

The HIV-1 Nucleocapsid Regulates Its Own Condensation by Phase-Separated Activity-Enhancing Sequestration of the Viral Protease during Maturation

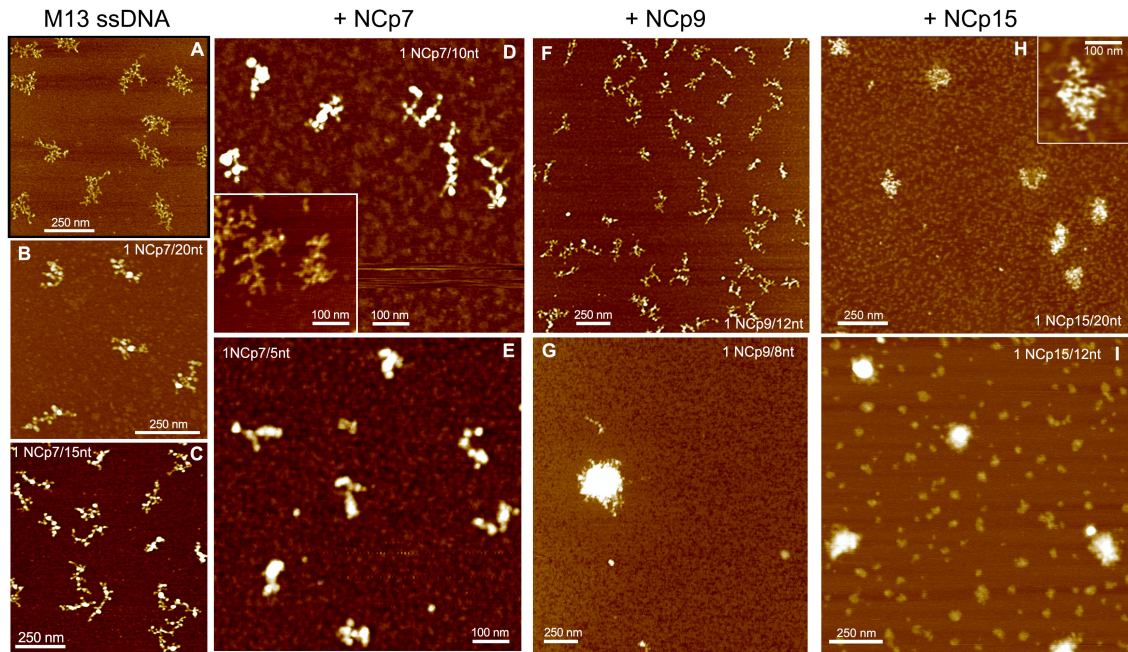
Supplementary Materials

Sébastien Lyonnais^{1,2,*,#}, S. Kashif Sadiq^{3,4,5,*,#}, Cristina Lorca-Oró¹, Laure Dufau⁶, Sara Nieto-Marquez¹, Tuixent Escribà¹, Natalia Gabrielli¹, Xiao Tan^{1,6}, Mohamed Ouizougoun-Oubari¹, Josephine Okoronkwo¹, Michèle Reboud-Ravaux⁶, José Maria Gatell^{1,7}, Roland Marquet⁸, Jean-Christophe Paillart⁸, Andreas Meyerhans^{3,9}, Carine Tisné¹⁰, Robert J. Gorelick¹¹ and Gilles Mirambeau^{1, 12,*,#}

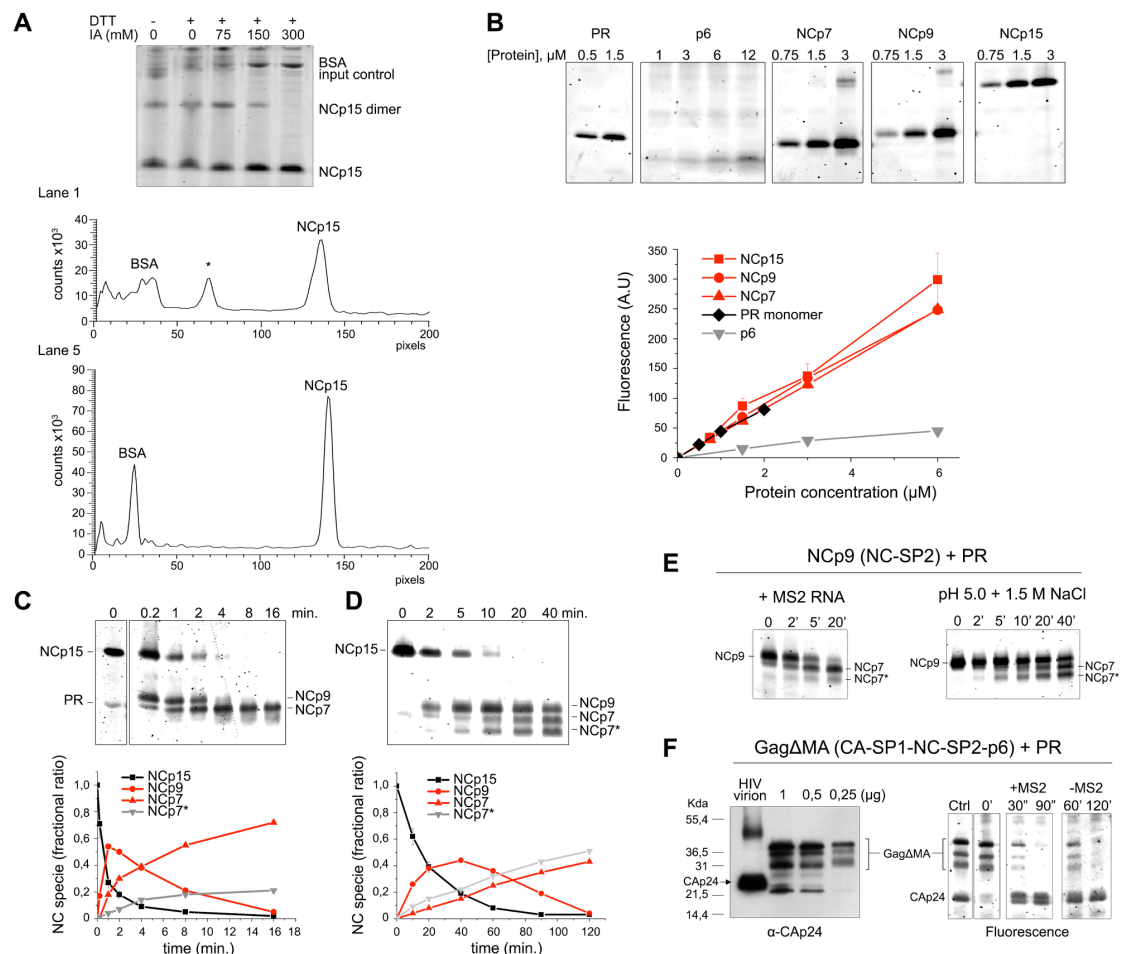
- 1 Infectious Disease & AIDS Research Unit, Institut d'Investigacions Biomèdiques August Pi i Sunyer (IDIBAPS), Villarroel 170, 08036 Barcelona, Spain;
- 2 Centre d'Etudes des Maladies Infectieuses et Pharmacologie Anti-Infectieuse (CEMIPAI), CNRS UAR 3725, Université de Montpellier, 1919 Route de Mende, 34293 Montpellier Cedex 05, France;
- 3 Infection Biology Laboratory, Department of Experimental and Health Sciences (DCEXS), Universitat Pompeu Fabra, Carrer Doctor Aiguader 88, 08003 Barcelona, Spain;
- 4 Molecular and Cellular Modeling Group, Heidelberg Institute for Theoretical Studies (HITS), Schloss-Wolfsbrunnenweg 35, 69118 Heidelberg, Germany;
- 5 Genome Biology Unit, European Molecular Biology Laboratory, Meyerhofstrasse 1, 69117 Heidelberg, Germany;
- 6 Biological Adaptation and Ageing (B2A), CNRS UMR 8256 & INSERM ERL U1164, Institut de Biologie Paris-Seine (IBPS), Faculté des Sciences et d'Ingénierie (FSI), Sorbonne Université, 7 Quai St Bernard, 75252 Paris Cedex 05, France;
- 7 Facultat de Medicina y Ciencias de la Salud, Universitat de Barcelona, Carrer de Casanova 143, 08036 Barcelona, Spain;
- 8 Architecture et Réactivité de l'ARN, CNRS UPR 9002, Université de Strasbourg, 2 Allée Conrad Roentgen, 67000 Strasbourg, France;
- 9 Institució Catalana de Recerca i Estudis Avançats (ICREA), Passeig de Lluís Companys 23, 08010 Barcelona, Spain;
- 10 Expression Génétique Microbienne, CNRS UMR 8261, Institut de Biologie Physico-Chimique (IBPC), Université de Paris, 13 Rue Pierre et Marie Curie, 75005 Paris, France;
- 11 AIDS and Cancer Virus Program, Leidos Biomedical Research, Inc., Frederick National Laboratory for Cancer Research, Frederick, MD 21701, USA;
- 12 Biologie Intégrative des Organismes Marins (BIOM), CNRS UMR 7232, Observatoire Océanologique de Banyuls (OOB), Faculté des Sciences et d'Ingénierie (FSI), Sorbonne Université, 1 Avenue Pierre Fabre, 66650 Banyuls-sur-Mer, France

[#] These authors contributed equally.

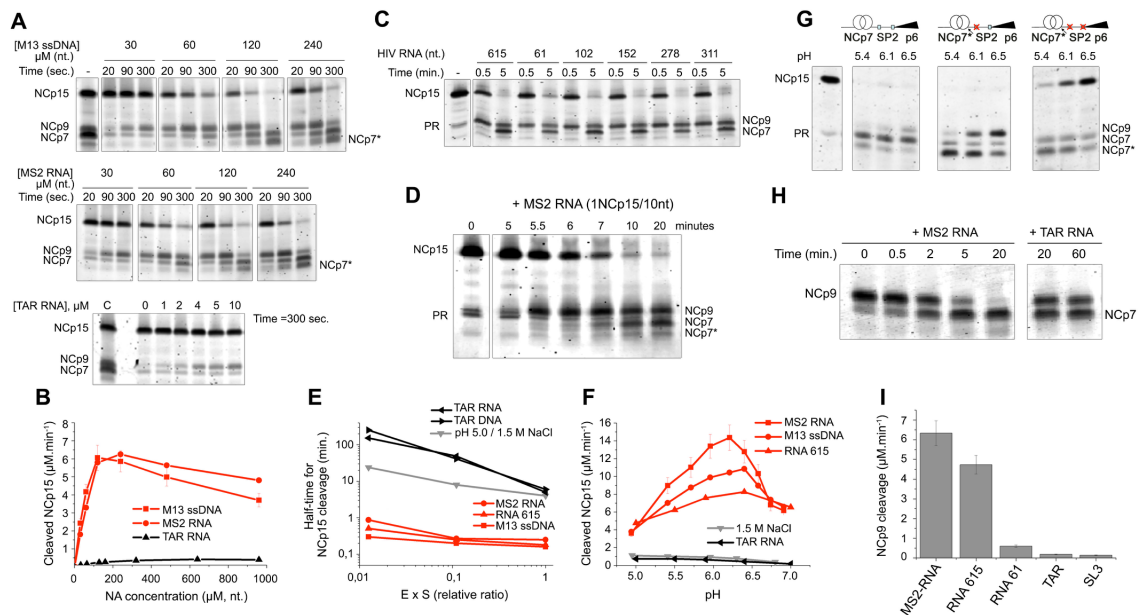
* Correspondence: sebastien.lyonnais@cemipai.cnrs.fr, kashif.sadiq@embl.de, gilles.mirambeau@sorbonne-universite.fr



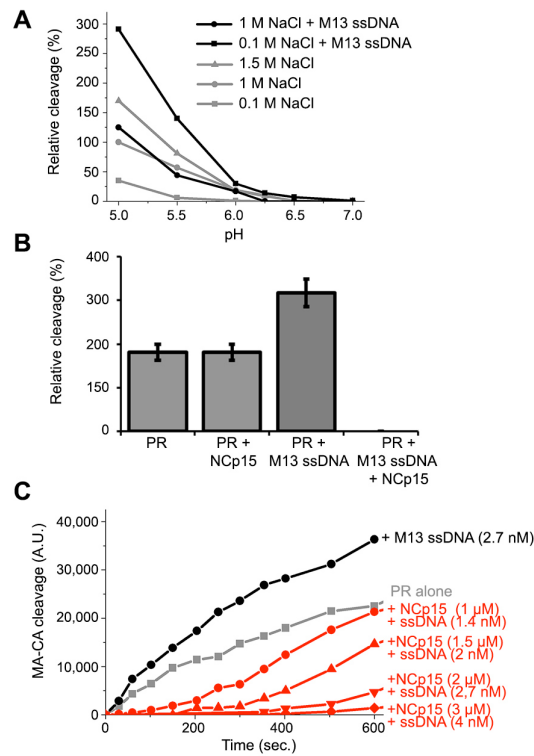
Supplementary Figure 2. Binding, condensation or aggregation of M13 ssDNA by NCp7, NCp9 and NCp15 followed by AFM imaging (A) free M13 ssDNA. (B-E) Binding and progressive condensation of M13 ssDNA for 1 NCp7/20 nt (B); 1 NCp7/15 nt (C); 1 NCp7/10 nt (D), 1 NCp7/5nt. The insert in (D) is the free ssDNA at the same scale to appreciate the compaction and the melting of secondary structures by NCp7. Maximum compaction is observed in (E). (F) Field of individual and condensed NP complexes obtained at 1 NCp9/12 nt in 5 mM magnesium show NP complexes comparable to those obtained with NCp7. (G) At 1 NCp9/8 nt, highly dense spheroids are formed, containing thousands of NCp9:ssDNA NP condensates joined together, while the mica surface appeared completely empty of individual complexes. (H-I) M13-ssDNA in complex with NCp15 on M13 ssDNA (1/20 nt. and 1/12 nt., respectively) show more passive NA binding without the DNA backbone bridging characteristic of NCp7 and NCp9 binding.



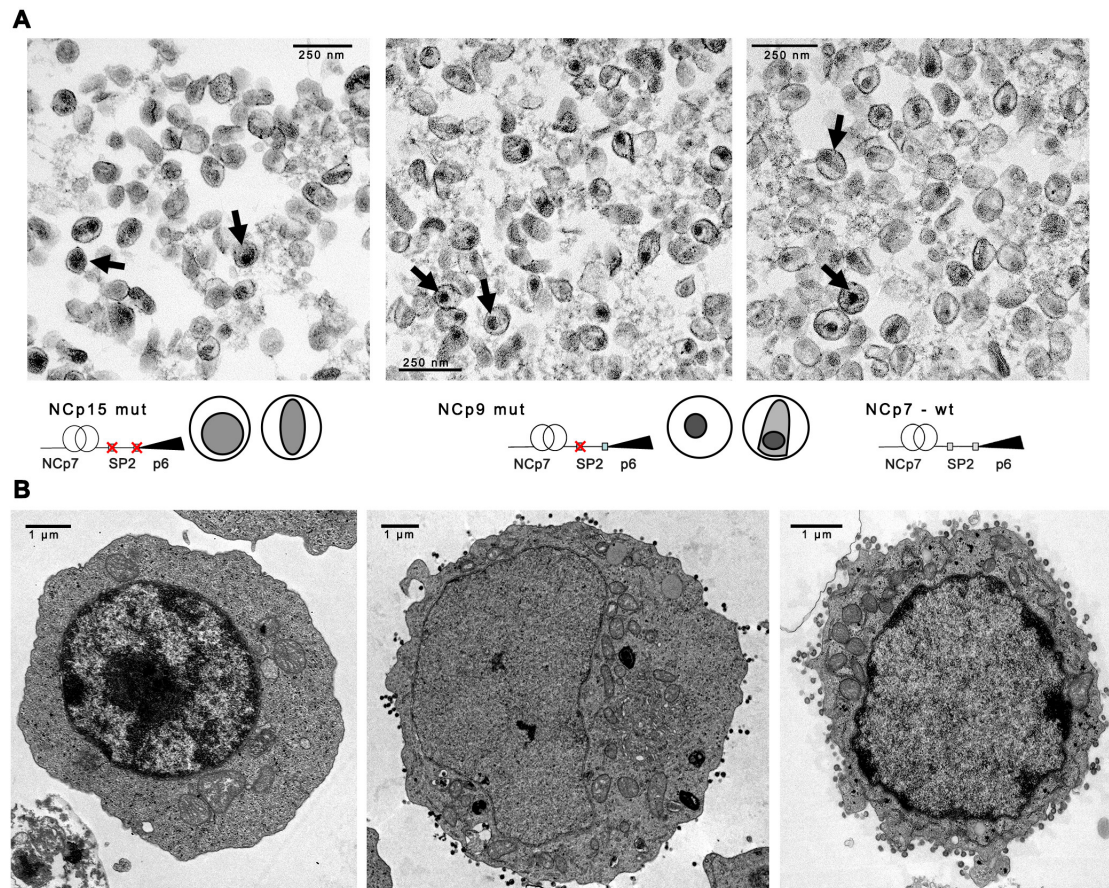
Supplementary Figure 3. The PR-driven cleavage of NCp *in vitro* is strongly activated by NA. (A) NCp15 separation by SDS-PAGE and fluorescent staining enables precise quantification of NC species. Boiling in presence of 0.1M DTT and Iodoacetamide (IA) resolved issues of NCp15 oxidization into dimers and increased the fluorescence intensity signal. Example densitograms are shown for lane 1 and lane 5. **(B)** SDS-PAGE separation of recombinant PR, p6, NCp7, NCp9 and NCp15. Fluorescence intensity was reported as a function of protein amounts and shows a linear signal enabling accurate protein quantification, augmented by the low sensitivity of p6 to the dye. The basic NCp7 migrates around the position of PR monomer. **(C-D)** Kinetics of NCp15 (6 μ M) cleavage in presence of limiting amounts of PR (0.6 μ M) show a strong NA-dependent activation at pH 6.25/0.1M NaCl **(C)** as compared with enzyme optimum at pH 5.0 / 1.5 M NaCl **(D)**. **(E)** NC-SP2 (NCp9, 6 μ M) cleavage by PR in presence of MS2 RNA (120 μ M, nt.) at pH 6.25 and 0.1M NaCl (left panel) or without NA at pH 5.0 in 1.5 M NaCl. **(F)** SP1-NC cleavage by PR in a Gag Δ MA construct. Left panel: Western blot with anti-Cap24 antibodies. The p6-containing Gag Δ MA protein was very difficult to produce and contained several discrete species. Right panel: products of cleavage (substrate 6 μ M) for the indicated times in presence or absence of MS2 RNA (120 μ M, nt.) at pH 6.25 and 0.1M NaCl. Despite substrate heterogeneity, we followed the discrete formation of 24-25 kDa CA-SP1/CA products that clearly reveal SP1-NC cleavage to be strongly activated in the presence of MS2 RNA.



Supplementary Figure 4. PR activation of NC proteolysis is modulated by NA length and NC:NA interactions. (A-B) Kinetics of NCp15 cleavage by PR for the indicated times at pH 6.25 and 0.1M NaCl in presence of increasing concentration of M13 ssDNA (A, top), MS2 RNA (A, middle) or TAR RNA stem-loop (A, bottom). In (B) the apparent reaction rates from the SDS-PAGE analysis are plotted against NA concentration. (C) NCp15 cleavage by PR for the indicated times at pH 6.25 and 0.1M NaCl in presence of HIV-1 genomic RNA fragments produced by in vitro transcription. (D) In an environment unfavorable for PR dimer stability (0.1 M NaCl, pH 6.25), NCp15 proteolysis is immediately activated after addition of MS2 RNA to a mixture containing both NCp15 and PR pre-incubated for 5 min. (RNCp15/nt=1/10). (E) The crowding effect of the NP complex formed between NCp15 and long ssNA sequesters PR. The half-time for NCp15 cleavage is plotted against the dilution of NCp15 and PR for a fixed NCp15:PR ratio of 10, in presence of M13 ssDNA, MS2 RNA, HIV-1 615-nt RNA, TAR RNA and cTAR DNA. A control reaction, performed at pH 5.0 / 1.5 M NaCl, is also plotted. (F) The rate of SP2-p6 cleavage was followed as a function of pH and shows a maximum for pH 6.25-6.5 in presence of long ssNA in comparison with TAR RNA or high salts. (G) SDS-PAGE analysis of Wt NCp15 and NCp15 with altered SP2 or SP2 + p6 cleavage sites, incubated with PR for 10 min in 0.1M NaCl and at the indicated pH. The additional cleavage after amino acid 49 is indicated as NCp7*. (H) Kinetics of NCp9 (NC-SP2) cleavage by PR at pH 6.25 and 0.1M NaCl in presence of MS2 RNA or TAR RNA (RNCp/nt=1/20 nt.). (I) Rate of NCp9 cleavage by PR at pH 6.25 and 0.1 M NaCl in presence of MS2 RNA, (1-615) HIV RNA, TAR and SL3 stem-loops RNA.



Supplementary Figure 5. PR is sequestered by the NCp15:ssDNA NP complex. (A) Cleavage of a DABCYL-MA-CA-EDANS (5.2 μ M) peptide by HIV-1 PR (50 nM) at 30°C followed by FRET in the presence or absence of M13 ssDNA (4 nM) in low (0.1M NaCl) or high (1 M NaCl) salts, as a function of pH. (B) Relative cleavage of the MA-CA probe by PR at pH 5.5 in 0.1 M NaCl for 5 min., in presence of NCp15 (3 mM), M13 ssDNA (4 nM) or M13 ssDNA+NCp15 (3 mM and 4 nM, respectively) . (C) MA-CA cleavage at pH 6.0 in 0.1 M NaCl in presence of increasing concentration of NCp15:ss M13 NP complex (1 NCp15/10 nt with the indicated concentrations).



Supplementary Figure 6. Nucleocapsid condensation is concomitant with budding. (A) TEM micrographs of representative fields of free HIV-1NL4-3 particles accumulating NCp15, NCp9 or NCp7 (wt virus), which show defects in the nucleocapsid condensation for NCp15-containing viruses. The schemes highlight the difference between diffuse cores (examples indicated by arrows in NCp15 image) and dense cores (examples indicated by arrows in NCp9 and NCp7 images). (B) Multiple washing and fast fixation with glutaraldehyde concentrate virus particles at the plasma membrane of latently infected ACH2 cells (left), producing mature (middle) or immature (right) viruses after 48h activation with Vorinostat.

Parameter	Value		
	Case1: PR + MACA	Case2: PR + MACA M13	Case3: PR + MACA NCp15 + M13
<i>Initial concentrations and NA size</i>			
n_l	-	7500	7500
$[n_l]$ (μM)	-	20.0	20.0
$[S_1^T]_0 ([MACA^T]_0)$ (μM)	3.0	3.0	3.0
$[S_2^T]_0 ([NCp15^T]_0)$ (μM)	-	-	2.0
$[E_{1/2}^T]_0 ([PR_{1/2}^T]_0)$ (μM)	0.1	0.1	0.1
<i>Enzyme decay</i>			
K_D (nM)	14.8	14.8	14.8
K_R (μM)	2.0	2.0	2.0
k_{cat}^* (s^{-1})	0.6	0.01	0.01
<i>Enzyme RNP absorption equilibrium</i>			
$K_E^0 (K_{PR}^0)$	0.1	0.1	0.1
c_{crit}	3	3	3
ξ	1.2	1.2	1.2
f	3	3	3
<i>Substrate RNP absorption equilibrium</i>			
$K_{S1} (K_{MACA})$	0.3	0.3	0.3
$K_{S2} (K_{NCp15})$	100	100	100

Supplementary Table 1. Parameters for RNP-modulated two-substrate kinetic model.

Supplementary Note: A theoretical model of RNP-modulated enzyme-substrate reaction kinetics

1: Theoretical Development

In order to account for the two phenomena of acceleration and sequestration observed in our *in vitro* biochemical assays (Figure 3 and Supplementary Figure 4), we developed a theoretical model of RNP-modulated enzyme kinetics. The model consists of a mass-action reaction kinetics approach to follow the competitive processing of two substrates (S1 and S2) by an enzyme (E) in the presence of NA, taking into account equilibrium absorption of each species into the volume pervaded by the NA. It thus enables us to follow the reactions in both the NA pervaded and unpervaded volumes individually whilst relating the transmission of each species across the two domains. We also derive an expression for the relation of the enzyme absorption equilibrium in terms of the experimentally observed phase change in reaction rate upon increased NA length (Figure 3B) in the non-competitive regime. Importantly, the enzyme absorption equilibrium constant is non-linearly dependent on the contiguity of non-specifically bound substrate to individual NA chains. This affords a change in enzyme absorption equilibrium constant as the reaction progresses and is what enables initial uptake of the enzyme by the NA followed by reduction in absorption in the processing-complete regime. We also include the decay of the enzyme due to self-processing. The viral PR is a dimer that can cleave its monomeric units. Therefore, the dimerization of the enzyme is important and described by a characteristic half-life. Including the effects of NA-modulation on the enzyme dimer equilibrium improves the accuracy of the model. We fit our one-substrate model to the experimental data to account for the basic feature of length-dependent acceleration and then re-fit parameters to solve the two-substrate model, which directly accounts for sequestration. The predicted reaction kinetics fit the experimental competitive substrate sequestration curves in all three cases (S1 + E, NA + S1 + E, NA + S1 + S2 + E), thus enabling the reaction time for completion of the NA bound substrate reaction to be calculated directly from the model. The model is recalculated for enzyme and substrate conditions characteristic of the *in virio* concentrations to provide an estimate of the timescale of core condensation, which occurs directly upon NCp15 cleavage.

1.1: Pervaded Volume

We first derive an expression for the pervaded volume of a concentration of NA in solution. We assume that we are in a regime of no chain entanglement (due to low concentration and net Coulombic inter-NA repulsion), as exhibited by our AFM experiments (Figure 1). Thus, each NA chain can be approximated as occupying an independent volume dependent on its polymer coil properties. Consider a solution of n_r single-stranded NA molecules each composed of n_l nucleotides such that the total nucleotide concentration is $[n_t]$. Thus:

$$n_r = N_A [n_t] V_t / n_l \quad (1)$$

where V_t is the total reaction volume and N_A is Avogadro's constant. We assume that the NA is a flexible Gaussian coil with a persistence length l_p , and where l_0 is the nucleotide length. The radius of gyration, R_g , of a single NA chain is approximated by:

$$R_g \sim \left(\frac{L \cdot l_p}{3} \right)^{1/2}, \quad (2)$$

where $L = n_l l_0$ is the linear chain length of each NA molecule (Ref. 113 in main manuscript). Thus, the pervaded volume of a single chain V_r can be written as:

$$V_r = \kappa \left(\frac{n_l l_0 l_p}{3} \right)^{3/2}, \quad (3)$$

and where κ is a volumetric constant. The total pervaded volume by all NA, V_p , is given by:

$$V_p = n_r V_r. \quad (4)$$

Substituting Equations 1 and 3 into Equation 4 and rearranging, we derive an expression for a dimensionless parameter α_P , which corresponds to the ratio of the NA-pervaded volume to the total reaction volume:

$$\alpha_P = V_p/V_t = \kappa N_A [n_t] (n_l)^{1/2} \left(\frac{l_0 l_p}{3} \right)^{3/2}. \quad (5)$$

Correspondingly, the fractional unpervaded volume α_U is simply:

$$\alpha_U = 1 - \alpha_P. \quad (6)$$

Our kinetic model thus consists of a pervaded and unpervaded region whose volume is governed by the polymer properties of the NA (Figure 4A). Although the model consists of multiple pervaded domains within an unpervaded region, these can be summed to represent just one pervaded domain, assuming volumetric homogeneity between individual pervaded volumes.

1.2: Enzyme-substrate reaction rate equations

Let us now consider two substrate species (S_1 and S_2) and one enzyme species (E) that are absorbed into and from the pervaded NA volume with equilibrium constants, K_{S1} , K_{S2} and K_E , respectively. We thus have the following relations:

$$K_E = [E_P]/[E_U], \quad (7a)$$

$$K_{S1} = [S_{1P}]/[S_{1U}], \quad (7b)$$

$$K_{S2} = [S_{2P}]/[S_{2U}], \quad (7c)$$

where the subscripts P and U refer to the corresponding concentrations of E , S_1 and S_2 in the pervaded (P) and unpervaded domains (U) respectively. We next consider that both S_1 and S_2 react competitively with E generating intermediate complexes ES_1 and ES_2 , respectively. Given a quasi-steady-state (QSS) approximation, the concentration of each intermediate complex can be

expressed in terms of the corresponding Michaelis constants (K_{M1} and K_{M2}) for S_1 and S_2 , respectively (see Supplementary Appendix). Then as this intermediate state is short-lived, we can assume that ES_1 and ES_2 do not cross between domains U and P . From this we derive (see Supplementary Appendix) an expression for the total enzyme concentration, $[E_P]$ and $[E_U]$ in domains P and U respectively:

$$[E_P] = [E_T]/[\alpha_P Q_P + \frac{\alpha_U}{K_E} Q_U], \quad (8)$$

$$[E_U] = [E_T]/[\alpha_P K_E Q_P + \alpha_U K_E Q_U], \quad (9)$$

where $[E_T]$ is the absolute total active enzyme concentration and where Q_P and Q_U are the reaction scaling factors in domains P and U , owing to the enzyme being distributed across all possible reactions, and are given by:

$$Q_P = 1 + \frac{[S_{1P}]}{K_{M1}} + \frac{[S_{2P}]}{K_{M2}}, \quad (10)$$

$$Q_U = 1 + \frac{[S_{1U}]}{K_{M1}} + \frac{[S_{2U}]}{K_{M2}}. \quad (11)$$

Similarly, approximate relations for the substrate concentrations in the pervaded ($[S_{1P}]$ and $[S_{2P}]$) and unpervaded ($[S_{1U}]$ and $[S_{2U}]$) volumes can be derived, ignoring negligible terms due to the Michaelis complexes with the assumption that $[ES_{1P}] \ll [S_{1P}]$ and $[ES_{1U}] \ll [S_{1U}]$ (see Supplementary Appendix):

$$[S_{1P}] \sim [S_{1T}]/(\alpha_P + \frac{\alpha_U}{K_{S1}}), \quad (12)$$

$$[S_{1U}] \sim [S_{1T}]/(\alpha_P K_{S1} + \alpha_U), \quad (13)$$

$$[S_{2P}] \sim [S_{2T}]/(\alpha_P + \frac{\alpha_U}{K_{S2}}), \quad (14)$$

$$[S_{2U}] \sim [S_{2T}]/(\alpha_P K_{S2} + \alpha_U), \quad (15)$$

where $[S_{1T}]$ and $[S_{2T}]$ are the total substrate concentrations of each species. We then follow the four rate equations, one for each of the two species in each of the two domains in terms of the corresponding concentrations within each domain:

$$\nu_{1P} = -\frac{d[S_{1P}]}{dt} = -\frac{k_{cat1}}{K_{M1}}[E_P][S_{1P}], \quad (16)$$

$$\nu_{1U} = -\frac{d[S_{1U}]}{dt} = -\frac{k_{cat1}}{K_{M1}}[E_U][S_{1U}], \quad (17)$$

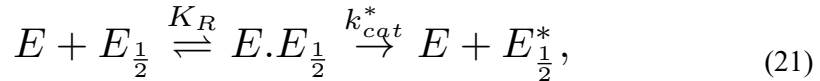
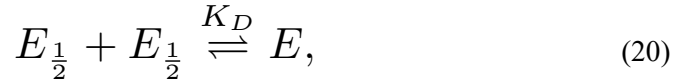
$$\nu_{2P} = -\frac{d[S_{2P}]}{dt} = -\frac{k_{cat2}}{K_{M2}}[E_P][S_{2P}], \quad (18)$$

$$\nu_{2U} = -\frac{d[S_{2U}]}{dt} = -\frac{k_{cat2}}{K_{M2}}[E_U][S_{2U}], \quad (19)$$

where k_{cat1} and k_{cat2} are the turnover numbers for S_1 and S_2 respectively.

1.3: Enzyme decay

Viral PR decays with a well-characterized half-life of 30 mins (Ref. 89 in main manuscript). This is due to recognition and inactivating cleavage of PR monomers by the active dimer. Therefore, the dimerization equilibrium of PR is expected to play a role in concentration of active PR available to be sequestered into the NA pervaded volume. Generalizing to enzyme E , consisting of a dimer of two monomers $E_{1/2}$, we have the following reaction schemes:



where K_D is the enzyme dimer dissociation constant and where K_R and k_{cat}^* are the Michaelis constant and turnover numbers of the enzyme inactivation reaction, $E \cdot E_{1/2}$ is the steady-state intermediate and $E_{1/2}^*$ is the cleaved monomer. We then have the following relations between each species:

$$[E] = \frac{[E_{\frac{1}{2}}]^2}{K_D}, \quad (22)$$

$$[E \cdot E_{\frac{1}{2}}] = \frac{[E_{\frac{1}{2}}]^3}{K_D K_R}. \quad (23)$$

The total monomer concentration ($[E^T_{1/2}]$) is given by:

$$[E^T_{\frac{1}{2}}] = \frac{3[E_{\frac{1}{2}}]^3}{K_D K_R} + \frac{2[E_{\frac{1}{2}}]^2}{K_D} + [E_{\frac{1}{2}}], \quad (24)$$

and the decay of viable monomers is then determined by the following rate law:

$$\frac{d[E_{\frac{1}{2}}]}{dt} = -\frac{k_{cat}^*[E_{\frac{1}{2}}]^3}{K_D K_R}. \quad (25)$$

1.4: Dependence of enzyme absorption equilibrium constant on RNP contiguity

We define the RNP contiguity number (c) as the average number of non-specifically bound substrate molecules within the pervaded volume (S_{2p}) per NA chain, consisting of n_l nucleotides. Then for $K_{S2} \gg 1$, the overwhelming majority of S_2 are in the pervaded volume and the contiguity can be expressed as:

$$c \sim \frac{[S_{2T}]n_l}{[n_t]}, \quad (26)$$

where $[S_{2T}]/[n_t]$ is the ratio of total RNP binding substrate concentration to total nucleotide concentration. Consider enzyme E absorbing into the RNP pervaded volume with absorption rate constant k_e and escaping with a rate constant k_{-e} to yield equilibrium constant $K_E = k_e / k_{-e}$. We stipulate that increases in contiguity result in an increased mean time for enzyme escape, thus a decrease in the escape constant k_{-e} , but not the absorption rate k_e . Then for NA with little or no bound substrate, that corresponds to a contiguity less than the critical threshold c_{crit} ($c < c_{crit}$), we have an invariant baseline escape constant $k_{-e} = k_{-e}^0$ and thus equilibrium constant $K_E^0 = k_e / k_{-e}^0$. Beyond the critical threshold ($c > c_{crit}$), k_{-e} becomes non-linearly dependent on contiguity c , with exponent ξ and constant f such that:

$$K_E = K_E^0 \left(f + \frac{c}{c_{crit}} \right)^\xi. \quad (27)$$

Thus, in our model, enzyme absorption is explicitly dependent on RNP-bound substrate number (contiguity) - then as this substrate is cleaved over time, we expect a dramatic decrease in the RNP's capacity to absorb the enzyme and this directly affects the enzyme reaction rate equations.

2: Computational implementation and model parameters

The theoretical model was implemented computationally in MATLAB. The coupled rate equations were numerically solved using the fourth-order Runge-Kutta ordinary differential equation (ODE) solver in MATLAB with a timestep of 1 second. Each integration timestep was split into two sequential components - first to compute the instantaneous active enzyme dimer concentration using the decay equation (Eq. 25) and secondly, this concentration was input into the enzyme-substrate reaction equations to compute the update on substrate concentrations.

3: Analysis of model features

The default general enzyme reaction parameters used in this study were $k_{cat1} = k_{cat(MACA)} = 7.07 \text{ s}^{-1}$, $k_{cat2} = k_{cat(NCp15)} = 0.65 \text{ s}^{-1}$, $K_{M1} = K_{M(MACA)} = 0.14 \text{ mM}$ and $K_{M2} = K_{M(NCp15)} = 0.03 \text{ mM}$. The polymer model parameters were $l_p = 75.0 \text{ nm}$, $l_0 = 1.0 \text{ nm}$ and $\kappa = 0.1$.

3.1: Sensitivity of pervaded volume to physical polymer properties

The variation of the pervaded volume with varying nucleotide chain length n_l was calculated for default polymer coil properties at a range of values for total nucleotide concentration $[n_t]$ in the approximate range (10 – 200 μM) carried out in the experimental assays. The effect of varying persistence length was also determined.

For a given nucleotide concentration, increasing chain length increases pervaded volume according to the $(n_l)^{1/2}$ dependency. However, in the relevant range of concentrations considered experimentally (10 – 200 μM), α_p does not increase beyond 0.1 even for NA approaching 1000 nucleotides. At 20 μM , α_p is below 0.02 for even 10,000 nucleotide chains. Increasing persistence length l_p by 2-fold and 10-fold for $[n_t] = 10 \mu\text{M}$, thus relaxing the polymer coiling propensity, still does not increase α_p beyond 0.02 and 0.1 for 10,000 and 1,000 nucleotide chain lengths, respectively. Therefore, according to our polymer model at moderate nucleotide concentration, the pervaded volume is always a small fraction of the overall reaction volume.

3.2: Matching a one-substrate rate model to experimental rate observables

We developed a one-substrate rate model to compare against the corresponding experimental assay at varying nucleotide length, n_l (Figure 4B). Nucleotide concentration was set to $[n_t] = 120 \mu\text{M}$, initial total NCp15 concentration was $[S_2^T]_0 = [\text{NCp15}^T]_0 = 6 \mu\text{M}$, initial total PR monomer structure was $[E^T]_0 = [\text{PR}^T]_0 = 1.2 \mu\text{M}$. Enzyme decay parameters were set to: $K_D = 14.8 \text{ nM}$, $K_R = 2.0 \mu\text{M}$ and $k_{cat}^* = 0.01 \text{ s}^{-1}$. The enzyme RNP absorption equilibrium parameters were set to: $K_E^0 = K_{PR}^0 = 0.54$. The NCp15 RNP absorption equilibrium parameter was set to $K_{S2} = K_{\text{NCp15}} = 100$.

The change of substrate concentrations in each of the separate reaction volumes (pervaded and unpervaded) was not followed directly by experiment. Therefore, to establish a direct comparison with experimental observables, it is necessary to reformulate Equations 16-19 in terms of the change in total substrate concentration ν_{1T} and ν_{2T} . In a one-substrate rate model consisting of just S_2 , in this case corresponding to NCp15, we recast Eq. 18 in terms of Eqs. 8 and 14 and rearranging we can write the total reaction rate of processing S_2 as:

$$\nu_{2T} = \frac{1}{A + (B/K_E)} \quad (28)$$

Where $A = \gamma\alpha_p Q_p$ and $B = \gamma\alpha_u Q_u$ and where $\gamma = K_{M2}/(k_{cat2} [E_T] [S_{2T}])$. We further express A and B in terms of α_p and the total substrate concentration $[S_{2T}]$ and the total enzyme concentration $[E_T]$:

$$A = \gamma\alpha_p \left(1 + \frac{[S_{2T}]K_{S2}}{(1 + (K_{S2} - 1)\alpha_p)K_{M2}} \right) \quad (29)$$

$$B = \gamma\alpha_p \left(1 + \frac{[S_{2T}]}{(1 + (K_{S2} - 1)\alpha_p)K_{M2}} \right). \quad (30)$$

Note, that this gives:

$$A + B = \frac{K_{M2}}{k_{cat2}} [E_T] [S_{2T}] \left(1 + \frac{[S_{2T}]}{K_{M2}} \right), \quad (31)$$

so that in the case that $K_E = 1$ we recapture the normal Michaelis-Menten equation from Equation 28. This formulation enables a direct handle on experimentally observable changes in total substrate concentration $[S_{2T}]$ in terms of the substrate equilibrium absorption properties, K_{S2} , as well as those of the enzyme, K_E . It also permits prediction of reaction rate in terms of nucleotide chain length-dependence, n_l , which controls α_p .

3.2.1: Estimation of initial enzyme absorption equilibrium

In the specific case that we are in the sub-critical contiguity regime ($c < c_{crit}$), then $K_E = K_E^0$ and if we assume $v = v_0 = 0.02$ mM/s corresponding to the n_l -independent regime of the experimental assay as shown in Figure 4B, then $K_E^0 = B/(1/v_0 - A)$. Although K_E^0 is formally dependent on α_p and thus n_l , it is extremely insensitive to variation of α_p . Therefore, even order of magnitude changes in NA persistence length and the volumetric constant do not affect K_E^0 , which can effectively be treated as constant. For n_l in the $n_l < n_{l,crit}$ regime, we obtain mean $K_E^0 = 0.545 \pm 0.005$, where we take $n_{l,crit} \sim 50$ as estimated from experiment ($n_{l,crit}$ is the critical NA chain length at c_{crit}). This provides a fitted estimate for K_E^0 in terms of the experimental data.

3.2.2: Fitting of acceleration assay to one-substrate rate model

Using the experimentally fitted value for K_E^0 and $n_{l,crit}$, which gives $c_{crit} \sim 1.1$, for the low- n_l constant rate (v_0) regime, we fit the n_l -dependent experimental data to Eq. 28 using the general definition of K_E in terms of Eq. 27, using parameters f and ξ and obtain $f = 3$, $\xi = 0.4$ (Figure 4B). Our one-substrate rate model is therefore consistent with the notion that beyond a critical threshold the enzyme absorption equilibrium becomes non-linearly dependent on the contiguity of substrate molecules within the NA-pervaded volume resulting in the acceleration of substrate processing, whilst below it a constant reaction rate is exhibited.

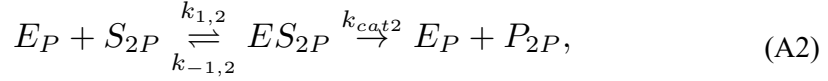
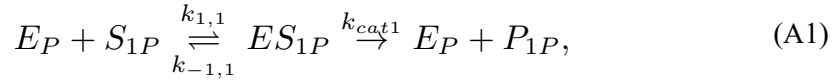
3.3: Two-substrate kinetic model

Given that the basic feature of NA-length-dependent acceleration is already captured in the one-substrate rate model, we next evaluated whether the same theoretical approach could account for the sequestration effect that was observed experimentally using an additional competitive substrate. Here, rather than fitting reaction rates at specific concentrations, we require computing the time evolution of substrate concentration of both substrates whilst also fitting the required parameters of our model. The fitted parameters used for the two-substrate kinetic model are provided in Supplementary Table 1.

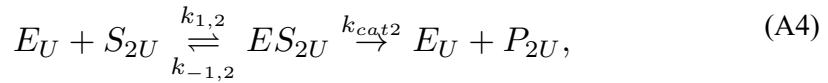
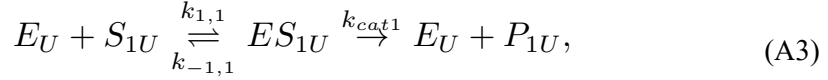
Supplementary Appendix

Consider a total reaction volume V_t partitioned into two effective sub-volumes V_p and V_u in each of which occur competitive enzymatic reactions between one enzyme E and two substrate species S_1 and S_2 . This constitutes four reactions described by the following scheme:

Reactions in sub-volume V_p :



Reactions in sub-volume V_u :



where subscripts P and U denote the corresponding species in each of the separate sub-volumes V_p and V_u , respectively. Similarly, ES_1 and ES_2 are the Michaelis complexes and P_1 and P_2 are the products. The total enzyme number E^T is then:

$$E^T = E^P + ES_1^P + ES_2^P + E^U + ES_1^U + ES_2^U \quad (A5)$$

where superscripts P and U denote the absolute number of each species. Dividing by the total volume we obtain:

$$\frac{E^T}{V_t} = \alpha_P \frac{E^P}{V_P} + \alpha_P \frac{ES_1^P}{V_P} + \alpha_P \frac{ES_2^P}{V_P} + \alpha_U \frac{E^U}{V_U} + \alpha_U \frac{ES_1^U}{V_U} + \alpha_U \frac{ES_2^U}{V_U} \quad (A6)$$

where $\alpha_P = V_p/V_t$ and $\alpha_U = V_u/V_t$ are the fractional sub-volumes for P and U respectively. Then we use subscripts P and U to represent the effective concentration of each species in its corresponding sub-volume (e.g. $[E_T] = E^T/V_t$, $[E_P] = E^P/V_p$, $[ES_{1P}] = ES_1^P/V_p$, etc.) and by factoring we obtain the total enzyme concentration $[E_T]$ in terms of the effective concentrations in each sub-volume:

$$[E_T] = \alpha_P([E_P] + [ES_{1P}] + [ES_{2P}]) + \alpha_U([E_U] + [ES_{1U}] + [ES_{2U}]) \quad (A7)$$

Assuming a quasi-steady state for each reaction in each of the sub-volumes and that the reaction transition is sufficiently short-lived that ES_1 and ES_2 do not cross between domain U and P, we can rewrite $[ES_{1P}]$, $[ES_{2P}]$, $[ES_{1U}]$ and $[ES_{2U}]$ in terms of the respective Michaelis constants ($[ES_{1P}] = [E_P][S_{1P}]/K_{M1}$, $[ES_{2P}] = [E_P][S_{2P}]/K_{M2}$, $[ES_{1U}] = [E_U][S_{1U}]/K_{M1}$ and $[ES_{2U}] = [E_U][S_{2U}]/K_{M2}$). Substituting these into the above equation and rearranging we have:

$$[E_T] = \alpha_P[E_P] \left(1 + \frac{[S_{1P}]}{K_{M1}} + \frac{[S_{2P}]}{K_{M2}} \right) + \alpha_U[E_U] \left(1 + \frac{[S_{1U}]}{K_{M1}} + \frac{[S_{2U}]}{K_{M2}} \right)$$

(A8)

Then by identifying the terms in brackets as Q_P and Q_U respectively as expressed in Equations 10 and 11 and substituting out either $[E_P]$ or $[E_U]$ using the enzyme equilibrium equation (7a) we obtain Equations 8 and 9 for the effective enzyme concentrations in each sub-volume.

Using a similar approach to above, the total substrate number, S_I^T , for S_I is:

$$S_1^T = S_1^P + ES_1^P + S_1^U + ES_1^U, \quad (\text{A9})$$

from which we can express the total concentration $[S_{1T}]$ as:

$$[S_{1T}] = \alpha_P([S_{1P}] + [ES_{1P}]) + \alpha_U([S_{1U}] + [ES_{1U}]). \quad (\text{A10})$$

By rewriting $[ES_{1P}]$ and $[ES_{1U}]$ in terms of the Michaelis constants and by making use of the substrate equilibrium equation (Equation 7b) to substitute out $[S_{1U}]$, and rearranging we obtain:

$$[S_{1T}] = [S_{1P}] \left(\alpha_P + \frac{\alpha_U}{K_{S1}} \right) + [S_{1P}] \left(\frac{\alpha_P [E_P]}{K_{M1}} + \frac{\alpha_U [E_U]}{K_{M1} K_{S1}} \right). \quad (\text{A11})$$

By further substituting out $[E_U]$ using the enzyme equilibrium equation (Equation 7a) we obtain:

$$[S_{1T}] = [S_{1P}] \left(\alpha_P + \frac{\alpha_U}{K_{S1}} \right) + \frac{[E_P][S_{1P}]}{K_{M1}} \left(\alpha_P + \frac{\alpha_U}{K_E K_{S1}} \right). \quad (\text{A12})$$

This yields an exact expression for $[S_{1P}]$ in terms of the total substrate concentration $[S_{1T}]$:

$$[S_{1P}] = [S_{1T}] / \left(\alpha_P + \frac{\alpha_U}{K_{S1}} + \frac{[E_P]}{K_{M1}} \left(\alpha_P + \frac{\alpha_U}{K_E K_{S1}} \right) \right). \quad (\text{A13})$$

Similarly, if one substitutes out $[S_{1P}]$ in terms of $[S_{1U}]$ and $[E_P]$ in terms of $[E_U]$ from Equation A10, one derives $[S_{1U}]$:

$$[S_{1U}] = [S_{1U}] / \left(\alpha_P K_{S1} + \alpha_U + \frac{[E_U]}{K_{M1}} (\alpha_P K_E K_{S1} + \alpha_U) \right). \quad (\text{A14})$$

Correspondingly for $[S_{2P}]$ and $[S_{2U}]$ these are:

$$[S_{2P}] = [S_{2T}] / \left(\alpha_P + \frac{\alpha_U}{K_{S2}} + \frac{[E_P]}{K_{M2}} \left(\alpha_P + \frac{\alpha_U}{K_E K_{S2}} \right) \right). \quad (\text{A15})$$

$$[S_{2U}] = [S_{2U}] / \left(\alpha_P K_{S2} + \alpha_U + \frac{[E_U]}{K_{M2}} (\alpha_P K_E K_{S2} + \alpha_U) \right). \quad (\text{A16})$$

Finally, if we assume that $[S_{1P}] \gg [ES_{1P}]$, then Equation A12 can be approximated to Equation 12 and similarly for $[S_{1U}]$, $[S_{2P}]$ and $[S_{2U}]$ this approximation leads directly to Equations 13-15.