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A Mathematical Model and Two Fuzzy Approaches Based on Credibility and Expected Interval for Project Cost-Quality-Risk Trade-Off Problem in Time-Constrained Conditions

Mohammad Hossein Haghighi and Seyed Meysam Mousavi *

Department of Industrial Engineering, Shahed University, Tehran 3319118651, Iran; mh.haghighi.ie@gmail.com
* Correspondence: sm.mousavi@shahed.ac.ir

Abstract: To successfully finalize projects and attain their determined purposes, it is indispensable to control all success criteria of a project. The time–cost trade-off (TCT) is known as a prevalent and efficient approach applied when the planned finish date of a project is not admitted by stakeholders, and consequently, the project duration must be decreased. This paper proposes a new mathematical model under fuzzy uncertainty to deal with the project cost–risk–quality trade-off problem (CRQT) under time constraints. Because of the unique nature of projects and their uncertain circumstances, applying crisp values for some project parameters does not seem appropriate. Hence, this paper employs fuzzy sets to resolve these weaknesses. In this study, two approaches are presented to handle proposed fuzzy multi-objective mathematical model. First, fuzzy credibility theory and then goal attainment method are used. Secondly, the model is solved by a fuzzy method based on expected interval and value and augmented ε-constraint method. A project from the literature review is adopted and solved by the presented methodology. The results demonstrate the accuracy and efficiency of the two proposed approaches for the introduced practical problem.

Keywords: time-cost trade-off (TCT); fuzzy credibility theory; multi-objective optimization; augmented ε-constraint method; goal attainment method

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1. Introduction and Literature Review

Project management plays a significant role in modern management as a usage of knowledge, skills, and techniques in project activities to attain project purposes [1]. In a project schedule, it is assumed that the activities are performed at their planned time. In some cases, the project may need to be completed even earlier than the planned duration. The new finish date is usually determined by the customers or senior managers in accordance with the project's various goals or policies. One way to achieve an earlier completion time is crashing the duration for a number of project activities [2]. Time–cost trade-off (TCT) is considered a very efficient and practical technique that is applied to attain the required completion time with minimal additional cost [3,4]. Indeed, the execution time of some activities must be crashed by allocating more resources (such as materials, workers, and equipment) [5].

Crashing time of activities may influence the quality of activities and, ultimately, the quality of the whole project [6]. Moreover, decreasing the time of activities can have risks that negatively influence project success [2]. Therefore, this problem is transformed from two dimensions of time and cost into four dimensions of time, cost, quality, and risk. On the other hand, projects are accomplished under uncertain circumstances and special features. As a result, there is often no historical data related to the projects. Consequently, using a fuzzy approach to manage real-world uncertainty is preferred.

Until today, various studies have been conducted on the TCT problem. Kim et al. [7] proposed a TCT approach considering the potential cost of lack of quality in terms of non-conformance risks. They prioritized the non-conformance risks by multiplying probability and impact criteria. Orm and Jeunet [8] provided an overview of time–cost–quality trade-off (TCQT) studies with a particular focus on how quality is assessed in previous studies. Salari et al. [9] considered the application of statistical modeling and earned value management (EVM) for the TCT problem in a fuzzy environment. He et al. [10] developed variable neighborhood search and tabu search algorithms in a discrete TCT problem minimizing the maximum difference between contractor input and output cash flows. Tran and Long [11] developed a time–cost–risk trade-off model. In this research, the concept of risk is introduced as a function of total float and resource fluctuations in project scheduling.

Moreover, Xu et al. [12] proposed a discrete time–cost–environmental trade-off for large-scale construction systems and considered various operating modes for activities. Paidar et al. [13] presented a time–cost–resource optimization model with different execution modes for project activities. In this research, the objective functions were project duration, contractor net present value, and renewable resources of the project. To solve the proposed model, a multi-objective gravitational search algorithm has been applied. Wood [14] introduced a new TCQT model under uncertain conditions in a construction project in the oil and gas industry. Khadem et al. [15] presented a quantitative risk analysis via Monte Carlo simulation in a gas injection project in Oman. Haghighi et al. [16] proposed a methodology under interval-valued fuzzy uncertainty for the TCT problem by considering the cost of quality loss. Additionally, Hamta et al. [17] proposed a goal programming model to address a TCQT problem in a real project in Iran. Panwar and Jha [18] provided a scheduling model taking into account cost, time, quality, and safety criteria and applied a genetic algorithm III to solve it. Furthermore, Banihashemi and Khalilzadeh [19] integrated a parallel data envelopment analysis (DEA) approach and a time–cost–quality–environmental impact trade-off to analyze and select the best execution mode of project activities for project scheduling. Lotfi et al. [20] presented a trade-off problem among time, cost, quality, energy, and environment criteria with resource constraints in a bridge construction project in Tehran. They solved their mathematical model via an augmented ε-constraint method.

In addition to the abovementioned papers, some recent studies on TCT problems have been presented. Mahmoudi and Feylizadeh [21] developed a grey mathematical model to crash the duration of projects while considering cost, time, risk, and quality criteria and the law of diminishing returns concept. Feylizadeh et al. [22] applied crashing and fast-tracking techniques, and developed a fuzzy multi-objective non-linear model paying attention to cost, time, risk, and quality criteria so as to reduce the duration of projects. Mahmoudi and Javed [23] incorporated the potential quality loss cost (PQLC) concept in TCT problems and then introduced two new models of project scheduling with PQLC. Liu et al. [24] presented a discrete symbiotic organism search (SOS) method to solve large-scale TCT problems for projects with 180 to 6300 activities. Mahdiraji et al. [25] proposed a new hesitant fuzzy TCQT model to identify the best implementation option for each project activity in an R&D real project in the food industry. Kebriyaii et al. [26] concentrated on solution approaches and applied three metaheuristic algorithms in a TCQT problem, taking into account the time value of money for construction projects. Tao et al. [27] proposed a stochastic programming model for TCT problems in the GERT-type network so as to minimize the mean duration within an appropriate on-time completion probability and under-budget probability. They solved a numerical example by a genetic algorithm-based approach to depict the performance of their presented model. Nguyen et al. [28] applied the fuzzy α-cut approach and SOS algorithm in a TCQT model to determine a set of Pareto-optimal solutions. They introduced two case studies of a repetitive construction project to depict the effectiveness of their model.

Despite the various studies, several research gaps are presented as follows:

- In addition to time, cost, and quality criteria, it is indispensable to consider the risk criterion in the traditional TCT problem; since by reducing the duration of activities, the risk associated with the activities and consequently the whole project is increased, which can even lead to project failure. There are very few studies that consider the risk criterion in the TCT problem. These studies mostly applied qualitative approaches or the multiplication of the probability and impact.

- Crashing time of activities can also influence the quality of activities and, ultimately, the quality of the whole project. Therefore, the creation of a new and efficient approach for considering the project reduction quality in TCT problems seems crucial.
- In previous studies on TCT problems, uncertainty in the activity parameters, such as the duration and cost, has rarely been considered. Nowadays, using crisp values for some project parameters in the actual conditions in which projects are carried out is not practical.

Table 1 also reviews the most recent studies on the TCT problem.

Table 1. The most recent studies on the TCT problem and the comparison with this paper.

Fuzzy Credibility Theory	Fuzzy Uncertainty	Risk	Quality	Cost	Time	Year	Author (s)
	✓			✓	✓	2011	Chen and Tsai [29]
	✓			✓	✓	2012	Xu et al. [12]
			✓	✓	✓	2012	Kim et al. [7]
			✓	✓	✓	2015	Monghasemi et al. [30]
				✓	✓	2017	He et al. [10]
		✓	✓	✓	✓	2018	Mahmoudi and Feylizadeh [21]
	✓	✓	✓	✓	✓	2018	Feylizadeh et al. [22]
		✓		✓	✓	2018	Tran and Long [11]
				✓	✓	2018	Ammar [31]
				✓	✓	2018	Albayrak and Özdemir [32]
				✓	✓	2019	Ballesteros-Perez et al. [33]
				✓	✓	2020	Liu et al. [24]
			✓	✓	✓	2020	Mahmoudi and Javed [23]
			✓	✓	✓	2020	Jeunet and Orm [34]
			✓	✓	✓	2021	Panwar and Jha [18]
			✓	✓	✓	2022	Sharma and Trivedi [35]
				✓	✓	2022	Tao et al. [27]
				✓	✓	2022	Dhawan et al. [36]
✓	✓	✓	✓	✓	✓		This paper

Research Motivation and Contribution

Through an extensive literature review regarding TCT problems, the limitations of the approaches were determined. At the same time, the merits of these approaches were identified. The following are the two main motivations for the proposed decision approach:

- The presented TCT mathematical model should have considered real-world uncertainty and then applied efficient fuzzy approaches to solve the fuzzy optimization model.
- The presented approach should have improved the computations of risk and quality criteria values in projects.

Given the mentioned research gaps and motivations, the main novelties of this paper are as follows:

- A new fuzzy multi-objective mathematical model is developed for cost–risk–quality trade-off (CRQT) under time constraints.
- An appropriate framework is provided to consider the effective risk and quality criteria in the TCT problem.
- In this paper, two efficient solution approaches from the fuzzy credibility theory and goal attainment method are employed; also, Jimenez et al. [37] and augmented epsilon constraint are presented, and the results are compared.

The main innovation of this paper is the development of a recent crisp mathematical model of TCT in terms of considering uncertainty in the mathematical model. Considering uncertainty in the parameters of time and crashing cost of activities in the mathematical model creates an extended model that can better model the real-world uncertain conditions. Indeed, the first objective function of the mathematical model and all the constraints, including the duration of activities, have created a new model that has distinguished the solution from the crisp model. On the other hand, presenting a fuzzy mathematical model requires the use of new and effective fuzzy solution approaches, so this research has applied two recent and efficient fuzzy approaches of fuzzy credibility theory and Jimenez et al. [37] for the presented fuzzy mathematical model. These two approaches are known as appropriate approaches to solve fuzzy mathematical models, because they have high efficiency in solving linear programming problems. Additionally, they do not increase the objective functions and model constraints. On the other hand, these approaches have been applied to other optimization problems, and the desired results have been obtained.

The rest of the paper is arranged as follows. Section 2 presents a description of the problem and the presented mathematical model. Then, an application of the methodology is proposed in Section 3. Finally, conclusions are drawn in the Section 4.

2. Description of the Problem and Presented Mathematical Model

The main purpose of this research is to develop a new fuzzy mathematical model for the classic TCT problem. Crashing the execution time of each activity can affect the success criteria of projects, such as time, cost, quality, and risk. Therefore, this paper proposes a new mathematical model under uncertainty addressing a project CRQT problem under time constraints. Moreover, a more comprehensive definition for the risk criterion is provided based on PMBOK [38], and the impact of a risk on the four objectives of a project is considered.

Total project time (project life cycle), total project cost (project life cost), product and project scope (requirements and criteria related to the project product), and product quality (product quality that is understandable to the customer) are considered. In this model, all the predecessor relationships between activities linking the start and finish times of activities with lag or lead are considered. These relationships include finish-to-finish (FF), start-to-start (SS), start-to-finish (SF), and finish-to-start (FS). After presenting the fuzzy mathematical model, an equivalent auxiliary model is constructed using fuzzy credibility theory. Then, the proposed multi-objective model is solved by the goal attainment method. Then, to evaluate the efficiency of the presented methodology as well as to show the accuracy of the obtained results, the proposed mathematical model is solved by the fuzzy method of Jimenez et al. [37] and the epsilon constraint method.

2.1. Mathematical Model

2.1.1. Cost

To decrease the total project duration, the duration of some activities must be crashed. This action can impose an extra cost on those activities. Given the uncertain circumstances of real-world projects, considering the uncertain crashing cost for activities in the mathematical model can lead to more accurate results. Consequently, the aim of the first objective function is to minimize the total extra cost of the project.

2.1.2. Risk

Time reduction of each activity may cause a risk creation/enhancement [2]. According to PMBOK [38], a more comprehensive definition for the risk criterion is provided, and the impacts of a risk on four objectives of a project such as total project time (project life cycle), total project cost (project life cost), product and project scope (requirements and criteria related to the project product), and product quality (product quality that is understandable to the customer) are considered. In addition to risk impacts, the risk probability should be considered to compute the final risk score of each activity time reduction option. Both presented parameters can be obtained by experts or historical data from similar projects. Consequently, the aim of the second objective function is to minimize the project total risk enhancement.

2.1.3. Quality

Time reduction of each activity may impose a quality reduction on an activity whose duration is crashed, and consequently, the overall project quality requirements will not be met [2]. To calculate the project's reduced quality, both the level of reduced quality and the quality impact of each activity on an overall project should be measured. The level of reduced quality of each activity can be determined according to the proportion of unmet quality components of an activity based on its quality checklist. In the third objective function of the presented mathematical model, a parameter as the weight of each activity has been applied. This parameter indicates the quality impact of an activity on the project quality requirements. It should be noted that each activity has various impacts on the overall project quality requirements. In real-world situations, there are several methods for calculating this parameter, such as expert opinions, pairwise comparisons of activities, cost of activities, time of activities, and the combination of cost and time of activities. In the presented project, the activities pairwise comparison approach has been applied.

Index:

i	Predecessor (s) of activity j
j	Activities of the project $j = 1, \ldots, n$
k	Successor (s) of activity j
r	Project goals indices that can be affected by reducing duration of activities. $r = 1, \ldots, m$
t	Number of units to reduce activity time
L	Last activity

Parameters:

$\widetilde{C}_j$	Additional fuzzy cost associated with reducing one unit of time of activity j
$\widetilde{d}_j$	Fuzzy duration of activity j
MR_j	Maximum crashing time of activity j
ft	Project completion date determined by senior managers or customers.
FS_{ij}^{min}	Finish-to-Start relationships between activities i and j with minimal lag/lead
SS_{ij}^{min}	Start-to-Start relationships between activities i and j with minimal lag/lead
FF_{ij}^{min}	Finish-to-Finish relationships between activities i and j with minimal lag/lead
SF_{ij}^{min}	Start-to-Finish relationships between activities i and j with minimal lag/lead
Pr_{jt}	Risk occurrence probability caused by reducing t unit of time from activity j
I_{rjt}	Impact of risk on the project objective r created by reducing t time unit of activity j
Q_{jt}	Reduced quality due to reduction t time unit of activity j
We_j	Weight of each activity j

Decision variables:

u_{jt}	A binary variable which is equivalent to 1 if activity j is crashed by t units, otherwise 0.
S_j^E	Early start time of activity j
f_j^L	Late finish time of activity j

The CRQT fuzzy model under time-constrained is presented as follows:

$$\min z_1 = \sum_{j=1}^{n} \sum_{t=1}^{MR_j} t \cdot \widetilde{c}_j \cdot u_{jt} \quad (1)$$

$$\min z_2 = \sum_{j=1}^{n} \sum_{t=1}^{MR_j} Pr_{jt} \cdot \left(\sum_{r=1}^{m} I_{rjt} \right) \cdot u_{jt} \quad (2)$$

$$\min z_3 = \sum_{j=1}^{n} We_j \sum_{t=1}^{MR_j} Q_{jt} \cdot u_{jt} \quad (3)$$

Subject to:

$$f_L^l = ft \tag{4}$$

$$\sum_{t=1}^{MR_j} u_{jt} \leq 1 \quad \forall j \tag{5}$$

$$f_j^l - s_j^E \geq (\widetilde{d}_j - \sum_{t=1}^{MR_j} tu_{jt}) \quad \forall j \tag{6}$$

$$s_j^E - s_i^E \geq (\widetilde{d}_i - \sum_{t=1}^{MR_i} tu_{it}) + FS_{ij}^{\min} \quad \forall i,j \in FS \text{ relationships} \tag{7}$$

$$f_k^l - f_j^l \geq (\widetilde{d}_k - \sum_{t=1}^{MR_k} tu_{kt}) + FS_{jk}^{\min} \quad \forall j,k \in FS \text{ relationships} \tag{8}$$

$$s_j^E - s_i^E \geq SS_{ij}^{\min} \quad \forall i,j \in SS \text{ relationships} \tag{9}$$

$$f_k^l - f_j^l \geq (\widetilde{d}_k - \sum_{t=1}^{MR_k} tu_{kt}) - (\widetilde{d}_j - \sum_{t=1}^{MR_j} tu_{jt}) + SS_{jk}^{\min} \quad \forall j,k \in SS \text{ relationships} \tag{10}$$

$$s_j^E - s_i^E \geq (\widetilde{d}_i - \sum_{t=1}^{MR_i} tu_{it}) - (\widetilde{d}_j - \sum_{t=1}^{MR_j} tu_{jt}) + FF_{ij}^{\min} \quad \forall i,j \in FF \text{ relationships} \tag{11}$$

$$f_k^l - f_j^l \geq FF_{jk}^{\min} \quad \forall j,k \in FF \text{ relationships} \tag{12}$$

$$s_j^E - s_i^E \geq SF_{ij}^{\min} - (\widetilde{d}_j - \sum_{t=1}^{MR_j} tu_{jt}) \quad \forall i,j \in SF \text{ relationships} \tag{13}$$

$$f_k^l - f_j^l \geq SF_{jk}^{\min} - (\widetilde{d}_j - \sum_{t=1}^{MR_j} tu_{jt}) \quad \forall j,k \in SF \text{ relationships} \tag{14}$$

$$s_j^E, f_j^l \geq 0 \quad \forall j \tag{15}$$

$$u_{jt} \in \{0,1\} \quad \forall j,t \tag{16}$$

Equations (1)–(3) contain the objective functions of the model by minimizing the total additional cost, the threatening project risk, and the reduced quality of the project, respectively. Constraint (4) indicates that the project must be completed by the specified delivery time. Equation (5) depicts a constraint selecting a specific time reduction mode for activities, i.e., one activity cannot have two crashing time modes. Equations (6)–(14) show constraints on the predecessor relationships of activities for project critical path analysis related to FS, SS, FF, and SF, respectively.

2.2. Constructing an Equivalent Auxiliary Model Using Fuzzy Credibility Theory for Presented CRQT Model under Time-Constrained with Trapezoidal Fuzzy Data

There are various methods for constructing the equivalent auxiliary model for solving a fuzzy model. Because of its appropriate adaptation to the presented problem, fuzzy credibility theory is applied. This theory is known as an appropriate approach because it can efficiently solve linear programming problems, and it does not increase the objective functions and model constraints [39]. Fuzzy credibility theory is described below.

Assume Y is a non-empty set representing the sample space, and $P(Y)$ consists of all possible subsets. Each element of $P(Y)$ is called an event. To provide a clear definition of credibility, it is essential to assign a value of $Cr(A)$ to event A, which represents the credibility of the occurrence of event A. In addition, the following four principles must be established to make sure *Cr (A)* has certain mathematical properties [40]:

$$Cr\{Y\} = 1 \tag{17}$$

$$Cr\{A\} < Cr\{B\} \text{ if } A \subset B \tag{18}$$

$$\text{If } A \in P(Y) \qquad Cr\{A\} + Cr\{A^c\} = 1 \tag{19}$$

$$\text{If } Cr\{A_i\} < 0.5 \quad Cr\{A_i U_i\} \wedge 0.5 = sup_i\, Cr\{A_i\} \tag{20}$$

Now, suppose ζ is a fuzzy variable with function μ. (For each $r \in R$, according to Liu and Liu [41] and Ebrahimi and Jirofti [40]):

$$Cr\left\{\widetilde{\zeta} \le r\right\} = \frac{1}{2}(sup\ \mu(x)_{x \le r} + 1 - sup\ \mu(x)_{x > r}) \tag{21}$$

$$E[\widetilde{\zeta}] = \int_0^{\infty} Cr\left\{\widetilde{\zeta} \ge r\right\} dr - \int_{-\infty}^{0} Cr\left\{\widetilde{\zeta} \le r\right\} dr. \tag{22}$$

$$EV[\widetilde{\zeta}] \frac{\widetilde{\zeta}_1 + \widetilde{\zeta}_2 + \widetilde{\zeta}_3 + \widetilde{\zeta}_4}{4}. \tag{23}$$

$$Cr\left\{\widetilde{\zeta} \le r\right\} \ge \alpha \ \ <==> \ \ r \ge (2 - 2\alpha)\ \widetilde{\zeta}_3 + (2\alpha - 1)\ \widetilde{\zeta}_4 \tag{24}$$

$$Cr\left\{\widetilde{\zeta} \ge r\right\} \ge \alpha \ \ <==> \ \ r \le (2\alpha - 1)\widetilde{\zeta}_1 + (2 - 2\alpha)\ \widetilde{\zeta}_2 \tag{25}$$

Now, the equivalent auxiliary model according to fuzzy credibility theory for the presented multi-objective model is constructed:

$$\min z_1 = \sum_{j=1}^{n} \sum_{t=1}^{MR_j} t \cdot \left(\frac{c_j^1 + c_j^2 + c_j^3 + c_j^4}{4}\right) \cdot u_{jt} \tag{26}$$

$$\min z_2 = \sum_{j=1}^{n} \sum_{t=1}^{MR_j} Pr_{jt} \cdot \sum_{r=1}^{m} I_{rjt} \cdot u_{jt} \tag{27}$$

$$\min z_3 = \sum_{j=1}^{n} We_j \sum_{t=1}^{MR_j} Q_{jt} \cdot u_{jt} \tag{28}$$

$$f_L^l = ft \tag{29}$$

Subject to:

$$\sum_{t=1}^{MR_j} u_{jt} \le 1 \quad \forall j \tag{30}$$

$$f_j^l - s_j^E \ge (((2 - 2\alpha)d_3 + (2\alpha - 1)d_4) - \sum_{t=1}^{MR_j} t u_{jt}), \forall j \tag{31}$$

$$s_j^E - s_i^E \ge (((2 - 2\alpha)d_3 + (2\alpha - 1)d_4) - \sum_{t=1}^{MR_i} t u_{it}) + FS_{ij}^{\min} \quad \forall i, j \in FS \text{ relationships} \tag{32}$$

$$f_k^l - f_j^l \ge (((2 - 2\alpha)d_3 + (2\alpha - 1)d_4) - \sum_{t=1}^{MR_k} t u_{kt}) + FS_{jk}^{\min} \quad \forall j, k \in FS \text{ relationships} \tag{33}$$

$$s_j^E - s_i^E \ge SS_{ij}^{\min} \quad \forall i, j \in SS \text{ relationships} \tag{34}$$

$$f_k^l - f_j^l \geq (((2-2\alpha)d_3 + (2\alpha-1)d_4) - \sum_{t=1}^{MR_k} tu_{kt}) - (((2-2\alpha)d_3 + (2\alpha-1)d_4) - \sum_{t=1}^{MR_j} tu_{jt}) + SS_{jk}^{\min} \forall j,k \in SS \text{ relationships} \tag{35}$$

$$s_j^E - s_i^E \geq (((2-2\alpha)d_3 + (2\alpha-1)d_4) - \sum_{t=1}^{MR_i} tu_{it}) - (((2-2\alpha)d_3 + (2\alpha-1)d_4) - \sum_{t=1}^{MR_j} tu_{jt}) + FF_{ij}^{\min} \forall i,j \in FF \text{ relationships} \tag{36}$$

$$f_k^l - f_j^l \geq FF_{jk}^{\min} \quad \forall j,k \in FF \text{ relationships} \tag{37}$$

$$s_j^E - s_i^E \geq SF_{ij}^{\min} - (((2-2\alpha)d_3 + (2\alpha-1)d_4) - \sum_{t=1}^{MR_j} tu_{jt}) \forall i,j \in SF \text{ relationships} \tag{38}$$

$$f_k^l - f_j^l \geq SF_{jk}^{\min} - (((2-2\alpha)d_3 + (2\alpha-1)d_4) - \sum_{t=1}^{MR_j} tu_{jt}) \forall j,k \in SF \text{ relationships} \tag{39}$$

$$s_j^E, f_j^l \geq 0 \quad \forall j \tag{40}$$

$$u_{jt} \in \{0,1\} \quad \forall j,t \tag{41}$$

Goal Attainment Method

The goal attainment method is considered a very efficient method for dealing with optimization problems with multiple objective functions like the model presented in this paper. Wo_i and B_i are the inputs of the goal attainment method, and they represent the weight and goal values determined for each goal, respectively. To solve the proposed multi-objective model, the model is transformed by the goal attainment method as follows:

$$\min G \tag{42}$$

Subject to:

$$\sum_{j=1}^{n} \sum_{t=1}^{MR_j} t \cdot \left(\frac{c_j^1 + c_j^2 + c_j^3 + c_j^4}{4}\right) \cdot u_{jt} - Wo_1 \cdot G \leq B_1 \tag{43}$$

$$\sum_{j=1}^{n} \sum_{t=1}^{MR_j} Pr_{jt} \cdot \sum_{r=1}^{m} I_{rjt} \cdot u_{jt} - Wo_2 \cdot G \leq B_2 \tag{44}$$

$$\sum_{j=1}^{n} We_j \sum_{t=1}^{MR_j} Q_{jt} \cdot u_{jt} - Wo_3 \cdot G \leq B_3 \tag{45}$$

Equations (29)–(41)

$$G \geq 0 \tag{46}$$

2.3. Constructing an Equivalent Auxiliary Model Using Fuzzy Method Based on Expected Interval and Value for Presented CRQT Model under Time-Constrained with Trapezoidal Fuzzy Data

In addition to the fuzzy credibility theory, there are other methods for constructing the equivalent auxiliary model and solving multi-objective function models, among which the method of Jimenez et al. [37] is applied because of its suitability for the problem. This method also has the advantages of the fuzzy credibility method, such as high efficiency in solving linear programming problems, as well as not increasing the objective functions and constraints of a model. The method of Jimenez et al. [37] is described below.

First, assume $\widetilde{d}$ as a trapezoidal fuzzy number; then, according to Jimenez et al. [37], two definitions of the expected interval (*EI*) and the expected value (*EV*) are presented in Equations (47) and (48).

$$EI\ (\widetilde{d}) = \left[E_1^d, E_2^d\right] = \left[\int_0^1 f_d^{-1}(x)dx, \int_0^1 g_d^{-1}(x)dx\right] = \left[\frac{1}{2}\left(d^1 + d^2\right), \frac{1}{2}\left(d^3 + d^4\right)\right] \quad (47)$$

$$EV\ (\widetilde{d}) = \frac{E_1^d + E_2^d}{2} = \frac{d^1 + d^2 + d^3 + d^4}{4} \quad (48)$$

Based on Jimenez ranking method [42], $\widetilde{a}$ is bigger than $\widetilde{b}$ and can be defined as follows ($\widetilde{a}$ and $\widetilde{b}$ are fuzzy numbers):

$$\mu_M\left(\widetilde{a}, \widetilde{b}\right) = \begin{cases} 0 & E_2^a - E_1^b < 0 \\ \frac{E_2^a - E_1^b}{E_2^a - E_1^b - \left(E_1^a - E_2^b\right)} & if\ 0 \in \left[E_1^a - E_2^b, E_2^a - E_1^b\right] \\ 1 & E_1^a - E_2^b > 0 \end{cases} \quad (49)$$

If $\mu_M\left(\widetilde{a}, \widetilde{b}\right) \geq \alpha$ is expressed with $\widetilde{a}$ bigger than or equivalent to $\widetilde{b}$ at least in the degree of α, it will be shown as $\widetilde{a} \geq \alpha\ \widetilde{b}$; the following fuzzy mathematical programming with trapezoidal fuzzy numbers is considered:

$$\begin{array}{c} \text{Min } Z = \widetilde{d}^t x \\ \text{Subject to} \\ \widetilde{a}_i x\ \geq\ \widetilde{b}_i\ ,\ i = 1, \ldots, l \\ \text{X } \geq 0 \end{array} \quad (50)$$

In accordance with Jimenez et al. [37], a decision vector $x \in \mathcal{R}^n$ in the degree of α is feasible if $\min\limits_{i=1,\ldots,l}\left\{\mu_M\left(\widetilde{a}_i x, \widetilde{b}_i\right)\right\} = \alpha$. Consequently, the first constraint of fuzzy mathematical programming will be equal to Equation (51):

$$\frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \geq \alpha \quad i = 1, \ldots, l \quad (51)$$

Equation (47) can be rewritten as follows:

$$\left[(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}\right]x \geq\ \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i} \quad i = 1, \ldots, l \quad (52)$$

Ultimately, the equivalent crisp α-parametric model of the presented mathematical programming (46) is presented by *EI* and *EV* definitions as follows:

$$\begin{array}{c} \text{Min } EV\ \left(\widetilde{d}\right)\ \text{x} \\ \text{Subject to} \\ \left[(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}\right]x \geq\ \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i}\ , \\ i = 1, \ldots, l \\ \text{X } \geq 0 \end{array} \quad (53)$$

Now, an equivalent auxiliary model according to the method of Jimenez et al. [37] for the presented multi-objective model is constructed:

$$\min z_1 = \sum_{j=1}^{n}\sum_{t=1}^{MR_j} t \cdot \left(\frac{c_j^1 + c_j^2 + c_j^3 + c_j^4}{4}\right) \cdot u_{jt} \quad (54)$$

$$\min z_2 = \sum_{j=1}^{n} \sum_{t=1}^{MR_j} Pr_{jt} \cdot \sum_{r=1}^{m} I_{rjt} \cdot u_{jt} \quad (55)$$

$$\min z_3 = \sum_{j=1}^{n} We_j \sum_{t=1}^{MR_j} Q_{jt} \cdot u_{jt} \quad (56)$$

Subject to:

$$f_L^l = ft \quad (57)$$

$$\sum_{t=1}^{MR_j} u_{jt} \leq 1 \quad \forall j \quad (58)$$

$$f_j^l - s_j^E \geq ((\alpha(\frac{d_j^3 + d_j^4}{2}) + (1-\alpha)(\frac{d_j^1 + d_j^2}{2})) - \sum_{t=1}^{MR_j} tu_{jt}) \quad (59)$$

$$s_j^E - s_i^E \geq ((\alpha(\frac{d_i^3+d_i^4}{2}) + (1-\alpha)(\frac{d_i^1+d_i^2}{2})) - \sum_{t=1}^{MR_i} tu_{it}) + FS_{ij}^{\min} \quad \forall\, i,\, j \in\ FS \text{ relationships} \quad (60)$$

$$f_k^l - f_j^l \geq ((\alpha(\frac{d_k^3+d_k^4}{2}) + (1-\alpha)(\frac{d_k^1+d_k^2}{2})) - \sum_{t=1}^{MR_k} tu_{kt}) + FS_{jk}^{\min} \quad \forall\, j, k \in\ FS \text{ relationships} \quad (61)$$

$$s_j^E - s_i^E \geq SS_{ij}^{\min} \ \forall\, i,\, j \in\ SS \text{ relationships} \quad (62)$$

$$f_k^l - f_j^l \geq ((\alpha(\frac{d_k^3+d_k^4}{2}) + (1-\alpha)(\frac{d_k^1+d_k^2}{2})) - \sum_{t=1}^{MR_k} tu_{kt}) - ((\alpha(\frac{d_j^3+d_j^4}{2}) + (1-\alpha)(\frac{d_j^1+d_j^2}{2})) - \sum_{t=1}^{MR_j} tu_{jt}) + SS_{jk}^{\min} \quad \forall\, j,\, k \in\ SS \text{ relationships} \quad (63)$$

$$s_j^E - s_i^E \geq ((\alpha(\frac{d_i^3+d_i^4}{2}) + (1-\alpha)(\frac{d_i^1+d_i^2}{2})) - \sum_{t=1}^{MR_i} tu_{it}) - ((\alpha(\frac{d_j^3+d_j^4}{2}) + (1-\alpha)(\frac{d_j^1+d_j^2}{2})) - \sum_{t=1}^{MR_j} tu_{jt}) + FF_{ij}^{\min} \quad \forall\, i,\, j \in FF \text{ relationships} \quad (64)$$

$$f_k^l - f_j^l \geq FF_{jk}^{\min} \quad \forall\, j,\, k \in FF \text{ relationships} \quad (65)$$

$$s_j^E - s_i^E \geq SF_{ij}^{\min} - ((\alpha(\frac{d_j^3+d_j^4}{2}) + (1-\alpha)(\frac{d_j^1+d_j^2}{2})) - \sum_{t=1}^{MR_j} tu_{jt}) \quad \forall\, i,\, j \in \text{SF relationships} \quad (66)$$

$$f_k^l - f_j^l \geq SF_{jk}^{\min} - ((\alpha(\frac{d_j^3+d_j^4}{2}) + (1-\alpha)(\frac{d_j^1+d_j^2}{2})) - \sum_{t=1}^{MR_j} tu_{jt}) \quad \forall\, j,\, k \in SF \text{ relationships} \quad (67)$$

$$s_j^E, f_j^l \geq 0 \quad \forall\, j \quad (68)$$

$$u_{jt} \in \{0,1\} \quad \forall\, j, t \quad (69)$$

Augmented Epsilon Constraint Method

One efficient method for solving multi-objective models is the augmented epsilon constraint method, which is applied in this research. In this approach, an objective function must remain, and other objective functions are transferred in constraints with a special pattern. The proposed multi-objective model is transformed as follows:

$$\min z_1 = \sum_{j=1}^{n} \sum_{t=1}^{MR_j} t \cdot \left(\frac{c_j^1 + c_j^2 + c_j^3 + c_j^4}{4}\right) \cdot u_{jt} + 0.00001 * (\vartheta_1 + \vartheta_2) \quad (70)$$

Subject to:

$$\sum_{j=1}^{n} \sum_{t=1}^{MR_j} Pr_{jt} \cdot \sum_{r=1}^{m} I_{rjt} \cdot u_{jt} + \vartheta_1 = e_1 \quad (71)$$

$$\sum_{j=1}^{n} We_j \sum_{t=1}^{MR_j} Q_{jt} \cdot u_{jt} + \vartheta_2 = e_2 \quad (72)$$

Equations (57)–(69)

3. Application

To depict the application and effectiveness of the proposed methodology, a construction project from a previous study [2] has been selected. This project includes 18 activities. The project manager tends to crash the duration of activities so as to attain the determined project duration. Information on project activities is provided and presented in Table 2.

Table 2. Project information.

Fuzzy Duration	Fuzzy Crashing Cost	Reduced Quality		Risk Value ($Pr*I$), u_{jt}=1,2, r = 1, . . . , 4								MR_j
		u_{jt}=1,2		1		2		3		4		
		1	2	1	2	1	2	1	2	1	2	
(3,4,5,6)	(670,690, 710,730)	0.1	0.25	0.04	0.12	0.01	0.02	0.01	0.02	0.04	0.32	2
(2,3,4,5)	(135,145,155,165)	0.2	-	0.01	-	0.01	-	0.02	-	0.02	-	1
(3,4,5,6)	(275,290,300,330)	0.5	-	0.01	-	0.01	-	0.01	-	0.08	-	1
(7,9,10,12)	(470,490,500,530)	0.25	0.4	0.01	0.02	0.02	0.12	0.01	0.06	0.02	0.08	2
(7,8,9,10)	(480,490,500,525)	0.25	0.4	0.01	0.02	0.02	0.12	0.01	0.06	0.02	0.08	2
(14,17,18,19)	(580,585,595,630)	0.2	0.4	0.01	0.02	0.02	0.12	0.01	0.06	0.02	0.08	2
(3,4,5,6)	(94,96,100,108)	0.4	-	0.02	-	0.02	-	0.01	-	0.08	-	1
(11,12,15,17)	(970,980,990,1050)	0.2	0.5	0.02	0.08	0.02	0.12	0.01	0.04	0.02	0.1	2
(16,18,19,24)	(570,580,610,635)	0.05	0.25	0.01	0.02	0.01	0.02	0.01	0.01	0.02	0.08	2
(13,14,15,16)	(380,385,410,420)	0.1	0.25	0.02	0.08	0.01	0.02	0.01	0.01	0.08	0.32	2
(2,3,4,5)	(125,140,155,160)	0.2	-	0.01	-	0.01	-	0.02	-	0.02	-	1
(3,4,5,6)	(280,300,305,315)	0.5	-	0.01	-	0.01	-	0.01	-	0.08	-	1
(6,8,10,11)	(490,495,500,515)	0.25	0.4	0.01	0.02	0.02	0.12	0.01	0.06	0.02	0.08	2
(7,8,9,10)	(475,485,505,520)	0.25	0.4	0.01	0.02	0.02	0.12	0.01	0.06	0.02	0.08	2
(14,15,17,22)	(575,580,600,630)	0.2	0.4	0.01	0.02	0.02	0.12	0.01	0.06	0.02	0.08	2
(3,4,5,6)	(85,95,100,110)	0.4	-	0.02	-	0.02	-	0.01	-	0.08	-	1
(17,18,19,24)	(590,595,610,615)	0.05	0.25	0.01	0.02	0.01	0.02	0.01	0.01	0.02	0.08	2
(13,14,15,16)	(370,385,400,420)	0.1	0.25	0.02	0.08	0.01	0.02	0.01	0.01	0.08	0.32	2

Initially, using available data, the mathematical model of the CRQT problem under time constraints is constructed. Now, the main purpose is to find the optimal set of activities to be crashed with minimal additional cost, threatening risk, and reduced quality.

The proposed multi-objective fuzzy model is separately solved with two approaches, namely, fuzzy credibility theory and the goal attainment method as well as the method of Jimenez et al. [37] and the augmented ε-constraint method using GAMS software. Pareto-optimal solutions from the two presented approaches are presented in Tables 3 and 4.

Table 3. A number of Pareto-optimal solutions by the fuzzy credibility method and the goal attainment method.

Wo_1	Wo_2	Wo_3	B_1	B_2	B_3	G	Z_1	Z_2	Z_3	u_{jt}=1
0.1	0.8	0.1	3700	0.6	0.065	0.512	3593	1.01	0.057	$\{u_{1,2}, u_{4,2}, u_{5,1}, u_{6,1}, u_{7,1}\}$
0.3	0.5	0.2	4200	0.55	0.07	0.36	4095	0.73	0.061	$\{u_{1,1}, u_{4,2}, u_{5,1}, u_{6,1}, u_{7,1}, u_{9,1}, u_{17,1}\}$
0.3	0.3	0.4	3500	0.635	0.043	1.78	3494	1.17	0.052	$\{u_{1,2}, u_{4,2}, u_{5,2}, u_{7,1}\}$
0.45	0.45	0.1	4000	0.735	0.063	0.344	3996	0.89	0.55	$\{u_{1,1}, u_{4,2}, u_{5,2}, u_{7,1}, u_{9,1}, u_{17,1}\}$
0.6	0.3	0.1	3900	0.65	0.06	0.867	3888	0.91	0.065	$\{u_{1,1}, u_{4,2}, u_{5,2}, u_{6,1}, u_{7,1}, u_{14,1}\}$

Table 4. A number of Pareto-optimal solutions by Jimenez et al. [37] method and augmented epsilon constraint method.

Z_1	Z_2	Z_3	u_{jt}=1
3600	1.01	0.056	$\{u_{1,2}, u_{4,2}, u_{5,1}, u_{6,1}, u_{7,1}\}$
4500	0.82	0.049	$\{u_{1,1}, u_{4,2}, u_{5,2}, u_{6,1}, u_{9,1}, u_{17,1}\}$
4600	1.14	0.044	$\{u_{1,2}, u_{4,2}, u_{5,2}, u_{9,1}, u_{17,1}\}$
4800	0.67	0.087	$\{u_{1,1}, u_{4,1}, u_{5,1}, u_{6,1}, u_{7,1}, u_{9,1}, u_{17,1}\}$
3500	1.17	0.052	$\{u_{1,2}, u_{4,2}, u_{5,2}, u_{7,1}\}$
4100	0.73	0.059	$\{u_{1,1}, u_{4,1}, u_{5,2}, u_{6,1}, u_{7,1}, u_{9,1}, u_{17,1}\}$
3900	0.91	0.064	$\{u_{1,1}, u_{4,2}, u_{5,2}, u_{6,1}, u_{7,1}, u_{14,1}\}$

In Table 3, some Pareto-optimal solutions through the fuzzy credibility method and goal attainment method are presented. The Wo_i and B_i represent the weights and goals determined for each goal, respectively. OF_i indicate the value of the project's objective functions (cost, risk, and quality criteria) for each Pareto-optimal solution. G is the optimal value of the goal attainment method. The last column also depicts the optimal sets of activities and their crashing values. In fact, in this methodology, various weight and goal values for objective functions can generate Pareto-optimal solutions. For example, one of the optimal solutions is that the first, fourth, and fifth activities must be crashed by two days, and the seventh activity must be crashed by one day. In this solution, the additional cost, threatening risk value, and reduced quality values are 3496, 1.17, and 0.052, respectively.

Table 4 presents some Pareto-optimal solutions for the given application through the method of Jimenez et al. [37] and the augmented epsilon constraint method. This table also depicts the optimal set of activities, their crashing values, and the values obtained for each objective function. By examining the results of both methods and existing common Pareto-optimal solutions, the correctness and efficiency of the two methods in the TCT problem are proven. It should be noted that both methods have advantages and disadvantages; the application of these methods in other issues requires their adaptation to the assumed problems.

The methodology of this research enables project managers to choose the optimal set of project activities for crashing so as to reduce the duration of their projects as long as they can. On the other hand, the results of this approach provide knowledge to project managers about changes in project status, such as additional costs imposed, threatening risk incurred, and reduced quality due to project time crashing. Access to this important information enables them to make indispensable provisions for project cost –risk and quality criteria at an appropriate time.

In order to demonstrate the accuracy of the obtained results as well as the advantages of the proposed fuzzy approaches, the introduced project is solved using the fuzzy α-cut method [43], which is regarded as another efficient fuzzy solution method, and the presented goal attainment method. The results are presented in Table 5. As can be seen, two Pareto-optimal solutions computed by the fuzzy α-cut method are compared with two Pareto-optimal solutions by the two suggested fuzzy approaches. The calculated values of the cost objective function (Z_1), which are obtained by the proposed approaches, are better than the values of the fuzzy α-cut method. Moreover, the fuzzy credibility theory and Jimenez et al. [37] method are known as efficient approaches to solve fuzzy mathematical models without increasing the objective functions and model constraints. However, some other fuzzy approaches increase objective functions and constraints, so the problem becomes more complicated, and the solution time increases.

Table 5. A comparison of the obtained results by two presented fuzzy approaches and fuzzy α-cut method.

Fuzzy α-Cut Method and Presented Goal Attainment Method											This Paper: Fuzzy Credibility theory & Goal Attainment Method			
Wo_1	Wo_2	Wo_3	B_1	B_2	B_3	G	Z_1	Z_2	Z_3	u_{jt}=1	G	Z_1	Z_2	Z_3
0.45	0.45	0.1	4000	0.735	0.063	0.389	3929	0.91	0.065	$\{u_{1,1}, u_{4,2}, u_{5,2}, u_{6,1}, u_{7,1}, u_{14,1}\}$	0.344	3888	0.91	0.065
											Jimenez et al. [37] method & Augmented epsilon constraint			
0.4	0.3	0.3	4200	0.65	0.07	0.267	4132	0.73	0.059	$\{u_{1,1}, u_{4,1}, u_{5,2}, u_{6,1}, u_{7,1}, u_{9,1}, u_{17,1}\}$	-	4100	0.73	0.059

4. Conclusions

Project scheduling is one of the most important problems in project management and control. In today's competitive world, one of the most common problems that project managers encounter is reducing the project's total time until the specified finish date and constructing a new schedule. Consequently, creating an appropriate and efficient approach to reducing the project's total time as well as paying attention to all effective criteria of a project in this crucial decision-making process is critical. In this paper, a new fuzzy approach for the cost–risk–quality trade-off (CRQT) problem under time constraints was developed. Indeed, this paper developed a multi-objective fuzzy mathematical model including three objective functions—namely, minimizing additional costs, threatening risk values, and reduced quality of a project. In order to solve the model, two various fuzzy solution approaches were proposed. First, the proposed model was solved by integrating fuzzy credibility theory and the goal attainment method. Next, the presented fuzzy method of Jimenez et al. [37] and the augmented epsilon constraint method were applied to solve the model. In order to evaluate the proposed methodology, a project from the literature has been adopted and solved. Both proposed fuzzy solution approaches were able to determine Pareto-optimal solutions, including the activities that must be crashed, the amount of crashing, and the values of objective functions. The results demonstrate that both methods have high efficiency and accuracy. These two approaches will give project managers the ability to manage the project and finish it by a specific date by controlling the time, cost, risk, and quality criteria that affect it, given the uncertainty of the real-world.

For future studies, other fuzzy solution algorithms can be used for the proposed multi-objective model, and the results can be compared. Considering other project success criteria, such as resources and environmental considerations, can further complement the existing model. In addition, project managers are always interested in knowing the amount of cost or profit for different project completion times. Indeed, a trade-off between the crashing cost and project delay cost can be accomplished. Considering the effect of the quality reduction of predecessor activities on the quality of the subsequent activities in the quality objective function equation can improve it. In order to better regard high uncertainty in the real-world, extended fuzzy sets, such as type-2 fuzzy sets, can be applied from the related literature [44–47]. To improve the results, a new approach for determining the weight of each project activity can be developed. Ultimately, new approaches for computing project quality and risk criteria can be introduced.

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References

1. Singh, G.; Ernst, A.T. Resource constraint scheduling with a fractional shared resource. *Oper. Res. Lett.* **2011**, *39*, 363–368. [CrossRef]
2. Mohammadipour, F.; Sadjadi, S.J. Project cost–quality–risk tradeoff analysis in a time-constrained problem. *Comput. Ind. Eng.* **2016**, *95*, 111–121. [CrossRef]
3. El-Sayegh, S.M.; Al-Haj, R. A new framework for time-cost trade-off considering float loss impact. *J. Financ. Manag. Prop. Constr.* **2017**, *22*, 20–36. [CrossRef]
4. Ke, H.; Ma, J. Modeling project time–cost trade-off in fuzzy random environment. *Appl. Soft Comput.* **2014**, *19*, 80–85. [CrossRef]
5. Kessler, E.H.; Chakrabarti, A.K. Speeding up the pace of new product development. *J. Prod. Innov.Manag. Int. Publ. Prod. Dev. Manag. Assoc.* **1999**, *16*, 231–247. [CrossRef]
6. Babu, A.J.G.; Suresh, N. Project management with time, cost, and quality considerations. *Eur. J. Oper. Res.* **1996**, *88*, 320–327. [CrossRef]
7. Kim, J.; Kang, C.; Hwang, I. A practical approach to project scheduling: Considering the potential quality loss cost in the time–cost tradeoff problem. *Int. J. Proj. Manag.* **2012**, *30*, 264–272. [CrossRef]
8. Orm, M.B.; Jeunet, J. Time-cost-quality trade-off problems: A survey exploring the assessment of quality. *Comput. Ind. Eng.* **2018**, *118*, 319–328. [CrossRef]
9. Salari, M.; Bagherpour, M.; Reihani, M.H. A time-cost trade-off model by incorporating fuzzy earned value management: A statistical based approach. *J. Intell. Fuzzy Syst.* **2015**, *28*, 1909–1919. [CrossRef]
10. He, Z.; He, H.; Liu, R.; Wang, N. Variable neighbourhood search and tabu search for a discrete time-cost trade-off problem to minimize the maximal cash flow gap. *Comput. Oper. Res.* **2017**, *78*, 564–577. [CrossRef]
11. Tran, D.H.; Long, L.D. Project scheduling with time, cost and risk trade-off using adaptive multiple objective differential evolution. *Eng. Constr. Archit. Manag.* **2018**, *25*, 623–638. [CrossRef]
12. Xu, J.; Zheng, H.; Zeng, Z.; Wu, S.; Shen, M. Discrete time–cost–environment trade-off problem for large-scale construction systems with multiple modes under fuzzy uncertainty and its application to Jinping-II Hydroelectric Project. *Int. J. Proj. Manag.* **2012**, *30*, 950–966. [CrossRef]
13. Paidar, F.; Ashtiani, B.; Tavakkoli Moghaddam, R. Time-cost-resource optimization of resource constrained project scheduling problem with discounted cash flows using multi-objective gravitational search algorithm. *Int. J. Ind. Eng. Prod. Manag.* **2017**, *28*, 1–14. Available online: https://www.sid.ir/en/journal/viewpaper.aspx?id=566050 (accessed on 18 May 2022).
14. Wood, D.A. Gas and oil project time-cost-quality tradeoff: Integrated stochastic and fuzzy multi-objective optimization applying a memetic, nondominated, sorting algorithm. *J. Nat. Gas Sci. Eng.* **2017**, *45*, 143–164. [CrossRef]
15. Khadem, M.M.R.K.; Piya, S.; Shamsuzzoha, A. Quantitative risk management in gas injection project: A case study from Oman oil and gas industry. *J. Ind. Eng. Int.* **2018**, *14*, 637–654. [CrossRef]
16. Haghighi, M.H.; Mousavi, S.M.; Antuchevičienė, J.; Mohagheghi, V. A new analytical methodology to handle time-cost trade-off problem with considering quality loss cost under interval-valued fuzzy uncertainty. *Technol. Econ. Dev. Econ.* **2019**, *25*, 277–299. [CrossRef]
17. Hamta, N.; Ehsanifar, M.; Sarikhani, J. Presenting a goal programming model in the time-cost-quality trade-off. *Int. J. Constr. Manag.* **2021**, *21*, 1–11. [CrossRef]
18. Panwar, A.; Jha, K.N. Integrating quality and safety in construction scheduling time-cost trade-off model. *J. Constr. Eng. Manag.* **2021**, *147*, 04020160. [CrossRef]
19. Banihashemi, S.A.; Khalilzadeh, M. Time-cost-quality-environmental impact trade-off resource-constrained project scheduling problem with DEA approach. *Eng. Constr. Archit. Manag.* **2021**, *28*, 1979–2004. [CrossRef]
20. Lotfi, R.; Yadegari, Z.; Hosseini, S.H.; Khameneh, A.H.; Tirkolaee, E.B.; Weber, G.W. A robust time-cost-quality-energy-environment trade-off with resource-constrained in project management: A case study for a bridge construction project. *J. Ind. Manag. Optim.* **2022**, *18*, 375. [CrossRef]

21. Mahmoudi, A.; Feylizadeh, M.R. A grey mathematical model for crashing of projects by considering time, cost, quality, risk and law of diminishing returns. *Grey Syst. Theory Appl.* **2018**, *8*, 272–294. [CrossRef]
22. Feylizadeh, M.R.; Mahmoudi, A.; Bagherpour, M.; Li, D.F. Project crashing using a fuzzy multi-objective model considering time, cost, quality and risk under fast tracking technique: A case study. *J. Intell. Fuzzy Syst.* **2018**, *35*, 3615–3631. [CrossRef]
23. Mahmoudi, A.; Javed, S.A. Project scheduling by incorporating potential quality loss cost in time-cost tradeoff problems: The revised KKH model. *J. Model. Manag.* **2020**, *15*, 1187–1204. [CrossRef]
24. Liu, D.; Li, H.; Wang, H.; Qi, C.; Rose, T. Discrete symbiotic organisms search method for solving large-scale time-cost trade-off problem in construction scheduling. *Expert Syst. Appl.* **2020**, *148*, 113230. [CrossRef]
25. Mahdiraji, H.A.; Sedigh, M.; Hajiagha, S.H.R.; Garza-Reyes, J.A.; Jafari-Sadeghi, V.; Dana, L.P. A novel time, cost, quality and risk tradeoff model with a knowledge-based hesitant fuzzy information: An R&D project application. *Technol. Forecast. Soc. Chang.* **2021**, *172*, 121068.
26. Kebriyaii, O.; Heidari, A.; Khalilzadeh, M.; Antucheviciene, J.; Pavlovskis, M. Application of Three Metaheuristic Algorithms to Time-Cost-Quality Trade-Off Project Scheduling Problem for Construction Projects Considering Time Value of Money. *Symmetry* **2021**, *13*, 2402. [CrossRef]
27. Tao, L.; Su, X.; Javed, S.A. Time-cost Trade-off Model in GERT-type Network with Characteristic Function for Project Management. *Comput. Ind. Eng.* **2022**, *169*, 108222. [CrossRef]
28. Nguyen, D.T.; Le-Hoai, L.; Tarigan, P.B.; Tran, D.H. Tradeoff time cost quality in repetitive construction project using fuzzy logic approach and symbiotic organism search algorithm. *Alex. Eng. J.* **2022**, *61*, 1499–1518. [CrossRef]
29. Chen, S.P.; Tsai, M.J. Time–cost trade-off analysis of project networks in fuzzy environments. *Eur. J. Oper. Res.* **2011**, *212*, 386–397. [CrossRef]
30. Monghasemi, S.; Nikoo, M.R.; Fasaee, M.A.K.; Adamowski, J. A novel multi-criteria decision making model for optimizing time–cost–quality trade-off problems in construction projects. *Expert Syst. Appl.* **2015**, *42*, 3089–3104. [CrossRef]
31. Ammar, M.A. Efficient modeling of time-cost trade-off problem by eliminating redundant paths. *Int. J. Constr. Manag.* **2020**, *20*, 812–821. [CrossRef]
32. Albayrak, G.; Özdemir, İ. Multimodal optimization for time-cost trade-off in construction projects using a novel hybrid method based on FA and PSO. *Rev. De La Construcción* **2018**, *17*, 304–318. [CrossRef]
33. Ballesteros-Pérez, P.; Elamrousy, K.M.; González-Cruz, M.C. Non-linear time-cost trade-off models of activity crashing: Application to construction scheduling and project compression with fast-tracking. *Autom. Constr.* **2019**, *97*, 229–240. [CrossRef]
34. Jeunet, J.; Orm, M.B. Optimizing temporary work and overtime in the Time Cost Quality Trade-off Problem. *Eur. J. Oper. Res.* **2020**, *284*, 743–761. [CrossRef]
35. Sharma, K.; Trivedi, M.K. AHP and NSGA-II-Based Time–Cost–Quality Trade-Off Optimization Model for Construction Projects. *Artif. Intell. Sustain. Comput.* **2022**, 45–63. [CrossRef]
36. Dhawan, A.; Sharma, K.; Trivedi, M.K. Simulated Annealing-Based Time–Cost Trade-Off Model for Construction Projects. *Artif. Intell. Sustain. Comput.* **2022**, 33–43. [CrossRef]
37. Jiménez, M.; Arenas, M.; Bilbao, A.; Rodrigues, M.V. Linear programming with fuzzy parameters: An interactive method resolution. *Eur. J. Oper. Res.* **2007**, *177*, 1599–1609. [CrossRef]
38. Project Management Institute. *Project Management Body of Knowledge*; PMI Standard Committee: Newtown Square, PA, USA, 2013.
39. Cheraghi, S.; Hosseini-Motlagh, S.M. Optimal blood transportation in disaster relief considering facility disruption and route reliability under uncertainty. *Int. J. Transp. Eng.* **2017**, *4*, 225–254.
40. Ebrahimi, S.; Jirofti, A. Investigating of using fuzzy credibility theory for measuring value at risk. *J. Quant. Econ.* **2016**, *13*, 1–24. Available online: https://www.sid.ir/en/journal/viewpaper.aspx?id=547700 (accessed on 18 May 2022).
41. Liu, B.; Liu, Y.K. Expected value of Fuzzy Variable and Fuzzy Expected Value Models. *IEEE Trans. Fuzzy Syst.* **2002**, *10*, 445–450.
42. Jiménez, M. Ranking fuzzy numbers through the comparison of its expected intervals. *Int. J. Uncertain. Fuzziness Knowl. Based Syst.* **1996**, *4*, 379–388. [CrossRef]
43. Sutthibutr, N.; Chiadamrong, N. Fuzzy multi-objective optimization with α-cut analysis for supply chain master planning problem. *Uncertain Supply Chain Manag.* **2019**, *7*, 635–664. [CrossRef]
44. Mohagheghi, V.; Mousavi, S.M.; Shahabi-Shahmiri, R. Sustainable project portfolio selection and optimization with considerations of outsourcing decisions, financing options and staff assignment under interval type-2 fuzzy uncertainty. *Neural Comput. Appl.* **2022**, 1–22. [CrossRef]
45. Dorfeshan, Y.; Mousavi, S.M.; Zavadskas, E.K.; Antucheviciene, J. A new enhanced ARAS method for critical path selection of engineering projects with interval type-2 fuzzy sets. *Int. J. Inf. Technol. Decis. Mak.* **2021**, *20*, 37–65. [CrossRef]
46. Aramesh, S.; Mousavi, S.M.; Mohagheghi, V. A new comprehensive project scheduling, monitoring, and management framework based on the critical chain under interval type-2 fuzzy uncertainty. *Iran. J. Fuzzy Syst.* **2021**, *18*, 151–170.
47. Mirnezami, S.A.; Mousavi, S.M.; Mohagheghi, V. A new interval type-2 fuzzy approach for multi-scenario project cash flow assessment based on alternative queuing method and dependency structure matrix with a case study. *Eng. Appl. Artif. Intell.* **2020**, *95*, 103815. [CrossRef]