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An Optimization Algorithm for the Design of an Irregularly-Shaped Bridge Based on the Orthogonal Test and Analytic Hierarchy Process

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Abstract: Irregularly-shaped bridges are usually adopted to connect the main bridge and ramps in urban overpasses, which are under significant flexion-torsion coupling effects and in complicated stress states. In irregular-shaped bridge design, the parameters such as ramp radius, bifurcation diaphragm stiffness, box girder height, and supporting condition could affect structural performance in different manners. In this paper, the influence of various parameters on three indices, including maximum stress, the stress variation coefficient, and the fundamental frequency of torsional vibration, is investigated and analyzed based on orthogonal test method. Through orthogonal analysis, the major influence parameters and corresponding optimal values for these indices are achieved. Combining with the analytic hierarchy process (AHP), the hierarchical structure model of the multi-indices orthogonal test is established and a comprehensive weight analysis method is proposed to reflect the parameter influence on overall mechanical properties of an irregularly-shaped bridge. Influence order and optimal values of parameters for overall mechanical properties are determined based on the weight of factors and levels calculated by the comprehensive weight analysis method. The results indicate that the comprehensive weight analysis method is superior to the overall balance method, which verifies the effectiveness and accuracy of the comprehensive weight analysis in the parameter optimization of the multi-indices orthogonal test for an irregularly-shaped bridge. Optimal parameters obtained in this paper can provide reference and guidance for parameter control in irregularly-shaped bridge design.

Keywords: irregular-shaped bridge; design optimization; orthogonal test; analytic hierarchy process; comprehensive weight analysis

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1. Introduction

With continuous economic development, urban traffic congestion is increasingly becoming a serious problem. Overpasses play an important role in solving the urban traffic problem. As the connecting structure of the main bridge and ramps in urban overpasses, an irregularly-shaped bridge, as shown in Figure 1a, can solve the conflict of bridge and ground transportation effectively to improve the driving efficiency and keep it in harmony with the environment [1]. However, an irregularly-shaped bridge usually adopts the thin-walled box girder, which suffers salient restrained torsion and shear lag effects [2]. The main bridge and ramp are contacted and restricted with each other at the crotch of the irregularly-shaped bridge, which results in a complicated strained condition and varying degrees of damage. In recent years, the collapse of irregular-shaped bridges have happened several times in China, as illustrated in Figure 1b [3]. The optimal design of such structures can be extremely useful to improve this situation. Foti et al. [4,5] had studied the dynamic behaviors of a pedestrian bridge before retrofitting, and after, using experimental and numerical methods. The dynamic interaction between pedestrians and the bridge and the damping factor were investigated. Due to the change of mass and stiffness of the footbridge caused by retrofitting using glass fiber-reinforced polymers (GFRPs), the dynamic behavior was indeed modified. Dynamic interaction still existed, and the modal damping factors for the main modes of bending vibration almost doubled after retrofitting. Therefore, the dynamic interaction problem highlights the need to perform specific dynamic analysis during the design phase of light and slender bridges to avoid resonance. For the design of irregularly-shaped bridges, many parameters affect its mechanical properties. One of the key aspects of an optimization study is the sensitivity analysis of influence parameters. How to determine the major influence parameters of the design indices and find the optimal parameter levels for an irregularly-shaped bridge are becoming the focus of attention and demand for engineers.

(**a**) (**b**)

Figure 1. Irregularly-shaped bridge. (**a**) Plan view; and (**b**) collapse accident.

The thin-walled box girder adopted in irregularly-shaped bridges usually suffers from the influence of torsion, distortion, warping, and shear lag, which causes a severely uneven stress distribution [6,7]. Many researchers have studied the calculation principles and analysis methods for the thin-walled box girder [8–12]. However, the mechanical property of an irregularly-shaped bridge is different from the conventional straight and curved thin-walled box girder bridge. The traditional analytical method is unsuitable to solve this problem. Finite element analysis (FEA) is a widely used numerical calculation method, which provides an effective way to analyze the static and dynamic properties of structures [13–17]. Through numerical simulation, the influence of design parameters on mechanical properties of irregularly-shaped bridges and curved bridges have been studied and demonstrated [1,18]. However, the investigation on multi-indices optimization for the mechanical properties of irregularly-shaped bridges is relatively scarce [19,20]. Due to numerous parameters affecting the strained condition of irregularly-shaped bridges, the influence of a single parameter is not able to reflect the effect of parameters in combination. Moreover, the comprehensive combination tests of influence parameters need repeated and complicated numerical modeling and analysis, which is difficult to be achieved due to the large degree of testing and calculation. The classical optimization procedures, such as back propagation neural networks (BPNN), support vector machines (SVM), and genetic algorithms (GA), need significant amounts of numerical simulation with different parameter combinations, which also makes the optimization design more complex. Even for a simple problem, the objective functions are calculated based on a large number of training samples. By utilizing the orthogonal test method to optimize the design process, the representative parameter combinations are selected to be analyzed, which reduces the complexity of tests to a large extent.

The orthogonal test method provides a scientific and efficient way to arrange and analyze the multi-factors test. It can infer the comprehensive test results based on a limited number of tests, and has been widely used for test design and parameter optimization [21–26]. Taking the test with three factors and three levels, for example, the number of comprehensive tests is $3^3 = 27$, while only nine orthogonal tests can analyze the sensitivity of factors and determine the preferred levels. As shown in Figure 2, nine test points regularly distributed throughout the comprehensive test points, which shows a strong representation and is able to better reflect the basic information in the optimal selection space. The optimum condition determined by part tests has the same trends as that obtained by comprehensive tests.

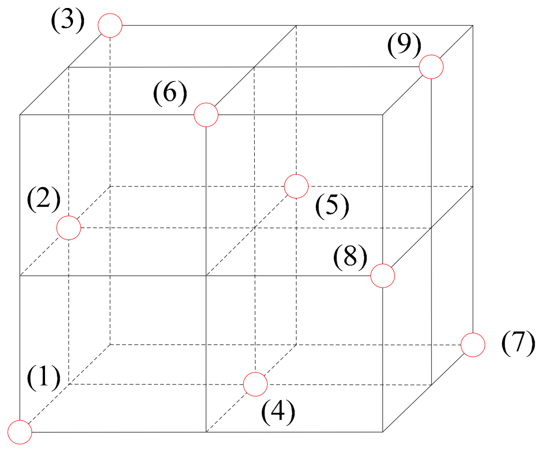


Figure 2. Uniform distribution of the orthogonal test.

The orthogonal test method can effectively reduce the complexity of testing analysis. However, there is still a large calculated amount, unreasonable weight, and human factor influence in the analysis process of the multi-indices orthogonal test [27]. In this paper, the index system for the mechanical properties of an irregularly-shaped bridge is established based on the AHP and orthogonal test methods. Maximum stress, the stress variation coefficient, and torsion vibration fundamental frequency are selected as test indices. The influence of the ramp radius, bifurcation diaphragm stiffness, box girder height, and supporting condition on each test index is analyzed by the orthogonal test method. On this basis, weights of factors and levels in the test indices for mechanical properties of an irregularly-shaped bridge are calculated by the proposed comprehensive weight analysis approach. Based on the weight value, the influence order and optimal values of parameters are determined. Experimental results indicate that the proposed comprehensive weight analysis method used for the analysis of the multi-indices orthogonal test can decrease the calculation amount, reasonably determine the weight, and reflect parameter influences objectively, which provides an effective way for parameter sensitivity analysis and design optimization. Additionally, a limitation of the method employed is that correlations and coupling effects between the variables are not considered and analyzed. The fundamental purpose is to investigate the effect of every variable on the overall mechanical properties and determine their weights. In the future, the correlation and coupling effects between variables will be paid more attention and intensively studied.

2. Comprehensive Weight Calculation Model Based on AHP and Orthogonal Test

2.1. Orthogonal Analysis

In an orthogonal array, every level occurs the same number of times for the factor in any column. Moreover, every combination of two levels occurs the same number of times for the two factors in any two columns [26]. This makes the test condition of every level in the same factor the same as each other, which excludes the interference of other factors and can comprehensively compare the influence of different levels in the same factor of the test index.

The orthogonal test can determine the influence order of factors and the optimal levels for a single index based on the orthogonal analysis with higher accuracy and a smaller workload. The larger the difference is, the greater the impact of the factor. On the contrary, the smaller the difference is, the less the impact [27]. The analysis of orthogonal array $L_4(2^3)$ is listed in Table 1, in which the numbers of factors and levels are $\alpha = 3$, $\beta = 2$, respectively.

Table 1. Analysis of the orthogonal test result.

Test No.	Factor			Index Value
	B$_1$	**B$_2$**	**B$_3$**	
1	1	1	1	y_1
2	1	2	2	y_2
3	2	1	2	y_3
4	2	2	1	y_4
K_{jl}	$K_{11} = y_1 + y_2$ $K_{12} = y_3 + y_4$	$K_{21} = y_1 + y_3$ $K_{22} = y_2 + y_4$	$K_{31} = y_1 + y_4$ $K_{32} = y_2 + y_3$	- -
m_j	2	2	2	-
k_{jl}	$k_{11} = K_{11}/m_1$ $k_{12} = K_{12}/m_1$	$k_{21} = K_{21}/m_2$ $k_{21} = K_{22}/m_2$	$k_{31} = K_{31}/m_3$ $k_{31} = K_{32}/m_3$	- -
R_j	R_1	R_2	R_3	-

Note: K_{jl} (j = 1, 2; l = 1, 2) is the summation of test index values corresponding to level l in factor j; k_{jl} is the average value of test index values corresponding to level l in factor j and represents the test index value at this level; m_j is the number of the same levels in factor j. The relationship between K_{jl} and k_{jl} satisfies $k_{jl} = K_{jl}/m_j$. R_j is the range of k_{jl} in factor j and $R_j = \max\{k_{j1}, \cdots, k_{j\beta}\} - \min\{k_{j1}, \cdots, k_{j\beta}\}$.

2.2. Influence Weight Calculation Based on AHP

The orthogonal test is able to reflect the influence of factors on the test index. For a multi-indices orthogonal test, the establishment of a hierarchical structure model to analyze the influence of factors and levels on mechanical properties of an irregularly-shaped bridge is a useful method to determine the influence order and preferred values of parameters.

2.2.1. Weight Vector of Test Index Layer

The AHP method, proposed by Satty [28], is a decision-making technique used to evaluate complex problems. Firstly, the two-level hierarchical structure including a target layer and an index layer with n test indices is constructed. Secondly, a comparison judgment matrix A is established as shown in Table 2, which is formed by a pairwise comparisons of the relative importance of the indices.

Table 2. Comparison judgment matrix.

Index	**A$_1$**	**A$_2$**	**···**	**A$_n$**
A$_1$	a_{11}	a_{12}	···	a_{1n}
A$_2$	a_{21}	a_{22}	···	a_{2n}
···	···	···	···	···
A$_n$	a_{n1}	a_{n2}		

Note: a_{ij} is the importance ratio of index i to index j in a given set $S = \{1, 2, 3, \ldots, 9\}$.

Then, the eigenvector equation can be constructed and solved by:

$$A \cdot W = \lambda \cdot W \tag{1}$$

where W is the eigenvector for comparison judgment matrix A and λ is the eigenvalue.

The normalized eigenvector corresponding to the maximum eigenvalue $\lambda_{\max}$, which can give the relative importance of each index, can be expressed by:

$$W^T = [w_1, w_2, \cdots, w_n] \; (\textstyle\sum_{i=1}^{n} w_i = 1) \tag{2}$$

where w_i represents the relative importance for index i by comparing with other indices.

In order to evaluate the consistency of the comparison judgment matrix A for determining the reliability of making the solved eigenvector W the weight vector, a consistency index (CI) and consistency ratio (CR) are defined as:

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{3}$$

$$CR = CI/RI \tag{4}$$

where n is the number of test indices, and RI is defined as a random consistency index.

The value of random consistency index (RI) is listed in Table 3.

Table 3. Value of a random consistency index.

n	1	2	3	4	5
RI	0	0	0.58	0.90	1.12

When $CR < 0.1$, the extent of inconsistency for a comparison judgment matrix is permitted, and the eigenvector W can be regarded as the weight vector.

2.2.2. Weights of Factors and Levels in the Orthogonal Test

Every test index is analyzed by the orthogonal test with α factors and β levels. The hierarchical structure of factors and levels for i-th index is shown in Figure 3.

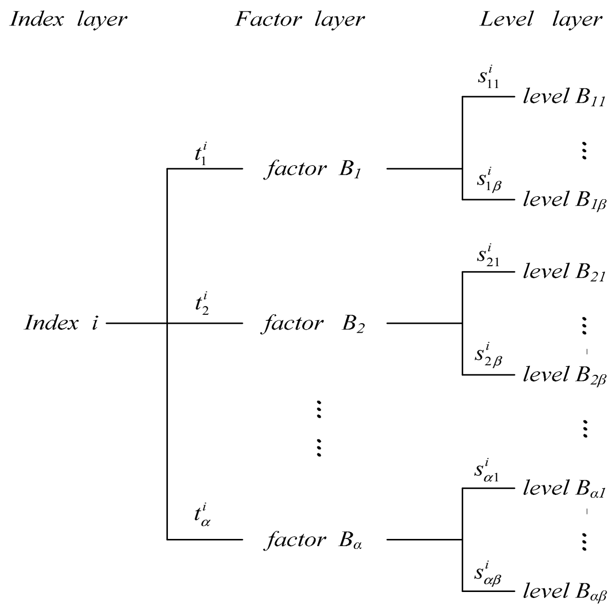


Figure 3. Hierarchical structure model of factors and levels for the i-th index.

The weight of j-th factor (t_j^i) represents the influence of the j-th factor on the i-th index, which is the ratio of range for the j-th factor to the summation of ranges for all factors:

$$t_j^i = R_j^i / \sum_{f=1}^{\alpha} R_f^i \tag{5}$$

where R_j^i is the range of the j-th factor. The weight matrix of the factor layer to the i-th index is defined as $T_i = \left[t_1^i, t_2^i, \cdots, t_\alpha^i\right]^T$.

The weight of the l-th level in the j-th factor (s_{jl}^i) represents the influence of the l-th level on the f-th factor relative to other levels. It has two calculation models. One is calculated by:

$$s_{jl}^i = k_{jl}^i / \sum_{g=1}^{\beta} k_{jg}^i \tag{6}$$

where k_{jl}^i is the test index value at the l-th level in the j-th factor for the i-th index. This model is applied to the situation "the larger k_{jl}^i is, the better the level".

For another calculation model, s_{jl}^i can be calculated by:

$$s_{jl}^i = (1/k_{jl}^i) / \sum_{g=1}^{\beta} (1/k_{jg}^i) \tag{7}$$

This model is applied to the situation "the smaller k_{jl}^i is, the better the level".

For the i-th test index, the weights of all levels in different factors to the i-th index are calculated by:

$$M_i = S_i \cdot T_i \tag{8}$$

where M_i is defined as the weight matrix of level layer to the i-th index. The weight matrix of the level layer to the factor layer is defined by Equation (9):

$$S_i = \begin{bmatrix} s_{11}^i & 0 & \cdots\cdots & 0 \\ s_{12}^i & 0 & \cdots\cdots & 0 \\ \vdots & \vdots & \cdots\cdots & \vdots \\ s_{1\beta}^i & 0 & \cdots\cdots & 0 \\ 0 & s_{21}^i & \cdots\cdots & 0 \\ 0 & s_{22}^i & \cdots\cdots & 0 \\ \vdots & \vdots & \cdots\cdots & \vdots \\ 0 & s_{2\beta}^i & \cdots\cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots\cdots & s_{\alpha 1}^i \\ 0 & 0 & \cdots\cdots & s_{\alpha 2}^i \\ \vdots & \vdots & \cdots\cdots & \vdots \\ 0 & 0 & \cdots\cdots & s_{\alpha\beta}^i \end{bmatrix} \tag{9}$$

2.2.3. Comprehensive Weight

Based on the weight vector of the test index layer on the object layer and the weight matrix of factors and levels in test i-th index, the weight matrix elements multiplied by each other can results in the influence weight of every factor and level of the object.

For the target value with n test indices, the influence weight of every level and factor to the target value are obtained by:

$$P = [M_1, M_2, \cdots, M_n] \cdot W \tag{10}$$

$$Q = [T_1, T_2, \cdots, T_n] \cdot W \tag{11}$$

where P is the weight matrix of the level layer to the object; T is the weight matrix of the factor layer to the object.

3. Structure Model of a Typical Irregularly-Shaped Bridge

A typical irregularly-shaped bridge is selected as the research object [1], which is shown in Figure 4. This bridge consists of a three-span straight continuous beam (main bridge) and a curved continuous beam (ramp). The support distribution is from C1–C6. Spans of the main bridge and ramp are 25 m + 30 m + 25 m, and 25 m + 25 m, respectively. The first span of the main bridge is a constant box girder section with double chambers and the others are constant box girder sections with a single chamber. The mid-span is a special structure to connect the main bridge and ramps. For the irregularly-shaped bridge, the influence parameters includes the ramp radius, box girder height, and bifurcation diaphragm thickness, which can be expressed as R, H, and D, respectively.

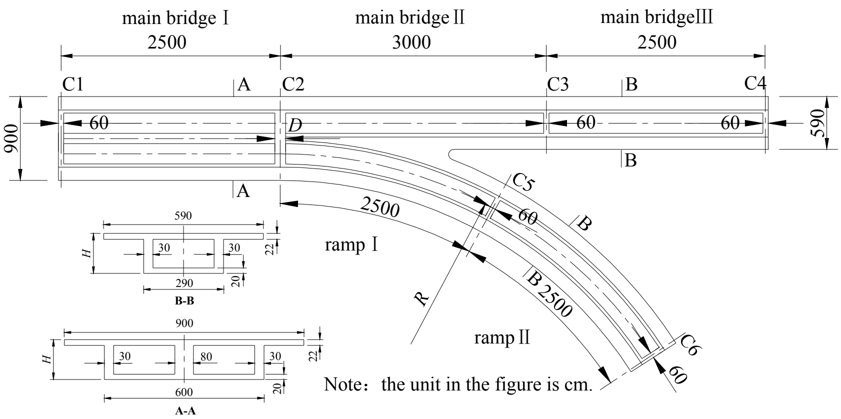


Figure 4. Plan view of a typical irregularly-shaped bridge.

The irregularly-shaped bridge is constructed of concrete, and the material properties are: elastic modulus $E = 3.5 \times 10^{10}\ Pa$, material density $\rho = 2500\ kg/m^3$, and Poisson's ratio $\mu = 0.2$, respectively. The model established by ANSYS 10.0 (Canonsburg, PA, USA) composed by solid 65 elements is shown in Figure 5.

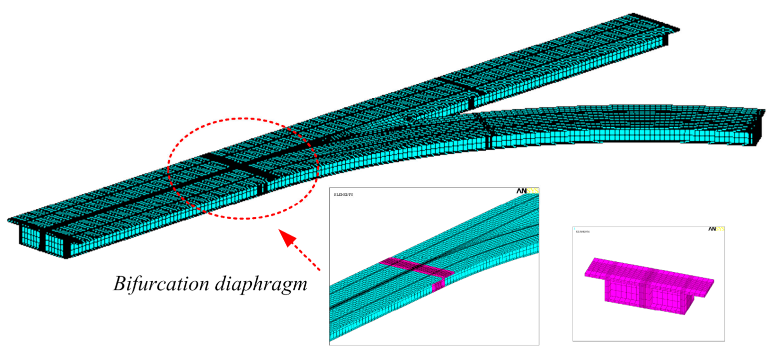
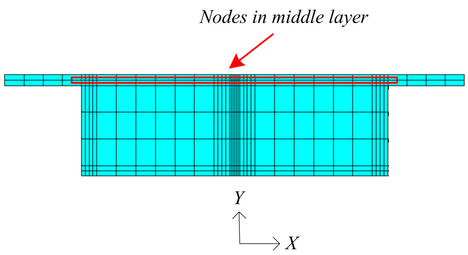


Figure 5. Finite element model of a typical irregularly-shaped bridge.

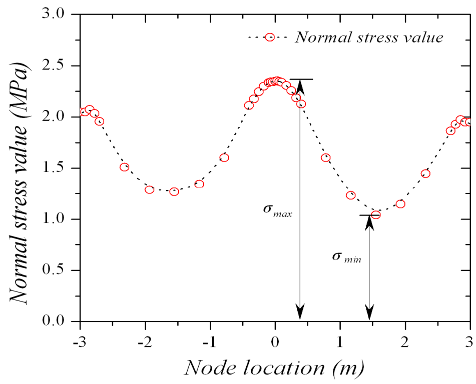
4. Construction of the Index System and the Determination of Factors and Levels

The irregularly-shaped bridge is a continuous structure with three spans. The top plate of the bifurcation diaphragm is under the effect of the hogging moment and tensile stress, which can easily result in cracking and breaking of the concrete in the top plate. Moreover, the bifurcation diaphragm is located at the crotch of main bridge and ramp. It is affected by the coupling effect of constraints, torsion, and shear lag, which makes the distribution of tensile stress seriously uneven. Research on the maximum stress and uneven stress distribution in the top plate of the bifurcation diaphragm is essential to improve the design security of irregularly-shaped bridges. Meanwhile, the torsion vibration fundamental frequency of the irregularly-shaped bridge should be greater than the external excitation frequency to avoid torsional resonance and improve its dynamic characteristics [5]. The middle section of the bifurcation diaphragm is selected and the nodes in the middle layer of the top plate are adopted as study objects, which are shown in Figure 6a. Taking into account the static and dynamic characteristics, the maximum stress ($\sigma_{\max}$), the stress variation coefficient (λ) of selected nodes under a dead load condition, and the torsion vibration fundamental frequency (f_d) of the bridge are regarded as the indices to reflect the parameter influences on the mechanical properties. Hierarchical structure and index weights are listed in Table 4. The stress variation coefficient (λ) is the ratio of the maximum normal stress ($\sigma_{\max}$) to the minimum normal stress ($\sigma_{\min}$), which can be calculated by Equation (12). The calculation results are shown in Figure 6b.

$$\lambda = \sigma_{\max}/\sigma_{\min} \tag{12}$$



(**a**)



(**b**)

Figure 6. Node selection and normal stress distribution in the top plate of the bifurcation diaphragm. (**a**) Node selected in the top plate; and (**b**) the normal stress distribution of selected nodes.

Based on the relative importance of maximum stress, the stress variation coefficient, and the torsion vibration fundamental frequency to the mechanical properties of the irregularly-shaped bridge, the comparative judgment matrix is established as follows:

$$A = \begin{bmatrix} 1 & 1 & 1/2 \\ 1 & 1 & 1/2 \\ 2 & 2 & 1 \end{bmatrix} \tag{13}$$

The eigenvector for matrix A can be calculated according to Equation (1), and is shown as follows:

$$W = \begin{bmatrix} 1/4 & 1/4 & 1/2 \end{bmatrix}^T \tag{14}$$

The consistency ratio CR is calculated by Equation (4). If $CR < 0.001$, then the extent of inconsistency for comparison judgment matrix is permitted.

Table 4. Hierarchical structure and index weights.

Target Layer	Mechanical Property of the Irregularly-Shaped Bridge		
Index layer	$\sigma_{\max}$	λ	f_d
Weight	0.25	0.25	0.50

The orthogonal test is carried out for every index to investigate the influence of parameters. In practice, the span is an important factor affecting the strained condition of the bridge structure.

However, it is restricted by topographical conditions. Therefore, the influence of other parameters on mechanical properties is demonstrated for the irregularly-shaped bridge with a certain span. In this paper, ramp radius, bifurcation diaphragm stiffness, box girder height, and supporting condition, which can strongly affect the mechanical properties of an irregularly-shaped bridge, are selected as the influence factors for mechanical properties in the irregularly-shaped bridge design [3]. The change of the bifurcation diaphragm stiffness is simulated by thickness variation.

The ramp is a curved beam with a small radius in numerical simulation of the irregularly-shaped bridge. Based on the standard requirements of urban overpass design and the rationality for irregularly-shaped bridge design [29], the variation ranges for design parameters are determined as ramp radius: 30–50 m, box girder height: 1.6–2.0 m, bifurcation diaphragm thickness: 1.2–2.0 m, and supporting conditions are listed in Table 5. The selected ranges of values are determined by the statistics for the design parameters of the irregularly-shaped bridge in practical engineering and are consistent with the standard requirements. Therefore, they are representative and reasonable for the optimization design of an irregularly-shaped bridge. The parameters are taken as the factors of the orthogonal test and every parameter is divided into three levels. The factors and levels are listed in Table 6.

Table 5. Support conditions of an irregularly-shaped bridge.

Support Position	Support Condition		
	①	②	③
C1	Double movable support	Double movable support	Single vertical support
C2	Double fixed support	Double fixed support	Double fixed support
C3	Single vertical support	Single vertical support	Single vertical support
C4	Double movable support	Double movable support	Single vertical support
C5	Single vertical support	Vertical eccentric support	Single vertical support
C6	Double movable support	Double movable support	Single vertical support

Note: the eccentric distance of vertical eccentric support in ② is e = 0.4 m.

Table 6. Factors and levels for the orthogonal test.

Level	Factor			
	Ramp Radius R (m)	Diaphragm Thickness D (m)	Box Girder Height H (m)	Supporting Condition U
1	30	1.2	1.6	①
2	40	1.6	1.8	②
3	50	2.0	2.0	③

5. Sensitivity Analysis and Design Optimization of Parameters

Based on the factors and levels in Table 6, an orthogonal test is adopted to arrange numerical simulation tests for analyzing the influence of parameters on test indices, including maximum stress, the stress variation coefficient, and the torsion vibration fundamental frequency. The orthogonal test is carried out utilizing an orthogonal array, $L_9(3^4)$, and test results are listed in Table 7.

Table 7. $L_9(3^4)$ orthogonal array testing for three indices.

Test No.	Factor				Test index		
	R **(m)**	***D*** **(m)**	***H*** **(m)**	***U***	σ_{max} **(MPa)**	λ	f_d **(Hz)**
1	30	1.2	1.6	①	2.95	1.76	7.7341
2	30	1.6	1.8	②	2.36	1.66	7.8733
3	30	2.0	2.0	③	1.99	1.57	5.2013
4	40	1.2	1.8	③	2.66	1.85	5.0639
5	40	1.6	2.0	①	2.14	1.78	8.3185
6	40	2.0	1.6	②	2.64	1.47	8.2158
7	50	1.2	2.0	②	2.35	1.90	8.3485
8	50	1.6	1.6	③	2.70	1.61	5.0858
9	50	2.0	1.8	①	2.24	1.51	8.4667

In order to determine the influence order of factors and optimal levels, orthogonal analysis is conducted for the test indices. The analysis results are listed in Table 8. k_i (i = 1, 2, 3) represents the corresponding index value at each parameter level and R denotes the difference between the largest and the smallest values of k_i. It is apparent that the parameter with the largest R value has the most significant influence on the test index.

Table 8. Analysis of orthogonal test results.

Test Index	Item	Factor			
		R **(m)**	***D*** **(m)**	***H*** **(m)**	***U***
Maximum stress	k_1	2.43	2.65	2.76	2.44
	k_2	2.48	2.40	2.42	2.45
	k_3	2.43	2.29	2.16	2.45
	R	0.05	0.36	0.60	0.01
Stress variation coefficient	k_1	1.66	1.84	1.61	1.68
	k_2	1.70	1.68	1.67	1.67
	k_3	1.67	1.52	1.75	1.68
	R	0.04	0.32	0.14	0.01
Torsion vibration fundamental frequency	k_1	6.9362	7.0488	7.0119	8.1731
	k_2	7.1994	7.0925	7.1346	8.1459
	k_3	7.3003	7.2946	7.2894	5.1170
	R	0.3641	0.2458	0.2775	3.0561

5.1. Result Analysis for a Single Index

5.1.1. Influence of Parameters on Maximum Stress

The influence of changing parameters on maximum stress is illustrated in Figure 7. As can be seen from Figure 7, the maximum stresses decrease with the increase of box girder height and bifurcation diaphragm thickness, while the influences of the ramp radius and supporting condition are negligible. The growth of the box girder height increases the flexural rigidity to make the vertical bending stress smaller. Vertical bending stress plays the most important role in the normal stress of the top plate, which results in the decrease of the maximum stress. With the bifurcation diaphragm thickness increasing, the torsional stiffness of the bridge increases to resist the torsion and distortion, which decreases the maximum stress. According to range R, it can be found that the influence degree of factors on the maximum stress is: box girder height > bifurcation diaphragm stiffness > ramp radius > supporting condition. The box girder height and bifurcation diaphragm stiffness are the major influencing factors for maximum stress. In the design of an irregularly-shaped bridge, the smaller maximum stress is conducive to bridge security. As can been seen from Table 7, the minimum value of the maximum stress index can be obtained when the box girder height is 2 m and the bifurcation diaphragm thickness is 2 m. For the maximum stress index, the ramp radius and the supporting condition can adopt reasonable values due to their negligible influence.

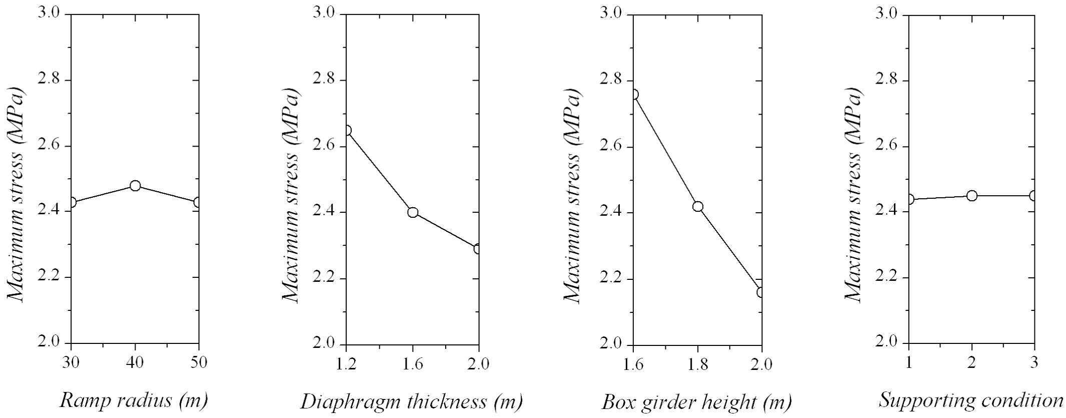


Figure 7. Influence of varying parameters on maximum stress.

5.1.2. Influence of Parameters on the Stress Variation Coefficient

The influence of parameters on the stress variation coefficient is shown in Figure 8. From Figure 8, it can be obtained that the stress variation coefficient decreases with the increase of the bifurcation diaphragm thickness and increases with the growth of the box girder height. The influence of the ramp radius and supporting condition are relatively insignificant. The increase of the bifurcation diaphragm thickness enhances the torsional stiffness, which improves the ability to resist torsion and distortion and reduces the uneven stress distribution. Increasing the box girder height increases the flexural rigidity and torsional stiffness of the bridge, simultaneously. The vertical bending stress in the total vertical stress quickly decreases, while distortion and warping stress reduce insignificantly [18]. This makes the impact of distortion and warping more significant. Therefore, the stress distribution is seriously uneven and the stress variation coefficient becomes larger. Based on the range of R values, it can be found that the influence of factors on the stress variation coefficient is: bifurcation diaphragm stiffness > box girder height > ramp radius > supporting condition. Bifurcation diaphragm stiffness and box girder height are the first two factors affecting the uneven stress distribution. The smaller the stress variation coefficient is, the more uniform the stress distribution. The optimal parameter combination for the stress distribution index is: a bifurcation diaphragm thickness of 2 m and box girder height of 1.6 m, while the ramp radius and supporting condition can adopt reasonable values due to their negligible influence.

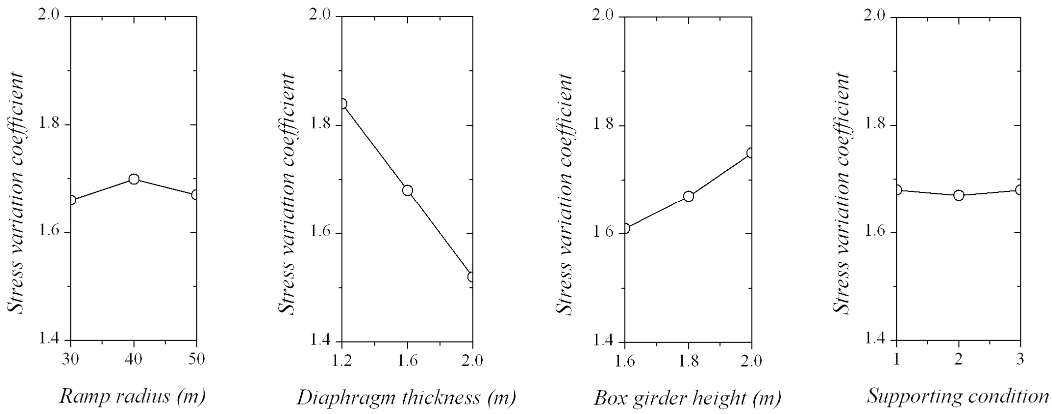


Figure 8. Influence of varying parameters on the stress variation coefficient.

5.1.3. Influence of Parameters on the Torsion Vibration Fundamental Frequency

The influence of parameters on the torsion vibration fundamental frequency is illustrated in Figure 9. The supporting condition plays the most significant role in the influence of parameters on the torsion vibration fundamental frequency. Supporting condition ③ can reduce the torsion vibration fundamental frequency by a great degree by comparing with others factors. Vertical single supports are set at the end of irregularly-shaped bridge in supporting condition ③. The torsional stiffness decreases and the ability to resist torsional vibration weakens, which leads to the serious decline of the torsion vibration fundamental frequency. According to the range of R values, the influences of factors on the torsion vibration fundamental frequency is in the following order: supporting condition > ramp radius > box girder height > bifurcation diaphragm stiffness. The supporting condition becomes the first factor to affect the torsional vibration of the irregularly-shaped bridge. The larger torsion vibration fundamental frequency is conducive to avoiding torsional vibration stimulated by the external environment. For driving safety, the parameter combination leading to the maximum torsion vibration fundamental frequency is: supporting condition ① or ②, a ramp radius of 50 m, a bifurcation diaphragm thickness of 2.0 m, and a box girder height 2.0 m.

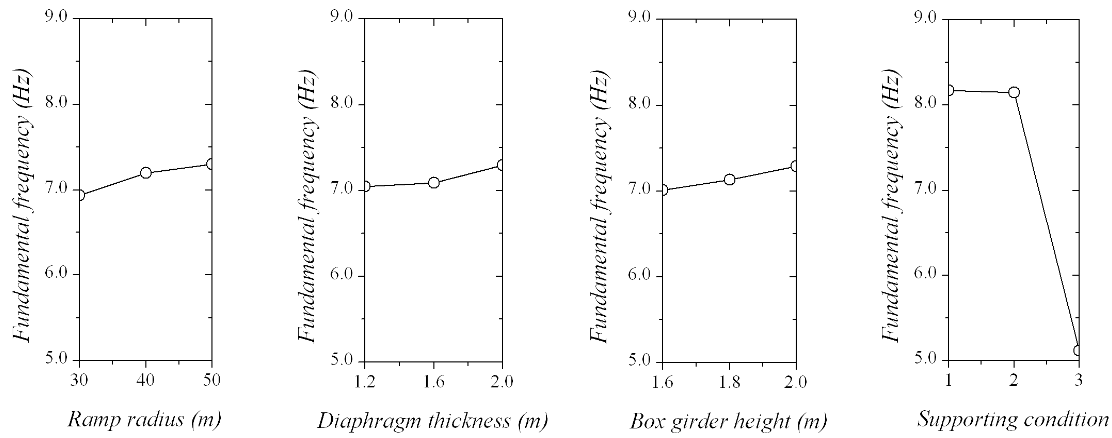


Figure 9. Influence of varying parameters on the torsion vibration fundamental frequency.

5.2. Result Analysis for the Mechanical Properties of an Irregularly-Shaped Bridge

5.2.1. Parameter Optimization Based on the Overall Balance Method

According to the orthogonal analysis results for indices, the overall balance method comprehensively considers the influence order of factors and the advantages of levels to determine the final optimal parameter combination. Ramp radius mainly affects the torsion vibration fundamental frequency, and the influence on maximum stress and the stress variation coefficient are negligible. Therefore, optimal ramp radius is selected based on the torsion vibration fundamental frequency. With increasing ramp radius, the torsion vibration fundamental frequency continues to increase. A ramp radius of 50 m is selected as the optimal level. Bifurcation diaphragm stiffness is the first influencing factor for the stress variation coefficient and the second influencing factor for maximum stress. The stress variation coefficient and maximum stress are all decreasing with an increase of the bifurcation diaphragm thickness. The optimal value of the bifurcation diaphragm thickness is 2.0 m. Box girder height affects maximum stress and the stress variation coefficient significantly. With the box girder height increasing, the maximum stress reduces and the stress variation coefficient becomes larger. Taking into account of optimization principles for both factors, the optimal box girder height is determined to be 1.8 m. Supporting condition mainly affects the torsion vibration fundamental frequency. In order to obtain the maximum torsion vibration fundamental frequency, ① is the optimal supporting condition selected. Comprehensively considering all of the above influences for indices, the optimal parameter combination for the mechanical properties of an irregularly-shaped bridge is: ramp radius R = 50 m, bifurcation diaphragm thickness D = 2.0 m, box girder height H = 1.8 m, and supporting condition ①.

5.2.2. Parameter Optimization Based on the Comprehensive Weight Analysis Method

Optimization conditions for a single index are inconsistent with each other. Overall balance methods determine the optimal parameter combination based on the influence of factors on multiple indices. However, it is seriously affected by subjective factors and is difficult to be selected as the actual optimal parameters. Therefore, it is a more favorable method of quantitative analysis to determine the factor influence order and the accurate optimal parameters for calculating the influence weights of factors and levels to the mechanical properties of the irregularly-shaped bridge based on the test index values.

Based on the influence weight matrixes ($T_{\sigma_{max}}$, T_λ, T_{f_d}) of factors to indices, the influence weight matrix Q of factors to mechanical properties of the irregularly-shaped bridge is shown in Table 9.

Table 9. Influence weight of factors.

Factor		R (m)	D (m)	H (m)	U
Weight	$T_{\sigma_{max}}$	0.0490	0.3529	0.5882	0.0098
	T_λ	0.0784	0.6275	0.2745	0.0196
	T_{f_d}	0.0923	0.0623	0.0704	0.7750
	Q	0.0780	0.2763	0.2509	0.3948

According to the influence weight matrixes ($M_{\sigma_{max}}$, M_λ, M_{f_d}) of levels to indices, the influence weight matrix P of levels to mechanical properties of the irregularly-shaped bridge is shown in Table 10.

Table 10. Influence weight of levels.

Factor		R (m)			D (m)			H (m)			U		
Level		30	40	50	1.2	1.6	2.0	1.6	1.8	2.0	①	②	③
Weight	$M_{\sigma_{max}}$	0.0165	0.0161	0.0165	0.1082	0.1195	0.1252	0.1721	0.1963	0.2199	0.0033	0.0033	0.0033
	M_λ	0.0264	0.0258	0.0262	0.1898	0.2079	0.2298	0.0952	0.0918	0.0876	0.0065	0.0066	0.0065
	M_{f_d}	0.0299	0.0310	0.0314	0.0205	0.0206	0.0212	0.0230	0.0234	0.0239	0.2955	0.2945	0.1850
	P	0.0257	0.0260	0.0264	0.0848	0.0922	0.0994	0.0783	0.0837	0.0888	0.1502	0.1497	0.0949

As can be seen from weight matrix Q in Table 9, the influence rank of factors is: supporting condition > bifurcation diaphragm stiffness > box girder height > ramp radius. Comparing the weights of levels in the same factor in matrix P listed in Table 10, the optimal mechanical properties of the irregularly-shaped bridge are obtained when ramp radius, bifurcation diaphragm thickness, box girder height, and supporting condition are R = 50 m, D = 2.0 m, H = 2.0 m, and supporting condition ①, respectively.

In order to compare the optimal parameter combinations for multiple indices obtained from the overall balance method and comprehensive weight analysis, mechanical properties of the irregularly-shaped bridge under two conditions are illustrated in Table 11.

As can be seen from Table 11, the comprehensive weight analysis method is superior to the overall balance method. For the comprehensive weight analysis method, maximum stress decreases by 13.4% compared with the overall balance method. There are also significant improvements for the torsion vibration fundamental frequency and the stress variation coefficient. The results indicate that the comprehensive weight analysis method is able to accurately determine the influence order and optimal values of parameters for mechanical properties of the irregularly-shaped bridge.

Table 11. Mechanical properties with different optimal parameters.

Analysis Method	Factor				Index		
	R (m)	D (m)	H (m)	U	$\sigma_{max\ (MPa)}$	λ	f_d (Hz)
Overall balance method	50	2.0	1.8	①	2.24	1.51	8.4667
Comprehensive weight analysis method	50	2.0	2.0	①	1.94	1.54	8.5642

6. Conclusions

In order to improve the mechanical properties of an irregularly-shaped bridge, a typical irregularly-shaped bridge is selected and its static and dynamic characteristics are optimized by the proposed comprehensive weight analysis method. According to the orthogonal analysis results, the influence of parameters, including ramp radius, bifurcation diaphragm stiffness, box girder height, and supporting conditions on maximum stress, stress variation coefficient, and torsion vibration fundamental frequency are investigated. Combining with AHP, the comprehensive weight analysis method is proposed to determine the influence order and preferred values of parameters for the mechanical properties of an irregularly-shaped bridge. Based on the test results, the following conclusions are obtained:

(1) Bifurcation diaphragm stiffness and box girder height are the main factors that affect the maximum stress and stress variation coefficient relative to the ramp radius and supporting condition. The increasing bifurcation diaphragm thickness decreases the maximum stress and stress variation coefficient. As box girder height increases, the maximum stress decreases and uneven stress distribution becomes serious.
(2) The supporting condition plays the most important role in the influence on torsion vibration fundamental frequency. Double torsional supports set at the end of an irregularly-shaped bridge can improve the ability of the irregularly-shaped bridge to resist torsional vibration, which is more beneficial to driving safety.
(3) Based on the influence weights of factors to the mechanical properties of an irregularly-shaped bridge, the influence order of parameters is: supporting condition > bifurcation diaphragm stiffness > box girder height > ramp radius. According to the influence weights of levels in each factor, the best parameters for an irregularly-shaped bridge design are obtained when the ramp radius, bifurcation diaphragm thickness, box girder height, and supporting conditions are R = 50 m, D = 2.0 m, H = 2.0 m, and ①, respectively.
(4) Comparative analysis with test results of optimal parameters obtained from the overall balance method reveals that the comprehensive weight analysis method is superior to the overall balance method. The comprehensive weight analysis method possesses more favorable accuracy and validity in the sensitivity analysis and design optimization of parameters for the mechanical properties of irregularly-shaped bridges.

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