 1600 kVA, 2500 kVA.
[10]	CMA-ES, SaDE	FEM	No information	4	Core-type, 3-phase.	150 kVA.
[11]	CSA, MFO, VOA, PSO, SL-PSO	FEM	3	No information	Core-type, Dry-type, 3-phase.	100 kVA.
[12]	MINLP	FEM	No information	1	Shell-type, 3-phase.	400 kVA.
[13]	MOHS, MHSR	Analytical	14	2	Shell-type, dry-type, 1-phase.	400 VA.
[14]	TPA-FEM	FEM and Experimentation	1	1	Core-type, 3-phase.	200 MVA.
[15]	GP- BBS	FEM	17	1	Autotransformer, core-type, 3-phase.	200 MVA.

Note: Covariance Matrix Adaptation Evolution Strategy (CMA-ES), Self-Adaptive Differential Evolution (SaDE), Crow Search (CSA), Moth-Flame Optimization (MFO), Vortex Optimization (VOA), Particle Swarm Optimization (PSO), Social Learning-Particle Swarm Optimization (SL-PSO), Mixed Integer Nonlinear Programming (MINLP), Multi-Objective Harmony Search (MOHS), Rick Maps harmony Search (MHSR), Tree Pruning Method-Finite Element Method (TPA-FEM), Geometric Programing-Branch and Bound Search (GP-BBS).

This paper is organized as follows: Section 2 provides the description of the procedure of obtaining the costs of the core and winding; in Section 3, the objective function and the corresponding constraints are obtained. We explain the simulation and the results obtained by the GA, PSO, and DE in Section 4. In Section 5, our observed conclusions and future work are presented.

2. Three-Phase Shell-Type Transformer Design Procedure

The design of the proposed shell-type three-phase distribution transformer has the following initial parameters: rated power 750 kVA, primary voltage 13,200 YT/7620 V with copper winding, secondary voltage 440 Y/254 V with an aluminum coil, and a core made of electrical M-3 steel. Figure 1 shows the active part of the transformer, consisting of a wound core and corresponding windings.

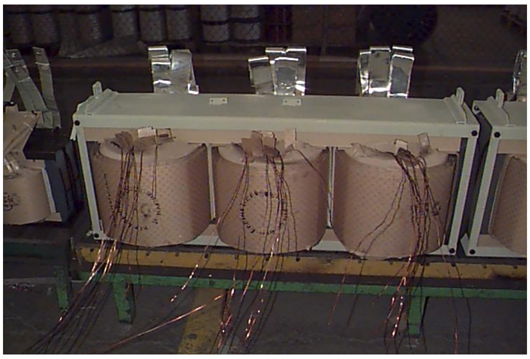

Figure 1. Transformer active part.

2.1. Windings

The mass of the conductors depends on the average length of the primary and secondary coil, number of turns, number of phases, the conductor cross-section, and the density of the conductor material [3], and is determined by the following expression:

$$M_{win} = V_m \cdot N_{coil} \cdot N_\theta \cdot S_{cond} \cdot \rho_{cond} \cdot 10^{-6} \tag{1}$$

where

M_{win} : Winding mass (kg)
V_m : Coil half-turn (mm)
N_{coil} : Number of coil turns
N_θ : Number of phases
S_{cond} : Winding conductor cross-section (mm^2)
ρ_{cond} : Conductor density (kg/mm^3)

The power losses in the primary and secondary windings are obtained as follows [3]:

$$W_{win} = J_{win}^2 \cdot M_{win} \cdot W_{d(cond)} \cdot W_k \tag{2}$$

where

J_{win} : Current density (A/mm^2)
$W_{d(cond)}$: Volumetric resistivity and density of the conductive material ($\Omega \cdot$ mm^4/kg)
W_k : Eddy current losses factor

For the cost of the winding, we get

$$C_{win} = k_{mat} \cdot M_{win} \tag{3}$$

where k_{mat} is the unit cost of the conductive material (copper or aluminum) [US$/kg].

2.2. Core

The magnetic circuit is the essential active part of a transformer that transfers energy from one electrical circuit to another. It is composed of laminations providing thereby a low reluctance path to the magnetic flux produced by the energized winding [28]. When the winding of the three-phase transformer is energized with a sine-form wave, the induced voltage then becomes:

$$V_{prim} = \sqrt{2} \cdot \pi \cdot N_{prim} \cdot \Phi_m \cdot f \tag{4}$$

where

V_{prim}:	Primary voltage
N_{prim} :	Number of primary turns
Φ_m :	Magnetic flux (Wb)
f :	Frequency (Hz)

The effective area in the core considering a frequency of 60 Hz can be obtained according to the formula:

$$A_{ef} = \frac{34{,}945.3281 \cdot (V_{sec}/N_{sec})}{B_m \cdot F_l} \tag{5}$$

where the number of volts per turn $V_{sec}/N_{sec} = V_{prim}/N_{prim}$ is the same in both windings and F_l is the lamination factor which quantifies the insulating material in the core [29]. From [3] the core thickness and mass are obtained as follows:

$$E = \frac{A_{ef}}{2 \cdot D}$$

$$M_{core} = \left(2 \cdot E \cdot (F + G) + E^2\right) \cdot F_l \cdot D \cdot \rho_{core} \cdot 10^{-6}$$

respectively. Here,

F :	Core window width (mm)
G :	Core window height (mm)
D :	Core sheet width (mm)
ρ_{core} :	Density of the magnetic core material (kg/mm^3)

Information about losses per unit weight in the core of the M-3 grade magnetic material is important in the transformer design. These losses depend on the magnitude of the magnetic flux density B_m. Such dependence is usually provided by the supplier in the form of curves "losses/kg vs. flux density". Nevertheless, an analytical form of such a dependence is preferable in the optimization procedure. Analytically, the relationship "losses/kg vs. flux density" can be modeled as a fifth-order polynomial [3] of the following form:

$$w_{kg} = 35.7028 - 15.2996 \cdot B_m + 2.5425 \cdot B_m^2 - 0.2037 \cdot B_m^3 + 0.007907 \cdot B_m^4 - 0.00011 \cdot B_m^5 \tag{6}$$

As a result, the no-load losses are calculated as follows

$$W_{core} = M_{core} \cdot w_{kg} \tag{7}$$

and the core cost takes the form:

$$C_{core} = k_{core} \cdot M_{core} \tag{8}$$

where k_{core} is the unit cost of the core material [US$/kg].

2.3. Operating Constraints

Constraints, such as power, efficiency, impedance, etc. ensure that the obtained parameters of the transformer correspond to all the necessary characteristics. The following specifies the limitations that are included in the transformer design optimization.

2.3.1. Excitation Current

Acquiring the excitation current of transformers involves conducting a no-load test under conditions where the high-voltage winding remains open. In contrast, the low voltage is connected to the rated voltage. The flow of excitation current through the transformer winding solely under the specified conditions stimulates the transformer core. Although numerous factors significantly influence the excitation current value, it is crucial to consider the following factors: the annealing process, mechanical process, operating conditions, magnetic material, assembly process, and core design process [30]. The excitation current is determined by

$$\%I = \frac{VA_{fe}}{10 \cdot S_T} \tag{9}$$

where:

S_T : Transformer rating (kVA)
VA_{fe} : Apparent core losses (VA)

The apparent core losses are given by

$$VA_{fe} = P_n \cdot VA/kg \cdot F_c \tag{10}$$

where:

P_n : Core weight
VA/kg : Volt Ampere per kilogram for a given magnetic flux density
F_c : Core empirical constant

The exciting current magnitude is usually about 1–5% of the rated current of the primary.

2.3.2. Total Losses

Total transformer losses are divided into two components: no-load and load losses. No-load losses refer to those that occur in the core when it is energized. They depend on six key factors that have been identified as critical [31]: (i) lamination insulation quality, (ii) silicon content, (iii) chemical purity, (iv) grain size, (v) crystal orientation, and (vi) core lamination thickness. Load losses are the power dissipated during a short-circuit test including losses in windings, copper eddy current losses, and stray losses in conducting parts. The genesis of the stray losses is rooted in the generation of eddy currents in the structural components of the transformer (transformer tanks). Thus, it is imperative to comprehend stray losses and their mitigation mechanisms for the enhancement of transformer design. The phenomenon of stray losses in transformers is dependent on a multitude of variables, encompassing the dimensions and structure of structural elements, as well as the properties of the materials employed in their construction. The phenomenon of stray losses exhibits an increase with the ascending magnitude of the transformer rating [32].

2.3.3. Impedance

The short-circuit impedance in a two-winding transformer is determined by shorting the secondary winding and energizing the primary winding with a reduced voltage until the rated current flows. In the laboratory, the way to calculate the $\%Z_{cc}$ is by

$$\%Z_{cc} = \frac{V_{cc}}{V_n} \tag{11}$$

where V_{cc} is the short circuit voltage and V_n is the rated voltage. The power transformer manufacturing guarantee includes transformer impedance. During the design phase, the typical short-circuit test is used to validate the impedance estimation. The ANSI/IEEE C57.12.00 standard [33] requires a tolerance of ±7.5% for two winding transformers with an impedance greater than 2.5%, and it must be satisfied by the difference between the measured impedance value and the value that the customer requested. The tolerance is ±10% for impedance levels under 2.5%.

The number of turns in a transformer winding, the material of the transformer core, the size of the transformer, and the transformer frequency are only a few of the variables that determine a transformer's impedance. The maximum level of short circuits allowed in a transformer is determined by the transformer impedance. A transformer impedance can be increased to reduce short circuit currents but also cause a drop in voltage. A low-impedance transformer, on the other hand, will have a higher short-circuit current. Depending on the application, high/low impedance transformers will be used. Transformer impedance consists of two components, the resistive and reactive parts. The real part of the impedance can be calculated as follows:

$$\%R = \frac{W_c}{10 \cdot kVA} \tag{12}$$

where:

$\%R$: Percentage resistance at 85 °C
W_c : Conductor losses (W)
kVA : Transformer rating

The imaginary part of the transformer impedance mainly represents the coil geometry contribution. The percentage of reactance is obtained according to the formula:

$$\%X = \frac{8\pi^2 \cdot f \cdot V_m \cdot 10^{-8}}{\gamma \cdot (V_{sec}/N_{sec})} \tag{13}$$

where:

γ : Average winding heights and thicknesses (mm)

The impedance percentage represents the percentage of the nominal voltage necessary to operate the transformer in short-circuit. The impedance percentage is calculated as follows:

$$\%Z = \sqrt{(\%R)^2 + (\%X)^2} \tag{14}$$

2.3.4. Efficiency

The performance and aging of a transformer are directly impacted by its efficiency. Customers have been placing high demands on transformer efficiency due to environmental concerns (the greenhouse effect) and growing energy bills. Even though a modern transformer has an efficiency of more than 99%, the loss cost is still substantial. The transformer core and windings experience the most significant losses, referred to as no-load and load losses, respectively. The amount of load connected to the transformer determines the load loss. Stray losses have been the focus of the majority of efforts to reduce load losses [34]. Introducing new materials, improved design, and manufacturing techniques are the key elements that increase efficiency [35]. Employing amorphous materials results in a substantial reduction in core losses, thereby enhancing overall efficiency significantly.

The definition of the efficiency of a transformer entails the ratio of the output power to the input power. However, it is unfeasible to measure the efficiency of a transformer via the output-to-input method. Instead, the efficiency is determined for full load at unity power factor using:

$$\eta = \frac{S_T}{W_{win} + W_{core} + S_T} \cdot 100 \tag{15}$$

2.4. Objective Function: Total Owning Cost

This section presents the optimization of the cost of the active part of the three-phase shell-type transformer, where the objective function which is to be minimized given by

$$\min(TOC(N_{sec}, B_m, cal, S_{sec}, D) = C_{win} + C_{core} + A \cdot W_{core} + B \cdot W_{win}) \tag{16}$$

subject to:

$$\begin{cases} I_{exc} < 1.5\% \\ 3.5 < \%Z < 5.5 \\ \text{core losses} < 1500\ \text{W} \\ \text{windings losses} < 9700\ \text{W} \\ \eta > 98.5\% \end{cases} \tag{17}$$

where

cal : Primary winding copper conductor size
S_{sec} : Cross-section of the aluminum conductor of the secondary winding
C_{win} : Cost of primary and secondary winding (US$)
C_{core} : Core cost (US$)
W_{core} : Core losses (W)
W_{win} : Primary and secondary winding losses (W)
A : No-load loss cost rate (US$/W)
B : Load loss cost rate (US$/W)

Table 2 defines the optimization variables used to minimize the TOC of the three-phase shell-type distribution transformer 750 kVA, 13.2/0.44 kV at 60 Hz.

Table 2. Optimization variables and upper and lower limit.

Variable	Unit	Lower Limit	Upper Limit	Value Alternative
N_{sec}	Turns	10	20	10
B_m	Teslas	1.7	1.9	Continuos
cal	AWG	7	16	10
S_{sec}	mm^2	34.29	452.12	4
D	mm	152.4	308.4	4

3. Optimization Methods

3.1. Genetic Algorithm

The Genetic Algorithm proposed in the early 1960s by John H. Holland is a metaheuristic method based on the generation of random solutions and survival of the fittest [36]. Initially, a population of NP individuals of dimension dim is randomly generated, each of which, in turn, has a certain fitness, which is evaluated using (16). Subsequently, the best ones (known as parents) are selected based on fitness, and the worst are not considered for breeding. Once the parents are selected in NP, the crossing operator generates NP offspring. In the next step, the mutation operator is applied to the offspring, which consists of randomly changing the genetic code of the offspring. The best solution for the current population is kept as all offspring will be the population for the next generation (i.e., elitism is considered). This process is repeated for a certain number of generations NGs. The criteria for parent selection, considering constraints are established [37] as follows:

T_1: When comparing two feasible individuals, the one with the better fitness value is selected.
T_2: When comparing two individuals, where one is feasible and the other is not, the feasible is selected.
T_3: If both individuals are infeasible, then the one who violates the constraints to a lesser extent is selected.

Algorithm 1 describes the GA pseudocode for minimizing the TOC of the active part of the transformer under study.

Algorithm 1 GA

Require: Material costs, initialize $NP,\ P_{cross},\ P_{mut}$
for i = 1 to NP **do**
  Create $X_i = [N_{sec}, B_m, cal, S_{sec}, D]$
end for
for i = 1 to NP **do**
  Evaluate (16)
end for
for gen = 1 to NGs **do**
  Selecting NP individuals based on the three criteria to obtain parents
  Apply the crossover operator to the selected parents to generate NP offspring
  Apply mutation operator to NP offspring
  Keep the NP offspring and discard the NP individuals in X,
   just keeping the best solution to replace the worst child
end for
Ensure: $min\ TOC$

3.2. Particle Swarm Optimization

The particle swarm optimization algorithm that emulates the social behavior of a bird flock was proposed by James Kennedy and Russell S. Eberhart in 1995 [38]. The particle with the best fitness value (known as leader), together with the experience of each particle, affects the movement (flight) of each particle in the swarm. The method consists in generating a certain number of NP particles which are placed in the domain of the objective function (16). Each particle ($Particle^g_{i,j}$) is a potential solution to the problem. Moving Particles remember their best position so far ($pbest$) and can identify their best position in the swarm (gbest). At each iteration, the velocity and position of the particle are updated with the following expressions:

$$v^g_{i,j} = K(v^{g-1}_{i,j} + C_1 r_1 (pbest^{g-1}_{i,j} - Particle^{g-1}_{i,j}) + C_2 r_2 (gbest^{g-1}_{i,j} - Particle^{g-1}_{i,j})) \quad (18)$$

$$Particle^g_{i,j} = Particle^{g-1}_{i,j} + v^g_{i,j} \quad (19)$$

Equation (18) is known as the constriction factor PSO [39], which is composed of three parts: the velocity multiplied by k, the cognitive component, which is the difference between the current position and the best position of the particle ($pbest$) and, finally, the social component which is the difference between the particle and the best position of the swarm ($gbest$). The criteria used in PSO to update the pbest and determine the gbest are the same as in the GA. The Algorithm 2 describes the PSO pseudocode for minimizing the TOC of the active part under study.

Algorithm 2 PSO

Require: Material costs, initialize $NP, \ K, \ C_1, \ C_2$

for i = 1 to NP **do**
  Create $Particle_i = [N_{sec}, B_m, cal, S_{sec}, D]$
end for
for i = 1 to NP **do**
  Evaluate (16)
end for
while g < NGs **do**
  for i = 1 to NP **do**
    for j = 1 to D=(6) **do**
      $r_1, r_2 = rand[0, 1]$
      Update velocity (18)
      Update position (19)
    end for
    if $Particle_i^g \leq pbest_i^{g-1}$ (based on the three selection criteria) **then**
      $pbest_i^g = Particle_i^g$
    end if
    if $f(Particle_i^g) \leq f(gbest_i^{g-1})$ **then**
      $gbest_i^g = Particle_i^g$
    end if
  end for
end while

Ensure: $min \ TOC$

3.3. Differential Evolution

Kenneth V. Price y R. Storn proposed the differential evolution algorithm in 1995 [40]. This is an evolutionary algorithm based on a special mutation operator applied to a linear combination of three different individuals and then recombined with the parent to be replaced [41]. The algorithm consists of generating a population of NP random individuals. Subsequently, each individual is evaluated with the objective function (16). The commonly used differential mutation is DE/rand/1/bin, which consists of randomly choosing three individuals from the population $(x_{r1} \neq x_{r2} \neq x_{r3})$ and adding to the first element (known as the base) a scaled difference between the other two individuals, where F is the scaling factor [42,43], yields:

$$u_i \leftarrow x_{r3} + F \cdot (x_{r1} - x_{r2}) \tag{20}$$

Algorithm 3 uses the criteria T_1, T_2, and T_3 (analogously to GA and PSO) on line 17 to update the solution vector (X_{G+1}).

Algorithm 3 DE/rand/1/bin

Require: Material costs, initialize $NP,\ F,\ CR$
for i = 1 to NP **do**
  Create $x_i = [N_{sec}, B_m, cal, S_{sec}, D]$
end for
for i = 1 to NP **do**
  Evaluate (16)
end for
for i = 1 to NP **do**
  Randomly select $r_1 \neq r_2 \neq r_3$
  $j_{random} = randint(1, D)$
  for j = 1 to D **do**
    if $rand_j[0,1) < CR\ or\ j = j_{random}]$ **then**
      $u^i_{j,G+1} = x^{r3}_{j,G} + F \cdot (x^{r1}_{j,G} - x^{r2}_{j,G})$
    else
      $u^i_{j,G+1} = x^i_{j,G}$
    end if
  end for
  if $u^i_{G+1} \leq x^i_G$ (based on the three selection criteria) **then**
    $x^i_{G+1} = u^i_{G+1}$
  else
    $x^i_{G+1} = x^i_G$
  end if
end for
Ensure: $min\ TOC$

4. Results

A critical problem in using metaheuristic algorithms, such as those studied in this paper, is the fine-tuning of the parameters. To ensure fair comparison and appropriate parameter calibration, the IRACE tool [44,45] was used to determine the parameter values for each compared algorithm. Table 3 shows some constant values required for transformer design. Table 4 shows the values of the adjusted parameters. In addition, the population and number of generations were fixed for the three algorithms so that they could use the same number of estimates.

Each optimization algorithm performs thirty runs. We used Python 3.9 and a computer with the following specifications: Intel core i5 processor, 3.10 GHz, 12 GB RAM, and Windows 11 64 bits. Feasible and infeasible solutions were obtained. Feasible solutions satisfy all the imposed design constraints, while infeasible solutions do not satisfy at least one constraint. Out of the 30 independent runs, where the final solution is the one reported, the GA and PSO obtained 23 feasible solutions, while 26 were found by DE/rand/1/bin. To validate the results obtained, the Wilcoxon rank-sum test with 95% confidence was used.

Table 3. Constants of transformer design.

Description	Value	Units
Lamination factor, (F_l)	0.95	dimensionless
Aluminum density, (ρ_{al})	2.7	g/cm^3
Copper density, (ρ_{cu})	8.9	g/cm^3
Volumetric resistivity and material density factor for aluminum, ($W_{d,al}$)	13.25	$\Omega \cdot$ mm^4/kg
Volumetric resistivity and material density factor for copper, ($W_{d,cu}$)	2.43	$\Omega \cdot$ mm^4/kg
Core empiric constant, (F_c)	1.35	dimensionless
Eddy current losses factor, (W_k)	1.25	dimensionless

Table 4. Adjusted values of the parameters with the use of IRACE for each metaheuristic algorithm.

	GA	**PSO**	**DE**
Population	40	40	40
Iterations	40	40	40
Crossover probability	0.8394	–	–
Mutation probability	0.5005	–	–
k	–	0.9896	–
C_1	–	0.9917	–
C_2	–	0.9585	–
F	–	–	0.9122
CR	–	–	0.4707

Table 5 represents the TOC of the active part of the transformer, obtained by each compared algorithm as well as the corresponding statistical values. It can be seen that the DE obtained better TOC, with respect to the GA and PSO. Such statistical differences are significant as indicated by the "+" sign in the Wilcoxon rank sum test [46,47]. It is worth mentioning that the GA obtained more robust results (i.e., lower standard deviation values with a better worst result), but the median and mean values are not as good as those found by DE.

Table 5. Statistical values obtained by each metaheuristic algorithm.

Methods / **Stat**	**DE**	**GA**	**PSO**
	TOC (*pu*)	**TOC (*pu*)**	**TOC (*pu*)**
Best	**0.9426**	0.9444	0.9549
Mean	**0.9463**	0.9543	0.9606
Medium	**0.9444**	0.9549	0.9598
Worst	0.9695	**0.9602**	0.9690
St. Dev.	0.0055	0.0044	0.0042
Wilcoxon rank-sum test with 95% confidence		+	+

Table 6 details the specifications of the core, the compliance with the guarantee values, the found values of input variables, the percentage reduction in the cost of the transformer design with respect to that provided by the manufacturer, and, finally, the time required to find the global optimum where the reduction in time is very significant concerning to the method used by the manufacturer.

Table 6. Comparison of the values provided by the manufacturer with respect to those obtained by the GA, PSO, and DE.

	Manufacturer	**GA**	**PSO**	**DE**
Warranties				
I_{exc} (%)	1.39	0.74	0.68	**0.3**
Impedance (%)	4.92	**4.02**	4.62	4.67
Total losses (W)	11,522.45	**10,581.92**	10,887.69	10,846.49
Efficiency (%)	98.49	**98.61**	98.57	98.57
Core				
E (mm)	60	100	**93**	96
F (mm)	85	85	85	85
G (mm)	280	280	280	280
D (mm)	304.8	203.2	203.2	203.2

Table 6. *Cont.*

	Manufacturer	GA	PSO	DE
Optimum values				
Secondary turns	15	**14**	15	15
B_m (T)	1.818	1.76	1.76	**1.7**
DP (AWG)	10	10	10	10
DS (mm^2)	254 × 1.78	254 × 1.78	254 × 1.78	254 × 1.78
Core width (mm)	304.8	203.2	203.2	203.2
TOC (*pu*)	1.0	0.9444	0.9549	**0.9426**
Cost reduction (%)	—	5.89	4.73	**6.08**
Time (seg)	347.23	**0.104**	0.171	0.195

Figure 2 depicts the convergence curves provided by the GA, PSO, and DE algorithms, of the run located in the median value out of those runs where feasible solutions were found. Although GA and PSO seem to have good results early in the search process, DE is the one that avoids local optimum and can find a lower TOC.

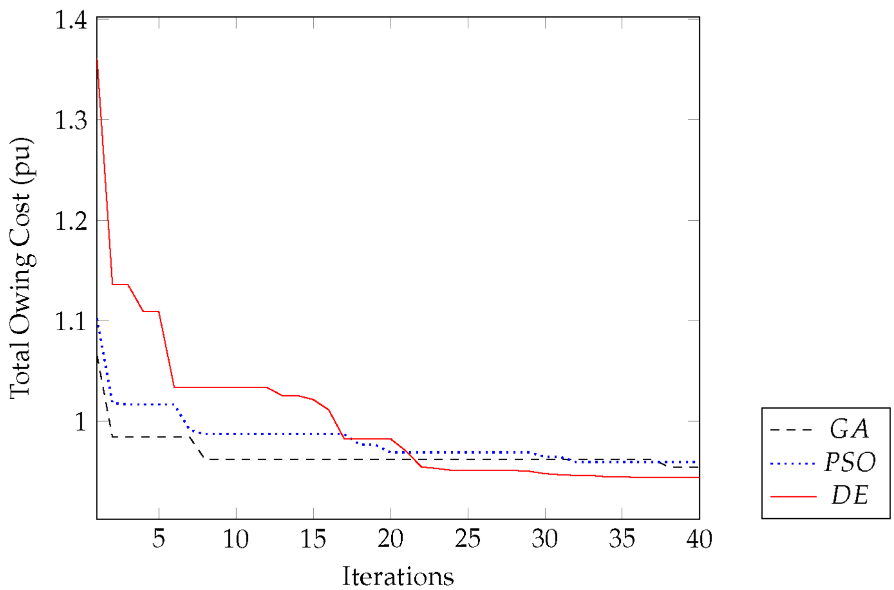

Figure 2. Convergence curves of the 750 kVA three-phase transformer by the GA, PSO, and DE.

5. Conclusions

In this work, the GA, PSO, and DE metaheuristic optimization algorithms were used to minimize the TOC of the active part of a three-phase shell-type distribution transformer, and their results and characteristics were compared. These algorithms provided better results of the transformer optimization problem satisfying the corresponding constraints than those provided by the manufacturer, reducing the TOC by 4.73–6.08%. Furthermore, it was observed that DE provided the most competitive results, and it is also straightforward to implement. In addition, the three algorithms required less than one second to find competitive solutions to the problem, which is significantly less than the transformer manufacturer 348 s. In general, the tested metaheuristic algorithms, particularly DE, can be an effective alternative for design engineers seeking to minimize the time to improve the design and performance of a transformer as the one tackled in this work. In the future, a lot of work remains to be conducted on the design of the transformer, for example, (a) include the calculation of the hottest spot in the program, (b) design the transformer including tank and mineral oil, (c) Design other types of transformers (pole, lined, submersible, for wind farms, step-up generators, shunt reactors, autotransformers, phase shifters, DC converters), and (d) use other, more advanced versions of the algorithms used in this study.

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Abbreviations

The following abbreviations are used in this manuscript:

GA	Genetic Algorithm
PSO	Particle Swarm Optimization
DE	Differential Evolution
TOC	Total Owning Cost

Nomenclature

M_{win} = Winding mass (kg); V_m = Coil half-turn (mm); N_{coil} = Number of coil turns; N_θ = Number of phases; S_{cond} = Winding conductor cross-section (mm^2); ρ_{cond} = Conductor density (kg/mm^3); J_{win} = Current density (A/mm^2); $W_{d(cond)}$ = Volumetric resistivity and density of the conductive material ($\Omega \cdot$ mm^4/kg); W_k = Eddy current losses factor; N_{sec} = Number of secondary turns; N_{prim} = Number of primary turns; Φ_m = Magnetic flux (Wb); f = Frequency (Hz); G = Core window height (mm); F = Core window width (mm); D = core sheet width (mm); ρ_{core} = Density of the magnetic core material (kg/mm^3); F_l = Lamination factor (%); S_T = Transformer rating (kVA); VA_{fe} = Apparent core losses (VA); F_c= Core empiric constant; $\%R$ = Percentage resistance at 85 °C; W_c = Conductor losses (W); kVA = Transformer rating; IN = Amperes-turn of transformer; γ = Average winding heights and thicknesses (mm); LL = Load Losses (W); NLL = No-Load Losses (W); B_m = Magnetic field density (T); cal = Primary winding copper conductor size (AWG); S_{sec} = Cross-section of the aluminum conductor of the secondary winding (mm^2); C_{core} = Core cost (US$); C_{win} = Cost of primary and secondary winding (US$); W_{core} = Core losses (W); W_{win} = Primary and secondary winding losses (W); A = No-load loss cost rate (US$/W); B = Load loss cost rate (US$/W).

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