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Article

Valuing Expansions of the Electricity Transmission Network under Uncertainty: The Binodal Case

Luis M. Abadie [1] and José M. Chamorro [2,]*

[1] Basque Centre for Climate Change (BC3), Alameda Urquijo 4,4ª-1ª, 48008 Bilbao, Spain; E-Mail: lm.abadie@bc3research.org

[2] University of the Basque Country, Av. Lehendakari Aguirre 83, 48015 Bilbao, Spain

* Author to whom correspondence should be addressed; E-Mail: jm.chamorro@ehu.es; Tel.: +34-946-013-769.

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Abstract: Transmission investments are currently needed to meet an increasing electricity demand, to address security of supply concerns, and to reach carbon-emissions targets. A key issue when assessing the benefits from an expanded grid concerns the valuation of the uncertain cash flows that result from the expansion. We propose a valuation model that accommodates both physical and economic uncertainties following the Real Options approach. It combines optimization techniques with Monte Carlo simulation. We illustrate the use of our model in a simplified, two-node grid and assess the decision whether to invest or not in a particular upgrade. The generation mix includes coal- and natural gas-fired stations that operate under carbon constraints. The underlying parameters are estimated from observed market data.

Keywords: electricity; transmission network; congestion; expansion; load; fuel prices; generation costs; emission allowances; EU Emissions Trading Scheme (ETS); GHG abatement

1. Introduction

Transmission and distribution networks account for 54% of the global capital assets of electric power [1]. These critical infrastructures face a growing problem of ageing across Europe and the

United States. Infrastructure investments are therefore needed to meet an increasing electricity demand, address security of supply concerns, and reach carbon-emissions targets. Typically transmission investments are also needed to accommodate the integration of renewable energies, e.g., large wind farms.

In a sense this is only a partial approach. Indeed it is possible (and perhaps more suitable) to start from a higher level, namely that of energy and economic development planning [2]. Power system planning is part of this more general problem. Within this in turn one can distinguish between power generation, transmission, and distribution. The objective of power system planning would thus be to determine a minimum cost strategy for long-range expansion of the generation, transmission, and distribution systems which were adequate to supply the load forecast within a set of technical, economic and political constraints. Traditionally, though, power system planning has been mainly related to generation expansion planning. This is primarily due to two reasons. At one level, investment in transmission lines is a relatively small fraction of the investment in the construction of power stations. On the other hand, investment in the distribution of electric energy to customers, although sizeable, is to a large extent independent of the generation and transmission system. Nonetheless, investments in the transmission grid and power generation are very related, so they should not be considered separately. This is so despite the fact that some of these endeavours are currently being deregulated. In this regard, Deb [3] explains the lower growth of the transmission grid in relation to that of generation as also a consequence of the lack of incentives and of regulatory uncertainty.

The objectives of transmission expansion planning in deregulated power systems include (Buygi *et al*. [4]): (i) Encouraging and facilitating competition among electric market participants, the more so when cross-border interconnections are opened up; (ii) Providing nondiscriminatory access to cheap generation for all consumers; as long as the costs include the price of carbon emissions (which will presumably grow over time according to market operators), a network with enough capacity will allow to replace the most polluting plants with more efficient ones more frequently (provided fuel prices do not plummet); (iii) Providing fair supply-side reserve for all generators and fair demand-side reserve for all consumers; (iv) Providing a robust transmission network against all uncertainties; (v) Being value based instead of cost or reliability based; (vi) Allowing the deployment of distributed generation by making possible to sell and transmit an amount of power which is higher than local demand; this in turn allows power plants to operate at a higher efficiency level; (vii) Allowing the efficient usage of power from stations physically located at more suitable sites according to the technology involved (e.g., hydro stations at remote valleys, coal plants close to mines or ports, gas turbines close to ports with LNG facilities, or connected to gas pipelines).

Expansion of transmission systems faces a number of uncertainties. Some of them are repeatable, and their statistics can be derived from past observations (e.g., uncertainty in load). However, this is not the case for others (e.g., uncertainty in generation expansion) [4].

A key issue in assessing the benefits from an expanded grid concerns the valuation of the cash flows that result from the expansion. To account properly for the uncertainties involved is no easy task. At one level, a transmission system upgrade increases reliability by decreasing the amount of unserved load in the event of a transmission line or generator failure. In this regard, these investments allow to reduce the probability of unserved loads. Yet network upgrades also create value for grid users by reducing congestion costs, thus lowering the bills paid by electricity consumers.

On the other hand, these investments display some other characteristics which render them suitable to being assessed following the so-called Real Options Aproach (ROA) [5,6]. Specifically, they are irreversible in a high degree, their returns are uncertain, and managers have several options at hand concerning the investment process. For example, they can wait for some time (until new information arrives) before commiting resources to the project, or they can alter the scale of the project on course in response to how the future unfolds.

Our valuation model rests on solving an optimization problem. In particular, at any time the total costs of electricity generation must be minimized. In this sense it draws on Bohn *et al.* [7]. Nonetheless, a distinctive feature of our model is that the optimization process is subject to the behavior of the stochastic variables (e.g., nodal loads, fuel prices); thus we deal with a problem of stochastic optimal control. In addition, traditional optimization models consider time frames of a few years at most, with optimization solved on an hourly (or shorter) basis. Instead, here we deal with the long-term valuation of a long-lived asset; consequently, optimization is undertaken less frequently. Last, we account for the possibility that a fraction of the demand is not served, though the unserved part has a significant cost.

The other stream of the literature relevant for our purposes goes along the ROA. Kurlinski [8] assesses users' willingness to pay for a transmission upgrade in a simplified electric system. Generation units are subject to failure in any period; the same holds for transmission lines. Future loads are also uncertain, *i.e.*, they can take on a whole range of values with each possible realization having a certain probability. Other variables that affect the system cost, however, are assumed constant (e.g., fuel prices or emission allowance prices). Blanco *et al.* [9] consider a test system consisting of three areas, represented by three nodes, linked by three transmission interconnections. They consider a thermal generation system with two generators using a (common) fossil fuel as primary energy source. The fuel price follows a stochastic mean-reverting process. They assess a potential investment in a new transmission line between two of the nodes and also the deployment of flexible alternating current transmission systems (FACTS). García *et al.* [10] use the original IEEE 24-bus reliability test system without two particular lines, and then consider three investment alternatives: only the first (absent) line is added, only the second one, and both lines are added simultaneously. The prices of oil, nuclear fuel, and coal are assumed to follow (correlated) mean-reverting processes. Load and installed generation capacity are unknown; their annual growth rates follow a generalized Wiener process. One of their main contributions is to estimate the NPV of the investment as a function of the investment timing.

Drawing on these papers we set up our valuation model. In principle, any network system considered in the electric engineering literature with deterministic load and fuel prices can accommodate uncertain load and fuel prices as we model them. This can be accomplisehd by nesting the simulation stage within the optimization stage. In other words, at each simulation run, nodal loads and input/output prices are set at a given level. Cost minimization is then undertaken subject to Physics Laws and technical constraints. Optimization should work equally well irrespective of whether loads and prices are assumed constant or change over time (whatever the number of nodes happens to be). All that matters is that they be known at the very beginning of the minimization process. Therefore, our proposal can be considered as an extension of traditional optimization problems. The main difference is that economic uncertainty plays a role which is more on a par with technical uncertainty; *i.e.*, not only physical assets can fail unpredictably, but loads, fuel and allowance prices can change unpredictably as well. A more balanced treatment of risk seems just consistent with current trends toward electricity markets deregulation. This methodological improvement is the main contribution of the paper.

To illustrate the nature of our model, we consider a specific, simplified grid and assess the decision whether to invest or not in a network upgrade under carbon constraints. Our generation mix includes coal- and natural gas-fired power plants at the same time. In this simplified setting, we take *ad hoc* locations and capacities. We proceed as follows. First we introduce some basic concepts on the subject. Then we present the simplified circuit which will serve as our base scenario. Physical infrastructures are going to be subject to random failures. Next we introduce the stochastic processes adopted for the other sources of uncertainty in our model: load, coal price, natural gas price, and carbon price. The objective function is to minimize total costs, which include both the cost of generation and that of unserved load. The next step involves the use of Monte Carlo simulation for constructing random samples. Then we analyze the behavior of the system before and after the expansion under different assumptions. We take account of different scenarios ranging from deterministic load to stochastic load, to changes in the load growth rate and in the price of emission allowances. On the basis of the resulting frequency distributions in each case it is possible to assess the potential benefits of the network expansion. Last, upon comparison with its attached cost it is easier to check whether this upgrade pays off or not.

2. Some Background

2.1. The Physical Environment

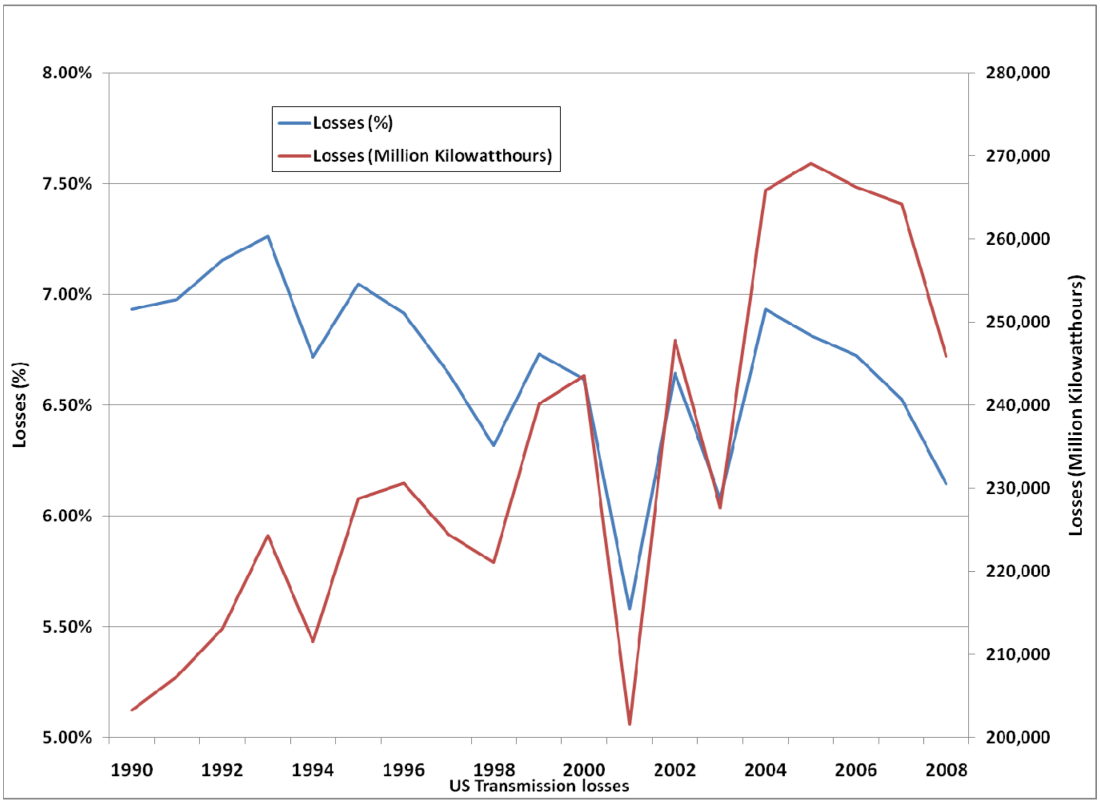
According to Stoft [11], electricity is like water, voltage is like water pressure, and a generator is like a water pump. Let us proceed by parts. The power law states that the time-t instantaneous electrical power flow $P(t)$ at any point in an electrical network is given by the product of the voltage $V(t)$ and the current $I(t)$. Voltage is the amount of electrical pressure that pushes current through electric appliances.

The physical flow through a meshed transmission network is complex. Power flows (which are measured in watts) are directed. The net injections at one node can thus be derived by summing up the incoming and outgoing power flows. In present transmission systems it is impossible to choose the path over which power will flow. These flows behave according to Kirchhoff's Laws.

Transmission lines oppose the flow of electricity. Ohm's law establishes that voltage $V(t)$ equals current $I(t)$ times resistance $R(t)$, which is measured in ohms. Combining both laws explains why transmission lines use high voltage (of 110 kV or more). Electricity is thus transmitted through high voltage lines (the so-called transmission grid). High voltage lines reduce losses, since these are inversely proportional to the square of voltage (*i.e.*, by multiplying voltage by 10 the losses are divided by 100 without any reduction in the power delivered). The "lost" power heats the power lines, causing copper to expand and the line to sag. Electrical current determines line losses, and thus the thermal limits on power lines.

The losses in networks are not negligible, and they add to other effects in the generation-transmission-distribution-usage chain. As a result, only a small part of the energy stored in fossil fuels is actually used at the final end-user point. For example, according to US data in 2008, the average percentage loss in that year was $p = 6.14\%$; see Figure 1. Losses in Spain over the same year amounted to 7.05% of the energy transmitted on the grid.

Figure 1. Electricity "losses" through the US transmission lines [12].



Power systems attempt to maintain a constant frequency; this is defined as the rate at which alternating current *AC* alternates (*i.e.*, changes direction) [13]. Similarly, power systems have a certain target voltage that they attempt to maintain. To avoid deliberate interruptions of service, power systems buy several kinds of operating reserves, that is, generators that are paid to be ready to provide power at a moment's notice. Together these reserves amount to approximately 10% of load at any given time [14]. Following Schweppe *et al*. [15], the term "operating reserves" denotes the generator reserve the utility has to maintain to prevent blackouts in case of a sudden loss elsewhere. "Spinning reserves" refer to generators which are connected (synchronized) to the network but which are not operated at their maximum output levels. Electric power availability, reliability, and quality are anticipated to have somewhat independently varying economic importance in different applications and industries [16].

2.2. The Economic Environment

Future demand is never known with certainty. It shows several patterns which obey to hour of the day, day of the week, and season of the year. However, in addition to these deterministic drivers, weather-related components arise and they are unpredictable for the most part.

Regarding the supply side, in a deregulated electricity market it is necessary to produce it at the lowest possible cost. Each generator is assumed to submit its particular bid to a common pool (which sets a single price for the relevant period of time). In the absence of strategic behavior by generators the bids take into account short-term marginal cost. This cost includes fuel costs, a profit margin (aimed at covering fixed costs and getting a reasonable rate of return), and, in a carbon-constrained environment, emission allowance costs. Generating units submit their bids, which are then ordered in an ascending order. Differences among generators have to do with the type of fuel used, the emission allowances required, the technical characteristics of the units (approached by their efficiency rates) [17], and current input and output prices. The system operator then solves the economic dispatch problem which is to find (whenever possible) the particular output levels for each available generator that minimize the total costs while meeting all of the loads plus line losses [18].

Below we will assume that the unit price of each commodity (coal, natural gas, emission permit) is the same for all generators and does not change with their physical location. At each instant, the units potentially in operation will be determined stochastically. Thus at each time we will derive a stochastic realization of the "merit order" or supply curve.

The optimal power flow (OPF) algorithm dispatches generation assets in merit (least-cost) order subject to the physical constraints of the electric network. In the electrical engineering sense, a line is "congested" when the flow of power is equal to the line's thermal capacity, as determined by various engineering standards. As Sauma and Oren [19] point out, transmission congestion increases the risk of blackouts, reduces the ability to import power from remote cheap generators (thus raising the cost of energy), and impedes trade and competition. This in turn makes consumers more vulnerable to the exercise of market power.

Under congestion, overall demand exceeds the supply ability of the "cheapest" generator; some consumers will thus have to buy power from other generators. Consequently, the marginal costs of production will differ between nodes. Each node represents a physical location on the transmission system including generators and loads. The price at each node reflects the locational value of energy. Differences of prices between nodes reflect the costs of transmission. Competitive locational prices are sometimes called locational marginal prices (LMPs), and also locational-based marginal prices. This is because they equal marginal costs at the relevant location [11]. Nodal prices are one of several important considerations in analyzing where to site additional generation, transmission and load. Nodal pricing has emerged a powerful and efficient tool of transmission pricing, both in theory and in practice [20].

As a matter of fact, electricty supply and demand change continuously over time, so prices are uncertain. When there is a chance of congestion, a generator trading with a remote load faces transmission-price risk. In principle, tradable physical transmission rights (TRs) could be used by a classic, decentralized market to solve this congestion problem. TRs are not bundled with energy but are traded separately. They confer a right to schedule and flow power. This market architecture is often referred to as "bilateral trading" [11]. Bilateral-trading prices are exactly those that nodal pricing is designed to produce. It does not matter who owns the TRs initially (beyond the issue of who would collect the congestion rent). All that matters is that they be tradable and the owners not exercise market power.

An alternative approach rests on financial transmission rights (FTRs). These financial instruments are issued by the transmission system operator; they entitle the holder to be reimbursed for the congestion charges paid when energy is sent (if it is a producer) from one location to another. In a bilateral transaction, if a generator sends energy from node A to B, it pays a congestion charge equal to the amount of energy flowing between A and B times the LMP difference between them. To hedge against this stochastic payment, the generator can purchase an FTR guaranteeing a revenue equal to the LMP difference times an agreed-in-advance contract flow [10].

Irrespective of whether the rights are financial instead of physical ones, their amount would be the same, and also their value [21]. The rights to use the lines have the same value in any competitive market no matter how they are defined.

3. The Basic Setup

We consider a two-node network topology that is given and fixed. This particular network is chosen merely for the sake of simplicity. Uncertainty in loads or commodity prices applies the same whatever the number of nodes. And there are a number of test networks of varying sizes and topologies any one of which can in principle make room for the approach here adopted (upon the required modifications); alternatively, our basic model can be further developed to account for more complex situations. After all, optimization takes the availability of infrastructures along with load and price levels as given. There is a transmission line connecting both nodes. It is possible to build a second line between them. We do not consider the time required for building it; we assume it has a useful life of 20 years [22]. We ignore inflation and efficiency targets. We abstract from a possible substitution between transmission expansion and generation expansion [23]; generator operating reserves are neglected. We also abstract from access-pricing problems for new generators, as well as from potential strategic behavior by generators.

3.1. Some Features of the Model to Be Developed

Our immediate objective is to value the positive impacts of investments in generation through the increase in reliability, the decrease of congestion and its effects on the prices paid by the users, and the reduction of environmental impacts from lower CO_2 emissions. The model must account for both physical and technical restrictions. Valuation must proceed in such a way that at each time, depending on the circumstances in place, generation is optimally dispatched subject to the network topology.

In our context, electricity prices do not reflect the primary source of uncertainty: as the total amount of installed capacity changes, the price of power changes, but also the parameters of the stochastic evolution of power price. So instead of assuming the price to be exogenously given, we model more fundamental variables as stochastic. These are external processes, physical as well as financial ones, whose effects on the supply of, and demand for electricity are reasonably well understood [24].The steps to be taken are the following:

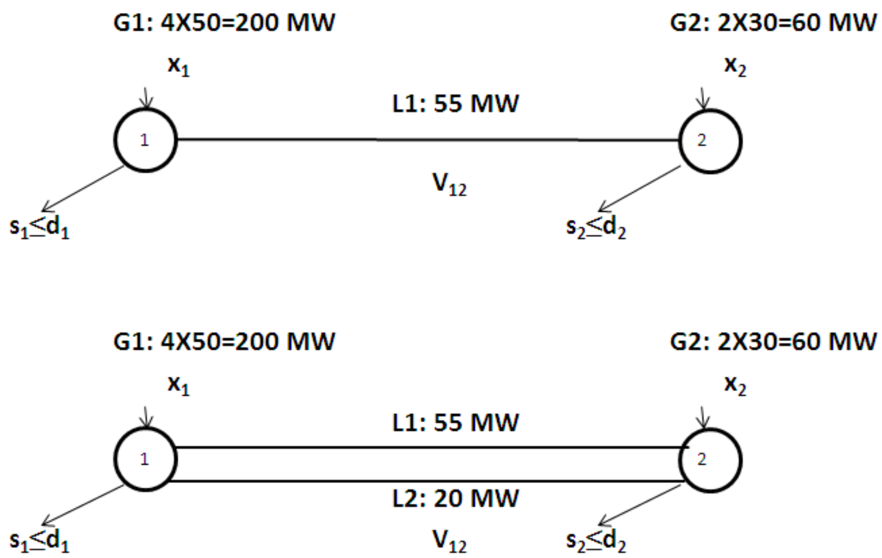
- We assume a stochastic behavior for the price of coal, natural gas, and carbon emission allowances. They evolve according to some parameters whose values are estimated from observed market prices.
- Each component of the network (generation unit, transmission line) has a certain probability of being out of service at any time (e.g., plants can be unavailable because of adverse weather conditions like high winds and low temperatures). So outages occur stochastically according to some rate.

- Given commodity prices and components availability, at any time the supply curve of generators is determined (the merit order), which is ordered from lower to higher bid price of electricity. The model must allow for the possibility that, on some occasions, the cheapest-to-run units be coal-fired plants while at some other times that role fall on natural gas combined cycles [25]. It depends on the stochastic evolution of prices along each path generated by Monte Carlo simulation(this numerical method will be adopted for solving this type of problem).
- At any time there is a stochastic demand at each node. This load is going to be correlated with the loads at adjacent nodes (e.g., because both are more or less affected by common factors such as outdoor temperature or sun light).
- From the intersection of supply and demand curves the (marginal) price of electricity results. In the case of a network under uniform pricing, this would be the price charged by all generators.
- At every time demand and supply must be balanced, and the Laws of Physics must apply in the network.
- We adopt the DC load flow model, so we do not model the impact of reactive power on the system. It provides an approximate solution for a network carrying AC power [15, Appendix D].

3.2. A Simple Transmission Model

In order to assess transmission investments as intuitively as possible we consider the simplified electric system depicted in Figure 2. Stochastic load at each node is assumed inelastic in that a given amount (in MWh) is demanded regardless of the price that the load pays for the electricity. Generator G1 is assumed to be a natural gas-fired power plant; it consists of four generation units each 50 MW in capacity size. They provide power to meet the demand at node #1; the excess capacity is used, at least initially, to serve load at node #2. Generator G2, instead, burns bituminous coal. It consists of two generators each with a capacity of 30 MW. In principle G1 is less polluting than G2. The ability of G1 to serve load at #2 depends, however, on the transmission capacity of the line L1, which connects both nodes.

Figure 2. Simple transmission model and potential expansion.



L1 has a given transmission capacity, which we assume to be 55 MW. However, transmission losses along L1 amount to p = 6.14 per cent (*i.e.*, only $1-p$ = 93.86 of each megawatt-hour transmitted reaches the opposite end). This parameter sets an upper bound on the power that G1 can effectively contribute (even if this generator is not operating at full capacity), namely 0.9386 × 55= 51.62 MWh at most. Thus L1 may act as a bottleneck under episodes of unexpectedly high load. And, needless to say, unserved or lost load has value (the value of lost load, or $VOLL$). Following Blumsack *et al*. [26], we make two simplifying assumptions: all consumers in the network have an identical $VOLL$, and the $VOLL$ is constant for any level of electricity use.

In the base case there are seven physical assets: four in G1, two in G2, and L1. We adopt a set of binary (Bernoulli) random variables for the possibility of any one contingency. Thus, regarding generation assets, we assume that, because of failures and/or maintenance works, each generation unit is out of service for 5 % of the year:

$$\mathrm{G1}_i, \mathrm{G2}_j = \begin{Bmatrix} 0, \text{ "off" state with probability } 0.05dt \\ 1, \text{ "on" state with probability } 0.95dt \end{Bmatrix}$$

Here $i \in \{1,2,3,4\}$ and $j \in \{1,2\}$. As for the transmission line, we assume:

$$\mathrm{L1} = \begin{Bmatrix} 0, \text{ "off" state with probability } 0.001dt \\ 1, \text{ "on" state with probability } 0.999dt \end{Bmatrix}$$

Therefore, depending on whether each asset is “on” or “off”, at any time there can be $2^7 = 128$ possible states. Later on, when we consider the potential availability of a second transmission line L2, the number of possible states rises to $2^8 = 256$.

In our model D_k denotes the demand for electricity at a given time at node $k \in \{1,2\}$; see Table 1. Regarding G1, which is composed of four units, there are five possible cases, namely that none, one, two, three, and four of them are operative; each of these cases corresponds to a different value of the availabilty parameter: $a_1 \in \{0,0.25,0.50,0.75,1\}$; similarly for G2, consisting of two units: $a_2 \in \{0,0.50,1\}$. Instead, the line L1 can only be either available or not: $b_1 \in \{0,1\}$.

Table 1. Notation.

	Node #1	**Node #2**	**Line L1**
Effective Generation	x_1	x_2	-
Maximum Generation	$\bar{x}_1$	$\bar{x}_2$	-
Availability {0,1}	a_1	a_2	b_1
Load Served	s_1	s_2	-
Load Demanded	D_1	D_2	-
Line capacity	-	-	l_{12}
Trans. Losses	-	-	$m_{12} = max[p(x_1 - s_1); p(x_2 - s_2); 0]$

We assume that initially $D_1 = 70$ MW and $D_2 = 100$ MW (below, local loads will be stochastic but correlated to each other: $\rho_{D1,D2} = 0.80$). We assume $\bar{x}_1$ = 200 and $\bar{x}_2$ = 60. If all the generation units and transmission lines are available at that time, at node #1 we will have an excess generation capacity of 130 MW, while at node #2 there will be a shortfall of 40 MW. Because of transmission losses, in order for 40 MW to reach node #2 a higher amount must be injected at node #1, namely

40/0.9386= 42.62 MW, which will be transmitted through L1. Therefore at node #1 a total of x_1 = 70 + 42.62 = 112.62 MW are generated, 70 of which will be consumed locally and the remaining 42.62 MW will become 40 MW at node #2. This 40 MW, when added to the other 60 MW locally generated, will allow to meet local load of 100 MW.

A plant burning natural gas has an emissions factor of 56.1 kgCO_2/GJ [27,28]. Since under 100% efficiency conditions 3.6 GJ would be consumed per megawatt-hour, we get

$$I_G = \frac{0.20196}{H_G} \frac{\text{tCO}_2}{\text{MWh}} \tag{1}$$

Similarly, a plant burning bituminous coal has an emission factor of 94.6 kgCO_2/GJ under 100% efficiency conditions; then:

$$I_C = \frac{0.34056}{H_C} \frac{\text{tCO}_2}{\text{MWh}} \tag{2}$$

Total (variable and fixed) costs of both stations thus amount to:

$$c(x_1, x_2) = x_1(M_m + \frac{G + 0.20196A}{H_G}) + x_2(M_m + \frac{C + 0.34056A}{H_C})$$

In what follows we assume thermal efficiencies of $H_G = 0.55$ and $H_C = 0.30$ [29]. Below, G and C will denote the price (in €/MWh) of natural gas and coal, respectively, while A will stand for the price (in €/tCO_2) of a carbon emission allowance. The fixed margin M_m is taken to be 13.30691 €/MWh. This is the long-term clean spark spread for natural gas-fired power plants derived in [30, Appendix A]. When coal-fired plants turn out to be the marginal plants, we assume that they earn the same margin.

In addition to the above explicit costs $c(x_1, x_2)$, there can also be an implicit cost. Specifically, unserved or lost load can be computed as $D_1 + D_2 - s_1 - s_2$. In our model this load has value; *VOLL* stands for the value of lost load per unit, *VOLL* = 500 €/MWh-interrupted [31]. In the end, the optimization process in our case includes both costs.

The optimization process developed henceforth draws on [7] but has two added characteristics: (a) the optimization process is subject also to the behavior of the stochastic variables, thus it is problem of Stochastic Optimal Control; (b) we allow that a fraction of the demand is not served, though the unserved part has a relevant cost.

We assume that the system operator dispatches generating resources to minimize the sum of generating costs and the costs associated with unserved energy (*VOLL*). We aim to find an optimal vector of power generated and power consumed that minimizes the sum of generation costs and unserved demand costs across all the nodes at any time:

$$\min_{x_1, x_2, s_1, s_2} c(x_1, x_2) + VOLL(D_1 + D_2 - s_1 - s_2)$$

s.t.

$$0 \le x_1 \le a_1 \overline{x}_1;\ 0 \le x_2 \le a_2 \overline{x}_2$$

$$0 \le s_1 \le D_1;\ 0 \le s_2 \le D_2$$

$$x_1 + x_2 = s_1 + s_2 + m_{12}$$

$$x_1 - s_1 \le b_1 l_{12};\ x_2 - s_2 \le b_1 l_{12}$$

$$dD = a(D,t)dt + b(D,t)dW;\ D = \{D_1, D_2\}$$

$$dX = a(X,t)dt + b(X,t)dZ;\ X = \{A, C, G\}$$

The first four restrictions set the environment as determined by the operation state of the physical assets. Thus, effective generation levels cannot surpass maximum available levels. Also, the power delivered is always equal to or less than the load stated at each node (thus, it is possible that some load is not met at the minimum possible cost). Besides, the electricity generated equals consumption plus transmission losses. In addition, the surplus generation at each node must be lower than the transmission capacity of the line (b_1 equals either 0 or 1).

The last two restrictions are the stochastic differential equations. In this case there is local demand at the two nodes; they evolve stochastically over time according to an Ito process. Similarly, the price of each commodity (emission allowance, coal, natural gas) follows another Ito process. Needless to say, the increments to a standard Wiener process dW and dZ are different; see next subsection. Further still, dZ is different for each commodity $\{A, C, G\}$ along with the terms $a(X,t)$ and $b(X,t)$. Instead, the nodal loads $\{D_1, D_2\}$ have the same $a(D,t)$, $b(D,t)$, and dW, but their initial values are different and their stochastic processes are correlated.

4. The Stochastic Model

Commodity prices and nodal loads are assumed to be governed by specific stochastic processes, with their corresponding cross correlations.

4.1. Future Loads

Regarding our two-node network, below we assume that the load (in MW) at node $k \in \{1, 2\}$ has a lognormal distribution. Its average grows at the specified rate per time period, and the standard deviation grows with the square root of time:

$$dD_{k,t} = \alpha_{k,D} D_{k,t} dt + \sigma_{k,D} D_{k,t} dW_t^{D_k}$$

We assume that initially $D_{1,0} =$ 70 and $D_{2,0} =$ 100. The correlation coefficient is $\rho_{D_1,D_2} = 0.80$.

4.2. Commodity Prices

Long-term prices of natural gas and coal are assumed to be governed by Inhomogeneous geometric Brownian motions (IGBM) with mean reversion. Natural gas prices also display a seasonal pattern; this is not so in the case of coal. Carbon prices, instead, are assumed to follow a standard geometric Brownian motion (GBM) [32]. Formally, in the risk-neutral world we have:

(a) Natural gas price:

$$dG_t = df_G(t) + [k_G G_m - (k_G + \lambda_G)(G_t - f_G(t))]dt + \sigma_G (G_t - f_G(t))dW_t^G$$

(b) Coal price:

$$dC_t = [k_C(C_m - C_t) - \lambda_C C_t]dt + \sigma_C C_t dW_t^C$$

(c) As for the emission allowance price, we adopt a non-stationary process:

$$dA_t = (\alpha - \lambda_A)A_t dt + \sigma_A A_t dW_t^A$$

The prices of natural gas (G) and coal (C) show mean reversion. G_m and C_m denote the long-term equilibrium levels; that is, current (deseasonalized) gas and coal prices tend toward them in the long run. $f_G(t)$ is a deterministic function that captures the effect of seasonality in gas prices. In general the function is defined by $f(t) = \gamma \cos(2\pi(t+\varphi))$, with the time t measured in years and the angle in radians; when $f(t = -\varphi) = \gamma$ the seasonal maximum value is reached. k_G and k_C are the speed of reversion towards the "normal" level of gas and coal prices. They can be computed as $k_G = \ln 2/t_{1/2}^G$, where $t_{1/2}^G$ is the expected half-life for (deseasonalized) natural gas, *i.e.*, the time required for the gap between $[G_0 - f_G(0)]$ and G_m to halve; similarly $k_C = \ln 2/t_{1/2}^C$. Regarding the price of the emission allowance, we adopt a standard GBM process. The parameter α stands for the instantaneous drift rate of carbon price. σ_G, σ_C and σ_A are the instantaneous volatility of natural gas, coal and carbon. λ_G, λ_C and λ_A denote the market price of risk for gas, coal, and carbon prices. dW_t^G, dW_t^C and dW_t^A are the increments to standard Wiener processes. They are normally distributed with mean zero and variance dt; besides:

$$dW_t^G dW_t^C = \rho_{GC}dt; \quad dW_t^G dW_t^A = \rho_{GA}dt; \quad dW_t^C dW_t^A = \rho_{CA}dt \tag{3}$$

5. Estimation of the Underlying Parameters

5.1. Drift and Volatility of the Demand

We assume a common drift rate for both demands $\alpha_{1,D} = \alpha_{2,D} = \alpha_D$ and also a common volatility rate $\sigma_{1,D} = \sigma_{2,D} = \sigma_D$. After discretization, the stochastic equation for the demand becomes:

$$\frac{D_{t+1} - D_t}{D_t} = (e^{\alpha_D \Delta t} - 1) + \sigma_e \sqrt{\Delta t}\,\varepsilon_t^D \tag{4}$$

where $\varepsilon_t^D : N(0,1)$. Now we rewrite this approximation as $Y_t = \beta_1 + u_t$. Using yearly data (U.S. Department of Energy) from 1990 through 2008 we get the ordinary least squares (OLS) estimation in Table 2.

Table 2. OLS estimation of the stochastic demand.

Coefficient	**Estimate**	**Std. Dev.**	***t*-statistic**	***p*-value**
$\hat{\beta}_1$	0.0176115	0.00365603	4.817	0.0002

Thus $\hat{\beta}_1 = 0.0176115$. Since $\hat{\beta}_1 \equiv e^{\alpha_D \Delta t} - 1$, this estimate implies ($\Delta t = 1$) that load grows at a rate $\hat{\alpha} = 0.017458$, or 1.7458 % annualy. Once we have an estimate of the drift rate we can easily derive the expected demand level at any time in the future. This way we can check how long it will take for congestion problems to arise in the initial setting; see Appendix B. The estimate of the standard deviation is $\hat{\sigma}_D$ = 0.015511.

5.2. Estimation of the Price Processes

Our sample includes daily prices of all futures contracts on natural gas and ARA coal available on the European Energy Exchange (EEX, Germany), irrespective of their maturity along with all futures contracts on EU emission allowances maturing in December that are traded on the Inter Continental Exchange (ICE, United Kingdom). Full details on the estimation procedure can be found in Abadie *et al*. [30]. The numerical estimates of the relevant (composite) parameters appear in Table 3.

Table 3. Parameter estimates of commodity prices.

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
$C_m^* \equiv \dfrac{k_C C_m}{k_C + \lambda_C}$	105.27	$G_m^* \equiv \dfrac{k_G G_m}{k_G + \lambda_G}$	25.04	ϕ (day) γ	−21.7 3.29	C_0 (\$/tCoal)	74.7898
$k_C + \lambda_C$	0.69	$k_G + \lambda_G$	0.85	$\alpha^* \equiv \alpha - \lambda_A$	0.054	G_0 (€/MWh)	7.2419
σ_C	0.4144	σ_G	0.6356	σ_A	0.20	A_0 (€/tCO_2)	13.18

Regarding the correlations we get:

$$\rho_{GC} = 0.2652;\ \rho_{GA} = 0.2572;\ \rho_{CA} = 0.2797$$

On the other hand, the risk-free interest rate is taken to be $r =$ 3.22%. As the ARA coal is traded on the EEX in US dollars per tonne, it is also necessary to transform the price units; note that natural gas is quoted in €/MWh [33].

6. Monte Carlo Simulation

We are going to check two possible configurations of the transmission grid, namely the initial one (with one line) and the expanded one (with a second line). Nodal loads and carbon prices, which are assumed to follow a GBM, are simulated by means of the following discrete-time approximations:

$$D_{i,t+\Delta t} = D_{i,t} e^{(\alpha_D - \frac{1}{2}\sigma_D^2)\Delta t + \sigma_D \sqrt{\Delta t} u_{D_i,t}}$$

$$A_{t+\Delta t} = A_t e^{(\alpha_A - \lambda_A - \frac{1}{2}\sigma_A^2)\Delta t + \sigma_A \sqrt{\Delta t} u_{A,t}}$$

Instead, for coal and natural gas we adopt the following approximations to the IGBMs in discrete time:

$$C_{t+\Delta t} = C_t + [k_C (C_m - C_t) - \lambda_C C_t]\Delta t + \sigma_C \sqrt{\Delta t} C_t u_{C,t}$$

$$G_{t+\Delta t} = G_t + (f_G(t+\Delta t) - f_G(t)) + [k_G G_m - (k_G + \lambda_G)(G_t - f_G(t))]\Delta t + +\sigma_G (G_t - f_G(t))\sqrt{\Delta t} u_{G,t}$$

We evaluate the benefits of a given enhancement to the grid through simulation. In this case we run 1000 simulations. Each simulation path consists of 1200 steps. Since 60 steps are taken for year (*i.e.*, five steps per month), this implies that the infrastructure is assumed to be in operation for 20 years. At each step the optimal dispatch problem is solved subject to the restrictions then in place; *i.e.*, we solve 1000 × 1200 = 1,200,000 optimization problems. Each problem consists in minimizing the sum of the costs of electricity generation and the cost of unserved load, subject to some linear and non-linear restrictions. The solution to each problem involves the levels of generation and the power effectively supplied. Hence we can compute the total production costs, the carbon emissions, and the emissions costs.

This procedure is first applied to our two-node network with only L1 available for transmission purposes. Later on, the same steps are followed when the network has been expanded with a second line L2 that runs parallel to L1; we assume the transmission capacity of L2 is just 20 MW. The comparison of these results allows to assess the benefits of that expansion.

7. Certainty and Uncertainty with a Single Transmission Line (L1)

A number of variables can be retrieved from the analysis, among them nodal load and generation, served/unserved load, transmission between nodes, transmission losses, nodal carbon emissions, allowance costs incurred, (bid-based) total cost at each node, and electricity price. For simplicity, we concentrate on the most relevant, aggregate ones.

7.1. The Base Case: No Growth in Loads and Only L1 under Certainty

After completing a number 1000 of simulations (and solving the 1,200,000 optimizations embedded in them) we get 1000 whole paths of a number of variables. Regarding generation first we compute the cumulative production by type of fuel and location. For example, we have 1000 levels of cumulative coal generation in node #2 over the next twenty years. We compute the average of these 1000 values. Then we divide the average by the number of years in the time horizon considered (20). Similarly for nodal loads and transmission levels. These yearly averages are shown in Table 4 when only line L1 is available in the base case (*i.e.*, with our numerical estimate of load volatility σ = 1.55%) and in the case with no load volatility (σ = 0).

Table 4. Average generation, load, transmission, and costs (L^1): Deterministic case *vs.* base case.

	No Volatility	**Base Case**
Coal generation (GWh)	493.63	493.27
Gas generation (GWh)	1201.64	1199.63
Load (GWh)	1785.44	1786.04
Unserved load (GWh)	118.83	121.69
Transmission (GWh)	466.60	464.59
Emissions (KtCO_2)	1000.61	1000.46
Allowance costs (M€)	15.88	15.84
Present cost (M€)	115.14	116.06

Both coal- and natural gas-based generation are slightly lower under uncertainty while demand is slightly higher. The small differences in generation levels have to do with the simulation error that Monte Carlo techniques entail but also with the increase in the amount of unserved load. Note the fundamental asymmetry that capacity constraints impose here [34]. When demand unexpectedly falls, generation units in the model can be adjusted to operate at a lower rate without any lower bound (other than zero). Instead, surges in demand cannot be fully matched because there are technical limits to the operation of the units. Thus, wider mismatches between supply and demand come naturally under uncertainty. In a fixed environment, however, the scope for load excursions is minimum. It is load volatility that underlies the increase in unmet load (uncertainty about generation and transmission infrastructures remains the same). Therefore, load uncertainty has a negative impact on the system reliability. An estimate of the reliability costs (or benefits) can be inferred from this amount and our above assumption on the value of lost load (VOLL). Table 4 also displays a minor change in load which has to do with simulation error. Indeed, the tiny size of the deviation can be interpreted as a signal of the accuracy of our model. It is well known that each individual evaluation of a Monte Carlo estimator, for a large number of individual draws of a random variable, behaves approximately like a normal variate. The standard error is inversely proportional to the square root of the number of drawings. This error is only of statistical nature. Thus, roughly every third simulation based on random numbers will have a result outside the standard error margin (*i.e.*, $\pm 1\ \sigma$) around the correct solution [35]. Further, the standard error itself is subject to a statistical error margin (though this shortcoming is less severe than the former one). Electricity transmission also falls, which is consistent with lower generation at both nodes.

On the other hand, a reduced operation of generating plants translates into less input fuels, lower carbon emissions and lower allowance costs. Thus reduced operation entails lower expenses in our model. Overall, though, the increase in (demand-side) reliability costs more than offsets the decrease in supply-side costs, and uncertainty pushes for higher system costs.

7.2. Impact of Higher Demand Volatitlity

Henceforth we adopt the base case ($\sigma = 1.55\%$) as a benchmark. Against this case we assess the effect of a more volatile load (specifically, double the original estimate). Table 5 displays the impact on select variables. The average load rises slightly but this is due to simulation error. Generation and transmission levels decrease noticeably. Note that the higher the volatility of demand, the higher the number of instances with low load and low transmission, but at the times of high demand the amount transmitted is limited by the capacity of the line. These changes bring about a small reduction in fuel consumption, carbon emissions, and allowances required because of the increase in unserved load. Yet the higher reliability costs outweighs the fall in supply-side costs, and the system performs worse.

Table 5. Average generation, load, transmission, and costs (L^1): Base case *vs.* higher load volatility.

	Base Case (σ = 1.55%)	σ = 3.1%
Coal generation (GWh)	493.27	491.63
Gas generation (GWh)	1199.63	1193.46
Load (GWh)	1786.04	1786.49
Unserved load (GWh)	121.69	129.57
Transmission (GWh)	464.59	458.56
Emissions ($KtCO_2$)	1000.46	996.34
Allowance costs (M€)	15.84	15.74
Present cost (M€)	116.06	118.45

7.3. Impact of Higher Demand Growth Rate

We concentrate on the base case once more, *i.e.*, we adopt the numerical estimate of load growth from our data sample (α = 1.745%). Against this case we assess the effect of a more steep load growth (specifically, double the original estimate). Table 6 displays the impact on our select variables. As expected, on this occasion the average load rises significantly. Generation tries to follow pace but falls short of it by a wide margin. Coal generation was already overstretched at node #2 so there is not much it can do. Instead, gas generation was in long supply at node #1; but when faced with a surging local demand (and fixed transmission infrastructure) the scope for exports falls. Indeed, transmission from node #1 to node #2 drops a bit (note that a rise in the drift rate entails a higher volatility due to the very nature of the stochastic process). As a consequence, unserved load more than doubles. These changes translate into a higher bill for input fuels and carbon emissions. Add these supply-side costs to higher demand-side costs and the result is unequivocally a steep rise in system costs.

Table 6. Average generation, load, transmission, and costs (L^1): Base case *vs.* higher load growth.

	Base Case (α = 1.74%)	α = 3.49%
Coal generation (GWh)	493.27	494.91
Gas generation (GWh)	1199.63	1341.73
Load (GWh)	1786.04	2163.30
Unserved load (GWh)	121.69	354.47
Transmission (GWh)	464.59	452.68
Emissions ($KtCO_2$)	1000.46	1054.50
Allowance costs (M€)	15.84	16.75
Present cost (M€)	116.06	193.30

7.4. Effect of a Higher Initial Allowance Price

Before leaving this Section, we check the base case with our numerical estimate of initial allowance price (A_0 = 13.18 €/tCO_2) against the case with double this amount. Table 7 shows the results. There is no longer any change attached to load so this on average remains the same. Both fossil fuels are adversely affected by a rise in carbon price, but the impact is relatively more severe on coal. As expected, coal generation falls and the slack is taken up by gas generation. Note, though, that it is gas generation that may be handycapped by the transmission capacity of line L1. There is also a rise in the electricity transmitted; unmet load increases slightly.

Table 7. Average generation, load, transmission, and costs (L^1): Base case *vs.* higher allowance price.

	Base Case $A_0 = 13.18$	**$A_0 = 26.36$**
Coal generation (GWh)	493.27	485.70
Gas generation (GWh)	1199.63	1206.98
Load (GWh)	1786.04	1786.04
Unserved load (GWh)	121.69	122.36
Transmission (GWh)	464.59	471.94
Emissions ($KtCO_2$)	1000.46	994.57
Allowance costs (M€)	15.84	31.32
Present cost (M€)	116.06	131.81

The substitution of filthy coal by cleaner gas entails lower carbon emissions. The assumed doubling in the initial allowance price, however, complicates the picture, and this impinges on supply-side costs. Higher carbon prices raise the system costs.

8. Expansion of the Transmission Network

8.1. The Case with Full Certainty

In principle, we would expect the expansion to allow for more trade opportunities. This would imply some substitution between cheap and expensive fuels (adjusting for their respective carbon emissions). This can also show up through enhanced transmission levels. Besides, the scope for unserved load should be reduced, and the overall system cost should be lower (remember that we only focus on assessing the benefits; the cost to expanding the network is left aside).

Table 8. No uncertainty in load with L^1 and $L^1 + L^2$.

	L^1	$L^1 + L^2$
Coal generation (GWh)	493.63	473.57
Gas generation (GWh)	1201.64	1301.98
Load (GWh)	1785.44	1785.44
Unserved load (GWh)	118.83	44.73
Transmission (GWh)	466.60	566.95
Emissions ($KtCO_2$)	1000.61	1015.68
Allowance costs (M€)	15.88	16.05
Present cost (M€)	115.14	95.23

As can be seen in Table 8, the picture fits pretty much these intuitions. While average load remains unchanged, the amount of unmet load is cut in more than half. This is accomplished through increased gas-based generation; indeed, gas units also drive some coal-based generation out of the system. Given the particular topology considered here, this increase in gas generation involves also higher flows of electricity transmitted from node #1 to #2. Carbon emissions and allowance costs increase slightly because of the increase in served load. However, the system cost fall noticeably after the expansion.

8.2. Expansion in the Base Case

Figure 3 shows the distribution of total system costs (explicit and implicit) under both grid configurations. Figure 4 refers to CO_2 emissions measured in tonnes, while Figure 5 shows the distribution of unserved load. All Figures refer to cumulative values over the 20-year period considered; the yearly average value is provided in the caption for comparison. Let us go by parts.

Figure 3. Cumulative generation costs plus cost of unserved load over twenty years. Yearly average (L^1): 116.06 M€; yearly average ($L^1 + L^2$): 96.51 M€.

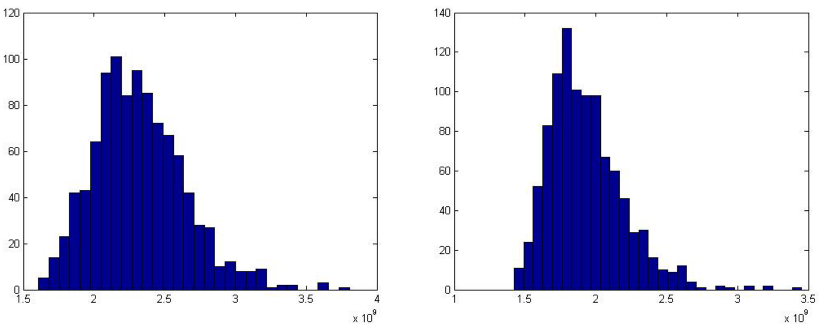


Figure 4. Cumulative CO_2 emisssions over twenty years. Yearly average (L^1): 1000.46 $KtCO_2$; yearly average ($L^1 + L^2$): 1012.57 $KtCO_2$.

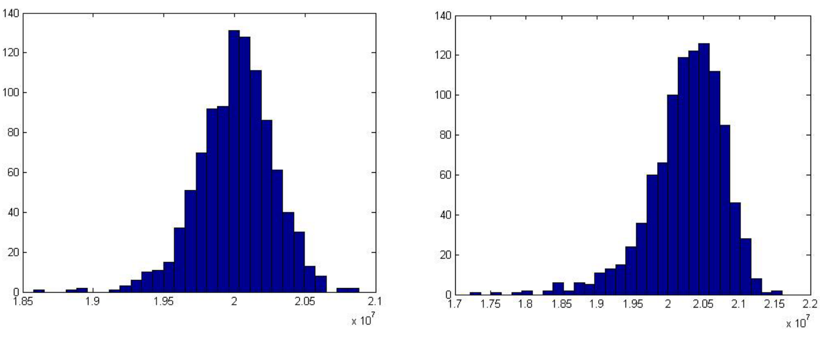
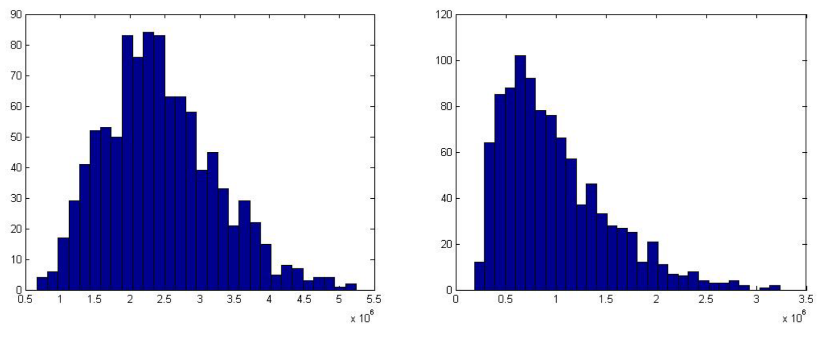


Figure 5. Cumulative unserved load over twenty years. Yearly average (L^1): 121.69 GWh; yearly average ($L^1 + L^2$): 49.71 GWh.



As shown in Figure 3, the distribution of system costs is skewed in both cases; there are a few simulation runs in which costs surge well over the average value. Apart from this, there are several differences between them. First, looking to the left and right extremes, the range of possible values under the single transmission line is wider than after the expansion. In other words, the new line L2 leaves no room for extreme (high and low) costs that were possible when only L1 was in place. The probability mass that is lost at the the tails of the distribution is obviously gained by more centered outcomes. This is shown in the middle part of the distribution, where new heights are now reached. But this gain does not seem to happen more or less equally around the average. Neglecting the right tails in both cases, the left distribution is relatively more symmetric than the right one; this is another change in the shape of the distribution of system costs that can be attributed to the expansion. The average (yearly) cost falls from 116.06 M€ before the expansion to 96.51 M€ after the expansion.

Now Figure 4 displays the distribution of carbon emissions under both topologies. Again, both profiles are skewed, but on this occasion it is the left tail that goes apart from the average. Another difference is that the range of potential values is wider after the network expansion. The new line L2 allows a more intense use of cleaner fuels (thus making lower emissions feasible) but also opens the way to enhanced generation (thus pushing emissions up). Indeed, the yearly average rises from 1000.46 thousand tonnes of carbon dioxide before the expansion to 1012.57 after the expansion. The probability mass that is now gained at the the tails of the distribution is lost in the middle part; a slight cut in the highest bars of the distribution can be observed. And the expansion makes the distribution relatively more asymmetric than it was before.

More in general, a greater decrease in carbon emissions can be expected if transmission expansions clear the way to greater operation of cleaner power plants or the penetration of renewable resources. At the same time, it can be important to check the effect on transmission losses; they could dampen the environmental benefits of clean technologies (or even more than offset them). Also, investing to relax a binding constraint somewhere on an interconnected network will facilitate economic power flows up to the point that some other constraint(s) become binding. Indeed, if congestion is relieved, prices converge to a common level, and thus areas with initially low prices may afterward face higher prices due to increased exports [36]. This means that distributional issues across space should also be considered.

As shown in Figure 5, the possible values that the unmet load can take on (horizontal axis) are lower after the network expansion. Besides, the distribution becomes more asymmetric in that higher values display lower frequencies (vertical axis). All this means that the average is going to be lower, and also that low values are going to be more likely. Consistent with both facts, the new line L2 makes the yearly average to fall from 121.69 GWh interrupted before the expansion to 49.71.57 GWh after the expansion. The reliability benefits of the expansion are thus rather clear.

Now, Table 9 displays the comparative results in terms of expected or average yearly values of a wider set of relevant variables. The results of adding a second line L2 to the system are very similar to those under load certainty. Average load is the same (as it should be), but unserved load falls by almost 60%. Gas-fired stations operate more intensely to meet demand at node #2 not only because now there is a new line in place, but to replace some local (coal) generation as well. Carbon emissions rise by 1.20% despite gas being cleaner than coal; note that generation changes in coal and gas are far from being one to one. But total generation rises by more (4.60%); thus carbon emissions per megawatt hour lower. Allowance costs rise by only 0.45%. However, the system cost fall by more than 16% after the expansion. Reliability benefits are certainly playing a role in this result, but mere comparison of the percentage reductions in unmet load and system cost (grossly 60% and 16%, respectively) suggests that supply-side costs have risen (gas is more expensive than coal, and gas stations are operating for longer than before the expansion).

Table 9. Base case with L^1 and $L^1 + L^2$.

	L^1	$L^1 + L^2$
Coal generation (GWh)	493.27	471.69
Gas generation (GWh)	1199.63	1299.32
Load (GWh)	1786.04	1786.04
Unserved load (GWh)	121.69	49.71
Transmission (GWh)	464.59	564.28
Emissions ($KtCO_2$)	1000.46	1012.57
Allowance costs (M€)	15.84	15.95
Present cost (M€)	116.06	96.51

Regarding total costs (which encompass those of generation and unserved load, $VOLL$ = 500 €/MWh interrupted), the expansion certainly brings an important reduction: 115.14 − 96.51 = 18.63 M€ per year. It is important to distinguish between the costs to generators, on one hand, and those of unserved load, on the other. Blumsack *et al*. [26] compute separately the "congestion cost" associated with line k in the network (which is defined as the difference in total system cost to serve identical demand profiles in a system with and without that line), and the "cost of unserved energy" (which is more subtle, and depends on the assumptions about $VOLL$). With the values in Table 9 we get:

$$\text{Cost Unserved Energy } (L^1) = \frac{121.69 \times 10^3 \text{ MWh} \times 500 \text{ €/MWh}}{1{,}000{,}000} = 60.84 \text{ M€}$$

$$\text{Cost Unserved Energy } (L^1 + L^2) = \frac{49.71 \times 10^3 \times 500}{1{,}000{,}000} = 24.85 \text{ M€}$$

Now subtracting these amounts from system costs we derive generation costs:

$$\text{Generation Cost } (L^1) = 115.14 - 60.84 = 54.3 \text{ M€}$$

$$\text{Generation Cost } (L^1 + L^2) = 96.51 - 24.85 = 71.66 \text{ M€}$$

Hence, the congestion cost of line L2 would be:

$$\text{Congestion Cost } (L^2) = 71.66 - 54.3 = 17.36 \text{ M€}$$

This definition of congestion cost is taken from Blumsack *et al*. [26]. Indeed they get a similar result in their case study: the congestion cost of the line considered turns out to be positive, "*but in theory this need not necessarily hold in more general networks*" [37].

Hence we can also learn a few more results. Line L2 entails a congestion cost of 17.36 M€. At the same time, though, it reduces the costs of unserved load from 60.84 M€ to 24.85 M€, that is, in 60.84 − 24.85 = 35.99 M€. Its net contribution is therefore the same that we got before: 35.99 − 17.36 = 18.63 M€ per year.

Of course the former amount (properly discounted and accumultated over time) must be weighted against the (fixed and variable) costs of the expansion and the environmental impacts (GHG emissions, land use, erosion risk, threats to wildlife, ...) [38]. These costs can obviously vary from one country to another. For a given set of line characteristics we could then assess whether the expansion should take place (on a now-or-never basis) or not [39].

8.3. Expansion under Higher Demand Volatility

Next we address the valuation of the network expansion under the assumption that load volatility is 3.10 % (*i.e.*, just double our estimate from observed data). The results are displayed in Table 10. This higher volatilility means a wider scope for load excursions that cannot be matched by generation units. Besides, the initial network with only L1 is more likely to be under a sudden, severe stress. The grid expansion effectively contributes to alleviating these shortcomings; unmet load falls by more than half. Gas-fired stations exploit the new infrastructure to their benefit, which is to the detriment of coal plants. Carbon emissions and allowance costs remain almost unchanged, but the system cost falls noticeably. Again, reliability benefits are the main driver of this reduction; indeed, they more than offset the rise in operation costs that take place after the expansion.

Table 10. Expansion from L^1 to $L^1 + L^2$ under higher load volatility.

	L^1	$L^1 + L^2$
Coal generation (GWh)	491.63	467.31
Gas generation (GWh)	1193.46	1291.60
Load (GWh)	1786.49	1786.49
Unserved load (GWh)	129.57	61.78
Transmission (GWh)	458.56	556.70
Emissions (KtCO_2)	996.34	1004.77
Allowance costs (M€)	15.74	15.75
Present cost (M€)	118.45	99.61

8.4. Expansion under Higher Demand Growth

Assume again that load grows at a rate 3.49% (*i.e.*, double our base case estimate). The results of expanding the grid now are displayed in Table 11.

Table 11. Expansion from L^1 to $L^1 + L^2$ under higher load growth.

	L^1	$L^1 + L^2$
Coal generation (GWh)	494.91	483.27
Gas generation (GWh)	1341.73	1452.84
Load (GWh)	2163.30	2163.30
Unserved load (GWh)	354.47	261.82
Transmission (GWh)	452.68	563.79
Emissions (KtCO_2)	1054.50	1082.09
Allowance costs (M€)	16.75	17.19
Present cost (M€)	193.30	166.25

The starting level of effective load is notoriously higher than before. Since the physical infrastructure has not changed, the scope for unserved load is also wider initially. When the new line L2 becomes available, minimization of system cost calls for enhaced operation of gas stations, even at the expense of coal stations. The expansion allows a better matching of (growing) needs and resources; this also shows up in terms of increased transmission to node #2. Carbon emissions and allowance costs show a minor increase; system cost, instead, fall much more. Once again, the reliability benefits of the expansion rise above congestion costs.

8.5. Expansion under Higher Allowance Price

Now we undertake a sensitivity analysis with respect to the initial allowance price; this is assumed to be 26.36 €/tCO_2 (*i.e.*, just double our estimate from observed data). Table 12 shows the results. The new setting clearly damages the competitive position of coal-fired stations. They cede ground to gas plants, which now meet a bigger proportion of load at node #2 through enhanced transmission reliability, thus bringing down unserved load significantly Consistent with the increase in gas-based generation's share, carbon emissions and allowance costs decrease slightly. A bigger drop is observed in the system cost. The main drivers of this reduction are again reliability benefits, which outweigh the rise in operation costs due to increased generation.

Table 12. Expansion from L^1 to $L^1 + L^2$ under higher initial allowance price.

	L^1	$L^1 + L^2$
Coal generation (GWh)	485.70	429.76
Gas generation (GWh)	1206.98	1343.30
Load (GWh)	1786.04	1786.04
Unserved load (GWh)	122.36	50.36
Transmission (GWh)	471.94	608.26
Emissions (KtCO_2)	994.57	981.12
Allowance costs (M€)	31.32	30.82
Present cost (M€)	131.81	112.16

To summarize, Table 13 provides two basic statistics of the distribution of unserved load in the different scenarios considered (annual values can be derived simply dividing by 20 years).

Table 13. Descriptive statistics of the distribution of (cumulative) unserved load across scenarios.

	L^1		$L^1 + L^2$	
	Average	**90% Percentile**	**Average**	**90% Percentile**
No load volatility	2,376,500	2,441,589	894,570	958,389
Base case	2,433,759	3,561,563	994,179	1,747,684
Double load volat.	2,591,459	4,854,215	1,235,570	2,793,873
Double load growth	7,089,456	8,678,301	5,236,445	6,943,400
Double CO_2 price	2,447,227	3,588,797	1,007,142	1,765,800

The average value is lowest under a deterministic load, both when only L1 is available and with $L^1 + L^2$. As expected, the values after the expansion are sizeably lower than those before it. Note, though, that there will be still a 10% of cases in which the unserved load will rise above a substantial level. For example, in the base case scenario, the amount unserved will surpass 1.75 times the average demand in one out of ten occasions. Before leaving this section, note that there are a number of possible comparisons that we do not undertake. In particular, restricting ourselves to the case with $L^1 + L^2$ in place, we can make cross comparisons of different scenarios. For example, we can compare the system performance (under $L^1 + L^2$) in the base case and in that with higher load volatitlity. The information provided in the above tables allows to do this comparison and others; nonetheless, they are left to the interested reader.

9. Concluding Remarks

Electricity transmission has been an essential part of power system planning traditionally. This is so because electricity transmission is critical to the efficient and reliable delivery of services. Certainly, delivered power is a bundle of many services: transmission, distribution, frequency control, and voltage support, as well as generation [11]. And, in the past, the main purpose of utility transmission planning was indeed to support system reliability. Examples of some reliability metrics (like the $N-k$ criterion, the Loss of Load Probability, and the Loss of Energy Expectation) can be found in Blumsack *et al*. [26]. However, some of these techniques for deciding what upgrades to build (and who should pay for them) have been proven to be inefficient [8].

At present, the challenges are even more daunting. The demand for electricity keeps on rising and it is expected to remain so in the foreseeable future. To the extent that a sizeable part of that electricity will be generated from fossil fuels there is ample room for climate change concerns. And geopolitics shows no sign of receding as a source of international tensions; rather, security of supply ranks ever higher in national energy policies [40].

Against this background, the current state of transmission infrastructure leaves little room for optimism. In particular, Europe and the United States are witness to a growing problem of ageing with regard to these critical capital assets. This fact can in turn be traced back to a deeper problem of underinvestment. Why would the pattern of investment in new infrastructures fall behind the optimal level? As it turns out, there are a number of reasons for this.

At one level, investors may not be getting a fair return on investments undertaken in the past. Indeed, according to Perez-Arriaga *et al*. [41], revenues from nodal electricity prices only recover 25% of total costs [42]. On the other hand, the problem of underinvestment is further aggravated by policy makers. This is simply because private investment decisions depend heavily on (current and expected) rules set by public bodies.

Other reasons run deeper. For one, Léautier [43] finds two effects of an increase in transmission capacity. First, there is a substitution effect: transmission expansion permits that cheaper power substitutes for more expensive power. There is also a second, strategic effect: competition in generation increases. Thus, incumbent generators may not always have incentives to carry out transmission expansion projects. The analysis of appropriate incentives for a transmission network is further complicated by additional special characteristics of electricity transmission.

Interactions in actual transmission networks give rise to externalities whereby a particular modification is likely to influence the performance of other parts of the network. These interactions are governed by Kirchhoff's laws. Likewise, energy cost and transmission costs are not independent. And competition might work in power generation and electricity marketing, but transmission and distribution are still natural monopolies [44].

In sum, as Schweppe *et al*. [15] put it, "the dominant issues regarding long-term planning are: no unique scalar criterion to define what is the optimum, massive uncertainty about the future, and multiple decision makers" [45]. As a result, "In practice, power system planning is an art, not a science!"

The absence of a universal guide has paved the way to a plethora of proposals. For example, Buygi *et al*. [4] identified up to six different market-based criteria for transmission planning. Hogan [46] suggested a second-best standard according to which "small" transmission projects would rely on a merchant approach, while large and lumpy projects would be developed through some type of incentive regulation.

We are thus left with at least one conclusion, namely that market forces alone are not up to the task regarding investment in electricity transmission infrastructures. This is recognized at the highest levels. For instance, the EU governments have acknowledged that market forces alone cannot be expected to gain the infrastructure investments that Europe needs if it is to secure its electricity supplies and meet its carbon emission targets [47]. Note also that, in general, low investments in transmission capacity favor high-cost fuels, which in turn may have some implications on the national energy policy. What can then be done to attract the investments needed for long-term expansion? Multiple obstacles will surely require multiple responses.

We have left aside a number of them. To name a few, we do not address the distributional impacts that typically appear with transmission expansions. This is important, since society as a whole may benefit from mitigating congestion but some parties may be adversely affected; the implications of the exercise of local market power by generators are analysed in Sauma and Oren [19]. We neither address the external effects of expanding the transmission grid. A thorough analysis requires a more complex network, with a higher number of generators, loads, and interconnections. Transmission investments also affect geographical development, social policy, and so on. These subjects are well beyond our paper. Even in our basic setup, we do not assess when to expand, where to expand, or how much to expand the transmission capacity.

Instead, we have focused on the valuation of the potential benefits of the expansion under uncertainty. As a matter of fact, the cost of these projects is readily known with great precision. Yet the cost is only half the story. Without a reasonable estimate of their value, decision making seems shaky.

We have considered a simplified two-node network with loads in both nodes and two fossil-fired stations operating under carbon constraints. We have assumed that transmission line capacities are static and deterministic. Uncertainty stems not only from the physical equipment (which can fail), but from demand and supply sides as well since load and prices can unpredictably change. We have adopted particular stochastic processes for describing the behavior of fundamental variables over time. We have estimated the underlying parameters in these processes from observed market data. Then we have resorted to Monte Carlo simulation to build a number of potential scenarios. The aim is to analyze the performance of the system in the base case (with one transmission line) and the expanded case (with two lines). The performance is assessed in terms of several metrics, among them total generation costs, carbon emissions, and unserved load. In all the cases and scenarios, optimal operation of the system is assumed in that we solve the problem of economic dispacth under the prevailing conditions at any time.

Uncertainty impacts adversely the system reliability. Wider gaps between supply and demand appear naturally as generation units can follow nodal loads in their way down but face capacity constraints when following nodal loads in their way up. In a fixed environment, however, the scope for load excursions is minimum. Therefore, introducing load uncertainty leads to reliability costs. The rise in these costs can indeed outweigh whatever fall happens to be in supply-side costs, so the system performs worse (assuming of course that lost load has value).

The addition of the new line L2 leaves less room for extreme (high and low) system costs than before (when only L1 was in place). Thus, central outcomes become more likely. Besides, the distribution of total costs after expansion is more asymmetric than before. Specifically, higher costs display lower frequencies. These changes bring about a reduction in the average system cost.

Regarding carbon emissions, the range of potential values is wider after the network expansion. The new line L2 allows a more intense use of cleaner fuels (thus making lower emissions feasible) but also opens the way to enhanced generation (thus pushing emissions up). Indeed, the yearly average increases in the base case. And the expansion makes the distribution relatively more asymmetric, with lower emission levels now displaying lower frequencies than before.

Regarding the distribution of unserved load, the possible values are lower after the network expansion. And the shape of the distribution changes; it becomes more asymmetric in that higher values display lower frequencies. In the base case, the new line L2 cuts the yearly average of unmet load in more than half. The reliability benefits of the expansion are thus rather clear. Indeed, they can even offset the rise in operation costs due to the increased generation that the grid expansion enables.

The model can be extended in a number of ways. Since this is a long-run analysis and since transmission expansion can well affect electricity prices, demand effects on electricity may be worth exploring (here electricity demand was assumed to be totally inelastic). Another possibility is to consider a network with at least three nodes. Still another option is to consider (in isolation or simultaneously) generation investments and transmission investments. On the other hand, new technologies (e.g., FACTS) allow to use the existing transmission lines more efficiently. These devices can also be suitably analyzed (against the alternative of building new lines) following the approach here adopted. Any of these alternatives, and others, would undoubtedly enrich our analysis in breadth and depth. Yet this is left for future research.

Appendix A. Estimation of the Long-Term Fixed Margin

From Abadie *et al*. [30], the estimation of the long-term margin M_m requires defining the margin in the first place. The margin at time t, M_t (in €/MWh), is computed as follows:

$$M_t = S_t - \frac{G_t + 0.20196 A_t}{H_G} \tag{5}$$

where S denotes electricity price (€/MWh), G is the price of natural gas (€/MWh), H_G is the net thermal efficiency of a gas plant, and A stands for the price of an EU emission allowance (€/tCO$_2$).

Now we assume that this clean spark spread (CSS) evolves stochastically over time. In order to get an estimate for M_m, we propose a (theoretical) model for the behavior of M_t; we then estimate this model by following standard econometric techniques. Specifically, the time path of the margin is assumed to follow an Ornstein-Uhlenbeck process. This stochastic process accounts for mean reversion in CSS values along with continuous unpredictable swings. It also allows the margin to take on negative and positive values:

$$dM_t = k_M(M_m - M_t)dt + \sigma_M dW_t^M \tag{6}$$

M_m is the value which the margin tends to in the long term, and k_M is the speed of reversion toward this value. σ_M denotes the instantaneous volatility of the margin. dW_t^M stands for the increment to a standard Wiener process. It can be shown that:

$$E(M_t) = M_0 e^{-k_M t} + M_m(1 - e^{-k_M t}) \tag{7}$$

where M_0 stands for the value of the margin at time $t = 0$. For high speeds of reversion ($k_M >> 0$) the model provides margins that are close to the long-term value. The same holds for times that are far away into the future (since $k_M t > 0$). In both cases we thus get: $E(M_t) \approx M_m$.

Full details on the estimation procedure can be found in [30, Appendix A1]. Here we merely show the numerical estimate that will be used in our (long-term) evaluation: $\hat{M}_m = 13.30691$. It will be taken as constant henceforth.

Appendix B. Load and Risk of Unserved Load

Consider one stochastic electricity demand in isolation. Our aim is to show how the behavior assumed impinges on the possibility of being unable to meet demand in the future, that is, on the potential for experiencing unserved load. If demand follows a geometric Brownian motion (GBM), the instantaneous change is:

$$dD_t = \alpha_D D_t dt + \sigma_D D_t dW_t^D$$

Now adopting a logarithmic transformation $X \equiv \ln D$ and applying Ito's lemma we get:

$$dX_t = (\alpha_D - \frac{\sigma_D^2}{2})dt + \sigma_D dW_t^D$$

This implies that:

$$\ln D_T : \varphi\left(\ln D_0 + (\alpha_D - \frac{\sigma_D^2}{2})T, \sigma_D\sqrt{T}\right)$$

If the initial demand is D_0 and it grows at the rate α_D then its expected value at time T will be:

$$E_0(D_T) = D_0 e^{\alpha_D T}$$

Let $\overline{D}$ denote the maximum demand that can be served (either because it equals maximum generation or because it is restricted by the transmission capacity). Intuitively the probability that this threshold will be reached increases with the passage of time (specifically, it will be lower at earlier times, and higher at later times). At some particular time T^C, that probability will be exactly equal to 50%:

$$T^C = \frac{\ln \overline{D} - \ln D_0}{\alpha_D}$$

This entails probabilistically that the threshold $\overline{D}$ is surpassed at T^C on 50% of the cases and, consequently, in half of the cases at time T^C there would be a penalty (the value of lost load, $VOLL$).

The initial value value is taken to be $D_0 = 50$ MW, which grows at an annual rate of α_D = 0.03 with a volatility σ_D = 0.20. If we want the penalty at time T^{VOLL} to take place only on 5% of the cases we get the results in the last two columns of Table 14. $L_{0.05}(t = 10)$ stands for the capacity that we should have at time $t = 0$ in order to serve total demand in 10 years' time on 95% of the cases. $E_0(T_{0.05})$ denotes the number of years beyond which the current generation capacity will be surpassed by demand in more than 5% of the cases.

Table 14. Expected load, 5%-upper bound, and passage time ($\overline{D} = 86.5$).

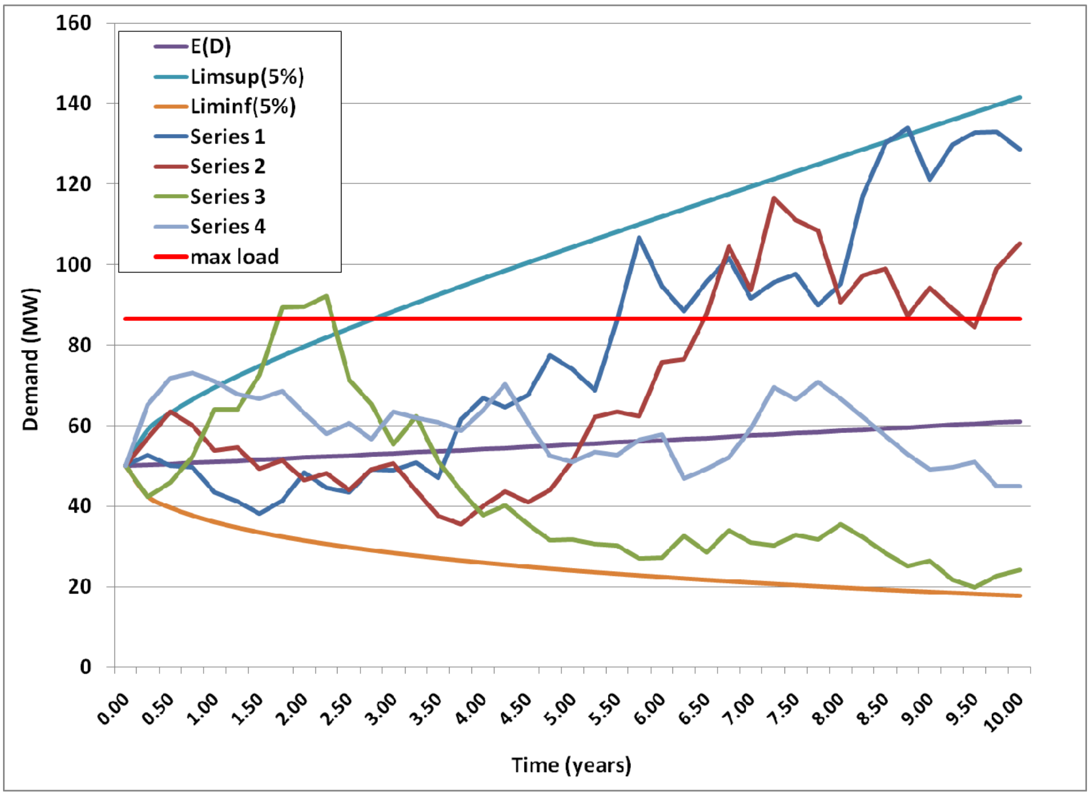
D_0	α_D	σ_D	$E_0\ (D_{10})$	$E_0\ (D_{20})$	$L_{0.05}\ (t = 10)$	$E_0\ (T_{0.05})$
50	0.03	0.20	67.49	91.11	156.38	2.53
50	0.02	0.20	61.07	74.59	141.50	2.78
50	0.01	0.20	55.26	61.07	128.04	3.10
50	0.03	0.10	67.49	91.11	108.00	5.92
50	0.02	0.10	61.07	74.59	97.73	7.17
50	0.01	0.10	55.26	61.07	88.43	9.30
30	0.03	0.20	40.50	54.66	93.83	8.72
30	0.02	0.20	36.64	44.75	84.90	10.36
30	0.01	0.20	33.16	36.64	76.82	13.08
30	0.03	0.10	40.50	54.66	64.80	16.02
30	0.02	0.10	36.64	44.75	58.64	20.70
30	0.01	0.10	33.16	36.64	53.06	30.40

Looking at the fourth and fifth columns, given the values of D_0, α_D, and σ_D, the expected load after 20 years is obviously higher than the level expected after only 10 years (note that the GBM is a non-stationary process). As could be inferred from the formula for $E_0(D_T)$, volatility plays no role in these values while the drift rate does matter.

Now comparing the fourth and the sixth columns (which share the same time horizon), it can be seen how far away the highest 5% of the loads can deviate from the average value. Thus, in the first row, the latter is 67.49 MW but in 5% of the cases the load would surpass 156.38 MW; obviously, as volatility decreases also this threshold value decreases.

Regarding the last column, given the values of D_0, α_D, and σ_D, it is easy to derive the 5% upper bound and the 5% lower bound of the stochastic process; see Figure 6. The numbers in this column show the number of years that pass on average until the upper bound is crossed by the actual load. As expected, when the volatility is higher the number of years is lower.

Figure 6. Stochastic demand and 95 per cent confidence interval.



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17. The inverse of efficiency is the so-called heat rate, which is the ratio between heat input into the plant (Btu/hr) and electrical power output (kW).
18. In practice, economic dispatch is recalculated every five to ten minutes. The economic dispatch problem is related to the unit commitment problem. The latter considers longer time scales; it specifies the daily on/off schedule of generators. The economic dispatch is a subproblem of unit commitment [15].
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23. The model allows addressing this issue but our main concern here is the alleged shortfall of investments in transmission.
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32. See Reference [57].
33. The interest rate corresponds to the rate of German government bonds in November 2009. On the other hand, we transform ARA coal prices from \$/tonne to €/tonne using an exchange rate of 1.4934 \$/€ (the rate on 27 November 2009, when the 15-year interest rates of the euro and the dollar were at similar levels). In a further step, we transform €/tonne into €/MWh considering 29.31 GJ/tonne and using the equivalence 1 GJ = 0.27777 MWh.
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38. If, in addition to GHG emissions, other environmental damages are encompassed (SO_2, NO_x, ...) then the objective function could include a third term with an additional cost per tonne emitted.
39. Regarding the costs of transmission lines, Blanco *et al*. [9] assume they have a fixed component (90,000 $/km) and a variable component (800 $/MW km); Rosellón and Weigt [36] choose a value of 100 €/km/MW; see also [51], Table 1.
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