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Approaches to Multiple-Attribute Decision-Making Based on Pythagorean 2-Tuple Linguistic Bonferroni Mean Operators

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Abstract: In this paper, we investigate multiple-attribute decision-making (MADM) with Pythagorean 2-tuple linguistic numbers (P2TLNs). Then, we combine the weighted Bonferroni mean (WBM) operator and weighted geometric Bonferroni mean (WGBM) operator with P2TLNs to propose the Pythagorean 2-tuple linguistic WBM (P2TLWBM) operator and Pythagorean 2-tuple linguistic WGBM (P2TLWGBM) operator; MADM methods are then developed based on these two operators. Finally, a practical example for green supplier selection is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords: multiple-attribute decision-making (MADM); Pythagorean 2-tuple linguistic numbers (P2TLNs); weighted BM (WBM) operator; WGBM operator; green supplier selection; green supply chain management (GSCM)

1. Introduction

The intuitionistic fuzzy set (IFS), developed by Atanassov [1], is an extension of fuzzy set theory [2]. IFS is constructed by a membership degree and a nonmembership degree, and can therefore depict the fuzzy character of data more comprehensively and in greater detail. In the past decades, many intuitionistic fuzzy information aggregation operators have been proposed [3–14]. More recently, the Pythagorean fuzzy set (PFS) [15,16] has appeared as an effective tool for depicting the uncertainty of multiple-attribute decision-making (MADM) problems. The PFS is also characterized by the membership degree and the nonmembership degree, whose sum of squares is less than or equal to 1; the PFS is thus more general than the IFS. In some cases, the PFS can solve problems that the IFS cannot—for example, if a DM problem gives the membership degree and the nonmembership degree as 0.8 and 0.6, respectively, then it is only valid for the PFS. In other words, all the IFS degrees are a part of the PFS degrees, which indicates that the PFS is more powerful for handling uncertain problems. Zhang and Xu [17] developed a Pythagorean fuzzy Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) for handling the MADM problem. Peng and Yang [18] developed the Pythagorean fuzzy superiority and inferiority ranking method to solve multiple-attribute group decision-making problems. Beliakov and James [19] focused on how the notion of “averaging” should be treated in the case of Pythagorean fuzzy numbers (PFNs). Reformat and Yager [20] used PFNs to handle a collaboration-based recommender system. Gou et al. [21] investigated the properties of continuous PFNs. Ren et al. [22] proposed the Pythagorean fuzzy TODIM (an acronym in Portuguese of interactive and multi-criteria decision making) method for MADM problems. Garg [23] proposed generalized Pythagorean fuzzy information aggregation based on the Einstein operations. Zeng et al. [24] developed a hybrid method for the Pythagorean fuzzy MADM method. Garg [25] investigated

the novel accuracy function of interval-valued PFNs for solving MADM problems. Liang et al. [26] developed a projection model for fusing the information of PFN multicriteria group decision-making based on the geometric Bonferroni mean. Peng et al. [27] defined some information measures of PFNs. Garg [28] proposed the generalized Pythagorean fuzzy geometric aggregation operators using the Einstein t-Norm and t-Conorm for MADM problems. Wei and Lu [29] proposed the Pythagorean fuzzy Maclaurin symmetric mean (PFMSM) operator and Pythagorean fuzzy weighted Maclaurin symmetric mean (PFWMSM) operator. Wei [30] developed some interaction aggregation operators for MADM with PFNs. Wei and Lu [31] proposed some Pythagorean fuzzy power aggregation operators: the Pythagorean fuzzy power average (PFPA) operator, Pythagorean fuzzy power geometric (PFPG) operator, Pythagorean fuzzy power weighted average (PFPWA) operator, Pythagorean fuzzy power weighted geometric (PFPWG) operator, Pythagorean fuzzy power ordered weighted average (PFPOWA) operator, Pythagorean fuzzy power ordered weighted geometric (PFPOWG) operator, Pythagorean fuzzy power hybrid average (PFPHA) operator, and Pythagorean fuzzy power hybrid geometric (PFPHG) operator. Wei and Lu [32] defined the concept of dual Pythagorean hesitant fuzzy sets and proposed some dual hesitant Pythagorean fuzzy Hamacher aggregation operators: the dual hesitant Pythagorean fuzzy Hamacher weighted average (DHPFHWA) operator, dual hesitant Pythagorean fuzzy Hamacher weighted geometric (DHPFHWG) operator, dual hesitant Pythagorean fuzzy Hamacher ordered weighted average (DHPFHOWA) operator, dual hesitant Pythagorean fuzzy Hamacher ordered weighted geometric (DHPFHOWG) operator, dual hesitant Pythagorean fuzzy Hamacher hybrid average (DHPFHHA) operator, and dual hesitant Pythagorean fuzzy Hamacher hybrid geometric (DHPFHHG) operator. Lu et al. [33] defined the concept of hesitant Pythagorean fuzzy sets and utilized Hamacher operations to develop some hesitant Pythagorean fuzzy aggregation operators: the hesitant Pythagorean fuzzy Hamacher weighted average (HPFHWA) operator, hesitant Pythagorean fuzzy Hamacher weighted geometric (HPFHWG) operator, hesitant Pythagorean fuzzy Hamacher ordered weighted average (HPFHOWA) operator, hesitant Pythagorean fuzzy Hamacher ordered weighted geometric (HPFHOWG) operator, hesitant Pythagorean fuzzy Hamacher hybrid average (HPFHHA) operator, and hesitant Pythagorean fuzzy Hamacher hybrid geometric (HPFHHG) operator. Wei et al. [34] defined the concept of Pythagorean 2-tuple linguistic sets and utilized arithmetic and geometric operations to develop some Pythagorean 2-tuple linguistic aggregation operators: the Pythagorean 2-tuple linguistic weighted average (P2TLWA) operator, Pythagorean 2-tuple linguistic weighted geometric (P2TLWG) operator, Pythagorean 2-tuple linguistic ordered weighted average (P2TLOWA) operator, Pythagorean 2-tuple linguistic ordered weighted geometric (P2TLOWG) operator, Pythagorean 2-tuple linguistic hybrid average (P2TLHA) operator, and Pythagorean 2-tuple linguistic hybrid geometric (P2TLHG) operator.

Obviously, these established Pythagorean 2-tuple linguistic operators cannot be utilized to fuse arguments which are correlated. Meanwhile, the Bonferroni mean (BM) [35–42] is a very practical tool with which to tackle arguments which are correlated. How to effectively extend the mature BM operator [35] and geometric BM (GBM) operator [36] to the P2TLN environment is a significant research task and is the focus of this paper.

The organization of this manuscript is given as follows. Section 2 reviews P2TLNs and some other basic definitions. Section 3 introduces the extended BM operator [35] and geometric BM (GBM) operator [36] which can be used to fuse the P2TLNs, and describes some properties of these operators. In Section 4, we study the MADM problem with P2TLNs based on the P2TLWBM and P2TLWGBM operators. Section 5 illustrates the functions of the proposed operators with an example for green supplier selection in the green supply chain management area. Section 6 concludes the paper.

2. Preliminaries

2.1. P2TLSs

Wei et al. [34] proposed the Pythagorean 2-tuple linguistic sets (P2TLSs) based on the PFSs [15,16] and 2-tuple linguistics [43–55].

Definition 1 [34]. A P2TLSs A in X is given by

$$P = \left\{ \left(s_{\theta(x)}, \rho \right), \left(\mu_P(x), \nu_P(x) \right), x \in X \right\} \quad (1)$$

where $s_{\theta(x)} \in S$, $\rho \in [-0.5, 0.5]$, $\mu_P(x) \in [0, 1]$, and $\nu_P(x) \in [0, 1]$, with the condition $0 \leq (\mu_P(x))^2 + (\nu_P(x))^2 \leq 1$, $\forall x \in X$. The numbers $\mu_P(x), \nu_P(x)$ represent, respectively, the degree of membership and degree of nonmembership of the element x with respect to linguistic variable $(s_{\theta(x)}, \rho)$.

Wei et al. [34] call $p = \langle (s_p, \rho), (u_p, v_p) \rangle$ a Pythagorean 2-tuple linguistic number (P2TLN).

Definition 2 [34]. Let $p_1 = \langle (s_{p_1}, \rho), (u_{p_1}, v_{p_1}) \rangle$ and $p_2 = \langle (s_{p_2}, \rho), (u_{p_2}, v_{p_2}) \rangle$ be two P2TLNs; $S(p_1) = \Delta \left(\Delta^{-1}(s_{\theta(p_1)}, \rho_1) \cdot \frac{1 + (\mu_{p_1})^2 - (\nu_{p_1})^2}{2} \right)$ and $S(p_2) = \Delta \left(\Delta^{-1}(s_{\theta(p_2)}, \rho_2) \cdot \frac{1 + (\mu_{p_2})^2 - (\nu_{p_2})^2}{2} \right)$ be the scores of p_1 and p_2 , respectively; and let $H(p_1) = \Delta \left(\Delta^{-1}(s_{\theta(p_1)}, \rho_1) \cdot \frac{(\mu_{p_1})^2 + (\nu_{p_1})^2}{2} \right)$ and $H(p_2) = \Delta \left(\Delta^{-1}(s_{\theta(p_2)}, \rho_2) \cdot \frac{(\mu_{p_2})^2 + (\nu_{p_2})^2}{2} \right)$ be the accuracy degrees of p_1 and p_2 , respectively.

If $S(p_1) < S(p_2)$, then $p_1 < p_2$.

If $S(p_1) = S(p_2)$, then

- (1) if $H(p_1) = H(p_2)$, then $p_1 = p_2$;
- (2) if $H(p_1) < H(p_2)$, then $p_1 < p_2$.

Wei et al. [34] defined some operational laws of P2TLNs.

Definition 3 [34]. Let $p_1 = \langle (s_{p_1}, \rho), (u_{p_1}, v_{p_1}) \rangle$ and $p_2 = \langle (s_{p_2}, \rho), (u_{p_2}, v_{p_2}) \rangle$ be two P2TLNs, then

$$\begin{aligned} p_1 \oplus p_2 &= \left\langle \Delta \left(\Delta^{-1}(s_{\theta(p_1)}, \rho_1) + \Delta^{-1}(s_{\theta(p_2)}, \rho_2) \right), \left(\sqrt{(\mu_{p_1})^2 + (\mu_{p_2})^2 - (\mu_{p_1})^2 (\mu_{p_2})^2}, \nu_{p_1} \nu_{p_2} \right) \right\rangle; \\ p_1 \otimes p_2 &= \left\langle \Delta \left(\Delta^{-1}(s_{\theta(p_1)}, \rho_1) \cdot \Delta^{-1}(s_{\theta(p_2)}, \rho_2) \right), \left(\mu_{p_1} \mu_{p_2}, \sqrt{(\nu_{p_1})^2 + (\nu_{p_2})^2 - (\nu_{p_1})^2 (\nu_{p_2})^2} \right) \right\rangle; \\ \lambda p_1 &= \left\langle \Delta \left(\lambda \Delta^{-1}(s_{\theta(p_1)}, \rho_1) \right), \left(\sqrt{1 - (1 - (\mu_{p_1})^2)^\lambda}, (\nu_{p_1})^\lambda \right) \right\rangle; \\ (p_1)^\lambda &= \left\langle \Delta \left(\left(\Delta^{-1}(s_{\theta(p_1)}, \rho_1) \right)^\lambda \right), \left((\mu_{p_1})^\lambda, \sqrt{1 - (1 - (\nu_{p_1})^2)^\lambda} \right) \right\rangle. \end{aligned}$$

2.2. BM Operator

Definition 4 [35]. Let $t, r > 0$ and a_i ($i = 1, 2, \dots, n$) be a set of nonnegative crisp numbers with weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, $\omega_i \in [0, 1]$, and $\sum_{i=1}^n \omega_i = 1$. The weighted Bonferroni mean (WBM) is

$$\text{WBM}_{\omega}^{t,r}(a_1, a_2, \dots, a_n) = \left(\sum_{i,j=1}^n \omega_i \omega_j a_i^t a_j^r \right)^{1/(t+r)}. \quad (2)$$

Definition 5 [36]. Let $t, r > 0$ and a_i ($i = 1, 2, \dots, n$) be a collection of nonnegative crisp numbers with weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, $\omega_i \in [0, 1]$, and $\sum_{i=1}^n \omega_i = 1$. If

$$\text{WGBM}_{\omega}^{t,r}(a_1, a_2, \dots, a_n) = \frac{1}{t+r} \prod_{i,j=1}^n (ta_i + ra_j)^{\omega_i \omega_j}, \quad (3)$$

then the $\text{WGBM}_{\omega}^{t,r}$ operator is called the weighted geometric BM (WGBM) operator.

3. The P2TLWBM Operator and P2TLWGBM Operator

This section extends WBM and WGBM to fuse the P2TLNs and proposes several new Pythagorean 2-tuple linguistic operators.

3.1. P2TLWBM Operator

Definition 6. Let $t, r > 0$ and $p_i = ((s_i, \rho_i), (\mu_i, v_i))$ ($i = 1, 2, \dots, n$) be a set of P2TLNs with weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, $\omega_i \in [0, 1]$, and $\sum_{i=1}^n \omega_i = 1$. If

$$\text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) = \left(\bigoplus_{i,j=1}^n \omega_i \omega_j (p_i^t \otimes p_j^r) \right)^{1/(t+r)}, \quad (4)$$

then the $\text{P2TLWBM}_{\omega}^{t,r}$ is called the Pythagorean 2-tuple linguistic WBM (P2TLWBM) operator.

The P2TLWBM operator has four properties.

Property 1. Let $t, r > 0$ and $p_i = ((s_i, \rho_i), (\mu_i, v_i))$ ($i = 1, 2, \dots, n$) be a collection of P2TLNs. The aggregated result of P2TLWBM is a P2TLN.

$$\begin{aligned} & \text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) \\ &= \left(\bigoplus_{i,j=1}^n \omega_i \omega_j (p_i^t \otimes p_j^r) \right)^{1/(t+r)} \\ &= \left(\Delta \left(\left(\sum_{i,j=1}^n \omega_i \omega_j (\Delta^{-1}(s_i, \rho_i))^t (\Delta^{-1}(s_j, \rho_j))^r \right)^{1/(t+r)} \right), \right. \\ & \quad \left(\left(\sqrt{1 - \prod_{i,j=1}^n (1 - \mu_i^{2t} \mu_j^{2r})^{\omega_i \omega_j}} \right)^{1/(t+r)}, \right. \\ & \quad \left. \left. \sqrt{1 - \left(1 - \prod_{i,j=1}^n (1 - (1 - v_i^2)^t (1 - v_j^2)^r)^{\omega_i \omega_j} \right)^{1/(t+r)}} \right) \right) \end{aligned} \quad (5)$$

Proof.

$$p_i^t = \left(\Delta \left((\Delta^{-1}(s_{p_i}, \rho_i))^t \right), \left(\mu_i^t, \sqrt{1 - (1 - v_i^2)^t} \right) \right) \quad (6)$$

$$p_j^r = \left(\Delta \left((\Delta^{-1}(s_{p_j}, \rho_j))^r \right), \left(\mu_j^r, \sqrt{1 - (1 - v_j^2)^r} \right) \right) \quad (7)$$

Thus,

$$p_i^t \otimes p_j^r = \left(\Delta \left((\Delta^{-1}(S_i, \rho_i))^t (\Delta^{-1}(S_j, \rho_j))^r \right), \left(\mu_i^t \mu_j^r, \sqrt{1 - (1 - v_i^2)^t (1 - v_j^2)^r} \right) \right) \quad (8)$$

Thereafter,

$$\begin{aligned} & \omega_i \omega_j (p_i^t \otimes p_j^r) \\ &= \left(\Delta \left(\omega_i \omega_j (\Delta^{-1}(S_i, \rho_i))^t (\Delta^{-1}(S_j, \rho_j))^r \right), \right. \\ & \left. \left(\sqrt{1 - (1 - \mu_i^{2t} \mu_j^{2r})^{\omega_i \omega_j}}, \left(\sqrt{1 - (1 - v_i^2)^t (1 - v_j^2)^r} \right)^{\omega_i \omega_j} \right) \right) \end{aligned} \quad (9)$$

Furthermore,

$$\begin{aligned} & \bigoplus_{i,j=1}^n \omega_i \omega_j (p_i^t \otimes p_j^r) \\ &= \left(\Delta \left(\sum_{i,j=1}^n \omega_i \omega_j (\Delta^{-1}(S_i, \rho_i))^t (\Delta^{-1}(S_j, \rho_j))^r \right), \right. \\ & \left. \left(\sqrt{1 - \prod_{i,j=1}^n (1 - \mu_i^{2t} \mu_j^{2r})^{\omega_i \omega_j}}, \left(\sqrt{\prod_{i,j=1}^n (1 - (1 - v_i^2)^t (1 - v_j^2)^r)} \right)^{\omega_i \omega_j} \right) \right) \end{aligned} \quad (10)$$

Therefore,

$$\begin{aligned} & \text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) \\ &= \left(\Delta \left(\left(\sum_{i,j=1}^n \omega_i \omega_j (\Delta^{-1}(S_i, \rho_i))^t (\Delta^{-1}(S_j, \rho_j))^r \right)^{1/(t+r)}, \right. \right. \\ & \left. \left(\left(\sqrt{1 - \prod_{i,j=1}^n (1 - \mu_i^{2t} \mu_j^{2r})^{\omega_i \omega_j}} \right)^{1/(t+r)}, \right. \right. \\ & \left. \left. \sqrt{1 - \left(1 - \prod_{i,j=1}^n (1 - (1 - v_i^2)^t (1 - v_j^2)^r)^{\omega_i \omega_j} \right)^{1/(t+r)}} \right) \right) \end{aligned} \quad (11)$$

Hence, (5) is maintained. Thereafter:

$$\omega_i \omega_j (\Delta^{-1}(S_i, \rho_i))^t (\Delta^{-1}(S_j, \rho_j))^r \leq \omega_i \omega_j (\Delta^{-1}(S_{\max}, \rho_{\max}))^{t+r}, \quad (12)$$

$$\begin{aligned} & \sum_{i,j=1}^n \omega_i \omega_j (\Delta^{-1}(S_i, \rho_i))^t (\Delta^{-1}(S_j, \rho_j))^r \leq \\ & \sum_{i,j=1}^n \omega_i \omega_j (\Delta^{-1}(S_{\max}, \rho_{\max}))^{t+r} = (\Delta^{-1}(S_{\max}, \rho_{\max}))^{t+r}. \end{aligned} \quad (13)$$

$$\Delta \left(\left(\sum_{i,j=1}^n \omega_i \omega_j (\Delta^{-1}(S_i, \rho_i))^t (\Delta^{-1}(S_j, \rho_j))^r \right)^{1/(t+r)} \right) \leq (S_{\max}, \rho_{\max}). \quad (14)$$

Similarly,

$$(S_{\min}, \rho_{\min}) \leq \Delta \left(\left(\sum_{i,j=1}^n \omega_i \omega_j (\Delta^{-1}(S_i, \rho_i))^t (\Delta^{-1}(S_j, \rho_j))^r \right)^{1/(t+r)} \right). \quad (15)$$

Thereafter,

$$0 \leq \left(\sqrt{1 - \prod_{i,j=1}^n (1 - \mu_i^{2t} \mu_j^{2r})^{\omega_i \omega_j}} \right)^{1/(t+r)} \leq 1, \quad (16)$$

$$0 \leq \sqrt{1 - \left(1 - \prod_{i,j=1}^n (1 - (1 - v_i^2)^t (1 - v_j^2)^r)^{\omega_i \omega_j} \right)^{1/(t+r)}} \leq 1. \quad (17)$$

Because $\mu_i^2 + v_i^2 \leq 1$,

$$\left(\sqrt{1 - \prod_{i,j=1}^n (1 - \mu_i^{2t} \mu_j^{2r})^{\omega_i \omega_j}} \right)^{1/(t+r)} \leq \sqrt{\left(1 - \prod_{i,j=1}^n (1 - (1 - v_i^2)^t (1 - v_j^2)^r)^{\omega_i \omega_j} \right)^{1/(t+r)}}. \quad (18)$$

Therefore,

$$\begin{aligned} & \left(\left(\sqrt{1 - \prod_{i,j=1}^n (1 - \mu_i^{2t} \mu_j^{2r})^{\omega_i \omega_j}} \right)^{1/(t+r)} \right)^2 + \\ & \left(\sqrt{1 - \left(1 - \prod_{i,j=1}^n (1 - (1 - v_i^2)^t (1 - v_j^2)^r)^{\omega_i \omega_j} \right)^{1/(t+r)}} \right)^2 \\ & \leq \left(1 - \prod_{i,j=1}^n (1 - (1 - v_i^2)^t (1 - v_j^2)^r)^{\omega_i \omega_j} \right)^{1/(t+r)} + 1 \\ & - \left(1 - \prod_{i,j=1}^n (1 - (1 - v_i^2)^t (1 - v_j^2)^r)^{\omega_i \omega_j} \right)^{1/(t+r)} = 1 \end{aligned} \quad (19)$$

□

Property 2 (Idempotency). If $p_i = p = ((s, \rho), (\mu, v))$, then

$$\text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) = p. \quad (20)$$

Proof.

$$\begin{aligned} & \text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) \\ & = \left(\bigoplus_{i,j=1}^n \omega_i \omega_j (p^t \otimes p^r) \right)^{1/(t+r)} \\ & = p \left(\bigoplus_{i,j=1}^n \omega_i \omega_j \right)^{1/(t+r)} \\ & = p \end{aligned} \quad (21)$$

□

Property 3 (Monotonicity). Let $p_i = ((S_{p_i}, \rho_{p_i}), (\mu_{p_i}, v_{p_i}))$. ($i = 1, 2, \dots, n$) and $q_i = ((S_{q_i}, \rho_{q_i}), (\mu_{q_i}, v_{q_i}))$ ($i = 1, 2, \dots, n$) be two sets of P2TLNs. If $(S_{p_i}, \rho_{p_i}) \leq (S_{q_i}, \rho_{q_i})$, and $\mu_{p_i} \leq \mu_{q_i}$ and $v_{p_i} \geq v_{q_i}$ hold for all i , then

$$\text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) \leq \text{P2TLWBM}_{\omega}^{t,r}(q_1, q_2, \dots, q_n). \quad (22)$$

Proof. Let $\text{P2TLWBM}^{t,r}(p_1, p_2, \dots, p_n) = ((S_p, \rho_p), (\mu_p, v_p))$ and $\text{P2TLWBM}^{t,r}(q_1, q_2, \dots, q_n) = ((S_q, \rho_q), (\mu_q, v_q))$. Given that $\Delta^{-1}(S_p, \rho_p) \leq \Delta^{-1}(S_q, \rho_q)$, we can obtain

$$\frac{(\Delta^{-1}(S_{p_i}, \rho_{p_i}))^t (\Delta^{-1}(S_{p_j}, \rho_{p_j}))^r}{(\Delta^{-1}(S_{q_i}, \rho_{q_i}))^t (\Delta^{-1}(S_{q_j}, \rho_{q_j}))^r} \leq \quad (23)$$

Therefore,

$$\frac{\left(\sum_{i,j=1}^n \omega_i \omega_j (\Delta^{-1}(S_{p_i}, \rho_{p_i}))^t (\Delta^{-1}(S_{p_j}, \rho_{p_j}))^r \right)^{1/(t+r)}}{\left(\sum_{i,j=1}^n (\Delta^{-1}(S_{q_i}, \rho_{q_i}))^t (\Delta^{-1}(S_{q_j}, \rho_{q_j}))^r \right)^{1/(t+r)}} \leq \quad (24)$$

Thus,

$$\frac{\Delta \left(\sum_{i,j=1}^n \omega_i \omega_j (\Delta^{-1}(S_{p_i}, \rho_{p_i}))^t (\Delta^{-1}(S_{p_j}, \rho_{p_j}))^r \right)^{1/(t+r)}}{\Delta \left(\sum_{i,j=1}^n (\Delta^{-1}(S_{q_i}, \rho_{q_i}))^t (\Delta^{-1}(S_{q_j}, \rho_{q_j}))^r \right)^{1/(t+r)}} \leq \quad (25)$$

That means that $(S_p, \rho_p) \leq (S_q, \rho_q)$, and we also can obtain

$$\mu_{p_i}^{2t} \mu_{p_j}^{2r} \leq \mu_{q_i}^{2t} \mu_{q_j}^{2r}, \quad (26)$$

$$\prod_{i,j=1}^n (1 - \mu_{p_i}^{2t} \mu_{p_j}^{2r}) \geq \prod_{i,j=1}^n (1 - \mu_{q_i}^{2t} \mu_{q_j}^{2r}), \quad (27)$$

$$1 - \prod_{i,j=1}^n (1 - \mu_{p_i}^{2t} \mu_{p_j}^{2r}) \leq 1 - \prod_{i,j=1}^n (1 - \mu_{q_i}^{2t} \mu_{q_j}^{2r}). \quad (28)$$

Therefore,

$$\left(\sqrt[n]{1 - \prod_{i,j=1}^n (1 - \mu_{p_i}^{2t} \mu_{p_j}^{2r})} \right)^{1/(t+r)} \leq \left(\sqrt[n]{1 - \prod_{i,j=1}^n (1 - \mu_{q_i}^{2t} \mu_{q_j}^{2r})} \right)^{1/(t+r)}. \quad (29)$$

Thus,

$$\left(\left(\sqrt[n]{1 - \prod_{i,j=1}^n (1 - \mu_{p_i}^{2t} \mu_{p_j}^{2r})} \right)^{1/(t+r)} \right)^2 \leq \left(\left(\sqrt[n]{1 - \prod_{i,j=1}^n (1 - \mu_{q_i}^{2t} \mu_{q_j}^{2r})} \right)^{1/(t+r)} \right)^2, \quad (30)$$

which means $\mu_p^2 \leq \mu_q^2$. Similarly, we can obtain $v_p^2 \geq v_q^2$.

If $(S_{p_i}, \rho_{p_i}) < (S_{q_i}, \rho_{q_i})$, and $\mu_{p_i}^2 < \mu_{q_i}^2$ and $v_{p_i}^2 > v_{q_i}^2$, then

$$\text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) < \text{P2TLWBM}_{\omega}^{t,r}(q_1, q_2, \dots, q_n);$$

If $(S_{p_i}, \rho_{p_i}) < (S_{q_i}, \rho_{q_i})$, and $\mu_{p_i}^2 < \mu_{q_i}^2$ and $v_{p_i}^2 > v_{q_i}^2$, then

$$\text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) < \text{P2TLWBM}_{\omega}^{t,r}(q_1, q_2, \dots, q_n);$$

If $(S_{p_i}, \rho_{p_i}) < (S_{q_i}, \rho_{q_i})$, and $\mu_{p_i}^2 = \mu_{q_i}^2$ and $v_{p_i}^2 > v_{q_i}^2$, then

$$\text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) < \text{P2TLWBM}_{\omega}^{t,r}(q_1, q_2, \dots, q_n);$$

If $(S_{p_i}, \rho_{p_i}) < (S_{q_i}, \rho_{q_i})$, and $\mu_{p_i}^2 = \mu_{q_i}^2$ and $v_{p_i}^2 = v_{q_i}^2$, then

$$\text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) < \text{P2TLWBM}_{\omega}^{t,r}(q_1, q_2, \dots, q_n);$$

If $(S_{p_i}, \rho_{p_i}) = (S_{q_i}, \rho_{q_i})$, and $\mu_{p_i}^2 < \mu_{q_i}^2$ and $v_{p_i}^2 > v_{q_i}^2$, then

$$\text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) < \text{P2TLWBM}_{\omega}^{t,r}(q_1, q_2, \dots, q_n);$$

If $(S_{p_i}, \rho_{p_i}) = (S_{q_i}, \rho_{q_i})$, and $\mu_{p_i}^2 < \mu_{q_i}^2$ and $v_{p_i}^2 = v_{q_i}^2$, then

$$\text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) < \text{P2TLWBM}_{\omega}^{t,r}(q_1, q_2, \dots, q_n);$$

If $(S_{p_i}, \rho_{p_i}) = (S_{q_i}, \rho_{q_i})$, and $\mu_{p_i}^2 = \mu_{q_i}^2$ and $v_{p_i}^2 > v_{q_i}^2$, then

$$\text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) < \text{P2TLWBM}_{\omega}^{t,r}(q_1, q_2, \dots, q_n);$$

If $(S_{p_i}, \rho_{p_i}) = (S_{q_i}, \rho_{q_i})$, and $\mu_{p_i}^2 = \mu_{q_i}^2$ and $v_{p_i}^2 = v_{q_i}^2$, then

$$\text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) = \text{P2TLWBM}_{\omega}^{t,r}(q_1, q_2, \dots, q_n).$$

Therefore, the proof of Theorem 3 is completed. $\square$

Property 4 (Boundedness). Let $p_i = ((S_i, \rho_i), (\mu_i, v_i)) (i = 1, 2, \dots, n)$ be a set of P2TLNs. If $p^+ = (\max_i(S_i, \rho_i), (\max_i(\mu_i), \min_i(v_i)))$ and $p^- = (\min_i(S_i, \rho_i), (\min_i(\mu_i), \max_i(v_i)))$, then

$$p^- \leq \text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) \leq p^+. \quad (31)$$

Proof. From Theorem 2, we can obtain

$$\text{P2TLWBM}_{\omega}^{t,r}(p^+, p^+, \dots, p^+) = p^+, \text{P2TLWBM}_{\omega}^{t,r}(p^-, p^-, \dots, p^-) = p^-. \quad (32)$$

From Theorem 2, we can also obtain

$$\begin{aligned} p^- &= \text{P2TLWBM}_{\omega}^{t,r}(p^-, p^-, \dots, p^-) \leq \\ &\text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) \leq \\ &\text{P2TLWBM}_{\omega}^{t,r}(p^+, p^+, \dots, p^+) = p^+ \end{aligned} \quad (33)$$

Therefore,

$$p^- \leq \text{P2TLWBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) \leq p^+. \quad \square \quad (34)$$

3.2. P2TLWGBM Operator

Thereafter, we extend WGBM to P2TLN and introduced the Pythagorean 2-tuple linguistic WGBM (P2TLWGBM) operator.

Definition 7. Let $t, r > 0$ and $p_i = ((s_i, \rho_i), \mu_i, v_i) (i = 1, 2, \dots, n)$ be a set of P2TLNs with weights $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, $\omega_i \in [0, 1]$, and $\sum_{i=1}^n \omega_i = 1$. If

$$\text{P2TLWGBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) = \frac{1}{t+r} \bigotimes_{i,j=1}^n (tp_i \oplus rp_j)^{\omega_i \omega_j}, \quad (35)$$

then the P2TLWGBM operator has four properties.

Property 5. Let $t, r > 0$ and $p_i = ((s_i, \rho_i), \mu_i, v_i) (i = 1, 2, \dots, n)$ be a set of P2TLNs. The aggregated result of P2TLWGBM is a P2TLN.

$$\begin{aligned}
 & \text{P2TLWGBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) \\
 &= \frac{1}{t+r} \bigotimes_{i,j=1}^n (tp_i \oplus rp_j)^{\omega_i \omega_j} \\
 &= \left(\Delta \left(\frac{1}{t+r} \prod_{i,j=1}^n (t\Delta^{-1}(S_i, \rho_i) + r\Delta^{-1}(S_j, \rho_j))^{\omega_i \omega_j} \right), \right. \\
 & \quad \left(\left(\sqrt{1 - \left(1 - \prod_{i,j=1}^n (1 - (1 - \mu_i^2)^t (1 - \mu_j^2)^r)^{\omega_i \omega_j}} \right)^{1/(t+r)}, \right. \right. \\
 & \quad \left. \left. \left(\sqrt{1 - \prod_{i,j=1}^n (1 - v_i^{2t} v_j^{2r})^{\omega_i \omega_j}} \right)^{1/(t+r)} \right) \right) \quad (36)
 \end{aligned}$$

Proof. Through Definition 3, we can obtain

$$tp_i = \left(\Delta(t\Delta^{-1}(S_i, \rho_i)), \left(\sqrt{1 - (1 - \mu_i^2)^t}, v_i^t \right) \right), \quad (37)$$

$$rp_j = \left(\Delta(r\Delta^{-1}(S_j, \rho_j)), \left(\sqrt{1 - (1 - \mu_j^2)^r}, v_j^r \right) \right). \quad (38)$$

Thereafter,

$$tp_i \oplus rp_j = \left(\Delta(t\Delta^{-1}(S_i, \rho_i) + r\Delta^{-1}(S_j, \rho_j)), \left(\sqrt{1 - (1 - \mu_i^2)^t (1 - \mu_j^2)^r}, v_i^t v_j^r \right) \right) \quad (39)$$

Thereafter,

$$\begin{aligned}
 & (tp_i \oplus rp_j)^{\omega_i \omega_j} \\
 &= \left(\Delta \left((t\Delta^{-1}(S_i, \rho_i) + r\Delta^{-1}(S_j, \rho_j))^{\omega_i \omega_j} \right), \right. \\
 & \quad \left. \left(\left(\sqrt{1 - (1 - \mu_i^2)^t (1 - \mu_j^2)^r} \right)^{\omega_i \omega_j}, \sqrt{1 - (1 - v_i^{2t} v_j^{2r})^{\omega_i \omega_j}} \right) \right) \quad (40)
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 & \bigotimes_{i,j=1}^n (tp_i \oplus rp_j)^{\omega_i \omega_j} \\
 &= \left(\Delta \left(\prod_{i,j=1}^n (t\Delta^{-1}(S_i, \rho_i) + r\Delta^{-1}(S_j, \rho_j))^{\omega_i \omega_j} \right), \right. \\
 & \quad \left. \left(\left(\sqrt{\prod_{i,j=1}^n (1 - (1 - \mu_i^2)^t (1 - \mu_j^2)^r)} \right)^{\omega_i \omega_j}, \sqrt{1 - \prod_{i,j=1}^n (1 - v_i^{2t} v_j^{2r})^{\omega_i \omega_j}} \right) \right) \quad (41)
 \end{aligned}$$

Thus,

$$\begin{aligned}
 & \text{P2TLWGBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) \\
 &= \frac{1}{t+r} \bigotimes_{i,j=1}^n (tp_i \oplus rp_j)^{\omega_i \omega_j} \\
 &= \left(\Delta \left(\frac{1}{t+r} \prod_{i,j=1}^n (t\Delta^{-1}(S_i, \rho_i) + r\Delta^{-1}(S_j, \rho_j))^{\omega_i \omega_j} \right), \right. \\
 & \quad \left(\sqrt{1 - \left(1 - \prod_{i,j=1}^n \left(1 - (1 - \mu_i^2)^t (1 - \mu_j^2)^r \right)^{\omega_i \omega_j} \right)^{1/(t+r)}} \right. \\
 & \quad \left. \left(\sqrt{1 - \prod_{i,j=1}^n \left(1 - v_i^{2t} v_j^{2r} \right)^{\omega_i \omega_j} \right)^{1/(t+r)}} \right) \right) \quad (42)
 \end{aligned}$$

Hence, (36) is maintained. Thereafter,

$$\left(t\Delta^{-1}(S_i, \rho_i) + r\Delta^{-1}(S_j, \rho_j) \right)^{\omega_i \omega_j} \leq \left((t+r)\Delta^{-1}(S_{\max}, \rho_{\max}) \right)^{\omega_i \omega_j}, \quad (43)$$

$$\begin{aligned}
 & \prod_{i,j=1}^n (t\Delta^{-1}(S_i, \rho_i) + r\Delta^{-1}(S_j, \rho_j))^{\omega_i \omega_j} \leq \\
 & \prod_{i,j=1}^n \left((t+r)\Delta^{-1}(S_{\max}, \rho_{\max}) \right)^{\omega_i \omega_j} = (t+r)\Delta^{-1}(S_{\max}, \rho_{\max}) \quad (44)
 \end{aligned}$$

$$\Delta \left(\frac{1}{t+r} \left(\prod_{i,j=1}^n (t\Delta^{-1}(S_i, \rho_i) + r\Delta^{-1}(S_j, \rho_j))^{\omega_i \omega_j} \right) \right) \leq (S_{\max}, \rho_{\max}). \quad (45)$$

Similarly,

$$(S_{\min}, \rho_{\min}) \leq \Delta \left(\frac{1}{t+r} \left(\prod_{i,j=1}^n (t\Delta^{-1}(S_i, \rho_i) + r\Delta^{-1}(S_j, \rho_j))^{\omega_i \omega_j} \right) \right). \quad (46)$$

Thereafter,

$$0 \leq \sqrt{1 - \left(1 - \prod_{i,j=1}^n \left(1 - (1 - \mu_i^2)^t (1 - \mu_j^2)^r \right)^{\omega_i \omega_j} \right)^{1/(t+r)}} \leq 1, \quad (47)$$

$$0 \leq \left(\sqrt{1 - \prod_{i,j=1}^n \left(1 - v_i^{2t} v_j^{2r} \right)^{\omega_i \omega_j} \right)^{1/(t+r)} \leq 1. \quad (48)$$

Because, $\mu_i^2 + v_i^2 \leq 1$

$$\left(\sqrt{1 - \prod_{i,j=1}^n \left(1 - v_i^{2t} v_j^{2r} \right)^{\omega_i \omega_j} \right)^{1/(t+r)} \leq \sqrt{\left(1 - \prod_{i,j=1}^n \left(1 - (1 - \mu_i^2)^t (1 - \mu_j^2)^r \right)^{\omega_i \omega_j} \right)^{1/(t+r)}}. \quad (49)$$

Therefore,

$$\begin{aligned} & \left(\sqrt{1 - \left(1 - \prod_{i,j=1}^n \left(1 - (1 - \mu_i^2)^t (1 - \mu_j^2)^r \right)^{\omega_i \omega_j} \right)^{1/(t+r)}} \right)^2 + \\ & \left(\left(\sqrt{1 - \prod_{i,j=1}^n \left(1 - v_i^{2t} v_j^{2r} \right)^{\omega_i \omega_j} \right)^{1/(t+r)}} \right)^2 \leq \\ & 1 - \left(1 - \prod_{i,j=1}^n \left(1 - (1 - \mu_i^2)^t (1 - \mu_j^2)^r \right)^{\omega_i \omega_j} \right)^{1/(t+r)} \\ & + \left(1 - \prod_{i,j=1}^n \left(1 - (1 - \mu_i^2)^t (1 - \mu_j^2)^r \right)^{\omega_i \omega_j} \right)^{1/(t+r)} = 1 \end{aligned} \quad (50)$$

This completes the proof. $\square$

Similar to P2TLWBM, the P2TLWGBM has the same properties. The proofs of these properties are similar to those of the properties of P2TLWGBM, Accordingly, the proofs are omitted to save space.

Property 6. Let $t, r > 0$ and $p_i = ((S_i, \rho_i), (\mu_i, v_i)) (i = 1, 2, \dots, n)$ be a set of P2TLNs.

(1) *Idempotency.* If $p_i = p = ((S, \rho), (\mu, v))$, then

$$\text{P2TLWGBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) = p. \quad (51)$$

(2) *Monotonicity.* Let $p_i = ((S_{p_i}, \rho_{p_i}), (\mu_{p_i}, v_{p_i})) (i = 1, 2, \dots, n)$ and $q_i = ((S_{q_i}, \rho_{q_i}), (\mu_{q_i}, v_{q_i})) (i = 1, 2, \dots, n)$ be two sets of P2TLNs. If $(S_{p_i}, \rho_{p_i}) \leq (S_{q_i}, \rho_{q_i})$, and $\mu_{p_i} \leq \mu_{q_i}$ and $v_{p_i} \geq v_{q_i}$ hold for all i , then

$$\text{P2TLWGBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) \leq \text{P2TLWGBM}_{\omega}^{t,r}(q_1, q_2, \dots, q_n). \quad (52)$$

(3) *Boundedness.* If $p^+ = (\max_i(S_i, \rho_i), (\max_i(\mu_i), \min_i(v_i)))$ and $p^- = (\min_i(S_i, \rho_i), (\min_i(\mu_i), \max_i(v_i)))$, then

$$p^- \leq \text{P2TLWGBM}_{\omega}^{t,r}(p_1, p_2, \dots, p_n) \leq p^+. \quad (53)$$

4. Models for MADM with P2TLNs

Based on the P2TLWBM (P2TLWGBM) operators, in this section, we shall propose the model for MADM with P2TLNs. Let $A = \{A_1, A_2, \dots, A_m\}$ be a discrete set of alternatives, $G = \{G_1, G_2, \dots, G_n\}$ be the set of attributes, and $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ be the weighting vector of the attribute $G_j (j = 1, 2, \dots, n)$, where $\omega_j \in [0, 1]$, and $\sum_{j=1}^n \omega_j = 1$. Suppose that $R = (r_{ij})_{m \times n} = \langle (s_{ij}, \rho_{ij}), (\mu_{ij}, v_{ij}) \rangle_{m \times n}$ is the P2TLN decision matrix, where the r_{ij} take the form of the P2TLNs, where μ_{ij} indicates the degree to which the alternative A_i satisfies the attribute G_j given by the decision-maker, v_{ij} indicates the degree to which the alternative A_i does not satisfy the attribute G_j given by the decision-maker, $\mu_{ij} \in [0, 1]$, $v_{ij} \in [0, 1]$, $(\mu_{ij})^2 + (v_{ij})^2 \leq 1$, $\pi_{ij} = \sqrt{1 - ((\mu_{ij})^2 + (v_{ij})^2)}$, $s_{ij} \in S$, $\rho_{ij} \in [-0.5, 0.5]$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$.

In the following, we apply the P2TLWBM (P2TLWGBM) operator to the MADM problems with P2TLNs.

Step 1. We utilize the decision information given in matrix R , and the P2TLWBM operator

$$\begin{aligned}
 \tilde{p}_l &= \text{P2TLWBM}_{\omega}^{t,r}(r_{l1}, r_{l2}, \dots, r_{ln}) \\
 &= \left(\bigoplus_{i,j}^n \omega_i \omega_j r_{li}^{\alpha} \otimes r_{lj}^{\beta} \right)^{1/(t+r)} \\
 &= \left(\Delta \left(\left(\sum_{i,j=1}^n \omega_i \omega_j (\Delta^{-1}(s_{li}, \rho_{li}))^t (\Delta^{-1}(s_{lj}, \rho_{lj}))^r \right)^{1/(t+r)} \right), l = 1, 2, \dots, m. \right. \\
 &\quad \left(\left(\sqrt{1 - \prod_{i,j=1}^n (1 - \mu_{li}^{2t} \mu_{lj}^{2r})^{\omega_i \omega_j}} \right)^{1/(t+r)}, \right. \\
 &\quad \left. \left. \sqrt{1 - \left(1 - \prod_{i,j=1}^n (1 - (1 - v_{li}^2)^t (1 - v_{lj}^2)^r)^{\omega_i \omega_j} \right)^{1/(t+r)}} \right) \right)
 \end{aligned} \tag{54}$$

or

$$\begin{aligned}
 \tilde{p}_l &= 2\text{TLWGBM}_{\omega}^{t,r}(r_{l1}, r_{l2}, \dots, r_{ln}) \\
 &= \frac{1}{t+r} \bigotimes_{i,j,k=1}^n (tp_{li} \oplus rp_{lj})^{\omega_i \omega_j} \\
 &= \left(\Delta \left(\frac{1}{t+r} \prod_{i,j=1}^n (t\Delta^{-1}(s_{li}, \rho_{li}) + r\Delta^{-1}(s_{lj}, \rho_{lj}))^{\omega_i \omega_j} \right), l = 1, 2, \dots, m. \right. \\
 &\quad \left(\left(\sqrt{1 - \left(1 - \prod_{i,j,k=1}^n (1 - (1 - \mu_{li}^2)^t (1 - \mu_{lj}^2)^r)^{\omega_i \omega_j} \right)^{1/(t+r)}}, \right. \right. \\
 &\quad \left. \left. \left(\sqrt{1 - \prod_{i,j=1}^n (1 - v_{li}^{2t} v_{lj}^{2r})^{\omega_i \omega_j}} \right)^{1/(t+r)} \right) \right)
 \end{aligned} \tag{55}$$

to derive the P2TLNs $\tilde{p}_l (l = 1, 2, \dots, m)$ of the alternative A_i .

Step 2. Calculate the scores $S(\tilde{p}_l) (l = 1, 2, \dots, m)$ of the overall P2TLNs $\tilde{p}_l (l = 1, 2, \dots, m)$ to rank all the alternatives $A_l (l = 1, 2, \dots, m)$ and then select the best one(s). If there is no difference between two scores $S(\tilde{p}_l)$ and $S(\tilde{p}_t)$, then we need to calculate the accuracy degrees $H(\tilde{p}_l)$ and $H(\tilde{p}_t)$ of the overall P2TLNs p_l and p_t , respectively, and then rank the alternatives A_l and A_t in accordance with the accuracy degrees $H(\tilde{p}_l)$ and $H(\tilde{p}_t)$.

Step 3. Rank all the alternatives $A_l (l = 1, 2, \dots, m)$ and select the best one(s) in accordance with $S(\tilde{p}_l) (l = 1, 2, \dots, m)$.

Step 4. End.

5. Numerical Example and Comparative Analysis

5.1. Numerical Example

In this section we shall present a numerical example to select green suppliers in green supply chain management with P2TLNs in order to illustrate the method proposed in this paper. There are five possible green suppliers in the green supply chain management $O_i (i = 1, 2, 3, 4, 5)$ to select. The experts select four attributes by which to assess the five possible green suppliers: ① C_1 is the product quality factor; ② C_2 is environmental factors; ③ C_3 is the delivery factor; and ④ C_4 is the price factor. The five green suppliers $O_i (i = 1, 2, 3, 4, 5)$ are to be assessed with P2TLNs according to the four attributes (with weighting vector $\omega = (0.2, 0.1, 0.3, 0.4)$), as shown in Table 1.

Table 1. Pythagorean 2-tuple linguistic numbers (P2TLN) decision matrix.

	C ₁	C ₂	C ₃	C ₄
O ₁	<(s ₄ , 0), (0.50, 0.80)>	<(s ₂ , 0), (0.60, 0.50)>	<(s ₂ , 0), (0.30, 0.60)>	<(s ₃ , 0), (0.60, 0.70)>
O ₂	<(s ₂ , 0), (0.70, 0.50)>	<(s ₄ , 0), (0.70, 0.40)>	<(s ₁ , 0), (0.60, 0.20)>	<(s ₂ , 0), (0.40, 0.60)>
O ₃	<(s ₆ , 0), (0.70, 0.50)>	<(s ₂ , 0), (0.50, 0.70)>	<(s ₅ , 0), (0.50, 0.30)>	<(s ₄ , 0), (0.60, 0.20)>
O ₄	<(s ₅ , 0), (0.80, 0.20)>	<(s ₅ , 0), (0.60, 0.30)>	<(s ₇ , 0), (0.40, 0.50)>	<(s ₁ , 0), (0.60, 0.60)>
O ₅	<(s ₃ , 0), (0.60, 0.40)>	<(s ₁ , 0), (0.40, 0.70)>	<(s ₃ , 0), (0.70, 0.50)>	<(s ₂ , 0), (0.60, 0.80)>

Then, we utilize the proposed algorithm to select green suppliers in GSCM.

Step 1. According to P2TLNs $r_{ij}(i = 1, 2, 3, 4, 5, j = 1, 2, 3, 4)$, we can aggregate all P2TLNs r_{ij} by using the P2TLWBM (P2TLWGBM) operator to get the P2TLNs $A_i(i = 1, 2, 3, 4, 5)$ of the green suppliers O_i . Suppose that $t = r = 3$; then, the aggregating results are in Table 2.

Table 2. The aggregating results of green suppliers by P2TLWBM (P2TLWGBM) operator.

	P2TLWBM	P2TLWGBM
O ₁	<(s ₃ , −0.0074), (0.5466, 0.1451)>	<(s ₃ , −0.2501), (0.1911, 0.6979)>
O ₂	<(s ₂ , 0.2572), (0.6090, 0.1754)>	<(s ₂ , −0.1848), (0.1808, 0.5302)>
O ₃	<(s ₅ , −0.2511), (0.6039, 0.1618)>	<(s ₄ , 0.423), (0.1865, 0.4975)>
O ₄	<(s ₅ , 0.2024), (0.6531, 0.1813)>	<(s ₃ , 0.4517), (0.1658, 0.5350)>
O ₅	<(s ₃ , −0.4388), (0.6320, 0.1415)>	<(s ₂ , 0.3488), (0.1744, 0.7056)>

Step 2. The scores, derived according to the information in Table 2, are shown in Table 3.

Table 3. The scores of the green suppliers.

	P2TLWBM	P2TLWGBM
O ₁	(s ₂ , −0.0882)	(s ₁ , −0.2445)
O ₂	(s ₂ , −0.4875)	(s ₁ , −0.3179)
O ₃	(s ₃ , 0.1782)	(s ₂ , −0.2589)
O ₄	(s ₄ , −0.3748)	(s ₁ , 0.2793)
O ₅	(s ₂ , −0.2335)	(s ₁ , −0.3746)

Step 3. The ordering of the suppliers, according to the information in Table 3, is listed in Table 4. The order of the green suppliers is slightly different, but the best green supplier is O₃ or O₄.

Table 4. Ordering of the green suppliers.

	Order
P2TLWBM	O ₄ > O ₃ > O ₁ > O ₅ > O ₂
P2TLWGBM	O ₃ > O ₄ > O ₁ > O ₂ > O ₅

From Table 4, we can easily see that these two operators may generate slightly different ranking results. The main reason causing this ranking result difference is that the P2TLWBM operator emphasizes the group influences; however, the P2TLWGBM operator emphasizes the individual influences.

5.2. Influence of the Parameter on the Final Result

The effects on the ranking results caused by changing parameters of $(t, r) \in [1, 10]$ in the GP2TLWBM (GP2TLWGBM) operators are listed in Tables 5 and 6.

Table 5. Ranking results for different operational parameters of the GP2TLWBM operator.

(t, r)	S(O ₁)	S(O ₂)	S(O ₃)	S(O ₄)	S(O ₅)	Ordering
(1, 1)	(s ₂ , −0.4113)	(s ₁ , 0.1968)	(s ₃ , −0.0685)	(s ₃ , −0.4157)	(s ₂ , 0.4669)	O ₃ > O ₄ > O ₁ > O ₅ > O ₂
(2, 2)	(s ₂ , −0.2156)	(s ₁ , 0.3525)	(s ₃ , 0.0627)	(s ₃ , 0.2016)	(s ₂ , −0.3252)	O ₄ > O ₃ > O ₁ > O ₅ > O ₂
(3, 3)	(s ₂ , −0.0882)	(s ₂ , −0.4875)	(s ₃ , 0.1782)	(s ₄ , −0.3748)	(s ₂ , −0.2335)	O ₄ > O ₃ > O ₁ > O ₅ > O ₂
(4, 4)	(s ₂ , 0.0032)	(s ₂ , −0.3322)	(s ₃ , 0.2805)	(s ₄ , −0.0706)	(s ₂ , −0.1699)	O ₄ > O ₃ > O ₁ > O ₅ > O ₂
(5, 5)	(s ₂ , 0.0749)	(s ₂ , −0.1888)	(s ₃ , 0.3718)	(s ₄ , 0.1607)	(s ₂ , −0.122)	O ₄ > O ₃ > O ₁ > O ₅ > O ₂
(6, 6)	(s ₂ , 0.1342)	(s ₂ , −0.0624)	(s ₃ , 0.4534)	(s ₄ , 0.3439)	(s ₂ , −0.0839)	O ₄ > O ₃ > O ₁ > O ₂ > O ₅
(7, 7)	(s ₂ , 0.1847)	(s ₂ , 0.0459)	(s ₄ , −0.4732)	(s ₄ , 0.4928)	(s ₂ , −0.0527)	O ₄ > O ₃ > O ₁ > O ₂ > O ₅
(8, 8)	(s ₂ , 0.2283)	(s ₂ , 0.1373)	(s ₄ , −0.4073)	(s ₅ , −0.3846)	(s ₂ , −0.0262)	O ₄ > O ₃ > O ₁ > O ₂ > O ₅
(9, 9)	(s ₂ , 0.2664)	(s ₂ , 0.2142)	(s ₄ , −0.3482)	(s ₅ , −0.2825)	(s ₂ , −0.0038)	O ₄ > O ₃ > O ₁ > O ₂ > O ₅
(10, 10)	(s ₂ , 0.2999)	(s ₂ , 0.2793)	(s ₄ , −0.295)	(s ₅ , −0.1959)	(s ₂ , 0.0155)	O ₄ > O ₃ > O ₁ > O ₂ > O ₅

Table 6. Ranking results for different operational parameters of the P2TLWGBM operator.

(t, r)	S(O ₁)	S(O ₂)	S(O ₃)	S(O ₄)	S(O ₅)	Ordering
(1, 1)	(s ₁ , −0.1432)	(s ₁ , −0.2053)	(s ₂ , 0.1454)	(s ₁ , 0.4929)	(s ₁ , −0.1851)	O ₃ > O ₄ > O ₁ > O ₅ > O ₂
(2, 2)	(s ₁ , −0.1993)	(s ₁ , −0.2756)	(s ₂ , −0.0883)	(s ₁ , 0.3561)	(s ₁ , −0.3002)	O ₃ > O ₄ > O ₁ > O ₂ > O ₅
(3, 3)	(s ₁ , −0.2445)	(s ₁ , −0.3179)	(s ₂ , −0.2589)	(s ₁ , 0.2793)	(s ₁ , −0.3746)	O ₃ > O ₄ > O ₁ > O ₂ > O ₅
(4, 4)	(s ₁ , −0.2793)	(s ₁ , −0.3433)	(s ₂ , −0.3763)	(s ₁ , 0.236)	(s ₁ , −0.4211)	O ₃ > O ₄ > O ₁ > O ₂ > O ₅
(5, 5)	(s ₁ , −0.3062)	(s ₁ , −0.3593)	(s ₂ , −0.4583)	(s ₁ , 0.2095)	(s ₁ , −0.4513)	O ₃ > O ₄ > O ₁ > O ₂ > O ₅
(6, 6)	(s ₁ , −0.3275)	(s ₁ , −0.3699)	(s ₁ , 0.4819)	(s ₁ , 0.1924)	(s ₁ , −0.4715)	O ₃ > O ₄ > O ₁ > O ₂ > O ₅
(7, 7)	(s ₁ , −0.3448)	(s ₁ , −0.3774)	(s ₁ , 0.4371)	(s ₁ , 0.1801)	(s ₁ , −0.4861)	O ₃ > O ₄ > O ₁ > O ₂ > O ₅
(8, 8)	(s ₁ , −0.3592)	(s ₁ , −0.3828)	(s ₁ , 0.4023)	(s ₁ , 0.1712)	(s ₁ , −0.497)	O ₃ > O ₄ > O ₁ > O ₂ > O ₅
(9, 9)	(s ₁ , −0.3713)	(s ₁ , −0.3869)	(s ₁ , 0.3746)	(s ₁ , 0.1641)	(s ₀ , 0.4947)	O ₃ > O ₄ > O ₁ > O ₂ > O ₅
(10, 10)	(s ₁ , −0.3816)	(s ₁ , −0.3902)	(s ₁ , 0.3519)	(s ₁ , 0.1585)	(s ₀ , 0.4879)	O ₃ > O ₄ > O ₁ > O ₂ > O ₅

5.3. Comparative Analysis

Now, we compare our method with the P2TLWA operator and P2TLWG operator [34]. The comparative results are in Table 7.

Table 7. Order of the green suppliers.

Order	
P2TLWA	O ₃ > O ₄ > O ₅ > O ₁ > O ₂
P2TLWG	O ₃ > O ₄ > O ₅ > O ₁ > O ₂

From the above, we can see that we get the same results, showing the effectiveness of our approaches. However, the existing aggregation operators, such as the P2TLWA operator and P2TLWG operator, do not consider the relationship between arguments being aggregated, and thus cannot eliminate the influence of unfair arguments. Our proposed P2TLWBM and P2TLWGBM operators consider the information about the relationship between arguments being aggregated.

6. Conclusions

In this paper, we focused on P2TLN information operators and their application to MADM. To aggregate the P2TLNs, the P2TLWBM and P2TLWGBM operators have been developed. We have conducted further research on these two operators' numerous desirable properties. In addition, we demonstrated the effectiveness of the P2TLWBM and P2TLWGBM operators in practical MADM problems. Finally, we use an example about green supplier selection in the green supply chain management process to elaborate the applicability of these two operators; meanwhile, the comparison between parameters of different values has also been analyzed. In the future, we shall expand the proposed models to unbalanced fuzzy linguistic information [56] and some other fuzzy and uncertain MADM problems [57–86].

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