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Growth and Yield Models for Teak Planted as Living Fences in Coastal Ecuador

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Abstract: Teak plantations cover a total area of about 4.35 million ha worldwide. The species is currently being planted in silvopastoral systems in the coastal lowlands of Ecuador. However, there are no growth and yield models for teak grown in silvopastoral systems, especially as living fences, in this region. The aim of the present study was to develop volume and yield models for teak grown as living fences in silvopastoral systems. For teak planted as living fences, the biological rotation age was estimated to vary between 15 and 26 years. The final yield in the silvopastoral system varied from 49 m^3 ha^{-1} at 26 years in the least productive sites to 225 m^3 ha^{-1} at 15 years in the most productive sites in the study area. The mean annual yield for the highest quality site was 15.3 m^3 ha^{-1} $year^{-1}$ at age 15 years, for a density of 160 trees ha^{-1}. For a base age of 10 years, height-based site indexes of nine to 23 m were established. The growth and yield model obtained may be useful to define the biological (optimal) rotation age and estimate the productivity of teak living fences in the coastal lowlands of Ecuador.

Keywords: site index; silvicultural models; silvopastoral systems; *Tectona grandis* L.

1. Introduction

Silvopastoral systems (SPS) represent a type of agroforestry system adapted to the conditions of small and medium landowners (i.e., usually less than 20 ha) [1]. This type of agroforestry combines fodder plants with shrubs and trees for animal nutrition and complementary uses [2]. In SPS, farmers thus obtain multiple products from the same area of land whilst increasing soil and phytomass carbon sequestration [3].

Living fences, which comprise an important type of SPS, are defined as linear dividing elements that separate pasture areas, cropped areas and some forest patches. This type of SPS is generally a conspicuous element in agricultural landscapes in tropical and subtropical regions. Growing commercially important timber species as living fences may help to increase land productivity while protecting natural resources, favouring diversity and storing additional amounts of carbon, thus also helping to offset CO_2 emissions from agricultural activities [4]. However, information about the abundance, distribution and functioning of these systems is scarce, and growth and yield models have not been significantly developed because the ecological and productive roles of living fences have generally been overlooked and poorly valued [5].

Little is known about the growth and yield of teak (*Tectona grandis* L.) planted as living fences in SPS or about the local ecological conditions required for use of this species as an SPS component. The growth equations generated in Costa Rica by Pérez and Kanninen [6] are used in Ecuador to estimate total volume of trees in teak plantations, despite being developed for regions with different edaphic, climatic and silvicultural conditions from those prevailing in Ecuador and not generally applicable to SPS because of differences in stand density and tree spacing, etc. As stands become denser, tree taper decreases and the height–diameter ratio increases because of the cumulative effects of competition from neighbouring trees [7]. Moreover, site index (SI) models for teak, especially when planted as living fences in SPS, are scarce in Ecuador. To our knowledge, SI models generated for an area of 600 ha in the lowlands of western Ecuador have only been published in conference proceedings [8].

Site index relates tree height or diameter to tree age and is used to evaluate tree growth and yield potential and to point out limits of usage of living fences. However, height growth is sensitive to incidents in the history of tree stands, such as origin (e.g., sprout or seed), initial suppression, changes in density, and interferences in the normal growth of the stand, e.g., animal, insect, frost damage, cutting, fire, and grazing. Moreover, the SI determination on the basis of height growth predicates nothing about these incidents [9,10].

SI curves should display certain properties [11,12], e.g., polymorphism, a sigmoid growth pattern with an inflection point, the capacity to reach a horizontal asymptote at advanced ages, a logical response (e.g., the dominant height should be zero at age zero and the curve must always increase), path invariance and base–age invariance [13–15]. Different methods are used to estimate SI, e.g., guide curve models and the generalized algebraic difference approach (GADA) [12]. The guide curve method assumes proportionality among curves of different SI. First, an average curve is modelled. A set of anamorphic or polymorphic SI curves can then be created [16]. However, GADA generates the best models because the base model has the above-mentioned curve properties, so that the families of curves obtained are more flexible than would otherwise occur [12,14,17–20].

The main objective of the present study was to develop provisional growth and yield models, including SI curves and volume models, for teak planted as living fences in the coastal lowlands of Ecuador. The null hypothesis assumes that there is no significant difference between the observed and the expected values in our models. Such models could be used as the basis for developing further silvicultural and economic studies for teak, as well as for identifying sites with suitable ecological conditions to improve the wood production of teak as living fences in the region.

2. Materials and Methods

2.1. Study Area

Teak (*Tectona grandis* L.) occurs naturally in India, Laos, Myanmar and Thailand [21]. The total area planted with this commercially very important tropical hardwood is about 4.35 million ha, distributed across at least 43 different countries [22]. Teak was first introduced to Ecuador more than 50 years ago. The province of Los Ríos was one of the niches where the species adapted and grew best and has therefore become the main source of seeds for establishing commercial plantations of this exotic timber species in the country [23]. In 2013, the Ministry of Agriculture, Livestock, Aquaculture and Fisheries (MAGAP) recommended the establishment of pure teak plantations in coastal and Amazonian Ecuador within a governmental programme of incentives for commercial purposes. However, progress in the suggested goals expressed as reforested area has been modest [24].

The study was carried out in the provinces of Santa Elena, Guayas Manabí, Los Ríos and Esmeraldas, on the west coast of Ecuador. Sampling plots were distributed in eleven SPS clusters (Figure 1). The SPS was characterised by living fences of teak planted in lines, with a spacing of 2.5 m, and the dominant forage species was *Megathyrsus maximus* (Jacq.) B. K. Simon & S. W. L. Jacobs a tufted perennial grass. Total annual precipitation in the study area ranges from 651 to 2646 mm and the mean annual temperature from 24 to 28 °C. The elevation ranges from 0 to about 500 m above sea level. The number of plots varied from 50–150 per province depending on the size of the living fences (Table 1).

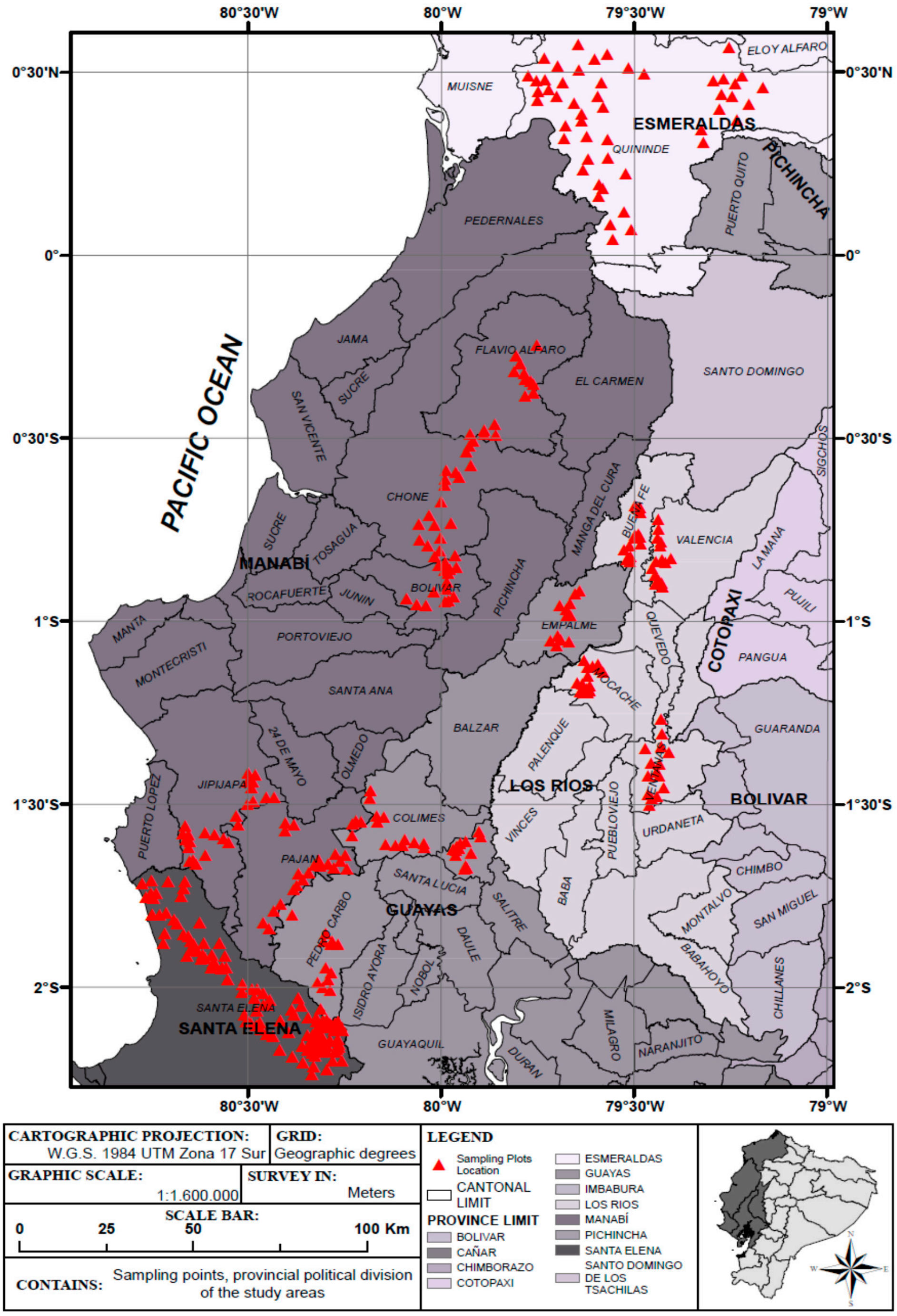

Figure 1. Geographic distribution of silvopastoral systems (SPS) and location of sampling plots for the use of teak as living fences in the coastal lowlands of Ecuador.

Table 1. Geographic location and general conditions of the silvopastoral systems under study in the coastal lowlands of Ecuador.

Province	Total Annual Precipitation (mm)	Mean Annual Temperature (°C)	Elevation (m a.s.l.)	Number of Plots
Manabí	798	24.6	20–300	50
Santa Elena	750	24.1	0–500	150
Guayas	1198	25.7	30–100	50
Los Ríos	2000	25.2	30–300	60
Esmeraldas	2646	25.6	190–300	50

2.2. Data Collection

The field data were collected in a total of 360 plots (50–300 m × 50–300 m size, Figure 2) in the five provinces under study in the coastal lowlands of Ecuador, in 2009, 2012 and 2016. Sixteen teak trees were randomly sampled in the rows in each SPS, thus providing data on a total of 64 trees per plot. The plot coordinates were recorded, along with establishment age, diameter at breast height (*DBH*) and total height (*H*) of each tree. *DBH* was measured with calliper and *H* was measured with a Haga hypsometer.

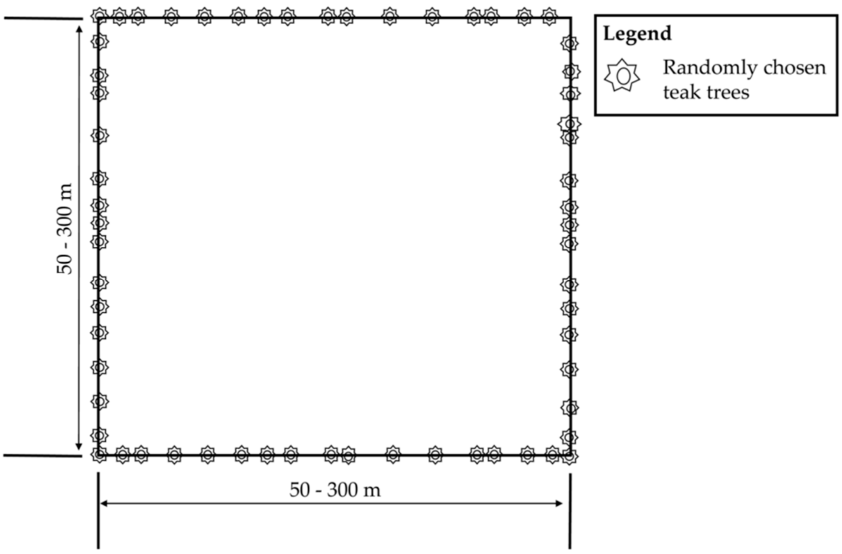

Figure 2. Experimental design of the sampling plots of living fences of teak planted in lines, with a spacing of 2.5 m, and the dominant forage species was *Megathyrsus maximus* (Jacq.) B. K. Simon & S. W. L. Jacobs in silvopastoral systems (SPS) in the coastal lowlands of Ecuador. Sixty-four teak trees per plot were randomly sampled.

2.3. Site Index

In order to determine the SI, tree age was determined by consulting the property records. The mean dominant height (H_d) and diameter (D_d) was calculated as the mean value of 30% of the tallest trees (i.e., 19 trees) for each sampling plot [25]. Three different models were used to develop the SI equations: the guide curve models developed by Chapman-Richards [26] (Equation (1)), by Hossfeld [27] (Equation (2)), and the generalized algebraic difference approach (GADA) [17] (Equations (3) and (4)) implemented in the R software (version 3.3.3) (R Foundation for Statistical Computing, Vienna, Austria):

$$H_d = a(1 - \exp^{(-bt)})^c \quad (1)$$

$$H_d = \frac{t^2}{a + bt + ct^2} \tag{2}$$

where H_d = dominant height (m); t = age (year); a, b, c = equation parameters.

The whole series of height–age data, with no differences between zones, was used to produce an equation describing the average pattern (i.e., the guide curve). Anamorphic curves were extracted from the guide curve, using the maximum height reached at 10 years as the reference age [28] for this study.

Models developed using the guide curve method were compared with the model obtained using the generalized algebraic difference approach (GADA) [12]. The GADA uses the Chapman-Richards equation (Equation (1)) as the base function for determining the site index. The parameters of the function selected are expressed as site functions defined by a variable X. X is a non-observable and independent variable that describes site productivity as the sum of different factors, including management, soil conditions, ecological and climatic factors and new parameters [19]. The initial bidimensional equation ($H_d = f(t)$) is expanded into an explicit tridimensional SI equation ($H_d = f(t, X)$), where X cannot be measured or defined. The GADA procedure involves determining the value of X from the initial site conditions, i.e., from the initial values of age and dominant height, t_0 and H_0 ($H_d = f(t, t_0, H_0)$). Thus, the model is implicitly defined and applicable in practice [18].

H_d represents the dominant height (m), t is the age in years, and a_1, a_2 and a_3 are the base model parameters; $b_1, b_2, \ldots, b_m$ are used as global parameters in subsequent GADA formulations. All GADA models have the general implicit form $H_d = f(t, t_0, H_0, b_1, b_2, \ldots, b_m)$, where Y is the value of the age function t and Y_0 is the reference variable defined by the value of the age function t_0. In order to derive the polymorphic model with multiple asymptotes from the Chapman-Richards model (Equation (1)), the parameters should be related to site productivity [18].

The following dynamic equation (Equation (3)) is obtained with GADA and provides polymorphic curves with multiple asymptotes.

$$H_1 = H_0 \left[\frac{1 - e^{-b_1 t_1}}{1 - e^{-b_1 t_0}} \right]^{(b_2 + b_3 / X_0)} \tag{3}$$

where H_0 is the dominant height at the initial age, t_0, and H_1 is the dominant height at age t_1. X_0 is derived from Equation (4).

$$X_0 = \frac{1}{2} \left\{ \ln H_0 - b_2 L_0 \pm \sqrt{[\ln H_0 - b_2 L_0]^2 - 4 b_3 L_0} \right\} \tag{4}$$

where $L_0 = \ln\left[1 - e^{-b_1 t_0}\right]$. Fitting this equation to real dominant height–age data enables estimation of the values of the global parameters b_1, b_2 and b_3. All of the families of curves obtained with the method of algebraic difference equations or their generalization are invariant in relation to the reference age and the simulation path [17,18,29].

Simultaneous fitting of the mean structure (given by the growth equation) and of the error structure (given by the autoregressive model) was carried out by GADA, implemented in the R software (version 3.3.3) (R Foundation for Statistical Computing, Vienna, Austria) [30]. The procedure was also used for *DBH* to generate the three mean dominant *DBH*—age models (Equations (1)–(4)).

Analysis of the model fitting performance for both H and *DBH* was based on comparison of graphs. The bias, the root mean square errors (RMSE) were calculated from the residuals obtained during the fitting stage. Graphical analysis was carried out to show that the curves fit the data across the whole range, by (1) overlaying the fitted curves on the trajectories of observed heights over time; (2) plotting the residuals against the values predicted by the model; and (3) analyzing the changes in bias and RMSE for the different age classes.

2.4. Tree Volume Assessment

A total of 760 dominant trees in the above-mentioned 360 sample plots of living fences (2–3 trees per plot) were randomly chosen for sampling. The *DBH* (mean 37.4 cm; range 5.0–76.4 cm) was measured before and *H* (mean 24.9 m; range 5.0–33.3 m) was measured after felling the trees. A diameter tape was used to measure over bark diameter (d_i) at ground level and at different heights: 0.3 m, 2.3 m, and every 2.0 m along the stem up to the top. The total tree volume overbark of every tree (V, m^3) was computed by measuring the diameter at each end of the section (d_i and d_{i+1}) and the length of sections (l) of felled specimens, by applying the following formula for the frustum:

$$V = \frac{l\pi}{3}\left(\left(\frac{d_i}{2}\right)^2 + \frac{(d_i d_{i+1})}{4} + \left(\frac{d_{i+1}}{2}\right)^2\right) \quad (5)$$

Several commonly used volume estimation models [31] were tested in the study to find the best regression model between *V* and *BHD* and *H* (Table 2); *a*, *b*, *c* and *d* are the parameters to be determined. The models were fitted using the generalized method of moments (GMM) implemented in SAS/ETS® which accurately estimates parameters under heteroscedastic conditions [32]. The mean squared error (MSE), standard error (SE) and adjusted determination coefficient ($R^2{}_{Adj}$) were used to estimate the goodness of fit of the volume models. The model with the smallest Akaike information criterion (AIC) was considered as the most appropriate [33].

Table 2. Models tested for fitting volume equations to diameter at breast height (DBH, cm) and total tree height (H, m) data of teak trees in the study.

Model	Expression	
Schumacher-Hall (allometric) [34]	$V = a \cdot DBH^b \cdot H^c$	(6)
Spurr [35]	$V = a \cdot DBH^2 \cdot H$	(7)
Spurr potential [35]	$V = a \cdot (DBH \cdot H)^b$	(8)
Spurr with independent term [35]	$V = a + b \cdot DBH^2 \cdot H$	(9)
Incomplete generalized combined variable [36]	$V = a + b \cdot H + c \cdot DBH^2 H$	(10)
Australian formula [37]	$V = a + b \cdot DBH^2 + c \cdot H + d \cdot DBH^2 \cdot H$	(11)
Honer [38]	$V = DBH^2/(a + b/H)$	(12)
Newnham [39]	$V = a + b \cdot DBH^c \cdot H^d$	(13)

2.5. Teak Production in SPS

The data used to determine wood production for teak grown as living fences corresponded to the results obtained in the previous steps. Volume per hectare (V_{ha}) was calculated as the product of the number of trees per hectare (N) and the modelled mean volume of tree per age and SI (V_i):

$$V_{ha} = NV_i \quad (14)$$

The model of mean annual increment (*MAI*) in V_{ha} was established as V_{ha} at harvesting divided by the stand age (t) at rotation length:

$$MAI = \frac{V_{ha}}{t} \quad (15)$$

The model of periodic annual increment (*PAI*) was defined as the change in *V* between the beginning and end of a growth period, divided by the number of years. V_1 is the volume per hectare ($V_{ha,1}$) at time one, and $V_{ha,2}$ the volume per hectare at time two and t_1 corresponds to the year starting the growth period, and t_2 to the end year.

$$PAI = \frac{V_{ha,2} - V_{ha,1}}{t_2 - t_1} \quad (16)$$

The values of *MAI* and *PAI* were used to estimate the biological (optimal) rotation age (when *PAI* and *MAI* are equal and *MAI* is maximal).

3. Results

3.1. Site Index

The p values for all SI models indicate significant relationships between dominant height and diameter, but the RMSE was higher for the GADA model (Equations (3) and (4)) than for the models derived from the Chapman-Richards (Equation (1)) and Hossfeld function (Equation (2); Table 3). The bias and the RMSE of the GADA model residuals for height varied less than those of the Chapman-Richards and Hossfeld models for all age classes included in the sample, but were similar for *DBH* (Figure 3).

Table 3. Estimated values of parameters, p values and goodness-of-fit statistics for the three mean dominant height (m)—age and diameter at breast height (*DBH*, cm)—age models for teak planted as living fences.

Variables	Base Model	Variable	Estimated Value	Standard Error (SE)	p Value	Root Mean Square Error
Height–age	Equation (1)	a_1	34.912	4.916	<0.0001	3.64
		a_2	0.044	0.016	0.0049	
		a_3	0.887	0.102	<0.0001	
	Equation (2)	a_1	−1.075	0.094	<0.0001	3.95
		a_2	0.748	0.038	<0.0001	
		a_3	0.012	0.002	<0.0001	
	Equation (3)	b_1	0.122	0.004	<0.0001	6.09
		b_2	−18.429	0.757	<0.0001	
		b_3	67.262	2.55	<0.0001	
DBH–age	Equation (1)	a_1	53.657	10.522	<0.0001	6.88
		a_2	0.043	0.019	0.028	
		a_3	0.956	0.141	<0.0001	
	Equation (2)	a_1	−0.778	0.085	<0.0001	7.18
		a_2	0.556	0.034	<0.0001	
		a_3	0.006	0.002	<0.0001	
	Equation (3)	b_1	0.115	0.012	<0.0001	9.75
		b_2	−16.173	0.812	<0.0001	
		b_3	65.855	3.121	<0.0001	

The Hossfeld and Chapman-Richards models of SI maintained the same height proportion at different ages, and therefore the curves appear to have the same form. This led to underestimation of tree height at young ages and overestimation at older ages. On the other hand, the dynamic GADA method always shows a skewed distribution around the zero line and a more stable RMSE for both height and diameter.

The GADA models developed for diameter and height estimation provided good fits. Both models explained approximately 99% of the total variance and residuals were randomly distributed around zero, with homogeneous variance and no obvious trends.

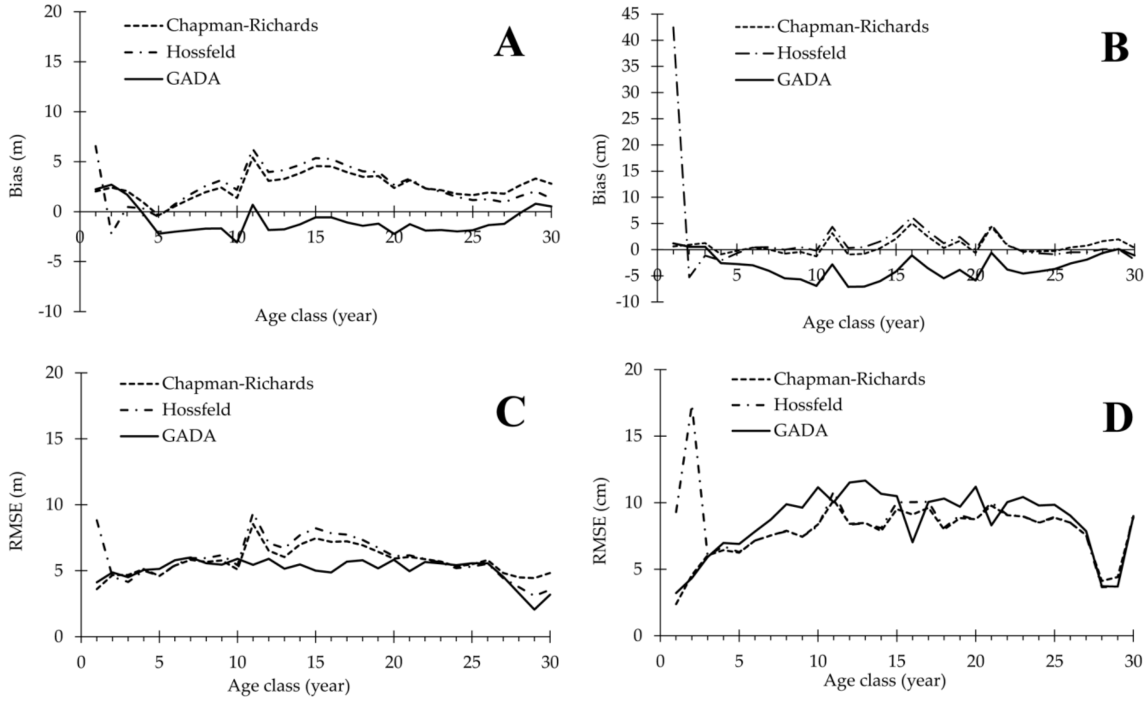

Figure 3. Bias and root mean square error (RMSE) for the mean dominant height and diameter at breast height (*DBH*) predictions yielded by the GADA (generalized algebraic difference approach) formulation (of the Chapman-Richards model) and by the Chapman-Richards and Hossfeld models ((**A**,**C**) for height and (**B**,**D**) for *DBH*).

In the GADA formulation of the Chapman-Richards base model (Equation (1)), parameters a_1 and a_3 are assumed to depend on site productivity, and the error structure is included by an interactive procedure:

Height

$$H_1 = H_0\left[\frac{1-e^{-0.122334t_1}}{1-e^{-0.122334t_0}}\right]^{(-18.428972+67.262380/X)} \tag{17}$$

Diameter

$$DBH_1 = DBH_0\left[\frac{1-e^{-0.114811t_1}}{1-e^{-0.114811t_0}}\right]^{(-16.17381+65.85742/X)} \tag{18}$$

where H_1 is the predicted height (m) at age t_1 (years), and H_0 and t_0 represent the initial dominant height and age.

Height

$$X = \frac{1}{2}\left\{\ln H_0 + 18.42897\ L_0 \pm \sqrt{[\ln H_0 + 18.42897\ L_0]^2 - 269.04952\ L_0}\right\}$$
$$L_0 = \ln\left[1 - e^{(-0.12233\ t_0)}\right]$$

Diameter

$$X = \frac{1}{2}\left\{\ln DBH_0 + 16.17381\ L_0 \pm \sqrt{[\ln DBH_0 + 16.17381\ L_0]^2 - 263.42968\ L_0}\right\}$$
$$L_0 = \ln\left[1 - e^{(-0.11481\ t_0)}\right]$$

The GADA model was used to fit SI curves for *H* (from 8.7 to 22.7 m) and *DBH* (from 10.0 to 35.0 cm) at a reference age of 10 years. The fitted curves follow the same trends (Figure 4).

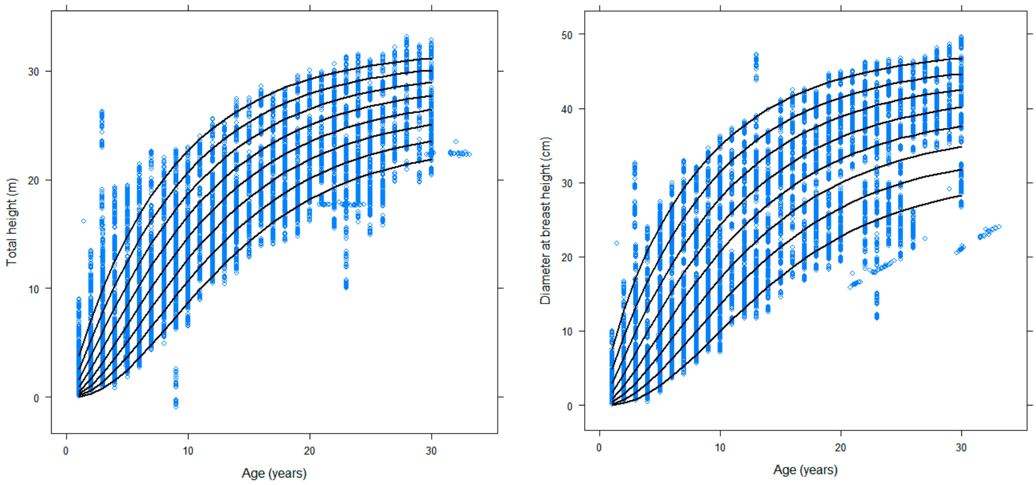

Figure 4. Curves fitted with the GADA model for the relationship between age and mean dominant total height and mean dominant diameter at breast height during development of growth models for *Tectona grandis* in living fences in coastal lowlands of Ecuador.

3.2. Volume Models

All volume models tested showed good fits to the data, with adjusted R^2 above 0.989 (Table 4). The tested models for volume estimation are ordered from the lowest to highest AIC values. Using this criterion, the best model was the Newnham model (Equation (13)) (Table 4), expressed as follows:

$$V = -0.02444 + 0.000054 \cdot DBH^{1.886436} \cdot H^{0.996266} \quad (19)$$

where V is the total tree volume of teak in m^3, for DBH over bark of 5 cm or more, and H is the total height (m). The variation in this parameter is due to the diverse ages of living fence plantations, as well as to competition, growth conditions, crown size and other factors.

Table 4. Goodness of fit of the models developed for determining the volume of teak in living fences in the coastal lowlands of Ecuador.

Model	MSE	R^2_{Adj}	Parameter	Value	SE	AIC
Newnham	0.00289	0.9987	a b c d	−0.024 0 1.886 0.996	0.00361 <0.0001 0.008 0.0148	−1921
Schumacher-Hall (allometric)	0.00307	0.9986	a b c	0 1.912 1.040	<0.0001 0.006 0.014	−1902
Australian formula	0.00333	0.9985	a b c d	−0.114 0 0.007 0	0.012 0 0.001 <0.0001	−1873
Incomplete generalized combined variable	0.00349	0.9984	a b c	−0.083 0.006 0	0.009 0.001 <0.0001	−1859
Spurr with independent term	0.00407	0.9982	a b	0.044 0	0.004 <0.0001	−1811
Honer	0.00485	0.9978	a b	125.853 25,812.210	13.329 375.400	−1753
Spurr	0.00624	0.9972	a	0	<0.0001	−1672
Spurr potential	0.02420	0.9892	a b	0 1.662	<0.0001 0.009	−1222

MSE = mean squared error; R^2_{Adj} = adjusted determination coefficient; SE = standard error; AIC = Akaike information criterion.

3.3. Mean Annual Increment in Volume

The curve for mean annual increment over time can be constructed for each SI obtained for the total volume within the study area (Figure 5), considering 160 trees per hectare within the SPS under study. The maximum *MAI* varied from 3 m^3 for an SI of 14.7 m, to 15 m^3 for an SI of 22.7 m. The age at which the maximum *MAI* occurs varies with site index, reaching higher values at early ages. In the most productive sites, the maximum *MAI* in volume occurred at 15 years, whereas in the least productive sites, the rotation age extended to 26 years for teak in these SPS.

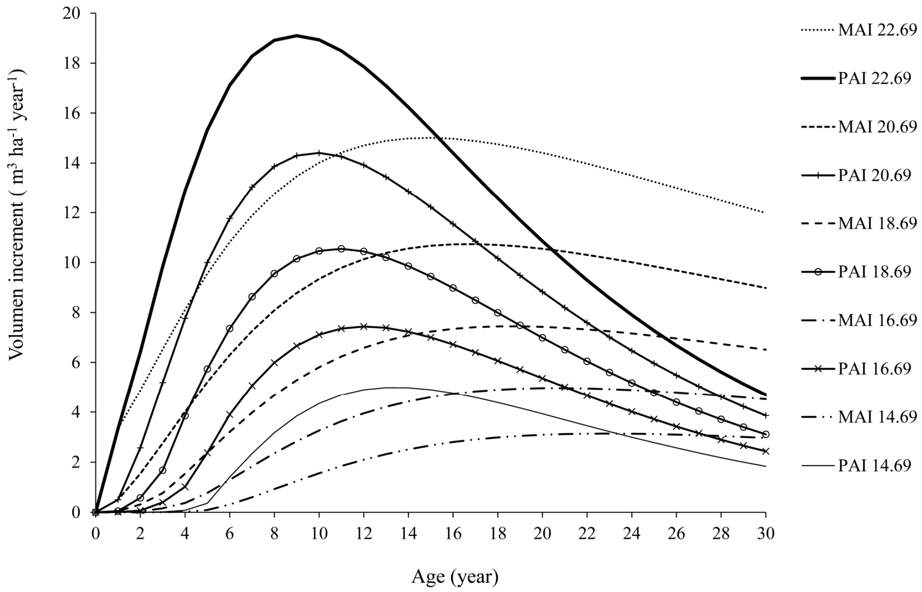

Figure 5. Mean annual increment (*MAI*) per hectare for different site indices considering a density of 160 teak trees ha^{-1} planted as living fences in SPS. The dashed line represents the periodic annual increment (*PAI*) per hectare. The solid line indicates the *MAI*. The local maximum *MAI* can be considered the optimal biological rotation age for each SI class.

4. Discussion and Conclusions

4.1. Height Projection Function and SI Classes

Several authors have used linear and log–log models to determine site indexes (SI) for teak grown in different parts of the world [40,41]. In these studies, height growth estimates did not show any limits (i.e., the SI models were asymptotic). The tree height growth at young ages in Costa Rica was higher than observed in the other studies [42–44]. Bermejo et al. [42] reported that the anamorphic form of the Hossfeld function was the best model for describing SI for teak growing in Costa Rica. Pérez and Kanninen [45] constructed anamorphic forms of the Richards function for diameter and height growth in teak plantations as a solution to the lack of sufficient data for stratification of soil, land and climate factors, which prevented construction of polymorphic curves.

The asymptotic value for height growth rate at young ages was fairly high, although this did not appear to have serious consequences for the quality of the GADA-derived predictions. Moreover, the curves appear reliable beyond the rotation age, as indicated by the site basal area values estimated from the trajectories of values observed over time (Figure 4). The main advantage of the GADA, introduced by Cieszewski and Bailey [12], is that the base equation can be expanded in accordance with the growth rate and the asymptote, so that more than one parameter in each model will depend on site quality, and the corresponding family of curves will therefore be more flexible [17–19]. This generalization enables families of curves that are both polymorphic and have multiple asymptotes to be generated [18,46].

In this study, the SI ranged from 15 m to 23 m in the most productive sites (Los Ríos province of Ecuador) at the reference age of 10 years. Bermejo et al. [42] reported SI from 19 to 23 m at a base age of 10 years for conventional teak plantations in the province of Guanacaste in Costa Rica, while Pérez and Kanninen [45] established an SI of 23 m at 20 years for teak in Costa Rica. Somarriba et al. [47] reported an SI of 23 m at nine years in teak planted in lines. Thus, the locations with SI for height below 17 m in our study may be considered as having marginal ecological conditions for growing teak for economic purposes.

4.2. Volume Equations for Teak in SPS

It has been suggested that when tree crowns compete for light, tree growth tends to focus on height, whereas crown growth and stem diameter increment appear to be more important in isolated trees [25]. Compared with conventional forestry, SPS are characterized by a low density of trees. The concept of a dominant crown is therefore not applicable, as the spacing reduces competition, generally resulting in a single layer of dominant and codominant trees [48]. Under these conditions, the stem form is more strongly influenced by low tree density than by the dominant height.

The volume equation developed in the present study for teak was fitted using a generalized method of moments to obtain good parameter estimates under heteroscedastic conditions, even without estimating the variances of the heteroscedastic errors. Tewari et al. [20] obtained similar results for calculating volumes of teak grown in India. Consideration of heteroscedastic conditions provided superior results to those obtained for teak in earlier studies [8,44,45,49,50]. These authors used polynomial methods (classic minimum squares methods) and non-linear regression techniques (Marquard's method) to establish the volume of teak and selected the best fit volume models (Schumacher-Hall) in accordance with statistics such as the coefficient of determination (R^2) and the statistical significance of the estimated parameters, without considering heteroscedastic conditions.

4.3. Teak Production in SPS

The rotation length for teak grown as living fences in Ecuador has not yet been defined, and trees are generally felled according to demand. In the present study, the biological rotation age was estimated to be 15 years for the most productive sites and 26 years for the least productive sites, for teak grown as living fences. These results show lower growth rates and productivity than those obtained by Somarriba et al. [47] in Costa Rica and Panama, establishing a 9-year harvesting cycle for line-planted teak for sites of intermediate to high productivity and an 11-year cycle for sites with low productivity potential. The maximum values of *PAI* and *MAI* for height were reached at a younger age (5 years) and the maximum value of the *PAI* for volume was reached at 8 years. Similar results have been reported by Bermejo et al. [42] for conventional teak plantations in Costa Rica under conditions of annual mean temperature 26–29 °C and precipitation of 1800–2450 mm year^{-1}. However, the teak trees in the study area are usually felled at *H* 30 m and *DBH* 50 cm, i.e., an approximate age of 30 years in the best SPS sites in the coastal lowlands of Ecuador.

The final production in the SPS sites under study in the coastal lowlands of Ecuador was 225 m^3 ha^{-1} at 15 years in the most productive sites and 35 m^3 ha^{-1} at 26 years in the least productive sites. Somarriba et al. [47] established a rotation length of 9 years with a yield of 215 m^3 ha^{-1} for teak grown by the line-planting method in Panama and Costa Rica. The results of the present study and those obtained by Somarriba et al. [47] can be compared with reference values established for teak grown in Panama, taking into account the prevailing conditions, i.e., mean annual precipitation of 3000–3500 mm year^{-1}, mean annual temperature of 26.7 °C and a dry period of 3–4 months (January–April). Griess and Knoke [51] calculated a yield of 250 m^3 ha^{-1} at 30 years. Quintero et al. [52] reported volume yields between 193 – 337 m^3 ha^{-1} for teak in Colombia at 60 years, and Stefanski et al. [50] presented yields of 67.5–88.6 m^3 ha^{-1} at 30 years. The *MAI* for the best SI in the study area was 15.3 m^3 ha^{-1} year^{-1} at an age of 15 years for the SPS and a density of 160 trees ha^{-1}. For 6 years old teak plantations, growth rates of 27.8 m^3 ha^{-1} year^{-1} have been recorded in Colombia [53] and between 3.4 and 11.5 m^3 ha^{-1} year^{-1} in Ivory Coast [54].

The information obtained in the present study regarding silviculture of the forest component of SPS in the coastal lowlands of Ecuador highlights the need to improve the management of these systems by creating local opportunities and strengthening silvicultural practices on small farms. For example, Midgley et al. [55] recommended carrying out field demonstrations to show small farmers the benefits of various different silvicultural practices, and Newby et al. [56] and Roshetko et al. [57] highlighted the need to support small farmers to encourage the adoption of silvopastoral systems for growing teak.

After model verification by independent samples, the site index model presented in the present study may contribute to the management of teak plantations in the coastal lowlands of Ecuador. For a base age of 10 years, height-based SI of 9 to 23 m, and diameter-based SI of 10 to 35 cm were established. De Sousa et al. [58] noted that the net value of the timber sold represents between 11% and 49% of the total income in agroforestry systems. However, this amount could be increased to 58% if the farmers were able to improve management of the forest component of agrosilvopastoral systems [5].

In summary, landowners can gain economic benefits from teak timber production, thereby also contributing to the economic status of farmers in the study area. Teak trees in areas with higher SI can be used as saw wood, whereas trees from SPS with lowest SI (9 m at 10 years) found in this study may still provide poles and fire wood for local use. MAGAP has not included planting trees in SPS in their development plans. However, public agencies and non-governmental organizations recognise that the production of merchantable timber may also provide multiple benefits associated with the intensification of land use, as in farming within SPS. Despite the good model fits to our data, we speculate that these models could get more resolution with dendroecological data using individual tree models based in other studies [10,15]. We recommend using more variables in the future that could help to better understand tree growth in response to competition or spatial position [10].

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