Article

# A Variable Block Insertion Heuristic for the Blocking Flowshop Scheduling Problem with Total Flowtime Criterion 

Mehmet Fatih Tasgetiren ${ }^{\text {1,* }}$, Quan-Ke Pan ${ }^{2}$, Damla Kizilay ${ }^{3}$ and Kaizhou Gao ${ }^{4}$<br>1 International Logistics Management Department, Yasar University, Izmir 35100, Turkey<br>2 Department of Industrial and Manufacturing System Engineering, School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, China; panquanke@hust.edu.cn<br>3 Industrial Engineering Department, Yasar University, Izmir 35100, Turkey; damla.kizilay@yasar.edu.tr<br>4 School of Electrical and Electronics, Nanyang Technological University, Singapore 639798, Singapore; kgao001@e.ntu.edu.sg<br>* Correspondence: fatih.tasgetiren@yasar.edu.tr; Tel.: +90-530-827-3715

Academic Editor: Javier Del Ser Lorente
Received: 30 June 2016; Accepted: 8 October 2016; Published: 20 October 2016


#### Abstract

In this paper, we present a variable block insertion heuristic (VBIH) algorithm to solve the blocking flowshop scheduling problem with the total flowtime criterion. In the VBIH algorithm, we define a minimum and a maximum block size. After constructing the initial sequence, the VBIH algorithm starts with a minimum block size being equal to one. It removes the block from the current sequence and inserts it into the partial sequence sequentially with a predetermined move size. The sequence, which is obtained after several block moves, goes under a variable local search (VLS), which is based on traditional insertion and swap neighborhood structures. If the new sequence obtained after the VLS local search is better than the current sequence, it replaces the current sequence. As long as it improves, it keeps the same block size. However, if it does not improve, the block size is incremented by one and a simulated annealing-type of acceptance criterion is used to accept the current sequence. This process is repeated until the block size reaches at the maximum block size. Furthermore, we present a novel constructive heuristic, which is based on the profile fitting heuristic from the literature. The proposed constructive heuristic is able to further improve the best known solutions for some larger instances in a few seconds. Parameters of the constructive heuristic and the VBIH algorithm are determined through a design of experiment approach. Extensive computational results on the Taillard's well-known benchmark suite show that the proposed VBIH algorithm outperforms the discrete artificial bee colony algorithm, which is one of the most efficient algorithms recently in the literature. Ultimately, 52 out of the 150 best known solutions are further improved with substantial margins.


Keywords: meta-heuristics; blocking flowshop; block insertion heuristic; variable local search; constructive heuristics

## 1. Introduction

There have been extensive studies about permutation flowshop scheduling (PFSP) in the literature with many important applications in manufacturing and service systems [1-4]. The traditional PFSP is concerned with scheduling $n$ jobs through $m$ machines in such a way that the same sequence is applied to each machine. An important assumption is such that work-in-process inventory is allowed since there are infinite buffer capacities amongst consecutive machines. Hence, allowing jobs to be waiting in front of machines for their next operations words. On the other hand, if any storage capacity amongst machines is not available, the traditional PFSP is said to be blocking flowshop scheduling problem
(BFSF) [5]. In this case, a job cannot leave the machine unless the next machine is free. Its industrial applications can be found in [5,6]. A comprehensive review on flowshop scheduling with blocking and no-wait constraint can be found in Hall and Siriskandarajah [7].

The makespan criterion for the BFSP is commonly used in the literature. This problem is also denoted as $F_{m} / b l o c k i n g / C_{\max }$ with the notation of Graham et al. [8] in the literature. Although Gilmore-Gomory's algorithm [9] can solve it optimally with two machine case ( $m=2$ ), it is proven to be NP-Hard by Hall and Sriskandarajah [7] when $m>2$. For this reason, efforts have been devoted on developing heuristic and meta-heuristic approaches for scheduling a large number of jobs, which is commonly needed for real-life problems.

Since the problem is NP-Hard, constructive heuristics and meta-heuristic algorithms have been attracted attention to solve the BFSP with the makespan criterion. Regarding the constructive heuristics, McCormick et al. [10] presented a profile fitting (PF) heuristic to solve BFSP with the performance measure of minimization of cycle time. The PF heuristic greedily establishes a sequence with the next job having minimum idle and blocking times on machines. Leisten [11] presented a similar heuristic for flowshop scheduling problems with finite and unlimited buffers in order to maximize the buffer usages and to minimize the machine blocking times. However, it was not able to obtain better results than the NEH heuristic, which is initially proposed in [12] to solve the traditional PFSP. Depending on the makespan properties defined by Ronconi and Armentano [13], Ronconi [6] presented a note on constructive heuristics and proposed three constructive heuristics, called MM, MM combined with NEH (MME), and PF combined with NEH (PFE), respectively for the BFSP with the makespan criterion. It was shown that the MME and PFE heuristics generated better results than the NEH heuristics up to 500 jobs and 20 machines. Abadi et al. [14] proposed an improvement heuristic to minimize cycle time and Ronconi and Henriques [15] considered the minimization of total tardiness in a flowshop with blocking and presented some constructive heuristics with promising results. Furthermore, Pan and Wang [16] developed some effective heuristics based on PF approach. These PF-based heuristics are inspired from LR heuristic proposed by Liu and Revees [17] for the PFSP with the total flowtime criterion. In their work, the PF heuristic is combined with the partial NEH implementation and they called the heuristics as PF_NEH(x), WPF_NEH(x) and PW_NEH(x), where $x$ is the number of sequences generated by considering the first $x$ number of jobs in the initial order of jobs. In order not to ruin good characteristics of the PF heuristic, the NEH heuristic is applied to only the last $\delta$ jobs. Their PW_NEH(x) heuristic with $x=5$ was substantially better than NEH, MME, PFE, WPFE and PWE heuristics from the literature. Regarding the meta-heuristic algorithms for BFSP with the makespan criterion, the literature can be found in [18-33].

The makespan criterion is a firm-oriented performance measure, which aims at minimizing the idle times on the machines, thus resulting in a maximization of machine utilization. However, total flowtime and total tardiness criteria are both customer-oriented performance measures, which aim at minimizing the waiting times of jobs amongst machines in order to finish the jobs as early as possible, thus resulting in a maximization of the customer satisfaction. Since the tardiness is measured by the difference between due date and flowtime, both performance measures are equivalent.

For the total tardiness criterion, a few papers can be found in [34,35]. Regarding the total flowtime criterion, a few papers can be found in the literature. A hybrid harmony search is presented in [36]. A discrete artificial bee colony algorithm is presented in [37]. An iterated greedy algorithm is developed in [38], and a branch and bound is presented in [39] for solving small size of instances. Very recently, a GRASP and a discrete artificial bee colony (DABC_RCT) algorithm are developed in [40,41], which outperformed the existing algorithms from the literature.

In this paper, we present a variable block insertion heuristic (VBIH) algorithm to solve the BFSP with the total flowtime criterion. Through extensive computational analyses on Taillard's well-known benchmark suite, we demonstrate that the proposed VBIH algorithm outperforms the recent best performing DABC_RCT algorithm from the literature. Ultimately, 52 out of 150 problem instances are further improved with substantial margins.

The rest of the paper is organized as follows. In Section 2, the blocking flowshop scheduling problem with speed-up method is formulated. Section 3 presents the components of the VBIH algorithm. Section 4 presents the design of experiment approach for parameter tuning. The computational results and comparisons are provided in Section 5. Finally, Section 6 gives the concluding remarks.

## 2. Blocking Flow Shop Scheduling Problem

In the blocking flowshop scheduling problem, $n$ jobs from the set $J=\{1,2, . ., n\}$ have to be processed on $m$ machines from the set $M=\{1,2, . ., m\}$ with the same permutation on each machine without any intermediate buffer. Each job $j$ has a processing time on machine $k$, which is denoted as $p_{j, k}$. The setup time is assumed to be included in the processing time. Only a single job can be processed on each machine. Since waiting times are not allowed in the flowshop due to no intermediate buffers, jobs cannot leave machines after completing their operations until next machines are free. In other words, if the next machine is busy, then the current job on the machine must be blocked since there is no intermediate buffer amongst machines. The goal is to obtain a permutation, which will be applied to each machine and the total flowtime (TFT) is to be minimized. Given a job sequence $\pi=\left\{\pi_{1}, \pi_{2}, . ., \pi_{n}\right\}$, the departure time $d_{\pi_{j}, k}$ of job $\pi_{j}$ on machine $k$ can be computed by following Ronconi [6] as follows:

$$
\begin{gather*}
d_{\pi_{1}, 0}=0  \tag{1}\\
d_{\pi_{1}, k}=d_{\pi_{1}, k-1}+p_{\pi_{1}, k} k=1, . ., m-1  \tag{2}\\
d_{\pi_{j}, 0}=d_{\pi_{j}-1,1} j=2, . . n  \tag{3}\\
d_{\pi_{j}, k}=\max \left\{d_{\pi_{j}, k-1}+p_{\pi_{j}, k}, d_{\pi_{j}-1, k+1}\right\} j=2, . ., n \text { and } k=1, . ., m-1  \tag{4}\\
d_{\pi_{j}, m}=d_{\pi_{j}, m-1}+p_{\pi_{j}, m} j=1, . ., n \tag{5}
\end{gather*}
$$

where $d_{\pi_{j}, 0}, j=1, . ., n$ denotes the starting time of job $j$ on the first machine. Since $C_{\pi_{j}, m}=d_{\pi_{j}, m}$ for all $j$, then the total flowtime of the sequence $\pi$ can be given as $T F T(\pi)=\sum_{j=1}^{n} C_{\pi_{j}, m}$. Briefly, the objective is to determine a sequence $\pi^{*}$ in the set of all sequences $\Pi$ such that $T F T\left(\pi^{*}\right) \leq T F T(\pi) \forall \pi \in \Pi$.

There are two neighborhood structures in scheduling problems in general. These are based on insertion and swap neighborhood structures. As known, the computational complexity of both neighborhood structures is $\mathrm{O}\left(n^{3} m\right)$. For the traditional PFSP with the total flowtime criterion, Lia et al. [42] proposed a speed-up method for the swap and insertion neighborhood structures. They showed that the proposed speed-up method makes a reduction in the CPU times up to 40 to 50 percent. Based on their idea, we develop a fast fitness function calculation for insertion and swap moves. Suppose that we have sequence $\pi$. We first calculate the departure times of each job on each machine and store them in a matrix denoted as $D_{\pi_{j}, k}$. Given the current sequence as $\pi=\{1,2,3,4,5\}$. Now, suppose that we would like to interchange job 3 at position 3 with job 4 at position 4 . Due to the fact that the departure times are stored in $D_{\pi_{j}, k}$ in advance until job 3 at position 3 , we do not need to re-calculate departure times until job 3 at position 3, again. In other words, after interchanging those two jobs, the departure times of the new sequence $\pi=\{1,2,4,3,5\}$ can be calculated as follows. First, we copy the departure times from $D_{\pi_{j}, k}$ matrix with $d_{\pi_{j}, k}=D_{\pi_{j}, k}$ for $j=1, . ., 3$ and $k=1, . ., m$. Then, we calculate the departure times starting from position 3 to 5 with $d_{\pi_{j}, k}, j=3, . .5, k=1, . ., m$.

In order to generalize it, we first determine two randomly chosen positons pos1 and pos2 such that $(\operatorname{pos} 1<\operatorname{pos} 2) \in(1, n)$. Our swap function, $\pi=\operatorname{Swap}\left(\pi^{0}\right.$, pos $\left.1, \operatorname{pos} 2\right)$, interchanges jobs $\pi_{j=p o s 1}^{0}$ and $\pi_{j=p o s 2}^{0} ;$ then, store the new sequence on a temporary sequence $\pi$. Finally, our fast fitness function, $f_{\text {fast }}(\pi, \operatorname{pos} 1)$, copies the departure times from $D_{\pi_{j}, k}$ matrix until position $j=p o s 1$ and then, re-calculates the departure times starting from position pos 1 to the end of the remaining part of the sequence. After swapping jobs and storing them in a sequence $\pi$, the fast fitness function is given in Figure 1.

The same fast fitness function can be used for insertion moves, too. Before we get into details, we need to clarify that our insertion move function utilizes forward or backward insertion move with an equal probability of 0.5 . We again determine two random positions pos 1 and pos 2 such that $(\operatorname{pos} 1<\operatorname{pos} 2) \in(1, n)$. Our insertion function, $\pi=\operatorname{Insert}\left(\pi^{0}, \operatorname{pos} 1, p o s 2\right)$, either removes job $\pi_{j=p o s 1}^{1}$ at position pos1 and inserts it into pos2 in a temporary sequence $\pi$ (indicating a forward insertion move) or removes job $\pi_{j=p o s 2}^{1}$ at position pos2 and inserts it into pos1 in $\pi$ (indicating a backward insertion move). In both cases, one does not need to re-calculate the departure times until position pos1, which are copied from the $D_{\pi_{j}, k}$ matrix. In other words, $f_{f a s t}(\pi, p o s 1)$ function in Figure 1 can also be used for insertion moves.

## Procedure FastFitness Calculation

```
Step1. if \((\operatorname{pos} 1=1)\) then compute fitness of whole permutation
    else go to Step 2
Step2. Copy departure times from \(D_{\pi_{j}, k}\) until pos1
    \(d_{\pi_{,}, k}=D_{\pi, k}\) for \(j=1, \ldots\), pos 1 and \(k=1, \ldots, m\).
Step3. Compute departure times from pos1 until n
    \(d_{\pi_{j}, k}=\max \left\{d_{\pi_{j, k-1}}+p_{\pi_{j}, k}, d_{\pi_{j}-1, k+1}\right\}\) for \(j=\operatorname{pos} 1, . ., n\) and \(k=1, . ., m-1\).
Step4. Compute departure times on the last machine \(m\)
        \(d_{\pi_{j}, m}=d_{\pi_{j}, m-1}+p_{\pi_{j}, m}\) for \(j=1, . ., n\)
Step5. Compute total flowtime
    \(T F T=0 ; \mathrm{TFT}=\mathrm{TFT}+d_{\pi_{j}, m}\) for \(j=1, . ., n\)
Step6. Return TFT and \(\pi\)
```

Figure 1. $f_{f a s t}(\pi, \operatorname{pos} 1)$ function.

## 3. Variable Block Insertion Heuristic

Traditional local search algorithms are based on swap and insertion neighborhood structures. The swap operator exchanges two jobs in a sequence, whereas the insertion operator removes a job from a sequence and inserts it into another position in the sequence. Recently, local search algorithms, which are based on block moves, are presented for the single machine scheduling problem in the literature. A block move of 2-edge exchange was presented in Kirlik and Oguz [43]. Then, Subramanian et al. [44] developed five different neighborhoods: swap, insertion, edge-insertion, 3-block insertion and 3-block reverse. The basic idea is to use larger neighborhood move operators rather than the swap and insertion move operators. Relying on this idea, Xu et al. [45] presented a neighborhood structure called a Block Move in which $l$ consecutive jobs (called a block) are inserted into another position in the sequence. They represent a block move by a triplet ( $i, k, l$ ), where $i$ represents the position of the first job of the block, $k$ denotes the target position of the block to be inserted and $l$ represents the size of the block. Note that one edge insertion, two edge-insertion and 3-block insertion corresponds to the block move neighborhoods with $l=1, l=2$, and $l=3$. Similarly, Gonzales and Vela [46] developed a variable neighborhood descent algorithm by using three block move neighborhoods were developed and used in a memetic algorithm.

Inspiring from the algorithms above, we propose a similar, but a different block insertion heuristic, which we call it a variable block insertion heuristic (VBIH) algorithm. We denote two block sizes as $b S_{\text {min }}$ and $b S_{\text {max }}$, where $b S_{\text {min }}$ is the minimum block size and $b S_{\max }$ is the maximum block size. In addition, we define a block move insertion size with a maximum insertion move size denoted as $m S_{\max }$. The VBIH algorithm removes a block of jobs with size, $b S$, from the current sequence starting from $b S_{\min }=1$, then it makes a number $m S_{\max }$ of block insertion moves randomly in the partial sequence, which we denote it as the block insertion move procedure, $(b I M())$. It chooses the best one amongst a number $m S_{\max }$ of block insertion moves. The sequence obtained after $b I M()$ procedure goes under a variable local search ( $V L S$ ), which is based on traditional insertion and swap neighborhood structures. If the new sequence obtained after the $V L S$ local search is better than the current sequence, it replaces the current sequence. As long as it improves, it keeps the same block size, i.e., $b S=b S$. Otherwise, the block size is incremented by one, i.e., $b S=b S+1$ and a simulated annealing type of
acceptance criterion is used to accept the inferior solution in order to escape from local minima. This process is repeated until the block size reaches at the maximum block size, i.e., $b S \leq b S_{\max }$. The outline of the VBIH algorithm is given in Figure 2. Note that $\pi^{R}$ is the reference sequence; $T$ is temperature parameter for the acceptance criterion, and $\mathrm{tPF}_{-} \mathrm{NEH}(\mathrm{x})$ is a constructive heuristic providing an initial sequence to the VBIH algorithm. They will be explained in detail in the subsequent sections.

Procedure VBIH

```
\(\pi=t P F_{-} N E H(x)\)
\(\pi_{\text {best }}=\pi^{R}=\pi\)
while (NotTermination) do
    \(b S=b S_{\text {min }}=1\)
    \(d o\{\)
        \(\pi_{1}=\operatorname{bIM}\left(\pi, b S, m S_{\text {max }}\right)\)
            \(\pi_{2}=V L S\left(\pi_{1}, f\left(\pi_{1}\right)\right)\)
            if \(\left(f\left(\pi_{2}\right)<f(\pi)\right)\) then do\{
                \(\pi=\pi_{2}\)
                \(b S=b S\)
                if \(f\left(\pi_{2}\right)<f\left(\pi_{\text {best }}\right)\) then do\{
                    \(\pi_{\text {best }}=\pi^{R}=\pi_{2}\)
                \}endif
            else\{
                \(b S=b S+1\)
                if \(\left.\left.\binom{r<\exp }{\pi=\pi_{2}}-\left(f(\pi)-f\left(\pi_{2}\right)\right) / T\right\}\right)\)
            \}endif
    \}while ( \(b S \leq b S_{\max }\) )
\}endwhile
return \(\pi_{\text {best }}\) and \(f\left(\pi_{\text {best }}\right)\)
endprocedure
```

Figure 2. Variable block insertion heuristic.

### 3.1. Initial Solution

The initial solution for the VBIH algorithm is generated by the $\mathrm{tPF}_{-} \mathrm{NEH}(\mathrm{x})$ heuristic, which is proposed for the first time in this paper in the literature. It is a novel modification of the $\mathrm{PF}_{-} \mathrm{NEH}(\mathrm{x})$ heuristic presented in Pan and Wang [16]. In the $\mathrm{PF}_{-} \mathrm{NEH}(\mathrm{x})$ heuristic, the cost function to determine the next job to be scheduled is comprised of the total idle and blocking times. Note that the cost function devised for any constructive heuristic has a significant impact on the results. For this reason, Tasgetiren et al. [47] developed a PFT_NEH(x) heuristic for the BFSP with the makespan criterion, which was inspired from the PF_NEH(x) heuristic in [16]. In [47], a new cost function was devised by adding the departure time of the last job on the last machine to the total idle and blocking times. In this paper, we extend the $\mathrm{PFT}_{-} \mathrm{NEH}(\mathrm{x})$ heuristic to the total flowtime criterion and denote it as the tPF_NEH(x) heuristic. As in the PFT_NEH(x) heuristic, we consider the total idle and blocking times as a part of the cost function. In addition, we also consider the sum of the departure times on all machines of the last job that could be inserted into the partial sequence, which is one of the main contribution of this paper.

To implement the tPF_ NEH(x) heuristic, an initial order of jobs should be determined. To construct the initial order, we use the front delay and total processing times of each job as in Ribas et al. [41]. The following measure is employed to construct the initial order of jobs of jobs:

$$
\begin{equation*}
i O(j)=\frac{2}{m-1}\left(\sum_{k=1}^{m}(m-k) p_{\pi_{j}, k}\right)+\sum_{k=1}^{m} p_{\pi_{j}, k} \tag{6}
\end{equation*}
$$

In order to establish the initial order of jobs, we sort the $i O(j)$ values with an ascending order. Now, the number $x$ of new sequences can be generated from the initial order of jobs as follows. Since the first job has an impact on the solution quality, the first job of the initial order is taken as the first job of the new sequence and the $\mathrm{tPF}_{-} \mathrm{NEH}(\mathrm{x})$ heuristic is applied to generate the new solution. Then, the second job of the initial order is taken as the first job of the new sequence and tPF_ NEH(x)
heuristic is applied to generate another new solution. This is repeated $x$ times and the number $x$ of new sequences will be generated. Of course, the best one amongst them is chosen as the initial solution for the VBIH algorithm.

The proposed constructive heuristic can be summarized as follows. Suppose that $\pi_{i-1}$ jobs have already been scheduled and a partial sequence, $\pi=\left\{\pi_{1}, \pi_{2, . .}, \pi_{i-1}\right\}$, is obtained. Clearly, job $\pi_{i}$ will be the next job to be inserted into the partial sequence $\pi_{i-1}$. It can be any job from all jobs in the set $U$ of the unscheduled jobs. To select the job $\pi_{i}$, we need a cost measure ( $C M$ ). For the BFSP with the total flowtime criterion, we propose a new cost function consisting of the total idle and blocking times as well as the sum of the departure times of job $\pi_{i}$ on all machines. We first calculate the total idle and blocking time as follows:

$$
\begin{equation*}
I T_{i}=\sum_{k=1}^{m}\left(d_{i, k}-d_{i-1, k}-p_{\pi_{i}, k}\right) \tag{7}
\end{equation*}
$$

Then, we calculate the indexed sum of the departure times of job $\pi_{i}$ on all machines as follows:

$$
\begin{equation*}
S D_{i}=\sum_{k=1}^{m} \frac{m d_{i, k}}{k+i(m-k) /(n-2)} \tag{8}
\end{equation*}
$$

Now, we define our cost function as follows:

$$
\begin{equation*}
C M_{i}=(1-\mu) \times I T_{i}+\mu \times S D_{i} \tag{9}
\end{equation*}
$$

Note that we also employ an index function, $m /(k+i \times(m-k) /(n-2))$, to give a weight to departure time on each machine. Then, the job with the smallest sum of $C M_{i}$ amongst all jobs in $U$ is determined as the $i$ th job to be inserted to the partial sequence $\pi_{i-1}$. Figure 3 outlines the procedure of the proposed heuristic. As seen in Figure 3, the $t P F\left(\pi^{*}, h\right)$ procedure takes the initial sequence as $\pi^{*}$ and the job-index as $y$ sent by the $\mathrm{tPF}_{-}$NEH $(\mathrm{x})$ heuristic. By using the job-index $y$, the $x$ number of sequences will be generated.

## Procedure $\operatorname{tPF}\left(\pi^{*}, y\right)$

```
Step 1. Do for \((j=1\) to \(n) b S(j)=\) false
Step2. Set nJob \(=\pi_{y}^{*}, \pi_{1}=n J o b, U=J-\left\{\pi_{1}\right\}\) and \(b S\left[\pi_{1}\right]=\) true
Step 2. Calcualte the departure times \(d_{1, k}\) of \(j\) job \(\pi_{1}\) for \(k=1, . ., m\)
for \((i=2\) to \(n) d o\)
        temp \(F=I N T_{-} M A X\)
        for \((j=1\) to \(n)\) do
            //Consider only jobs in \(U\) with bS(j) flags are false
            - if \((b S(j)=\) true \()\) then continue
            - \(\pi_{i}=j\)
            - Calculate the departure times \(d_{i, k}\) for \(k=1,2, . . m\)
            - Calculate the sum of idle and blocking times
                \(I T_{i}=\sum_{k=1}^{m}\left(d_{i, k}-d_{i-1, k}-p_{\pi_{i}, k}\right)\)
            - Calculate the indexed sum of departure times on all machines
            - \(S D_{i}=\sum_{k=1}^{m} \frac{m d_{i, k}}{k+i(m-k) /(n-2)}\)
            - Now, calculate the cost function
                \(C M_{i}=(1-\mu) \times I T_{i}+\mu \times S D_{i}\)
            - Determine the best job in \(U\)
                if \(\left(C M_{i}<t e m p F\right)\)
                    tempF \(=C M_{i}\)
                    \(n J o b=\pi_{i}\)
                endif
        Endfor
    \(\pi_{i}=n J o b\)
    \(f\left(\pi_{i}\right)=t e m p F\)
    //Remove job \(\pi_{i}\) from \(U\)
    \(U=J-\left\{\pi_{i}\right\}\)
    \(b S\left[\pi_{i}\right]=\) true
EndFor
Return \(\pi\) and \(f(\pi)\)
```

Figure 3. tPF heuristic.

Pan and Wang [16] showed that the PF heuristic is very effective, but applying the NEH heuristic to the whole sequence can worsen the objective function, i.e., total flowtime. To avoid it, they applied the NEH heuristic to only the last $\delta$ jobs. As can be seen above, the $\mathrm{tPF}_{-} \mathrm{NEH}(\mathrm{x})$ heuristic has two parameters, namely, $\delta$ and $\mu$. Since the number $x$ of sequences that will be generated are not so costly until the number of jobs is less than 200, i.e., $n \leq 200$, in terms of CPU times, we determined it as if $(n \leq 200)$ then $x=n$; else $x=20$. Another distinction between $\mathrm{PF}_{-} \mathrm{NEH}(\mathrm{x})$ and $\mathrm{tPF}_{-} \mathrm{NEH}(\mathrm{x})$ is about how to choose the best one amongst $x$ number of sequences. Since even the partial NEH implementation may result in an inferior solution when compared to tPF heuristic, we check both solutions and keep the better one. Note that in all figures throughout the paper, $f(\pi)$ and $r$ corresponds to the total flowtime (TFT) value of a sequence $\pi$ and a uniform random number between 0 and 1 , respectively. The $\mathrm{tPF}_{-} \mathrm{NEH}(\mathrm{x})$ heuristic is outlined in Figure 4.

## Procedure tPF_NEH(x)

Step1. Sort the jobs with $i O(j)$ values with an ascending order
Step2. Denote initial solution as $\pi^{1}$
Step3 if $(n \leq 200)$ then $x=n$ else $x=20$.
Step4. For $(h=1$ to $x)$ do

- Take $\pi_{h}$ as the first job
- Generate a sequence by tPF heuristic $\pi^{2}=t P F\left(\pi^{1}, h\right)$
- $\operatorname{For}(i=n-\delta+1$ to $n)$ do
- Remove job $\pi_{i}$ from sequence $\pi^{2}$
- Evaluate all possible position by inserting $\pi_{i}$ in $\pi^{2}$
- Insert $\pi_{i}$ in $\pi^{2}$ with the lowest total flowtime (TFT)
- EndFor
- Obtain the sequence $\pi^{3}$
- if $\left(f\left(\pi^{2}\right)<f\left(\pi^{3}\right)\right) \pi^{h}=\pi^{2}$; else $\pi^{h}=\pi^{3}$
- Record the sequence as $\pi^{h}$
- Return the best one with the lowest TFT amongst $\left\{\pi^{1}, \pi^{2}, \ldots, \pi^{x},\right\}$

Figure 4. tPF_NEH(x) heuristic.

### 3.2. Block Insertion Move Procedure

$b I M\left(\pi, b S, m S_{\max }\right)$ procedure is a core function in the VBIH algorithm. It takes the sequence $\pi$, the block size, $b S$ and the maximum insertion move size, $m S_{\max }$ as parameters. The procedure randomly removes a block of jobs with size, $b S$ from the current sequence $\pi$, which is denoted as $\pi^{b}$. Then, the partial sequence after removal will be denoted as $\pi^{p}=J-\left\{\pi^{b}\right\}$. Then, the procedure carries out a number $m S_{\max }$ of block insertion moves with the block size, $b S$. In other words, the block $\pi^{b}$ is inserted in the partial sequence $\pi^{p}$ randomly in which the maximum size of the insertion moves is $m S_{\max }$. Finally, it chooses the best one amongst a number $m S_{\max }$ of block insertion moves. The $b I M\left(\pi, b S, m S_{\text {max }}\right)$ procedure is given in Figure 5.

In order to ease the understanding of the block insertion move procedure, we give the following example. Suppose that we have a current sequence $\pi=\{7,4,1,8,2,6,5,3\}$. In addition, suppose that the block size is $b S=3$ and maximum insertion move size is $m S_{\max }=3$. Suppose that we randomly choose a block $\pi^{b}=\{8,2,6\}$. Then, the partial sequence will be $\pi^{p}=\{7,4,1,5,3\}$. Without considering the total flowtime of the sequence $\pi$, we randomly choose three positions, for instance, 1 , 2,5 , and insert the block in these positions. Hence we have three sequences as $\pi_{1}=\{8,2,6,7,4,1,5,3\}$, $\pi_{2}=\{7,8,2,6,4,1,5,3\}$, and $\pi_{3}=\{7,4,1,5,8,2,6,3\}$. Amonst these three sequences we take the one with the lowest total flowtime criterion. Note that the same speed up methods explained before can be used to accelerate the insertion procedure. For example, after removing the block $\pi^{b}=\{8,2,6\}, D_{j, k}$ matrix of partial sequence $\pi^{p}$ is once calculated, and then the fast fitness calculation procedure is used to accelerate the insertion procedure.

Procedure $\operatorname{bIM}\left(\pi, b S, m S_{\text {max }}\right)$

```
\(t e m p F=I N T_{-} M A X\)
\(\pi_{1}=\pi\)
\(\pi^{b}=\) remove the block of jobs with size bS from \(\pi_{1}\)
\(\pi^{p}=\) partial solution after removal
for \(\left(i=1\right.\) to \(\left.m S_{\text {max }}\right)\) do
    \(p t=\) Choose a position in \(\pi^{p}\) randomly
    \(\pi_{2}=\) Insert block \(\pi^{b}\) in position pt of \(\pi^{p}\)
    if ( \(f\left(\pi_{2}\right)<\) tempF) then do
        \(\pi_{1}=\pi_{2}\)
        temp \(F=f\left(\pi_{1}\right)\)
    endif
endfor
\(f\left(\pi_{1}\right)=\) temp \(F\)
return \(\pi_{1}\) and \(f\left(\pi_{1}\right)\)
endprocedure
```

Figure 5. Block insertion procedure.

### 3.3. Variable Local Search

The traditional variable neighborhood search (VNS) algorithm was developed in [48] and successfully applied to scheduling problems in [49-53]. Very recently, a variable local search (VLS) algorithm is developed by Ribas et al. [32,40,41]. The VLS algorithm is a novel and different from the traditional VNS algorithms in such a way that when a sequence is improved by any randomly chosen neighborhood structure, it switches to another neighborhood structure. For example, if a sequence is improved by the LS1 local search (i.e., swap neighborhood), it systematically switches to LS2 local search (i.e., insertion neighborhood), which is the difference between traditional VNS and VLS algorithms. As seen in Figure 6, if a sequence is improved by the local search type " $L S$ ", it switches to $L S=1-L S$. Suppose that the $L S$ is randomly chosen as 1 , then if the sequence is improved by the insertion neighborhood, the variable local search type is switched to 0 by $L S=1-1=0$. This is different from the traditional VNS algorithms, where if a neighborhood improves, that neighborhood is kept for the search process and it switches to the second neighborhood if it fails, which is a common sense to follow. Note that, at least one time, both neighborhoods are ensured to be applied by using a Counter. The VLS local search is given in Figure 6.

```
Procedure VLS \((\pi, f(\pi))\)
    Counter \(=0\)
    if \((r<0.5)\) then do
        \(L S=0\)
    else
        \(L S=1\)
    endif
    //Apply variable local search as long as it improves
    do\{
        Counter \(=\) Counter +1
        \(C_{T F T}^{0}=f(\pi)\)
        if \((L S=0)\) then do
            \(\pi_{1}=L S 1(\pi)\)
        else
            \(\pi_{1}=L S 2(\pi)\)
        endif
        if \(\left(f\left(\pi_{1}\right)<C_{T F T}^{0}\right.\) or Counter \(\left.=1\right)\) then do
                \(L S=1-L S\)
                \(\pi=\pi_{1}\)
                \(C_{T F T}^{0}=f\left(\pi_{1}\right)\)
        endif
    \}while(true)
return \(\pi\) and \(f(\pi)\)
```

Figure 6. Variable local search (VLS) algorithm.

Note that the VLS algorithm employs two powerful local search algorithms in the VBIH algorithm, namely, referenced insertion scheme (RIS) and referenced swap scheme (RSS). Briefly, in the VLS algorithm, the RIS local search is used as LS1 whereas the RSS local search is employed as LS2. The RIS and RSS local search algorithms are outlined in Figures 7 and 8, respectively.

$$
\text { Procedure } \operatorname{RIS}\left(\pi, \pi^{R}, f(\pi)\right)
$$

```
Counter \(=1\)
\(p o s=1, p t 1=-1, p t 2=-1\)
while (Counter \(\leq n\) )do
    \(k=1\)
    while \(\left(\pi_{k}!=\pi_{\text {pos }}^{R}\right) k=k+1\); endwhile
    pos \(=(\) pos +1\()(\) mod \() n\)
    \(\pi_{1}=\) remove job \(\pi_{k}^{R}\) from \(\pi\)
    update \(D_{j, k}\) matrix, \(i=1, . ., n-1 ; k=1 ., . . m\)
        for \(i=0\) to \(n\)
        \(\pi_{2}=\operatorname{Insert}\left(\pi_{1}, i, k\right)\)
        if \(\left(f_{\text {fast }}\left(\pi_{2}, i\right)<f(\pi)\right)\) then
            \(p t 1=k\)
            \(p t 2=i\)
            \(f(\pi)=f\left(\pi_{2}\right)\)
    endfor
    if ( \(p t 1>-1\) ) do
        \(\pi=\) insert job \(\pi_{p t 1}\) at position pt2 of \(\pi_{1}\)
        Counter \(=1\)
    else
        Counter \(=\) Counter +1
    endif
endwhile
return \(\pi\) and \(f(\pi)\)
endprocedure
```

Figure 7. Referenced insertion scheme (RIS) local search.

The RIS local search is an insertion local search neighborhood. The details can be found in [47,54-59]. Note that the fast fitness calculation is used in the RIS local search.

$$
\text { Procedure } \operatorname{RSS}\left(\pi, \pi^{R}, f(\pi)\right)
$$

```
Counter \(=1\)
\(p o s=1, p t 1=-1, p t 2=-1\)
while (Counter \(\leq n\) )do
    \(k=1\)
        while \(\left(\pi_{k}!=\pi_{\text {pos }}^{R}\right) k=k+1\); endwhile
        pos \(=(\) pos +1\()(\) mod \() n\)
        update \(D_{j, k}\) matrix, \(i=1, . ., n ; k=1 ., . . m\)
        for \(i=0\) to \(n\)
            \(\pi_{2}=\operatorname{swap}(\pi, i, k)\)
            if \(\begin{gathered}\left(f_{\text {fast }}\left(\pi_{2}, i\right)<f(\pi)\right) \text { then } \\ p t 1=k\end{gathered}\)
                \(p t 1=k\)
            \(p t 2=i\)
            \(f(\pi)=f\left(\pi_{2}\right)\)
    endfor
    if \((p t 1>-1)\) do
        \(\pi=\) swap \(j o b \pi_{p t 1}\) with the one at position pt 2 of \(\pi\)
        Counter \(=1\)
    else
        Counter \(=\) Counter +1
        endif
endwhile
return \(\pi\) and \(f(\pi)\)
endprocedure
```

Figure 8. Referenced swap scheme (RSS) local search.

As seen in Figure 8, the RSS local search designates the job to be swapped by using the reference sequence $\pi^{R}$ as in the RIS algorithm. Then, for each job $\pi_{k}$, it exchanges job $\pi_{k}$ with each $\pi_{i}$ until the last job in the sequence. As long as the sequence improves after a number $n$ of swap moves, the counter is fixed to 1 so that the search starts from the beginning again. Note that the fast fitness calculation is used in RSS local search, too.

After the local search phase, it should be decided if the new sequence is accepted as the incumbent sequence for the next iteration. A simple simulated annealing type of acceptance criterion is used with a constant temperature, which is suggested by Osman and Potts [60], as follows:

$$
\begin{equation*}
T=\frac{\sum_{j=1}^{n} \sum_{k=1}^{m} p_{j, k}}{10 \times n \times m} \times \tau P \tag{10}
\end{equation*}
$$

where $\tau P$ is a parameter to be adjusted.

## 4. Parameter Tuning

In this section, we first determine the parameters of the PFT_NEH(x) heuristic through a design of experiment (DOE) approach [61]. Then, we again make a design of experiment for the proposed VBIH algorithm in order to determine its parameters.

### 4.1. Parameter Tuning of PFT_NEX(x) Heuristic

As mentioned before, since the number $x$ of sequences generated are not so costly until $n \leq 200$ in terms of CPU times, we determined it as if $(n \leq 200)$ then $x=n$ else $x=20$. Then, PFT_NEX( x ) heuristic has two important parameters. Namely, $\mu$ and $\delta$. To determine $\mu$ and $\delta$ parameters, we carry out a design of experiments (DOE) [61]. To do it, we generate random instances with the method proposed in [20]. In other words, random instances are generated for each combination of $n \in\{20,50,100,200,500\}$ and $m \in\{5,10,20\}$. Five instances are generated for each job and machine combination, respectively. Ultimately, we obtained 75 instances for each pair of job and machine size. We consider two parameters, namely, the weight of the cost function $\mu$ and the size of the partial NEH heuristic $\delta$. We have taken $\mu$ with 21 levels as $\mu=0.00,0.05, . ., 1.00$ and $\delta$ with 10 levels as $\delta=5,10,15,20,25,30,35,40,45,50$. We conducted a full factorial design of experiments resulting in $21 \times 10=210$ treatments. For each job and machine combination, each instance is run for 210 treatments ( $210 \mathrm{PFT}_{-}$NEX(x) heuristics with $\mu$ and $\delta$ values). The relative percent deviation is calculated as follows:

$$
\begin{equation*}
R P D=\sum_{i=1}^{210}\left(\frac{C_{i}-C_{\min }}{C_{\min }}\right) * 100 \tag{11}
\end{equation*}
$$

where $C_{i}$ is the total flowtime generated by each treatment and $C_{\text {min }}$ is the minimum total flowtime found amongst 210 treatments. This is repeated for five instances and RPD values are averaged for each treatment. Then the response variable is obtained by averaging the RPD values of 15 different job and machine combinations for each treatment. The DOE is coded in Visual C++13 and carried out on an Intel(R) Core(TM) i7-2600 CPU with 3.40 GHz PC with 8.00 GB memory.

Once the response variable was determined for each treatment, we analyze the main effects plot of parameters, which is given in Figure 9. Figure 9 suggests that $\mu$ (parW) and $\delta$ (parNEH) should be taken as 0.35 and 15, respectively.

However, Figure 9 suggests that $\mu$ values with $0.25,0.30,0.35,0.40,0.55$ and 0.60 with $\delta=15$ generate very good and diversified results on random benchmark instances. Table 1 summarizes the computational results of those $\mu$ values as well as the CPU times in seconds. It can be seen in Table 1 that $\mu=0.30$ and $\mu=0.35$ provided the best results as Figure 9 suggested. However, $\mu=0.25,0.40,0.55$ and 0.60 provide some diversified solutions, too. Note that the PFT_NEH(x) heuristics are substantially better than the HPF2 heuristic presented in [41] since it generated the ARPDs of 1.473 and 1.468 , respectively when compared to the ARPD of 3.287. In addition, the same authors proposed GRASP-based algorithms as well as NHPF1 and NHPF2 heuristics for the initial
solution procedures in [40]. PFT_ NEH(x) heuristics are also substantially better than NHPF1 and NHPF2 heuristics since the ARPDs of 1.473 percent and 1.468 percent are much better than those ARPDs of 3.018 percent and 2.797 percent. In addition to above, $\mathrm{PFT}_{-} \mathrm{NEH}(\mathrm{x})$ heuristics were able to further improve some larger instances in about 0.760 s on overall average.


Figure 9. Main effects plot.

### 4.2. Parameter Tuning of VBIH Algorithm

In this section, we again present a DOE approach for parameter setting of the VBIH algorithms. In order to carry out experiments, we generate random instances with the method proposed in [20]. In other words, random instances are generated for each combination of $n \in\{20,50,100,200,500\}$ and $m \in\{5,10,20\}$. Five instances are generated for each job and machine combination, respectively. Ultimately, we obtained 75 instances for each pair of job and machine size. We consider three parameters in the DOE approach. These are maximum block size $\left(b S_{m a x}\right)$, maximum block insertion move size $\left(m S_{\max }\right)$; and temperature adjustment parameter $(\tau P)$. We have taken the maximum block size with four levels as $b S_{\max } \in(4,8,12,16)$; the maximum block insertion move size with three levels as $m S_{\max } \in\left(0.1 *\left(n-b S_{\max }\right), 0.2 *\left(n-b S_{\max }\right), 0.3 *\left(n-b S_{\max }\right)\right)$; and the temperature adjustment parameter with five levels as $\tau P \in(0.1,0.2,0.3,0.4,0.5)$. We conducted a full factorial design of experiments resulting in $4 \times 3 \times 5=60$ treatments. For each job and machine combination, each instance is run for 60 treatments with a maximum CPU time equal to $T_{\max }=10 \times n \times m$ milliseconds. The relative percent deviation is calculated as follows:

$$
\begin{equation*}
R P D=\sum_{i=1}^{60}\left(\frac{C_{i}-C_{\min }}{C_{\min }}\right) * 100 \tag{12}
\end{equation*}
$$

where $C_{i}$ is the total flowtime generated by each treatment and $C_{\text {min }}$ is the minimum total flowtime found amongst 60 treatments. This is repeated for five instances and RPD values are averaged for each treatment. Then the response variable is obtained by averaging the RPD values of 15 different job and machine combinations for each treatment. The DOE is coded in Visual C ++13 and carried out on an Intel (R) Core (TM) i7-2600 CPU with 3.40 GHz PC with 8.00 GB memory.

Table 1. Average relative percentage deviations for PFT_NEH(x) heuristics.

| $\mu$ Values and CPU (s) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{n} \times \mathbf{m}$ | HPF2 [41] | NHPF1 [40] | NHPF2 [40] | 0.25 | CPU (s) | 0.30 | CPU (s) | 0.35 | CPU (s) | 0.40 | CPU (s) | 0.55 | CPU (s) | 0.60 | CPU (s) |
| $20 \times 5$ | 4.038 | 2.928 | 3.059 | 1.368 | 0.002 | 1.190 | 0.000 | 1.158 | 0.003 | 1.014 | 0.003 | 1.264 | 0.000 | 1.190 | 0.002 |
| $20 \times 10$ | 3.156 | 2.719 | 2.340 | 1.069 | 0.000 | 1.228 | 0.003 | 1.005 | 0.002 | 1.154 | 0.000 | 1.175 | 0.000 | 1.005 | 0.003 |
| $20 \times 20$ | 3.989 | 2.857 | 2.766 | 1.011 | 0.002 | 0.942 | 0.003 | 1.016 | 0.003 | 1.063 | 0.003 | 1.009 | 0.002 | 1.047 | 0.003 |
| $50 \times 5$ | 3.929 | 3.903 | 3.528 | 3.029 | 0.022 | 2.775 | 0.023 | 2.774 | 0.024 | 3.204 | 0.025 | 2.881 | 0.022 | 2.915 | 0.025 |
| $50 \times 10$ | 3.664 | 4.191 | 3.665 | 2.718 | 0.039 | 2.651 | 0.039 | 2.608 | 0.041 | 2.671 | 0.039 | 2.655 | 0.041 | 2.871 | 0.041 |
| $50 \times 20$ | 5.318 | 4.398 | 4.237 | 2.453 | 0.078 | 2.409 | 0.077 | 2.274 | 0.073 | 2.310 | 0.075 | 2.210 | 0.080 | 2.258 | 0.078 |
| $100 \times 5$ | 3.816 | 3.757 | 3.668 | 2.989 | 0.116 | 2.743 | 0.114 | 2.702 | 0.125 | 2.692 | 0.116 | 2.822 | 0.116 | 2.913 | 0.120 |
| $100 \times 10$ | 4.087 | 4.450 | 3.964 | 2.611 | 0.211 | 2.783 | 0.216 | 2.598 | 0.213 | 2.569 | 0.213 | 2.755 | 0.223 | 2.863 | 0.217 |
| $100 \times 20$ | 5.554 | 4.428 | 4.539 | 2.223 | 0.431 | 2.030 | 0.434 | 2.125 | 0.433 | 2.064 | 0.433 | 2.353 | 0.434 | 2.207 | 0.436 |
| $200 \times 10$ | 2.362 | 2.505 | 1.915 | 1.057 | 1.730 | 0.944 | 1.740 | 1.053 | 1.737 | 1.070 | 1.736 | 1.160 | 1.731 | 1.336 | 1.736 |
| $200 \times 20$ | 2.811 | 2.676 | 2.478 | 0.777 | 3.553 | 0.669 | 3.580 | 0.889 | 3.594 | 0.780 | 3.552 | 1.000 | 3.552 | 1.122 | 3.555 |
| $500 \times 20$ | 1.595 | 1.464 | 1.533 | -0.177 | 2.352 | -0.158 | 2.402 | -0.111 | 2.358 | -0.082 | 2.349 | 0.066 | 2.350 | 0.048 | 2.352 |
| $200 \times 5$ | 2.394 | 2.260 | 1.936 | 1.400 | 0.917 | 1.292 | 0.920 | 1.264 | 0.917 | 1.314 | 0.919 | 1.727 | 0.917 | 1.833 | 0.920 |
| $500 \times 5$ | 1.191 | 1.330 | 1.027 | 0.545 | 0.755 | 0.356 | 0.758 | 0.317 | 0.750 | 0.475 | 0.775 | 0.825 | 0.761 | 0.916 | 0.758 |
| $500 \times 10$ | 1.398 | 1.399 | 1.307 | 0.272 | 1.194 | 0.247 | 1.195 | 0.353 | 1.189 | 0.248 | 1.188 | 0.477 | 1.191 | 0.564 | 1.191 |
| Average | 3.287 | 3.018 | 2.797 | 1.556 | 0.760 | 1.473 | 0.767 | 1.468 | 0.764 | 1.503 | 0.762 | 1.625 | 0.761 | 1.673 | 0.762 |

[^0]Once the response variable was determined for each treatment, we analyze the main effects plots of the parameters, which are given in Figure 10. Figure 10 suggests that the $b S_{\max }$ should be taken as $b S_{\max }=16 ; m S_{\max }$ should be taken as $m S_{\max }=0.1 *\left(n-b S_{\max }\right)$; and $\tau P$ should be taken as $\tau P=0.2$.


Figure 10. Main Effects Plots of Parameters.

The main effects plot might not be meaningful when there are significant interactions between the parameters. For this reason, the ANOVA table should be analyzed to see whether interactions are significant or not. The ANOVA results are given in Table 2. From Table 2, it can be seen that $b S_{\max } * m S_{\max }$ interaction is found to be significant because of the very high magnitude of $F$ ratio and the $p$-value being less than $\alpha=0.05$ level. For these reasons, we look at the $b S_{\max } * m S_{\max }$ interaction plot, which is given in Figure 11.

As seen in Figure 11, $b S_{\max }=16$ with $m S_{\max }=0.3 *\left(n-b S_{\max }\right)$ generated the lowest ARPD. For this reason, we decided to take the parameters as follows: $b S_{\max }=16 ; m S_{\max }=0.3 *\left(n-b S_{\max }\right)$; and $\tau P=0.2$.

Table 2. ANOVA table.

| Source | DF | Seq SS | Adj SS | Adj MS | $\boldsymbol{F}$ | $\boldsymbol{p}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $b S_{\max }$ | 3 | 0.095370 | 0.095370 | 0.031790 | 98.55 | 0.00 |
| $m S_{\max }$ | 2 | 0.019873 | 0.019873 | 0.009937 | 30.80 | 0.00 |
| $t P$ | 4 | 0.000676 | 0.000676 | 0.000169 | 0.52 | 0.72 |
| $b S_{\text {max }} * m S_{\max }$ | 6 | 0.039930 | 0.039930 | 0.006655 | 20.63 | 0.00 |
| $b S_{\text {max }} * t P$ | 12 | 0.002395 | 0.002395 | 0.000200 | 0.62 | 0.81 |
| $m S_{\max } * t P$ | 8 | 0.001585 | 0.001585 | 0.000198 | 0.61 | 0.76 |
| Error | 24 | 0.007742 | 0.007742 | 0.000323 | - | - |
| Total | 59 | 0.167569 | - | - | - | - |



Figure 11. Interaction plot.

## 5. Computational Results

To test the performance of the VBIH algorithms proposed, extensive experimental evaluations and comparisons with other powerful methods are provided based on the well-known flowshop benchmark suite of Taillard [20]. The benchmark set is composed of 15 groups of the given problems with the size ranging from 20 jobs and 5 machines to 500 jobs and 20 machines, and each group consists of ten instances. However, in [62], these benchmark problems are extended to $200 \times 5,500 \times 5$ and $500 \times 10$ sizes with each containing 10 instances. Ultimately, we employ 150 instances as in the DABC_RCT algorithm in [41]. We treat them as blocking flow shop scheduling problems with the total flowtime criterion. In the experimental tests, all algorithms are coded in Visual C++13 and carried out on an $\operatorname{Intel}(\mathrm{R})$ Core(TM) i7-2600 CPU with 3.40 GHz PC with 8.00 GB memory. Note that the maximum CPU time is fixed at $T_{\max }=100 \times n \times m$ milliseconds for all algorithms compared. We compare to the following best performing algorithms from the literature:

1. DABC_RCT in [41]. The DABC_RCT algorithm is a very efficient algorithm and has three phases. In the employed bee phase, the TNO procedure is employed with the VLS local search. In the onlooker bee phase, the path-relinking approach is employed to generate the onlooker bees. In the scout bee phase, HPF2 is used to generate the scout bees. We refer to [41] for the details. We have coded the DABC_RCT algorithm in Visual C+13 to have a fair comparison. Note that the DABC_RCT algorithm uses HPF2 heuristic as an initial solution. Since our PFT_NEH(x) heuristic is substantially better than HPF2 heuristic, we employ the PFT_NEH(x) heuristic with $\mu=0.35$ and $\delta=15$ as one of the solution in the population. The rest of the population individuals are constructed randomly as suggested in the DABC_RCT algorithm and we denote it as the DABC*_RCT algorithm to have a fair comparison. The same parameters are also used which are suggested in the DABC_RCT algorithm [41]. Note that fast fitness calculation is employed to accelerate the insertion and swap neighborhood structures in the VLS local search they employed in the TNO procedure.
2. IG_RIS algorithm in [47,54-59]. To be fair again, we employ PFT_NEH(x) heuristic with $\mu=0.35$ and $\delta=15$ as an initial solution in the IG_RIS algorithm. IG_RIS algorithm relies on the destruction and construction procedure, where $d S$ number of jobs is removed from a solution and they are reinserted to the partial solution sequentially. Then RIS local search is applied to the solution obtained after destruction and construction procedure. Note that fast fitness calculation is employed to accelerate the RIS insertion local search as in this paper. It is also employed in the destruction and construction procedure, too.
3. VBIH algorithms in this paper. Since the PFT_NEH(x) heuristics provide very diversified initial solutions, we run the VBIH algorithm with $\mu$ values with $0.25,0.30,0.35,0.40,0.55$ and 0.60 with $\delta=15$. Then we denote them as VBIH1, VBIH2, VBIH3, VBIH4, VBIH5 and VBIH6. In addition, when the maximum block size is equal to 1 , the VBIH algorithm becomes an iterated local search. In other words, the current solution is perturbed with several insertion moves and then the VLS local search is applied to the solution after perturbation. Then, the acceptance criterion is imposed to the solution obtained. We denote this variant of the VBIH algorithm as IVLS algorithm.

Each instance is run for five (R) independent replications and the relative percentage deviation. RPD is computed as follows:

$$
\begin{equation*}
A R P D=\sum_{i=1}^{R}\left(\frac{H-B K S}{B K S}\right) * 100 / R \tag{13}
\end{equation*}
$$

where $H, B K S$, and $R$ are the total flowtime value generated by the algorithms in each run, the most recent best-known solution value reported in [41], and the number of runs, respectively. In addition, the ARPD for each instance, totally 150, is recorded to make statistical analyses.

The computational results are given in Table 3. The first observation from Table 3 that the IG_RIS and DABC_RCT algorithms are not competitive to the DABC*_RCT and VBIH variants. Especially, even though the fast fitness calculation is employed in the IG_RIS algorithm, which is known as one of the best algorithms in the scheduling literature, it could not be able to generate competitive results. In fact, its local search is based on only the insertion neighborhood, which is very effective for the makespan criterion. However, the results in Table 3 indicate that the swap neighborhood should be used in algorithms designed for the total flowtime criterion. The second observation is the performance of the DABC*_RCT algorithm, which is quite competitive to the VBIH variants. The overall ARPD is decreased from 0.593 percent to 0.376 percent due to the use of the PFT_NEH(x) heuristic as a solution in the initial population. It indicates that the proposed PFT_NEH $(x)$ heuristic was so effective on the results of the DABC*_RCT algorithm. Amongst the VBIH variants, the first four variants were able to generate results ranging from 0.303 percent to 0.285 percent. As can be seen from Table 3, especially, the larger instances with sizes $500 \times 20,500 \times 5$ and $500 \times 10$ were further improved by the first four variants of the VBIH algorithms.

Table 3. Average relative percentage deviations for algorithms compared.

| $\mathbf{n} \times \mathbf{m}$ | IG_RIS | DABC_RCT | IVLS | DABC*_RCT | VBIH1 | VBIH2 | VBIH3 | VBIH4 | VBIH5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $20 \times 5$ | 0.049 | 0.004 | 0.068 | 0.006 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| $20 \times 10$ | 0.016 | 0.021 | 0.151 | 0.031 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $20 \times 20$ | 0.010 | 0.015 | 0.032 | 0.002 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $50 \times 5$ | 0.901 | 0.788 | 0.713 | 0.490 | 0.418 | 0.354 | 0.346 | 0.376 | 0.296 |
| $50 \times 10$ | 0.819 | 0.752 | 0.916 | 0.752 | 0.562 | 0.455 | 0.457 | 0.424 | 0.463 |
| $50 \times 20$ | 0.521 | 0.561 | 0.652 | 0.490 | 0.318 | 0.378 | 0.316 | 0.355 | 0.330 |
| $100 \times 5$ | 1.555 | 1.135 | 1.133 | 0.937 | 0.650 | 0.764 | 0.709 | 0.650 | 0.696 |
| $100 \times 10$ | 1.639 | 1.301 | 1.358 | 1.285 | 1.070 | 1.179 | 1.126 | 1.098 | 1.112 |
| $100 \times 20$ | 1.147 | 1.279 | 0.927 | 0.983 | 0.759 | 0.833 | 0.799 | 0.903 | 0.796 |
| $200 \times 10$ | 0.633 | 0.559 | 0.236 | 0.312 | 0.427 | 0.243 | 0.377 | 0.293 | 0.297 |
| $200 \times 20$ | 0.337 | 0.581 | 0.193 | 0.201 | 0.146 | 0.227 | 0.211 | 0.255 | 0.430 |
| $500 \times 20$ | -0.229 | 0.426 | -0.381 | -0.274 | -0.353 | -0.374 | -0.395 | -0.303 | -0.192 |
| $200 \times 5$ | 0.854 | 0.470 | 0.373 | 0.358 | 0.392 | 0.325 | 0.341 | 0.372 | 0.388 |
| $500 \times 5$ | 0.259 | 0.397 | -0.014 | -0.022 | 0.150 | -0.012 | -0.053 | -0.042 | 0.299 |
| $500 \times 10$ | 0.234 | 0.607 | -0.033 | 0.092 | 0.000 | -0.031 | 0.035 | -0.046 | 0.174 |
| Average | 0.583 | 0.593 | 0.422 | 0.376 | 0.303 | $\mathbf{0 . 2 8 9}$ | $\mathbf{0 . 2 8 5}$ | $\mathbf{0 . 2 8 9}$ | 0.308 |

Bold: better results.

In order to see the statistical difference between algorithms, we provide the interval plot of the algorithms compared in Figure 12. Since the $95 \%$ confidence intervals of DABC*_RCT and VBIH variants do not overlap, we can conclude that the DABC*_RCT and the VBIH variants generated results, which are statistically significant to those results generated by the IG_RIS and DABC_RCT algorithms. When we look at the confidence intervals of the $\mathrm{DABC}^{*} \_$RCT and VBIH variants, there are overlaps between the algorithms. However, an overlap does not mean that there is no difference. There may be statistically significant difference even if there is an overlap [63]. To determine the difference, the paired $t$-tests should be used [63].


Figure 12. Interval plot of algorithms compared.

The results of the paired $t$-tests are given in Table 4. If the $95 \%$ confidence interval for the mean difference between the two compared algorithms does not include zero values, it indicates that there is a difference between the two algorithms compared. In addition, $p$-values, which are smaller than $\alpha=0.05$ level, further suggest that the compare algorithms perform differently. As can be seen from Table 4, the IVLS, VBIH1, VBIH2, VBIH3 and VBIH4 algorithms are statistically better than the DABC*_RCT algorithm. However, the VBIH5 and VBIH 6 algorithms are statistically equivalent.

Table 4. Paired $t$-test for variable block insertion heuristic (VBIH) variants versus discrete artificial bee colony (DABC)*_RCT algorithm.

| Algorithm vs. algorithm | 95\% CI for Mean Difference | $p$-Value |
| :---: | :---: | :---: |
| IVLS - DABC*_RCT | (0.0041, 0.0875) | 0.032 |
| VBIH1 - DABC*_RCT | (-0.1162, -0.0298) | 0.001 |
| VBIH2 - DABC*_RCT | (-0.1242, -0.0486) | 0.000 |
| VBIH3 - DABC*_RCT | (-0.1193, -0.0639) | 0.000 |
| VBIH4 - DABC*_RCT | (-0.1258, -0.0474) | 0.000 |
| VBIH5 - DABC*_RCT | (-0.0843, 0.0117) | 0.137 |
| VBIH6 - DABC*_RCT | (-0.0671, 0.0252) | 0.371 |

Finally, in Table 5, we provide the best solutions found by each algorithm. 52 out of 150 problems instances are further improved together with 15 solutions being equal during in this study. All the results and permutations are available based on request.

Table 5. Best-known solutions.

| BKS [41] | IVLS | VBIH1 | VBIH2 | VBIH3 | VBIH4 | VBIH5 | VBIH6 | DABC_RCT | DABC*_RCT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Problem Set $20 \times 5$ |  |  |  |  |  |  |  |  |  |
| 14953 | 14953 | 14953 | 14953 | 14953 | 14953 | 14953 | 14953 | 14953 | 14953 |
| 16343 | 16349 | 16343 | 16343 | 16343 | 16343 | 16343 | 16343 | 16343 | 16343 |
| 14297 | 14297 | 14297 | 14297 | 14297 | 14297 | 14297 | 14297 | 14297 | 14297 |
| 16483 | 16483 | 16483 | 16483 | 16483 | 16483 | 16483 | 16483 | 16483 | 16483 |
| 14212 | 14212 | 14212 | 14212 | 14212 | 14212 | 14212 | 14212 | 14212 | 14212 |
| 14624 | 14624 | 14624 | 14624 | 14624 | 14624 | 14624 | 14624 | 14624 | 14624 |
| 14936 | 14938 | 14936 | 14936 | 14936 | 14936 | 14936 | 14936 | 14938 | 14938 |
| 15193 | 15240 | 15193 | 15193 | 15193 | 15193 | 15193 | 15193 | 15193 | 15193 |
| 15544 | 15544 | 15544 | 15544 | 15544 | 15544 | 15544 | 15544 | 15544 | 15544 |
| 14392 | 14392 | 14392 | 14392 | 14392 | 14392 | 14392 | 14392 | 14392 | 14392 |
| Problem Set $20 \times 10$ |  |  |  |  |  |  |  |  |  |
| 22358 | 22537 | 22358 | 22358 | 22358 | 22358 | 22358 | 22358 | 22358 | 22358 |
| 23881 | 23881 | 23881 | 23881 | 23881 | 23881 | 23881 | 23881 | 23881 | 23881 |
| 20873 | 20873 | 20873 | 20873 | 20873 | 20873 | 20873 | 20873 | 20873 | 20873 |
| 19916 | 20020 | 19916 | 19916 | 19916 | 19916 | 19916 | 19916 | 19916 | 19916 |
| 20196 | 20196 | 20196 | 20196 | 20196 | 20196 | 20196 | 20196 | 20196 | 20196 |
| 20126 | 20126 | 20126 | 20126 | 20126 | 20126 | 20126 | 20126 | 20126 | 20126 |
| 19471 | 19471 | 19471 | 19471 | 19471 | 19471 | 19471 | 19471 | 19471 | 19471 |
| 21330 | 21369 | 21330 | 21330 | 21330 | 21330 | 21330 | 21330 | 21330 | 21330 |
| 21585 | 21585 | 21585 | 21585 | 21585 | 21585 | 21585 | 21585 | 21585 | 21585 |
| 22582 | 22582 | 22582 | 22582 | 22582 | 22582 | 22582 | 22582 | 22582 | 22582 |
| Problem Set $20 \times 20$ |  |  |  |  |  |  |  |  |  |
| 34683 | 34683 | 34683 | 34683 | 34683 | 34683 | 34683 | 34683 | 34683 | 34683 |
| 32855 | 32855 | 32855 | 32855 | 32855 | 32855 | 32855 | 32855 | 32855 | 32855 |
| 34825 | 34825 | 34825 | 34825 | 34825 | 34825 | 34825 | 34825 | 34825 | 34825 |
| 33006 | 33006 | 33006 | 33006 | 33006 | 33006 | 33006 | 33006 | 33006 | 33006 |
| 35328 | 35328 | 35328 | 35328 | 35328 | 35328 | 35328 | 35328 | 35328 | 35328 |
| 33720 | 33720 | 33720 | 33720 | 33720 | 33720 | 33720 | 33720 | 33720 | 33720 |
| 33992 | 33992 | 33992 | 33992 | 33992 | 33992 | 33992 | 33992 | 33992 | 33992 |
| 33388 | 33388 | 33388 | 33388 | 33388 | 33388 | 33388 | 33388 | 33388 | 33388 |
| 34798 | 34798 | 34798 | 34798 | 34798 | 34798 | 34798 | 34798 | 34798 | 34798 |
| 33174 | 33174 | 33174 | 33174 | 33174 | 33174 | 33174 | 33174 | 33174 | 33174 |
| Problem Set $50 \times 5$ |  |  |  |  |  |  |  |  |  |
| 72672 | 72758 | 72672 | 72696 | 72672 | 72696 | 72758 | 72827 | 73135 | 72768 |
| 78140 | 78707 | 78254 | 78332 | 78181 | 78181 | 78181 | 78284 | 78327 | 78295 |
| 72913 | 73211 | 73096 | 73224 | 73101 | 73224 | 72913 | 72994 | 72913 | 73224 |
| 77399 | 77711 | 77513 | 77571 | 77547 | 77586 | 77547 | 77547 | 77582 | 77607 |
| 78353 | 78705 | 78627 | 78579 | 78544 | 78363 | 78511 | 78511 | 78767 | 78620 |
| 75402 | 75402 | 75661 | 75606 | 75475 | 75593 | 75402 | 75514 | 76122 | 75615 |
| 73842 | 74322 | 73952 | 74202 | 73952 | 73890 | 73952 | 73891 | 73954 | 73890 |
| 73442 | 73964 | 73945 | 73442 | 73834 | 73442 | 73549 | 73549 | 73858 | 73442 |
| 70871 | 71360 | 70905 | 70871 | 70871 | 70883 | 70883 | 70883 | 71096 | 71105 |
| 78729 | 79271 | 78773 | 78729 | 78729 | 78807 | 78729 | 78729 | 78773 | 79093 |
| Problem Set $50 \times 10$ |  |  |  |  |  |  |  |  |  |
| 99674 | 100508 | 100373 | 99674 | 100299 | 100059 | 99721 | 100410 | 99900 | 100325 |
| 95608 | 95669 | 95907 | 96157 | 95669 | 96047 | 95876 | 95876 | 96565 | 96367 |
| 91791 | 92760 | 91956 | 92090 | 91791 | 92090 | 92090 | 92276 | 92588 | 92524 |
| 98454 | 98767 | 98475 | 98689 | 98454 | 98454 | 98507 | 98507 | 98692 | 98576 |
| 98164 | 98286 | 98243 | 98164 | 98230 | 98164 | 98228 | 98228 | 98610 | 98228 |
| 97246 | 97826 | 97637 | 97779 | 97431 | 97530 | 97558 | 97333 | 98029 | 97625 |
| 99953 | 100142 | 100030 | 99965 | 99965 | 99953 | 99971 | 100116 | 100440 | 100584 |
| 98027 | 98231 | 98149 | 98271 | 98476 | 98436 | 98543 | 98270 | 98723 | 98521 |
| 96708 | 97248 | 96708 | 96996 | 96996 | 96708 | 97142 | 96708 | 96978 | 97634 |
| 98019 | 99012 | 98316 | 98316 | 98316 | 98316 | 98053 | 98053 | 98316 | 98362 |

Table 5. Cont.

| BKS [41] | IVLS | VBIH1 | VBIH2 | VBIH3 | VBIH4 | VBIH5 | VBIH6 | DABC_RCT | DABC*_RCT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Problem Set $50 \times 20$ |  |  |  |  |  |  |  |  |  |
| 136865 | 137240 | 136881 | 137075 | 137075 | 136968 | 137005 | 137005 | 136958 | 137161 |
| 129958 | 130333 | 130115 | 129975 | 130292 | 130244 | 130248 | 129975 | 130176 | 130303 |
| 127617 | 128784 | 127617 | 127957 | 127617 | 128073 | 127617 | 127617 | 128033 | 128393 |
| 131889 | 132452 | 132270 | 132169 | 132270 | 132103 | 131943 | 131943 | 132283 | 132169 |
| 130967 | 131159 | 130979 | 131217 | 131233 | 130967 | 131196 | 130979 | 131351 | 131064 |
| 131760 | 132121 | 131985 | 131760 | 131760 | 131929 | 131926 | 131921 | 131593 | 132007 |
| 134217 | 134857 | 134222 | 134534 | 134572 | 134534 | 134726 | 134451 | 134715 | 134636 |
| 132990 | 133502 | 132990 | 133210 | 133210 | 133210 | 133309 | 133309 | 133526 | 133210 |
| 132599 | 132819 | 132757 | 132715 | 132901 | 132757 | 132599 | 132599 | 132599 | 133120 |
| 135710 | 136166 | 135985 | 136162 | 136224 | 136363 | 136248 | 136146 | 136483 | 136473 |
| Problem Set $100 \times 5$ |  |  |  |  |  |  |  |  |  |
| 288332 | 289888 | 288446 | 289216 | 288765 | 288854 | 290346 | 288904 | 289463 | 288807 |
| 280491 | 282897 | 281066 | 280743 | 280853 | 280073 | 280873 | 280929 | 282857 | 282563 |
| 276228 | 277904 | 275863 | 276229 | 276322 | 277751 | 276589 | 276695 | 278117 | 277968 |
| 259596 | 262968 | 261462 | 261867 | 261601 | 261985 | 261715 | 261231 | 263990 | 261478 |
| 273086 | 275571 | 274651 | 274335 | 274451 | 274690 | 274005 | 274817 | 274804 | 275092 |
| 267381 | 271534 | 269418 | 267899 | 270406 | 269506 | 270408 | 269194 | 268899 | 269356 |
| 274744 | 277150 | 275884 | 277009 | 277342 | 276656 | 275491 | 276609 | 277163 | 277535 |
| 269689 | 272776 | 271187 | 270939 | 270945 | 270774 | 270668 | 271001 | 271916 | 271719 |
| 284816 | 286683 | 285308 | 285238 | 284901 | 284652 | 284952 | 284755 | 287494 | 284856 |
| 282005 | 282659 | 282969 | 283292 | 282814 | 282366 | 282367 | 282719 | 283596 | 283939 |
| Problem Set $100 \times 10$ |  |  |  |  |  |  |  |  |  |
| 354083 | 357361 | 354586 | 355794 | 356308 | 354892 | 353321 | 354624 | 354570 | 356911 |
| 333379 | 335775 | 335738 | 336636 | 335905 | 335268 | 336469 | 336601 | 337164 | 336403 |
| 343957 | 346543 | 344337 | 345157 | 345524 | 345069 | 344824 | 345654 | 344863 | 345889 |
| 359259 | 362441 | 361621 | 363410 | 360537 | 361230 | 360709 | 359680 | 361328 | 364095 |
| 338537 | 339455 | 339573 | 339976 | 338941 | 341468 | 341261 | 340741 | 340771 | 341207 |
| 327254 | 331594 | 329327 | 328482 | 330769 | 328075 | 328693 | 329377 | 330296 | 329454 |
| 335366 | 339300 | 338219 | 337094 | 339001 | 338948 | 338091 | 338091 | 337997 | 338643 |
| 343174 | 346305 | 345286 | 344609 | 344905 | 346013 | 345217 | 343843 | 344417 | 344886 |
| 344563 | 357006 | 354676 | 357664 | 356781 | 356050 | 355165 | 354659 | 356177 | 356628 |
| 347845 | 349966 | 349546 | 348910 | 350290 | 349351 | 350879 | 350580 | 350674 | 350693 |
| Problem Set $100 \times 20$ |  |  |  |  |  |  |  |  |  |
| 425224 | 427753 | 427810 | 426032 | 427688 | 426582 | 427549 | 426845 | 427899 | 426093 |
| 435289 | 438146 | 436000 | 436495 | 437478 | 436409 | 437908 | 436205 | 439020 | 437380 |
| 430634 | 431419 | 432953 | 433663 | 431713 | 434502 | 431905 | 432067 | 435222 | 434241 |
| 432314 | 436203 | 435708 | 435999 | 435001 | 437256 | 435401 | 435880 | 437725 | 438501 |
| 426405 | 429524 | 428737 | 429044 | 428845 | 430388 | 428200 | 429321 | 429886 | 429472 |
| 430308 | 434216 | 432099 | 431680 | 432345 | 433041 | 432139 | 434523 | 434105 | 431202 |
| 436642 | 440171 | 440174 | 441664 | 438725 | 441037 | 437488 | 439062 | 440081 | 442024 |
| 440930 | 445900 | 443867 | 445170 | 445219 | 443812 | 443648 | 444357 | 444299 | 445169 |
| 432876 | 435264 | 434452 | 435075 | 434655 | 433631 | 434701 | 434806 | 437318 | 435536 |
| 437286 | 440819 | 440918 | 438439 | 440992 | 438621 | 440484 | 438530 | 443779 | 439693 |
| Problem Set $200 \times 10$ |  |  |  |  |  |  |  |  |  |
| 1281633 | 1281292 | 1286077 | 1283314 | 1281947 | 1281401 | 1280745 | 1282445 | 1285819 | 1281919 |
| 1283164 | 1279913 | 1285268 | 1279947 | 1280799 | 1280548 | 1280432 | 1281886 | 1279240 | 1280107 |
| 1277933 | 1280412 | 1281905 | 1279882 | 1282228 | 1282447 | 1281843 | 1284700 | 1282812 | 1276690 |
| 1271502 | 1275865 | 1280417 | 1280027 | 1274711 | 1273979 | 1274071 | 1277414 | 1278833 | 1280871 |
| 1275901 | 1282570 | 1276110 | 1286907 | 1279126 | 1280511 | 1275710 | 1279823 | 1277954 | 1285519 |
| 1251213 | 1252110 | 1258463 | 1248655 | 1252536 | 1254724 | 1252145 | 1248058 | 1258054 | 1244667 |
| 1304158 | 1307602 | 1303545 | 1311626 | 1305541 | 1305201 | 1306954 | 1302585 | 1309672 | 1309810 |
| 1298900 | 1296591 | 1301184 | 1303103 | 1297501 | 1295844 | 1302459 | 1299302 | 1296469 | 1302829 |
| 1277801 | 1270883 | 1278023 | 1273946 | 1270118 | 1277145 | 1278259 | 1277646 | 1278808 | 1272519 |
| 1273794 | 1281472 | 1279452 | 1284374 | 1281680 | 1278887 | 1282278 | 1282763 | 1280799 | 1283774 |

Table 5. Cont.

| BKS [41] | IVLS | VBIH1 | VBIH2 | VBIH3 | VBIH4 | VBIH5 | VBIH6 | DABC_RCT | DABC*_RCT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Problem Set $200 \times 20$ |  |  |  |  |  |  |  |  |  |
| 1499623 | 1505301 | 1509409 | 1506809 | 1503861 | 1508265 | 1516354 | 1511393 | 1512033 | 1508941 |
| 1541253 | 1538208 | 1540131 | 1531179 | 1538989 | 1538626 | 1538036 | 1536971 | 1538890 | 1534481 |
| 1546279 | 1546915 | 1553963 | 1557771 | 1550237 | 1553555 | 1558080 | 1558445 | 1559908 | 1558470 |
| 1540822 | 1542961 | 1537852 | 1545112 | 1544472 | 1544429 | 1548607 | 1547062 | 1541447 | 1544097 |
| 1514600 | 1515692 | 1514609 | 1520799 | 1515470 | 1517899 | 1520051 | 1521375 | 1518989 | 1520058 |
| 1528885 | 1535006 | 1533907 | 1532007 | 1535114 | 1534306 | 1533721 | 1533357 | 1543380 | 1528532 |
| 1532090 | 1535822 | 1534243 | 1532903 | 1536909 | 1534423 | 1540592 | 1540200 | 1539144 | 1529204 |
| 1543229 | 1537885 | 1540539 | 1542855 | 1540221 | 1538906 | 1541537 | 1542151 | 1543159 | 1543637 |
| 1524293 | 1522771 | 1517701 | 1516868 | 1522958 | 1525068 | 1524214 | 1519558 | 1524550 | 1514130 |
| 1535329 | 1533492 | 1528995 | 1534456 | 1534830 | 1535504 | 1538711 | 1536245 | 1540038 | 1534682 |
| Problem Set $500 \times 20$ |  |  |  |  |  |  |  |  |  |
| 8719682 | 8706102 | 8701976 | 8693435 | 8705502 | 8691397 | 8704588 | 8720565 | 8730777 | 8702888 |
| 8849228 | 8823698 | 8804700 | 8813687 | 8825577 | 8816790 | 8820512 | 8825084 | 8908220 | 8814543 |
| 8789777 | 8745145 | 8746129 | 8742781 | 8728918 | 8750421 | 8759822 | 8746826 | 8811943 | 8739388 |
| 8828454 | 8791325 | 8807711 | 8793936 | 8793088 | 8795242 | 8807649 | 8803254 | 8862426 | 8795162 |
| 8796337 | 8735014 | 8722394 | 8741122 | 8736988 | 8755137 | 8781374 | 8756332 | 8791516 | 8774397 |
| 8837577 | 8804938 | 8821880 | 8793126 | 8803399 | 8820331 | 8809748 | 8824192 | 8870293 | 8791898 |
| 8729909 | 8718042 | 8734168 | 8734722 | 8719324 | 8737033 | 8750115 | 8748411 | 8802463 | 8733839 |
| 8800506 | 8766044 | 8756165 | 8773136 | 8772308 | 8774052 | 8761218 | 8787988 | 8796375 | 8772084 |
| 8782791 | 8739864 | 8751027 | 8727721 | 8743741 | 8743940 | 8761953 | 8747533 | 8799104 | 8773353 |
| 8849551 | 8791671 | 8802420 | 8780952 | 8788594 | 8796622 | 8810047 | 8818606 | 8878995 | 8805877 |
| Problem Set $200 \times 5$ |  |  |  |  |  |  |  |  |  |
| 1071652 | 1073055 | 1072760 | 1076925 | 1071889 | 1073065 | 1073348 | 1071170 | 1071705 | 1070790 |
| 1026640 | 1026510 | 1021433 | 1025826 | 1025310 | 1023515 | 1028051 | 1024432 | 1019431 | 1025138 |
| 1059120 | 1064728 | 1062714 | 1064025 | 1064244 | 1060982 | 1065879 | 1064569 | 1062759 | 1061449 |
| 1044074 | 1048420 | 1051225 | 1042391 | 1048350 | 1049298 | 1042292 | 1047066 | 1048212 | 1044776 |
| 1064274 | 1064019 | 1064175 | 1060847 | 1064649 | 1061081 | 1060213 | 1062069 | 1060420 | 1062216 |
| 1021482 | 1024893 | 1027578 | 1029903 | 1025409 | 1027670 | 1029104 | 1030626 | 1026561 | 1029891 |
| 1082018 | 1081107 | 1079945 | 1082921 | 1081033 | 1081780 | 1083320 | 1084464 | 1083668 | 1081968 |
| 1043921 | 1047490 | 1048609 | 1050141 | 1045691 | 1048150 | 1051041 | 1048321 | 1050936 | 1049380 |
| 1057482 | 1057673 | 1058438 | 1056355 | 1058281 | 1058369 | 1056063 | 1055705 | 1058199 | 1059175 |
| 1037496 | 1043719 | 1043777 | 1039727 | 1042310 | 1045628 | 1039183 | 1043266 | 1036938 | 1039695 |
| Problem Set $500 \times 5$ |  |  |  |  |  |  |  |  |  |
| 6389122 | 6375325 | 6381517 | 6371100 | 6371117 | 6366762 | 6397170 | 6387506 | 6403589 | 6369864 |
| 6415066 | 6413469 | 6433713 | 6392620 | 6400361 | 6396807 | 6432279 | 6436552 | 6421048 | 6392856 |
| 6460745 | 6426771 | 6426591 | 6435399 | 6434882 | 6434628 | 6440953 | 6454047 | 6478507 | 6430821 |
| 6334201 | 6303859 | 6323236 | 6305175 | 6306089 | 6318146 | 6337465 | 6334555 | 6364065 | 6318682 |
| 6373873 | 6383164 | 6392640 | 6369007 | 6383774 | 6355801 | 6408507 | 6413732 | 6397351 | 6369219 |
| 6282522 | 6275452 | 6281428 | 6283594 | 6277932 | 6274362 | 6292695 | 6301826 | 6302507 | 6283635 |
| 6244926 | 6262136 | 6262957 | 6262423 | 6260089 | 6261444 | 6285376 | 6273743 | 6261620 | 6258574 |
| 6352627 | 6367281 | 6395544 | 6370417 | 6370566 | 6377367 | 6376438 | 6392365 | 6350755 | 6366156 |
| 6328390 | 6335967 | 6342425 | 6336154 | 6335220 | 6343154 | 6342796 | 6359365 | 6354617 | 6330549 |
| 6309180 | 6309639 | 6314591 | 6297997 | 6307224 | 6300714 | 6320912 | 6323148 | 6346828 | 6307074 |
| Problem Set $500 \times 10$ |  |  |  |  |  |  |  |  |  |
| 7552404 | 7514159 | 7534854 | 7522904 | 7519846 | 7523259 | 7533739 | 7549934 | 7577869 | 7541901 |
| 7665025 | 7632382 | 7633377 | 7649655 | 7635561 | 7622243 | 7642501 | 7652010 | 7658541 | 7643615 |
| 7626599 | 7590037 | 7599850 | 7580415 | 7588202 | 7590780 | 7622675 | 7622445 | 7652497 | 7603273 |
| 7626405 | 7618385 | 7600996 | 7633161 | 7619058 | 7615308 | 7645654 | 7623706 | 7635679 | 7635154 |
| 7479900 | 7484025 | 7468087 | 7472600 | 7468703 | 7478446 | 7464923 | 7496738 | 7504574 | 7476013 |
| 7537299 | 7548071 | 7546071 | 7563456 | 7551273 | 7551039 | 7572887 | 7566574 | 7586150 | 7562912 |
| 7510712 | 7505921 | 7502693 | 7490848 | 7504096 | 7482595 | 7478959 | 7514666 | 7534649 | 7491561 |
| 7562013 | 7577902 | 7599263 | 7598437 | 7588036 | 7598947 | 7599345 | 7635525 | 7607737 | 7599718 |
| 7550242 | 7538219 | 7537118 | 7533874 | 7539127 | 7547730 | 7577922 | 7574227 | 7581486 | 7536618 |
| 7549596 | 7577156 | 7596351 | 7588889 | 7580750 | 7562898 | 7611269 | 7589683 | 7662823 | 7594287 |

Bold: better results, Bold Italic: equal results.

## 6. Conclusions

This paper presents a VBIH algorithm to solve the blocking flowshop scheduling problem with the total flowtime criterion. To the best of our knowledge, this is the first reported application of the VHIH algorithm to the blocking flowshop scheduling problem with the total flowtime criterion. Once an initial solution is constructed, the VBIH algorithm begins with a minimum block size of one. It removes the block from the current sequence and inserts the block into the partial sequence with a predetermined move size. Then, a variable local search (VLS) is applied to solution obtained after several block insertions. As long as the solution improves, it keeps the same block size. However, if the solution does not improve, the block size is increased by one. This process is repeated until the block size reaches at the maximum block size. In addition, we present a novel constructive heuristic based on profile fitting heuristic from the literature with results improving especially some larger instances in a few seconds. Parameters of the constructive heuristic and the VBIH algorithm are determined through a design of experiment approach. Extensive computational results on Taillard's well-known benchmark suite show that the proposed VBIH algorithm outperforms the discrete artificial bee colony algorithm, which has recently been proposed with the new best known solutions. Ultimately, 52 out of the 150 best known solutions are further improved with substantial margins.

Supplementary Materials: The supplementary materials are available online at http://www.mdpi.com/19994893/9/4/71/s1.

Author Contributions: M. Fatih Tasgetiren and Quan-Ke Pan and Damla Kizilay have written the C++ codes. Damla Kizilay and Gao Kaizhou have carried out all the DOE runs and prepared the analyses of DOE runs. M. Fatih Tasgetiren and Quan-Ke Pan have written the paper. All authors have read and approved the final manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

## References

1. Błażewicz, J.; Ecker, K.H.; Pesch, E.; Schmidt, G.; Weglarz, J. Handbook on Scheduling: From Theory to Applications; Springer: Berlin/Heidelberg, Germany, 2007.
2. Pan, Q.K.; Ruiz, R. An estimation of distribution algorithm for lot-streaming flow shop problems with setup times. Omega 2012, 40, 166-180. [CrossRef]
3. Vallada, E.; Ruiz, R. Genetic algorithms with path relinking for the minimum tardiness permutation flowshop problem. Omega 2010, 38, 57-67. [CrossRef]
4. Ruiz-Torres, A.J.; Ho, T.J.; Ablanedo-Rosas, J.H. Makespan and workstation utilization minimization in a flowshop with operations flexibility. Omega 2011, 39, 273-282. [CrossRef]
5. Grabowski, J.; Pempera, J. Sequencing of jobs in some production system. Eur. J. Oper. Res. 2000, 125, 535-550. [CrossRef]
6. Ronconi, D.P. A note on constructive heuristics for the flowshop problem with blocking. Int. J. Prod. Econ. 2004, 87, 39-48. [CrossRef]
7. Hall, N.G.; Sriskandarajah, C. A survey of machine scheduling problems with blocking and no-wait in process. Oper. Res. 1996, 44, 510-525. [CrossRef]
8. Graham, R.L.; Lawler, E.L.; Lenstra, J.K.; Rinnooy Kan, A.H.G. Optimization and approximation in deterministic sequencing and scheduling: A survey. Ann. Discret. Math. 1979, 5, 287-362.
9. Gilmore, P.C.; Lawler, E.L.; Shmoys, D.B. Well-solved special cases. In The Traveling Salesman Problem: A Guided Tour of Combinatorial Optimization; Lawler, E.L., Lenstra, K.L., Rinooy Kan, A.H.G., Shmoys, D.B., Eds.; John Wiley \& Sons: Hoboken, NJ, USA, 1985; pp. 87-143.
10. McCormick, S.T.; Pinedo, M.L.; Shenker, S.; Wolf, B. Sequencing in an assembly line with blocking to minimize cycle time. Oper. Res. 1989, 37, 925-936. [CrossRef]
11. Leisten, R. Flowshop sequencing problems with limited buffer storage. Int. J. Prod. Res. 1990, 28, 2085-2100. [CrossRef]
12. Nawaz, M.; Enscore, E.E.J.; Ham, I. A heuristic algorithm for the $m$-machine, $n$-job flow shop sequencing problem. Omega 1983, 11, 91-95. [CrossRef]
13. Ronconi, D.P.; Armentano, V.A. Lower bounding schemes for flowshops with blocking in-process. J. Oper. Res. Soc. 2001, 52, 1289-1297. [CrossRef]
14. Abadi, I.N.K.; Hall, N.G.; Sriskandarajh, C. Minimizing cycle time in a blocking flowshop. Oper. Res. 2000, 48, 177-180. [CrossRef]
15. Ronconi, D.P.; Henriques, L.R.S. Some heuristic algorithms for total tardiness minimization in a flowshop with blocking. Omega 2009, 37, 272-281. [CrossRef]
16. Pan, Q.K.; Wang, L. Effective heuristics for the blocking flowshop scheduling problem with makespan minimization. Omega 2012, 40, 218-229. [CrossRef]
17. Liu, J.Y.; Reeves, C.R. Constructive and composite heuristic solutions to the $\mathrm{P} / / \Sigma \mathrm{Ci}$ scheduling problem. Eur. J. Oper. Res. 2001, 132, 439-452. [CrossRef]
18. Caraffa, V.; Ianes, S.; Bagchi, T.P.; Sriskandarajah, C. Minimizing makespan in a blocking flowshop using genetic algorithms. Int. J. Prod. Econ. 2001, 70, 101-115. [CrossRef]
19. Ronconi, D.P. A branch-and-bound algorithm to minimize the makespan in a flowshop problem with blocking. Ann. Oper. Res. 2005, 138, 53-65. [CrossRef]
20. Taillard, E. Benchmarks for basic scheduling problems. Eur. J. Oper. Res. 1993, 64, 278-285. [CrossRef]
21. Grabowski, J.; Pempera, J. The permutation flow shop problem with blocking. A tabu search approach. Omega 2007, 35, 302-311. [CrossRef]
22. Wang, L.; Zhang, L.; Zheng, D. An effective hybrid genetic algorithm for flow shop scheduling with limited buffers. Comput. Oper. Res. 2006, 33, 2960-2971. [CrossRef]
23. Liu, B.; Wang, L.; Jin, Y. An effective hybrid PSO-based algorithm for flow shop scheduling with limited buffers. Comput. Oper. Res. 2008, 35, 2791-2806. [CrossRef]
24. Qian, B.; Wang, L.; Huang, D.X.; Wang, X. An effective hybrid DE-based algorithm for flow shop scheduling with limited buffers. Int. J. Prod. Res. 2009, 47, 1-24. [CrossRef]
25. Liang, J.J.; Pan, Q.; Tiejun, C.; Wang, L. Solving the blocking flow shop scheduling problem by a dynamic multi-swarm particle swarm optimizer. Int. J. Adv. Manuf. Technol. 2011, 55, 755-762. [CrossRef]
26. Wang, L.; Pan, Q.K.; Tasgetiren, M.F. A Hybrid Harmony Search Algorithm for the Blocking Permutation Flow Shop Scheduling Problem. Comput. Ind. Eng. 2011, 61, 76-83. [CrossRef]
27. Wang, L.; Pan, Q.K.; Suganthan, P.N.; Wang, W.H.; Wang, Y.M. A novel hybrid discrete differential evolution algorithm for blocking flow shop scheduling problems. Comput. Oper. Res. 2010, 37, 509-520. [CrossRef]
28. Ribas, I.; Companys, R.; Tort-Martorell, X. An iterated greedy algorithm for the flowshop scheduling with blocking. Omega 2011, 39, 293-301. [CrossRef]
29. Wang, C.; Song, S.; Gupta, J.N.D.; Wu, C. A three-phase algorithm for flowshop scheduling with blocking to minimize makespan. Comput. Oper. Res. 2012, 39, 2880-2887. [CrossRef]
30. Lin, S.W.; Ying, K.C. Minimizing makespan in a blocking flowshop using a revised artificial immune system algorithm. Omega 2013, 41, 383-389. [CrossRef]
31. Pan, Q.K.; Wang, L.; Sang, H.Y.; Li, J.Q.; Liu, M. A high performing memetic algorithm for the flowshop scheduling problem with blocking. IEEE Trans. Autom. Sci. Eng. 2013, 10, 741-756.
32. Ribas, I.; Companys, R.; Tort-Martorell, X. A competitive variable neighbourhood search algorithm for the blocking flow shop problem. Eur. J. Ind. Eng. 2013, 7, 729-754. [CrossRef]
33. Ding, J.-Y.; Song, S.; Gupta, N.D.J.; Wang, C.; Zhang, R.; Wu, C. New block properties for flowshop scheduling with blocking and their application in an iterated greedy algorithm. Int. J. Prod. Res. 2015, 54, 4759-4772. [CrossRef]
34. Armentano, V.A.; Ronconi, D.P. Minimização do tempo total de atraso no problema de flowshop com buffer zero através de busca tabu. Gestao Produçao 2000, 7, 352-362. (In Portuguese) [CrossRef]
35. Ribas, I.; Companys, R.; Tort-Martorell, X. An efficient iterated local search algorithm for the total tardiness blocking flow shop problem. Int. J. Prod. Res. 2013, 51, 5238-5252. [CrossRef]
36. Wang, L.; Pan, Q.K.; Tasgetiren, M.F. Minimizing the total flow time in a flow shop with blocking by using hybrid harmony search algorithms. Expert Syst. Appl. 2010, 37, 7929-7936. [CrossRef]
37. Deng, $\mathrm{G} . ; \mathrm{Xu}, \mathrm{Z} . ; \mathrm{Gu}, \mathrm{X}$. A discrete artificial bee colony algorithm for minimizing the total flow time in the blocking flow shop scheduling. Chin. J. Chem. Eng. 2012, 20, 1067-1073. [CrossRef]
38. Khorasanian, D.; Moslehi, G. An iterated greedy algorithm for solving the blocking flowshop scheduling problem with total flow time criteria. Int. J. Ind. Eng. Prod. Res. 2012, 23, 301-308.
39. Moslehi, G.; Khorasanian, D. Optimizing blocking flow shop scheduling total completion time criterion. Comput. Oper. Res. 2013, 40, 1874-1883. [CrossRef]
40. Ribas, I.; Companys, R. Efficient heuristic algorithms for the blocking flow shop scheduling problem with total flow time minimization. Comput. Ind. Eng. 2015, 87, 30-39. [CrossRef]
41. Ribas, I.; Companys, R. Xavier Tort-Martorell, An efficient Discrete Artificial Bee Colony algorithm for the blocking flow shop problem with total flowtime minimization. Expert Syst. Appl. 2015, 42, 6155-6167. [CrossRef]
42. Lia, X.; Wang, Q.; Wu, C. Efficient composite heuristics for total flowtime minimization in permutation flowshops. Omega 2009, 37, 155-164. [CrossRef]
43. Kirlik, G.; Oguz, C. A Variable Neighborhood Search for Minimizing Total Weighted Tardiness with Sequence Dependent Setup Times on a Single Machine. Comput. Oper. Res. 2012, 39, 1506-1520. [CrossRef]
44. Subramanian, A.; Battarra, M.; Potts, C.N. An Iterated Local Search heuristic for the single machine total weighted tardiness scheduling problem with sequence-dependent setup times. Int. J. Prod. Res. 2014, 52, 2729-2742. [CrossRef]
45. Xu, H.; Lü, Z.; Cheng, T.C.E. Iterated Local Search for single-machine scheduling with sequence dependent setup times to minimize total weighted tardiness. J. Sched. 2014, 17, 271-287. [CrossRef]
46. González, M.A.; Vela, C.R. An efficient memetic algorithm for total weighted tardiness minimization in a single machine with setups. Appl. Soft Comput. 2015, 37, 506-518. [CrossRef]
47. Tasgetiren, M.F.; Kizilay, D.; Pan, Q.K.; Suganthan, P.N. Iterated Greedy Algorithms for the Blocking Flowshop Scheduling Problem with Makespan Criterion. Comput. Oper. Res. 2016. [CrossRef]
48. Mladenovic, N.; Hansen, P. Variable neighborhood search. Comput. Oper. Res. 1997, 24, 1097-1100. [CrossRef]
49. Tasgetiren, M.F.; Liang, Y.-C.; Sevkli, M.; Gencyilmaz, G. A particle swarm optimization algorithm for makespan and total flowtime minimization in the permutation flowshop sequencing problem. Eur. J. Oper. Res. 2007, 177, 1930-1947. [CrossRef]
50. Pan, Q.-K.; Tasgetiren, M.F.; Liang, Y.-C. A Discrete Particle Swarm Optimization Algorithm for the No-Wait Flowshop Scheduling Problem with Makespan and Total Flowtime Criteria. Comput. Oper. Res. 2008, 35, 2807-2839. [CrossRef]
51. Pan, Q.-K.; Wang, L.; Tasgetiren, M.F.; Zhao, B.-H. A hybrid discrete particle swarm optimization algorithm for the no-wait flow shop scheduling problem with makespan criterion. Int. J. Adv. Manuf. Technol. 2008, 38, 337-347. [CrossRef]
52. Tasgetiren, M.F.; Liang, Y.-C.; Sevkli, M.; Gencyilmaz, G. Particle swarm optimization and differential evolution for single machine total weighted tardiness problem. Int. J. Prod. Res. 2006, 44, 4737-4754. [CrossRef]
53. Tasgetiren, M.F.; Sevkli, M.; Liang, Y.-C.; Yenisey, M.M. A particle swarm optimization and differential evolution algorithms for job shop scheduling problem. Int. J. Oper. Res. 2006, 3, 120-135.
54. Tasgetiren, M.F.; Pan, Q.K.; Suganthan, P.N.; Buyukdagli, O. A variable iterated greedy algorithm with differential evolution for the no-idle permutation flowshop scheduling problem. Comput. Oper. Res. 2013, 40, 1729-1743. [CrossRef]
55. Pan, Q.K.; Tasgetiren, M.F.; Liang, Y.C. A discrete differential evolution algorithm for the permutation flowshop scheduling problem. Comput. Ind. Eng. 2008, 55, 795-816. [CrossRef]
56. Tasgetiren, M.F.; Pan, Q.K.; Liang, Y.C. A discrete differential evolution algorithm for the single machine total weighted tardiness problem with sequence dependent setup times. Comput. Oper. Res. 2009, 36, 1900-1915. [CrossRef]
57. Tasgetiren, M.F.; Pan, Q.K.; Suganthan, P.N.; Chen, A.H.L. A discrete artificial bee colony algorithm for the total flowtime minimization in permutation flow shops. Inf. Sci. 2011, 181, 3459-3475. [CrossRef]
58. Tasgetiren, M.F.; Pan, Q.K.; Suganthan, P.N.; Oner, A. A Discrete Artificial Bee Colony Algorithm for the No-Idle Permutation Flowshop Scheduling Problem with the Total Tardiness Criterion. Appl. Math. Model. 2013, 37, 6758-6779. [CrossRef]
59. Tasgetiren, M.F.; Pan, Q.K.; Suganthan, P.N.; Chua, T.J. A Differential Evolution Algorithm for the No-Idle Flowshop Scheduling Problem with Total Tardiness Criterion. Int. J. Prod. Res. 2011, 49, 5033-5050. [CrossRef]
60. Osman, I.; Potts, C. Simulated annealing for permutation flow-shop scheduling. Omega 1989, 17, 551-557. [CrossRef]
61. Montgomery, D.C. Design and Analysis of Experiments; John Wiley \& Sons: Hoboken, NJ, USA, 2008.
62. Pan, Q.K.; Ruiz, R. Local search methods for the flowshop scheduling problem with flowtime minimization. Eur. J. Oper. Res. 2012, 222, 31-43. [CrossRef]
63. Schenker, N.; Gentleman, J.F. On judging the significance of differences by examining the overlap between confidence intervals. Am. Stat. 2001, 55, 182-186. [CrossRef]
© 2016 by the authors; licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC-BY) license (http:/ /creativecommons.org/licenses/by/4.0/).

[^0]:    Bold: better results

