Article

A Comparative Study of Open and Closed Heat-Engines for Small-Scale CHP Applications

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Abstract: In this paper the authors compare and contrast open and closed-cycle heat engines. First of all, by way of example and to aid discussion, the performance of proprietary externally heated closed-cycle Stirling engines is compared with that of internally heated open Otto cycle engines. Both types of engine have disadvantages and merits and this suggested that in order to accommodate the best of both engine types an externally-heated open-cycle engine might offer a more satisfactory solution for small-scale combined heat and power (CHP) systems. To investigate this possibility further the paper goes on to compare the performance of externally-heated and recuperated Joule hot-air cycle engines with that of an externally-heated closed Stirling cycle engines. The results show that an externally heated recuperated open Joule cycle engine can exceed that of a closed cycle Stirling engine operating between the same heat source and sink temperatures when a variable temperature heat source is used.

Keywords: heat engine cycles; recuperated Joule cycle; Stirling cycle; thermodynamic performance; thermodynamic efficiency; combined heat and power

1. Introduction

The paper begins by comparing the performance of proprietary externally-heated closed-cycle reciprocating Stirling engines with proprietary internally-heated reciprocating open Otto cycle engines. Disadvantages and merits for both engine types are discussed and from this it is concluded that some form of externally-heated open-cycle engine might offer a practical solution to the problem of selecting an engine cycle suitable for small-scale power generation. To support this view the paper goes on to compare the performance of a recuperated reciprocating open Joule cycle engine with that of an externally-heated reciprocating closed Stirling cycle engine.

Over recent years there has been an increase in the amount being written on the various types of Stirling cycle engine, particularly for applications in advanced combined heat and power energy systems. The cycle was invented by Robert Stirling in 1816 [1] and two recent papers [2,3] provide useful summaries of progress in this area. The constant pressure cycle too has a long history. In 1791 John Barber patented the first engine of this type, which incorporated an air compressor, a combustion chamber and an early type of turbine [4]. However, it was James Prescott Joule (1850) that provided the first theoretical description of the constant pressure cycle [5], more than 20 years before George Brayton invented his improvements [6]. It is interesting to note that both Joule's and Brayton's engines were reciprocating piston types and both the original work of Barber and the improvements by Brayton were based on internal combustion engines but that Joule's own test engine was externally heated.

More recently Moss, Roskilly and Nanda proposed a reciprocating Joule-cycle engine for domestic combined heat and power systems [7].

This paper compares and contrasts the performance of close and open cycle engines. The former being externally heated whilst the latter are almost all internally heated; and are commonly referred to as internal combustion engines.

2. Externally Heated *Versus* Internally Heated Engine Cycles

2.1. Comparison of Thermal Efficiency

For a given cycle temperature ratio there is no doubt that the thermodynamic efficiency of a fully reversible Carnot cycle engine cannot be bettered. In practice, however, the low value of indicated mean-effective-pressure (imep) of the Carnot cycle precludes any practical application. In contrast a fully-reversible Stirling cycle engine offers practical imep values with thermodynamic efficiencies equal to those of the Carnot cycle engine with a similar cycle temperature ratio [8]. The same maybe said of both Atkinson and Ericsson cycle engines [9].

Today Stirling cycle engines are used to drive relatively small electricity generators in combined heat and power systems. Being positive displacement machines drive-shaft speeds tend to be low and easily matched to those of electricity generators without the need for large gear boxes. Importantly too, Stirling engines are externally heated and, therefore, can be powered by a wide range of variable temperature heat sources, for example low-grade renewable fuels such as wood chip and other derived fuels that for internally heated engines would be impracticable.

Stirling engines operate in a closed-cycle; meaning that the same working fluid circulates between the hot-side and cold-side of the engine. Therefore both sides of a Stirling cycle engine require a heat exchanger: one to transfer heat from the high temperature heat source to the engine's working fluid and a second to transfer heat from the working fluid to a cooling medium outside the engine. This is a disadvantage of all closed cycle engines. Because heat must be transferred within a finite-time there must be temperature differences at the hot- and cold-sides of the engine cycle (ΔT_H and ΔT_L), in order to achieve the necessary heat rates. An effect of these necessary temperature differences is to reduce the maximum possible thermodynamic efficiency of an engine. Equation (1) shows thermodynamic efficiency is reduced as ΔT_L and ΔT_H increase:

$$\eta_{th} = 1 - \frac{T_L}{T_H} > 1 - \frac{T_L + \Delta T_L}{T_H - \Delta T_H} \tag{1}$$

Increasing these temperature differences reduces the required heat exchange area, which in turn reduces the physical size of any externally heated and externally cooled engine but at a cost of reduced thermodynamic efficiency. A further disadvantage of closed cycle engines is the often proposed use of non-benign working fluids, such as iso-propane (organic Rankine cycle), hydrogen (Stirling cycle). Neglecting any environmental or safety issues with some of these working fluids, leakage can problematic and require specialist maintenance on a regular basis, which may be an unwanted additional expense for domestic combined heat and power systems intended to replace conventional water heaters. Even Rankine cycle steam engines, using water as their working fluid, can suffer loss of vacuum pressure at the condenser, which harms their performance. On the other hand open cycle, (air breathing), engines do not suffer this disadvantage and so may be more suited to small scale combined heat and power applications.

For open cycle internal combustion engines, such as the Diesel engine, the ΔT_H and ΔT_L terms in Equation (1) are removed and so the potential maximum thermodynamic efficiencies of an internally-heated open cycle engines tend to be greater than that of a similar closed cycle engine operating with the same heat source and sink temperatures. It is interesting that the empirical data listed in Table 1, which compares the performance of (gas-fuelled) open Otto cycle engines with (gas-fuelled) closed Stirling cycle engines, (which theoretically at least should have a greater efficiency),

also seems to supports this view. In addition to the data listed in Table 1, Conroy, Duffy and Ayompe reported an overall efficiency for electricity generation of 7.9% [3], for the WhisperGen Stirling engine combined heat and power system, which is lower than others listed.

Table 1. A comparison between the thermodynamic efficiencies of open-cycle Otto and closed-cycle Stirling engines.

| Manufacturer	System Name	Engine Cycle	Output	Thermo Efficiency	Reference
asjaGen	TOTEM 10	Otto	10 kW	30%	[10]
Helec Ltd	Energimizer	Otto	7.5 kW	25%	[11]
Helec Ltd	Powerbox 7500QSE	Stirling	7.5 kW	18%	[12]
Baxi Ltd	Ecogen	Stirling	1 kW	13%	[12–14]

In addition, the physical size of open cycle engines tends to be smaller and lighter in weight than closed cycle machines with similar outputs. This potential for small, light-weight engines led to their general adoption for motorised transport systems. For combined heat and power applications, however, physical size and weight are probably not so important and therefore closed cycle engines have found useful applications: Rankine cycle steam turbine engines for large scale power generation and Stirling cycle engines for small-scale systems.

Open-cycle internal combustion engines are limited to burning refined fuels such as gasoline, liquefied petroleum gas and natural gas. This may be a disadvantage for power generation in areas of the world where the burning of low grade fuels (wood chips, pellets, peat, selected waste or poor quality bio-oils) is the most practical and economic source of high grade heat.

Therefore, an engine which can be both externally heated and based on an open cycle, so that it can run on any fuel or mixture of fuel type(s) and does not require a low-temperature heat exchanger, may offer some advantages over currently available technologies. One such engine-cycle is the constant pressure cycle.

2.2. The Recuperated Constant Pressure Heat Engine Cycle

Figure 1a shows a schematic view of an externally-heated, recuperated, constant pressure Joule cycle (RJC) engine whilst Figure 1b provides a T-s diagram for a fully reversible RJC engine. It is well known that the constant pressure open Joule cycle also provides the theoretical bases for the gas-turbine engine, the analysis of which is well documented in text books [15].

(a) **(b)**
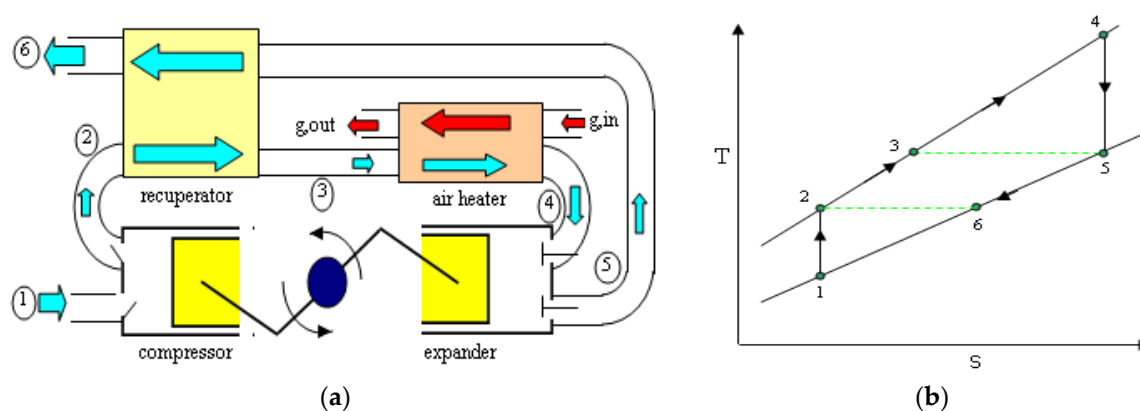
Figure 1. (**a**) Schematic view of an externally heated RJC (recuperated joule cycle) engine; (**b**) An ideal RJC engine in a temperature-entropy co-ordinate diagram. State-points defined in Table 2.

Referring to Figure 1b the processes shown are as listed in Table 2.

The RJC (recuperated Joule cycle) engine does not require external cooling as the working fluid discharges directly to the environment at state-point 6 in Figure 1b. For combined heat and power applications the hot airflow leaving the engine at state-point 6 may be used to; provide space heating, drive a thermally activated cooling cycle or power an organic Rankine cycle (ORC) engine. Any of these additions would further increase the usefulness of the heat source. In addition the waste heat in the combustion gas may also be captured as it leaves the air-heater at state-point g,out, shown in Figure 1a, thus increasing the thermal efficiency of the CHP system as a whole.

Table 2. Processes shown in Figure 2.

State-Points	Process
1–2	Compression of ambient air
2–3	Recuperative heating
3–4	External heating
4–5	Expansion
5–6	Recuperative cooling

3. Comparison of the Externally Heated RJC (Recuperated Joule Cycle) and Stirling Cycle

Figure 2 shows T-s diagrams for an internally reversible RJC engine and internally reversible Stirling cycle engine both with finite-time heat transfer between source and sink.
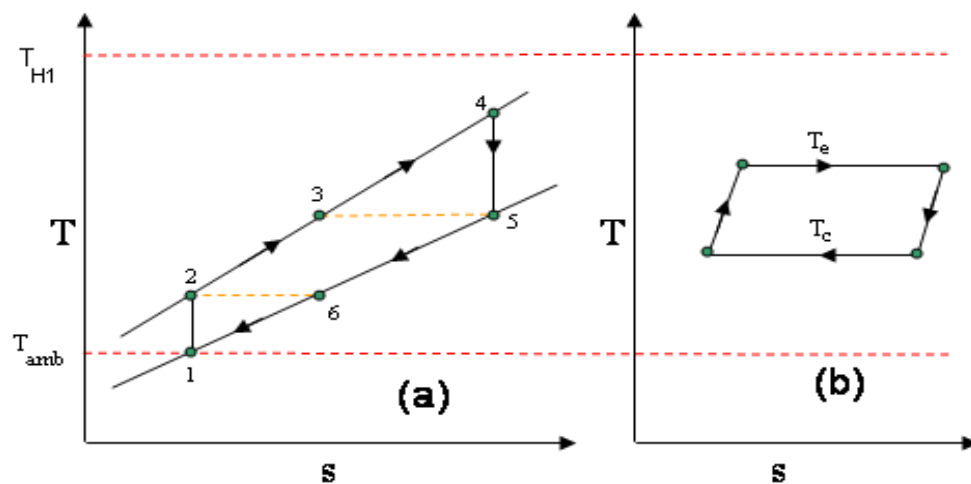
Figure 2. Showing T-s diagrams for: (**a**) An internally ideal recuperated Joule cycle with external heat addition; (**b**) An externally heated and cooled ideal Stirling cycle.

For the purpose of analysis the following assumptions were made:

(1) Both the RJC and Stirling cycles are internally reversible.
(2) Both cycles are heated externally via identical heat exchangers using hot combustion flue-gas: a variable temperature heat source.
(3) In both engines the combustion gases are assumed to enter the high-temperature heat exchanger at the same temperature (T_{H1}) and leave at the same temperature (T_{H2}).
(4) Both cycles are assumed to absorb heat at the same rate (J/s).
(5) Either $\Delta T_{m,H,RJC} = \Delta T_{m,H,SC}$ or $ATD_{H,RJC} = ATD_{H,RC}$ (both possibilities were investigated).
(6) For the RJC the heat capacity rate, (C_H), of its working fluid equals that of the combustion gases.
(7) The working fluid within both cycles is dry-air, which is assumed to be a perfect gas.

(8)   The Stirling cycle is assumed to reject heat to its environment (low-temperature sink) via an air-cooled heat exchanger and the ambient air entering this heat exchanger has the same temperature as the air entering the compressor of the RJC engine: $T_1 = T_{amb}$ in Figure 2a $= T_{L1}$ in Figure 5.

(9)   The temperature of the air leaving the Stirling cycle's low-temperature heat exchanger equals that leaving the recuperator of the RJC: T_6 in Figure 2a equals T_{L2} in Figure 5. This is thought to be a reasonable assumption if the waste heat from both cycles is to be utilized for heating purposes because it would mean its temperature in both cases would be equal.

(10)   For the purpose of analysis the ambient air temperature is assumed to be 300 K.

In order to compare the thermodynamic efficiencies of otherwise internally reversible RJC and SC engines with finite-time heat transfer it is necessary to define a suitable temperature difference between the heat sources and the engine. For heat exchangers there are two well know temperature differences that might be used: Approach-temperature-difference (ATD) and log-mean or area-weighted-temperature-difference (ΔT_m). The following analysis shows equal values of ΔT_m probably gives the fairest comparison but the difference appears not to be significant to the conclusion reached. ATD and ΔT_m are related by:
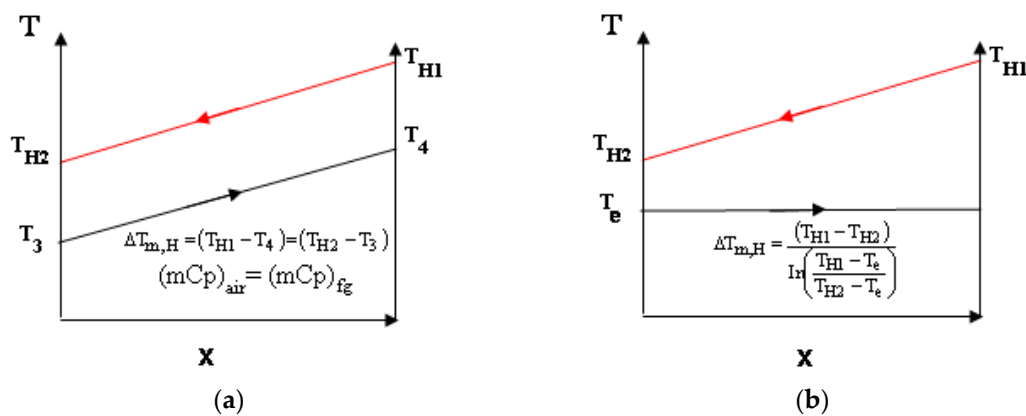
$$Q = C_{min} \varepsilon \text{ATD} = \text{UA} \Delta T_m \tag{2}$$

where the minimum capacity rate of the hot and cold streams, $C_{min} = (mC_p)_{min}$.

3.1. Comparison of Efficiency Based on Equal $\Delta T_{m,H}$ Values

Figure 3a,b show the variation in temperature through the high-temperature heaters of the RJC and SC engines. Defining the number-of-heat-transfer units as $\text{NTU} = \dfrac{\text{UA}}{C_{min}}$, from Equation (2) it may be shown:

$$\text{ATD} = \frac{\text{NTU}}{\varepsilon} \Delta T_m \tag{3}$$

(**a**)   (**b**)

Figure 3. (**a**) Variation in temperature through the RJC engine high-temperature heater; (**b**) Variation in temperature through the SC engine high-temperature heater.

For the RJC air heater, shown in Figure 3a, assuming the heat capacity rates (*C*) of both hot and cold streams are equal ($C_{min} = C_{max}$) then heat exchanger effectiveness (ε_{RJC}) is be given by:

$$\varepsilon_{RJC} = \frac{\text{NTU}}{\text{NTU} + 1} \tag{4}$$

For the SC heater, the cold stream of which is isothermal, then heat exchanger effectiveness is given by:

$$\varepsilon_{SC} = 1 - \exp(-NTU) \tag{5}$$

Figure 4 shows the variation in ε with NTU (number of heat transfer units) for both engine's high-temperature heat exchangers.
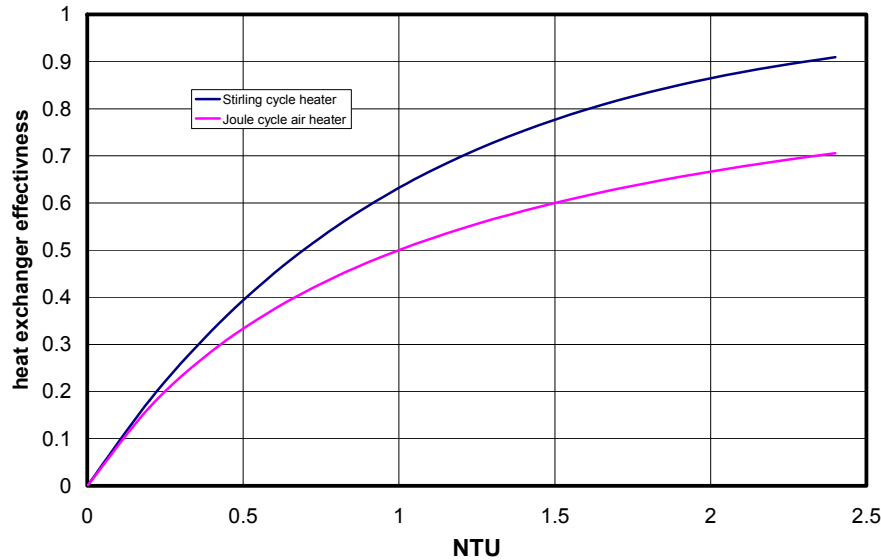
Figure 4. Showing the variation in heat-exchanger-effectiveness (ε) with NTU for the hot-side heat exchangers of SC (in blue) and RJC (in red) engines.

If it is assumed that the NTU and ΔT_m values are the same for both cycles. Assuming NTU = 1.5 then from Figure 4, or by substitution in to Equations (4) and (5), the heat-exchanger-effectiveness values for the RJC and SC heaters are 0.6 and 0.78. If we assume an equal $\Delta T_{m,H}$ value of 200 K and an NTU value of 1.5 in both cases then from Equation (2), $ATD_{SC} = 386$ K, $ATD_{RJC} = 500$ K. In other words, if the flue-gas temperature at the outlet of both engine heaters has the same value, the temperature of the SC engine expander inlet temperature, T_e, will be greater than that of the RJC engine heater air entry temperature, T_3 in Figure 3a. This appears to offer advantages for the performance of the SC engine. However, because the flue-gas temperature felt by the SC expander flow increases as it moves from left-to-right in Figure 3b the exergy loss is larger than in the case of the RJC engine heater, in which the temperature of the air stream increases as it flows the heater.

If the flue-gas temperature at entry to the high-temperature heat exchanger is assumed to be 1400 K then referring to the heat exchange process described in from Figure 3a:

$$T_4 = T_{H1} - \Delta T_{m,H} = 1400 - 200 = 1200\,\text{K}$$

and $T_3 = T_{H1} - ATD = 1400 - 500 = 900\,\text{K}$.

Therefore, $T_{H2} = T_3 + \Delta T_{m,H} = 900 + 200 = 1100\,\text{K}$

Referring to Figure 3b, for the SC engine the $\Delta T_{m,H}$ is given by:

$$\Delta T_{m,H} = \frac{(T_{H1} - T_{H2})}{\ln\left(\dfrac{T_{H1} - T_e}{T_{H2} - T_e}\right)}$$

Solving the above for T_e gives:

$$T_e = \frac{\exp\left(\dfrac{T_{H1} - T_{H2}}{\Delta T_{m,H}}\right) T_{H2} - T_{H1}}{\exp\left(\dfrac{T_{H1} - T_{H2}}{\Delta T_{m,H}}\right) - 1} \tag{6}$$

As the heat rate (Q_H) and the capacity rate (C_{min}) are assumed to be equal those of the RJC engine then by substituting known results in to Equation (6) gives:

$$T_e = \frac{\exp\left(\dfrac{1400 - 1100}{200}\right) 1100 - 1400}{\exp\left(\dfrac{1400 - 1100}{200}\right) - 1} = 1013.8\,\text{K}$$

According to Rogers *et al.* [15] the thermodynamic efficiency of the internally reversible RJC engine is given:

$$\eta_{th,RJC} = 1 - \frac{\alpha}{\theta^*_{RJC}} \tag{7}$$

Given that $T_4 = 1200$ K and assuming an ambient temperature of 300 K then the cycle temperature ratio, $(T_4/T_1) = \theta^*_{RJC} = 4$. From Equation (7) it is clear that the thermodynamic efficiency of the ideal RJC engine approaches that of a Carnot engine with the same cycle temperature ratio as α approaches unity, and in fact becomes for practical purposes the RJC engine equates to an Atkinson cycle. However, in order to permit a fair comparison between the Stirling cycle and the RJC it is necessary to assign a practical value to α in order to solve to Equation (7). By following a analysis described by Goodger [16], and by assuming reasonable component efficiencies for the expander (85%), compressor (75%) and recuperator (80%), it can be shown that $\alpha = 1.4$ gives the optimum thermodynamic efficiency when $\theta^*_{RJC} = 4$. The assignment of process irreversibilities to the RJC calculation of course is against the first assumption list above that both cycles are internally reversible. However, not to do so would place the RJC at a clear advantage. Then, for present case the thermodynamic efficiency for an externally heated, internally reversible RJC engine is given by:

$$\eta_{th,RJC} = 1 - \frac{1.4}{4.} = 0.65 \text{ or } 65\%$$

According to Wright-Barker [17], the thermodynamic efficiency of the internally reversible SC engine is given by:

$$\eta_{th,SC} = 1 - \frac{1}{\theta^*_{SC}} \tag{8}$$

where $\theta^*_{SC} = \dfrac{T_e}{T_c}$.

To solve Equation (8) and to provide a fair comparison between the RJC and the SC engines is assumed that the SC engine is air cooled and T_{L1} in Figure 5 equals to an ambient air temperature (T_{amb}) of 300 K, which equals T_1 in Figure 2a. Furthermore, if both engines were used to power a CHP systems it would be fair to assume that T_{L2} should equal the air temperature recuperator outlet, T_6 in Figure 2a, which in this case equals the compressor air discharge temperature, T_2 in the same diagram, which for a value of α of 1.4, $T_{L2} = T_6 = T_2 = 1.4 \times 300 = 420$ K.
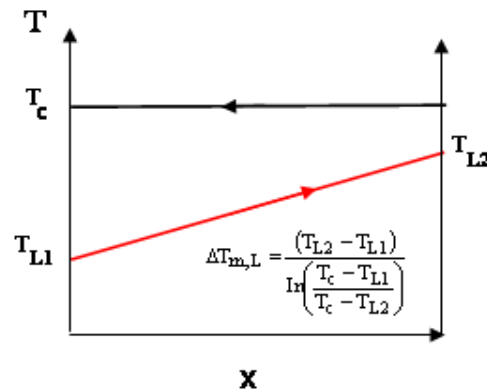
Figure 5. Showing the variations in working fluid and coolant air temperatures through the low-temperature heat exchanger of the internally reversible Stirling cycle engine.

Assuming that $T_C = T_{L2}$:

$$\eta_{th,SC} = 1 - \frac{420}{1013.8} = 0.58 \text{ or } 58\%$$

Because T_C must in practice be greater 420 K and T_{L1} then the thermodynamic efficiency of the SC engine would be less than the 58% calculated. In order for an internally reversible SC engine to equal that of an internally reversible RJC engine then the value of T_C would have to be reduced to 354 K (82 °C), which in practice may compromise the use of such an engine in a CHP system and require a significant increase in coolant flow (approximately 220% increase) with an associated increase pump/fan power requirement. An alternative to assuming both engines have the same high-temperature heater $\Delta T_{m,H}$ value is to assume that the ATD_H are equal.

3.2. Comparison of Efficiency Based on Equal ATD_H Values

From Equation (2), if the heat rates, Q_H, are to be the same for both engines and ATD_H and C_{min} are also equal then also the heat exchanger effectiveness values, ε_H must be equal. Rearranging Equation (2) gives:

$$\Delta T_{m,H} = ATD \frac{\varepsilon}{NTU}$$

Therefore, in this case the NTU values for each engine cycle cannot be equal. Solving Equations (4) and (5) for NTU then for the RJC engine heater:

$$NTU_{RJC} = \frac{\varepsilon_{RJC}}{1 - \varepsilon_{RJC}} \tag{9}$$

and, for the Stirling cycle heater:

$$NTU_{SC} = -Ln(1 - \varepsilon_{SC}) \tag{10}$$

Assuming an economic heat exchanger effectiveness value of 0.7 then $NTU_{RJC} = 2.33$ and $NTU_{SC} = 1.2$. If the heat source temperature, T_{H1}, of 1400 K is again assumed and with an equal approach temperature difference, ATD_H, of 500 K then:

$$\Delta T_{m,H,RJC} = 500 \frac{0.7}{2.33} = 150 \text{ K}, \ \Delta T_{m,H,SC} = 500 \frac{0.7}{1.2} = 292 \text{ K}$$

Therefore, for the RJC engine heater, with $C_{min} = C_{max}$:

$$\Delta T_{m,H,RJC} = T_{H1} - T_4$$

Therefore, $T_4 = 1400 - 150 = 1250$ K and the RJC cycle temperature ratio:

$$\theta^*_{RJC} = 1250/300 = 4.166$$

Assuming a compressor isentropic temperature ratio, ε, of 1.4 as used previously then:

$$\eta_{th,RJC} = 1 - \frac{\alpha}{\theta^*_{RJC}} = 1 - \frac{1.4}{4.166} = 0.66 \text{ or } 66\%$$

For the SC engine heater:

$$T_e = \frac{\exp\left(\dfrac{T_{H1} - T_{H2}}{\Delta T_{m,H}}\right) T_{H2} - T_{H1}}{\exp\left(\dfrac{T_{H1} - T_{H2}}{\Delta T_{m,H}}\right) - 1}$$

Inserting values gives:

$$T_e = \frac{\exp\left(\dfrac{1400 - 1100}{292}\right) 1100 - 1400}{\exp\left(\dfrac{1400 - 1100}{292}\right) - 1} = 933 \text{ K}$$

Assuming the same low-temperature heat exchanger temperature as previously calculated: $T_c = 420$ K the thermodynamic efficiency of the internally reversible SC engine is,

$$\eta_{th,St} = 1 - \frac{420}{933} = 0.0.55 \text{ or } 55\%$$

A conclusion at this point must be that due to finite-time heat transfer the internally reversible RJC engine has potentially a higher thermodynamic efficiency than the equivalent SC engine regardless of whether the comparison is carried out with equal ATD_H or $\Delta T_{m,H}$ values at the air heater.

3.3. Some Results and Discussion

Subject to the assumptions 1 to 10 previously listed, the results given in the previous example calculation show that the thermodynamic efficiency of the RJC cycle is about 10% more efficient than the Stirling cycle whether or not the heat is assumed to be added with either equal $\Delta T_{m,H}$ or ATD_H value.

Calculations described in Section 3.2 were repeated over a range of heat source temperatures and the results are shown in Figure 6. This data, based on an equal $\Delta T_{m,H}$ for the finite-time heat transfer calculations, show the variation in thermodynamic efficiency for both RJC and Stirling cycle engines. Curve A in gives the variation in thermodynamic efficiency for a fully-reversible Stirling engine whilst Curve C gives the results for an internally-reversible Stirling engine with finite-time heat transfer. The effect of finite-time heat transfer on thermodynamic efficiency is clearly seen from the results. What is interesting in the present case are the comparative results in between the internally-reversible RJC engine with finite-time heat transfer (Curve B) and the Stirling engine (Curve C). These data show that the thermodynamic efficiency of the RJC engine is more efficient than an air cooled Stirling cycle engine over the range of heat source temperatures investigated.

The results indicate that when heat is supplied at the same maximum temperature, T_{H1}, to internally-reversible cycles, and the heat source is of a variable temperature type, then the thermodynamic efficiency of the RJC is greater than that of the Stirling engine when this is cooled by ambient air which is then used for heating purposes. This result may be useful in the design of heat engines for small-scale CHP applications. A limitation to the application of reciprocating RJC engines may be their low imep, which in turn increases the physical size of an engine for a given

power. However, if an engine is required to supply small amounts of powers for domestic or light commercial applications this should not be a barrier. Also, by using multi-stage compression with inter-stage cooling it is possible to increase network output, but at greater capital expense introduced by the need for additional heat exchangers. Similar arguments may be put forward for inducing muli-stage expansion with inter-stage reheating. A detailed analysis of the effects on performance and particularly imep of multi-stage expansion and compression lies outside the scope of the article, however, the pros and cons of multi-staging processes are discussed in most standard text book of which Rogers and Mayhew [8] and Googer [16] are but two.
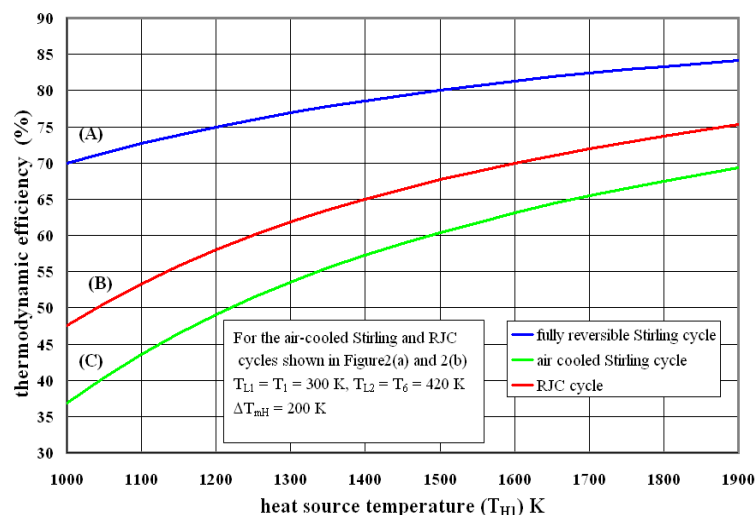
Figure 6. Showing a comparison between the thermodynamic efficiencies of: (**A**) Fully reversible; (**B**) RJC engine as described in Figure 2a; (**C**) Air cooled SC engine, as described in Figure 2b for an equal value of $\Delta T_{m,H}$.

4. Conclusions

The micro or small-scale CHP systems based on IC engine technology are invariably four-stroke Otto cycle machines with thermal electrical efficiencies of about 30%. The use of well-developed vehicle engine technology to power CHP systems is believed to have an initial-cost advantage at this time; however, IC engines are limited to burning highly refined fuels, such as petroleum, LPG or natural-gas.

External combustion engine types, such as the Stirling, Ericsson or Rankine cycle machines, can run on almost and mix of low-grade fuel, such as wood pellets or low grade bio-oil. However, these closed-cycle machines have the disadvantage that they need to reject heat to the environment via a low-temperature heat exchanger and this both reduces their overall thermodynamic efficiency and increases their capital cost.

By comparing the relative performance of internally heated open cycle engines with externally heated closed cycle machines the authors conclude that an externally heated open cycle engine might offer the advantages of both types. To investigate this further the performance of an externally heated RJC engine was compared with that of an externally heated Stirling cycle engine, assuming a variable temperature heat source, equal source and sink temperatures and an air-cooled Stirling engine. As shown in Figure 6, the results show that an externally heated RJC engine has the potential for greater thermodynamic efficiency than the Stirling machine for the same heat source and sink temperatures when applied to a typical CHP system.

powered by burning low-grade fuels common to the developing world. The subject had been of interest to the authors for several years. Ian W. Eames was the main author of the article and of the ideas behind the article: Kieran Evans carried out a detailed literature review of which a condensed version is included in this article and Stephen Pickering provided much useful advice and guidance on the direction of the project and this article.

Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

A	heat transfer area (m^2)
ATD	approach temperature difference (K)
C	heat capacity rate = mCp (W/K)
CHP	combined heat and power
C_p	specific heat capacity at constant pressure (J/kg·K)
m	mass flow (kg/s)
NTU	number of heat transfer units
P_r	RJC cycle pressure ratio (−)
Q_{input}	heat input rate (W)
RJC	recuperated Joule cycle
s	specific entropy (J/kg·K)
T	temperature (K)
T_c	isothermal compression temperature (K)
T_e	isothermal expansion temperature (K)
ΔT_m	area-weighted (or log-mean) temperature difference (°C)
U	overall heat transfer coefficient (W/m^2·K)

Greek letters

α	isentropic compression temperature ratio (−)
ε	heat exchange effectiveness (−)
θ	cycle temperature ratio, T_{H1}/T_{L1} (−)
θ^*	cycle temperature ratio for an internally reversible engine
η_c	isentropic compression efficiency (−)
η_e	isentropic expansion efficiency (−)
η_{th}	thermodynamic efficiency (−)

Subscripts

Amb	ambient air
SC	Stirling cycle
fg	flue-gas
H	heat source temperature
H1	heat source inlet temperature
L	heat sink temperature
L1	heat sink inlet temperature
opt	optimum value
r	recuperator
RJC	recuperated Joule cycle
SC	Stirling cycle

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