


## Article

# Stiffness Analysis to Predict the Spread Out of Fake Information

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**Abstract:** This work highlights how the stiffness index, which is often used as a measure of stiffness for differential problems, can be employed to model the spread of fake news. In particular, we show that the higher the stiffness index is, the more rapid the transit of fake news in a given population. The illustration of our idea is presented through the stiffness analysis of the classical SIR model, commonly used to model the spread of epidemics in a given population. Numerical experiments, performed on real data, support the effectiveness of the approach.

**Keywords:** fake news; SIR model; stiffness ratio



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## 1. Introduction

The birth of the Internet, and in particular of social media, has radically changed the way we transmit news. In fact, nowadays, the majority of the population considers the Internet and social media as the main channels of transmitting news. However, at the same time, social media are also believed to be the main sources of transmission of disinformation and fake news [1–18].

Fake news is created by completely ignoring the editorial rules and processes used to ensure its compliance and truthfulness [19]. Actually, it is created for different purposes; the most common is certainly the electoral one in order to discredit a political opponent by conditioning public opinion. The other purpose is the profit that is obtained online in proportion to the number of visitors to the article.

The problem of the spread of fake news has always existed, but what seems to have clearly changed, compared to the past, is the quantity of fake news present in the information and the weight they take on. Therefore, the truthfulness of the news we receive online is a problem that requires great attention [20].

In recent years, many authors have tried to create adequate mathematical models capable of predicting the spread of fake news, in order to limit in advance the effects that the spread of these are having on our society: the interested reader can refer, for instance, to [12,21–29] and references therein.

In this article, we use a famous model for the spread of a disease, i.e., the SIR model, as a model for fake news diffusion. SIR model, well known in the existing literature of mathematical epidemiology (see, for instance, the very first contributions on compartmental models [30–34], as well as the monographs [35–42] and references therein), describes the effects of the spread out of an epidemics in a population ideally divided into three subgroups (susceptible, infected and recovered people). As is visible in the literature (see, for instance, [19,23,43,44]), epidemiological models can be profitably used to describe the diffusion of fake information as an infectious disease. Most of these models are given by

nonlinear differential equations, whose dynamics can be understood first looking at the eigenvalues of the Jacobian of the linearized problem. Here, we show how the spread of fake news is closely linked to the stiffness ratio (i.e., the ratio between the largest and the smallest moduli of the eigenvalues of the aforementioned Jacobian matrix) that characterizes the underlying differential problem. We highlight that, in the spirit of this manuscript, the SIR model is only the channel we use to provide our idea: the stiffness ratio can help us understand the requested time to recover the truth in a given country exposed to a certain fake news. To some extent, we aim to provide a novel element to understand the effectiveness of modeling for the diffusion of fake news, as well as to the re-establishment of the truth: the higher the stiffness ratio, the faster the re-establishment of the truth after the diffusion of fake information. This kind of argument is not advisable in the literature, to the best of our knowledge. The work is organized as follows: Section 2 will describe the stiffness analysis of the SIR model for fake news diffusion; Section 3 is dedicated to numerical tests, performed on real data, confirming our thesis; and some concluding remarks are given in Section 4.

## 2. Stiffness Analysis of the SIR Model for the Diffusion of Fake Information

As aforementioned, the SIR model was first introduced in 1927 by Kermack and McKendrick [31], even if seminal contributions on compartmental models are also advisable in [30,32–34]. It represents an extremely simple mathematical model for describing the transmission of an infectious disease. This model describes the mutual interactions of three populations of individuals: the population  $S$  of susceptible people, i.e., the healthy individuals who can contract the disease; the population  $I$  of the infected, i.e., individuals who have contracted the disease and are able to transmit it; and the population  $R$  of the recovered, that is, individuals who are healed.

In our model for the spread of fake information, above populations are described as follows:

- $S(t)$ : potentially authoring the spreading of fake news;
- $I(t)$ : the wide variety of authors highly active in posting fake information;
- $R(t)$ : authors who are inactive to the spreading of fake news.

The model is based on the continuous interaction between susceptible and infected individuals along time. The corresponding system of ODEs is then given by:

$$\begin{cases} \frac{dS(t)}{dt} = -\beta S(t)I(t), \\ \frac{dI(t)}{dt} = \beta S(t)I(t) - \alpha I(t), \\ \frac{dR(t)}{dt} = \alpha I(t), \end{cases} \quad (1)$$

where  $\alpha$  is the rate of recovery and  $\beta$  the contact rate. Specifically, since the purpose of our method is to compare the impact of fake news in different countries, the parameters  $\beta$  and  $\alpha$ , are related to two important indices, commonly used to describe the social, economic and cultural performance of our society. In particular,

$$\beta = \frac{i}{10}, \quad \alpha = \frac{h}{100},$$

where  $i$  is the *internet penetration index* of the country and  $h$  is the human development index of the same country. These values are commonly provided in the annual report of United Nations Development Programme [45] (also see the projects [46,47]). In general, the value of  $\alpha$  is smaller than that of  $\beta$  because it is easier to spread a lie than reaffirming the truth. Table 1 shows, for instance, the values of  $\alpha$  and  $\beta$  for selected countries, i.e., Australia, Brazil, France, India, Italy, Mexico, Mozambique and the USA, referring to the year 2019.

**Remark 1.** It is crucial to highlight the different power of the denial of a fake news and that of the fake news itself [48]. Fake information circulates online much faster than true one and is more prone to be shared by users who encounter it; on the other hand, the re-assessment of the truth is nowhere viral and reaches far fewer people than those who have read or spread fake information. Due to this intrinsic characteristic of true news compared to false news, the former require a much greater commitment on the part of individual users than that related to the spread of fake news. Above all, the spread out of fake information does not necessarily require a particularly strong human presence: often fake news are circulated by bots created specifically by someone or shared by fake accounts, so they do not correspond to real people. Real people, on the other hand, are necessary and represent the only option to restore the truth. This motivates the choice of the recovery rate  $\alpha$  linked to human development index per country and the contact rate  $\beta$  to the spread of the internet in the same country.

**Table 1.** Values of the constants  $\alpha$ ,  $\beta$ , for France, India, Italy, Mexico and the United States, referring to 2019.

	$\alpha$	$\beta$
Australia	0.009	0.087
Brazil	0.008	0.072
France	0.009	0.089
India	0.006	0.035
Italy	0.009	0.061
Mexico	0.008	0.064
Mozambique	0.005	0.021
United States	0.009	0.075

In our analysis, it is relevant to consider a linearization around the initial value vector

$$\begin{bmatrix} S_0 \\ I_0 \\ R_0 \end{bmatrix} = \begin{bmatrix} S(0) \\ I(0) \\ R(0) \end{bmatrix},$$

leading to

$$\begin{cases} \frac{dS(t)}{dt} = \beta S_0 I_0 - \beta I_0 S(t) - \beta S_0 I(t) + \text{high order terms}, \\ \frac{dI(t)}{dt} = -\beta S_0 I_0 + \beta I_0 S(t) + (\beta S_0 - \alpha) I(t) + \text{high order terms}, \\ \frac{dR(t)}{dt} = \alpha I(t), \end{cases} \quad (2)$$

Correspondingly, let us compute the Jacobian matrix of the linear part of the vector field in (2), i.e.,

$$J_{\alpha,\beta}(S_0, I_0) = \begin{bmatrix} -\beta I_0 & -\beta S_0 & 0 \\ \beta I_0 & \beta S_0 - \alpha & 0 \\ 0 & \alpha & 0 \end{bmatrix},$$

whose spectrum consists in one eigenvalue equal to 0 and two real eigenvalues  $\lambda_{\alpha,\beta}^{\min}(S_0, I_0)$  and  $\lambda_{\alpha,\beta}^{\max}(S_0, I_0)$ , with  $|\lambda_{\alpha,\beta}^{\min}(S_0, I_0)| < |\lambda_{\alpha,\beta}^{\max}(S_0, I_0)|$ . Correspondingly, the ratio

$$\sigma_{\alpha,\beta}(S_0, I_0) = \frac{|\lambda_{\alpha,\beta}^{\max}(S_0, I_0)|}{|\lambda_{\alpha,\beta}^{\min}(S_0, I_0)|}, \quad (3)$$

meaningful in the analysis of stiff problems [49], provides the so-called *stiffness ratio* of (1). Table 2 reports the stiffness ratio for each country, related to the initial value

$$\begin{bmatrix} S_0 \\ I_0 \\ R_0 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 0.1 \\ 0 \end{bmatrix}, \quad (4)$$

i.e., assuming that 70% of the initial population is susceptible, 10% are infected and there are no recovered people. The results reveal that, the higher the internet penetration index  $i$ , the bigger the stiffness ratio. As a consequence, the corresponding model (1) is more stiff and the spread of fake news should be more damped in time. In other terms, the more (1) is stiff, the more the corresponding country exhibits a faster transit of fake news. Countries with a lower internet penetration index  $i$  are characterized by a less stiff model and, as a consequence, the transit of fake information is slower and circulates for much more time.

**Table 2.** Values of the stiffness ratios (3) in France, India, Italy, Mexico and the United States, referring to 2019, assuming the vector (4) as initial point.

	$S_{\alpha,\beta}(S_0, I_0)$
Australia	20.03
Brazil	20.85
France	23.07
India	8.38
Italy	12.35
Mexico	17.13
Mozambique	4.39
United States	17.00

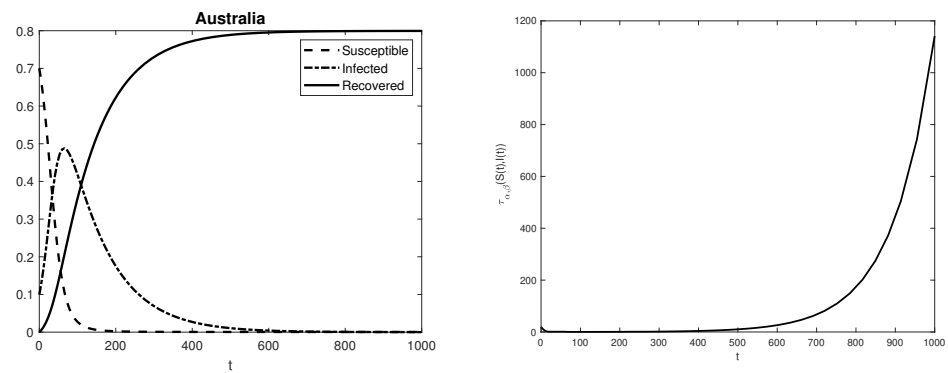
### 3. Numerical Experiments and Conclusions

In this section, we aim to give numerical evidence of the arguments contained in the previous section, i.e., the spread of fake news is closely linked to the stiffness ratio of Equation (1). For each listed country, Figures 1–8 show the solution of problem (1) in the interval  $[0,1000]$ , computed by the standard Matlab built-in function `ode15s`, and the pattern of the ratio  $\tau_{\alpha,\beta}(S(t), I(t))$  between the maximum and minimum moduli of the non-zero eigenvalues of the matrix

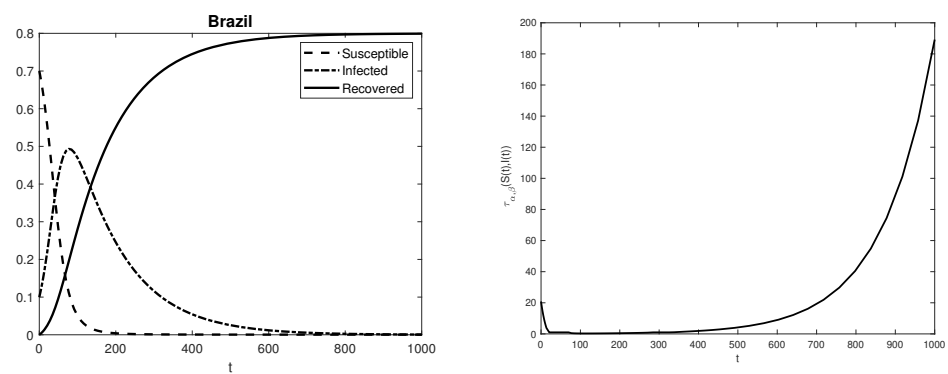
$$J_{\alpha,\beta}(S(t), I(t)) = \begin{bmatrix} -\beta I(t) & -\beta S(t) & 0 \\ \beta I(t) & \beta S(t) - \alpha & 0 \\ 0 & \alpha & 0 \end{bmatrix},$$

that corresponds to the Jacobian of the problem (1), frozen at time  $t$ . To some extent, we aim to check the evolution in time of the stiffness ratio  $\sigma_{\alpha,\beta}(S_0, I_0)$ .

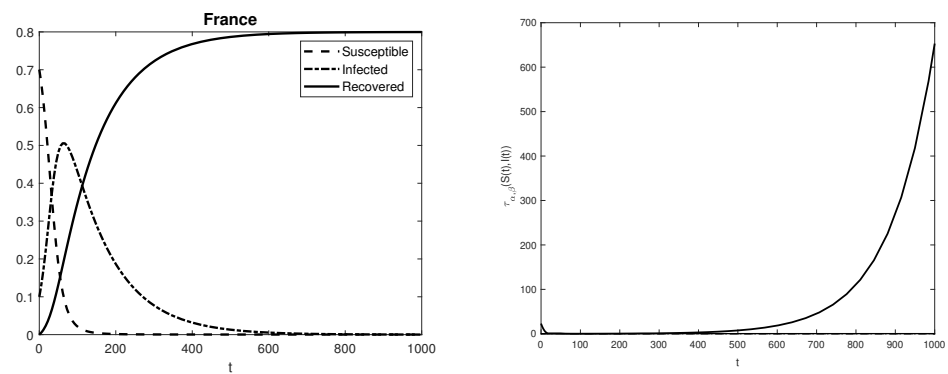
Each figure confirms that the higher the stiffness ratio, as listed in Table 2, the faster the transit of fake news will be. In some countries, such as India or Mozambique, where the internet penetration index is small, the function  $\tau_{\alpha,\beta}(S(t), I(t))$  grows much than in the other cases (corresponding to countries with higher internet penetration indices). As a consequence, smaller values of the stiffness ratio correspond to a slower achievement of the maximum number of infected people and, consequently, to a slower dispersion of fake news. The observed number of time units needed to achieve the maximum number of infected is listed in Table 3: one can observe that the number of time units is coherent with the stiffness ratio, so the smallest value is for France, while the largest is for Mozambique.



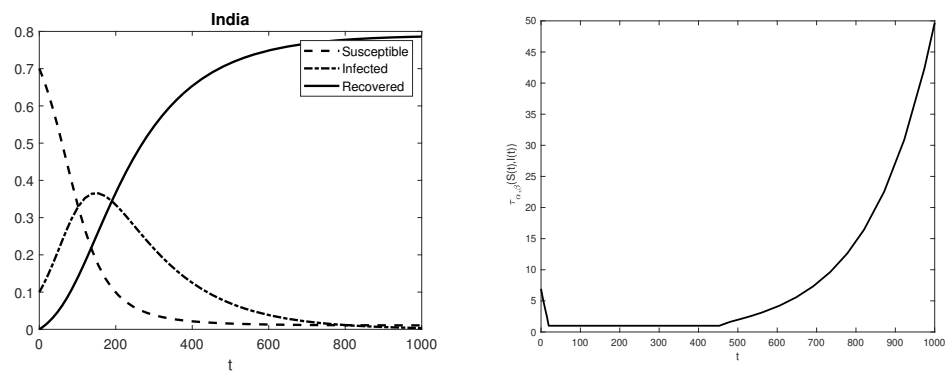
**Figure 1.** Solution to the SIR model (1), with initial value given by the vector (4), for Australia (left) and corresponding pattern of  $\tau_{\alpha,\beta}(S(t), I(t))$  (right).



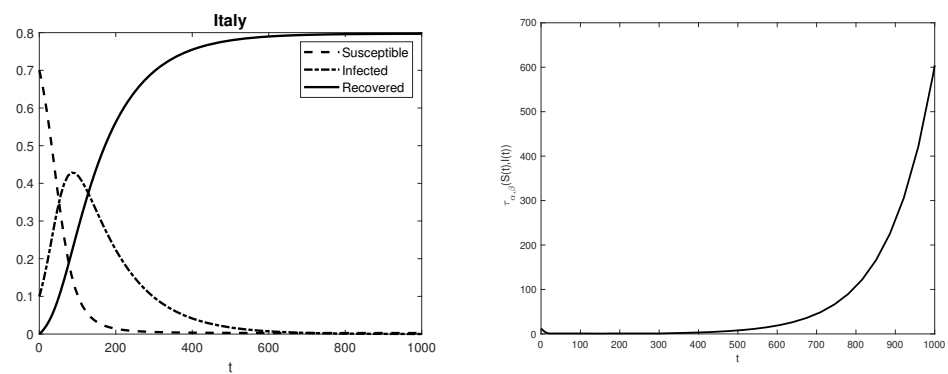
**Figure 2.** Solution to the SIR model (1), with initial value given by the vector (4), for Brazil (left) and corresponding pattern of  $\tau_{\alpha,\beta}(S(t), I(t))$  (right).



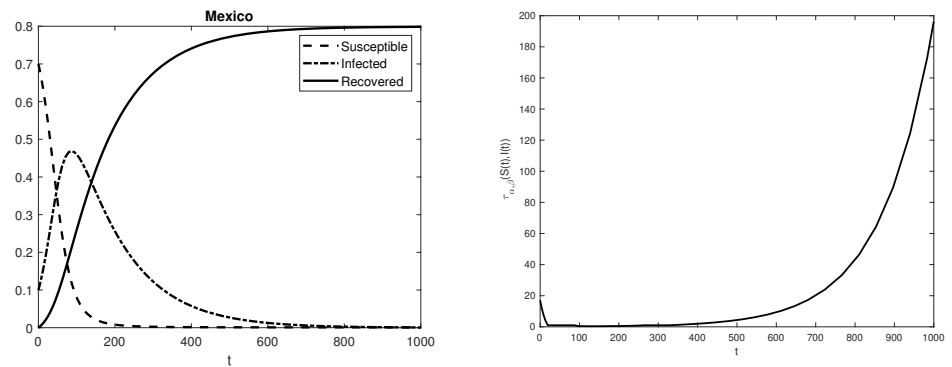
**Figure 3.** Solution to the SIR model (1), with initial value given by the vector (4), for France (left) and corresponding pattern of  $\tau_{\alpha,\beta}(S(t), I(t))$  (right).



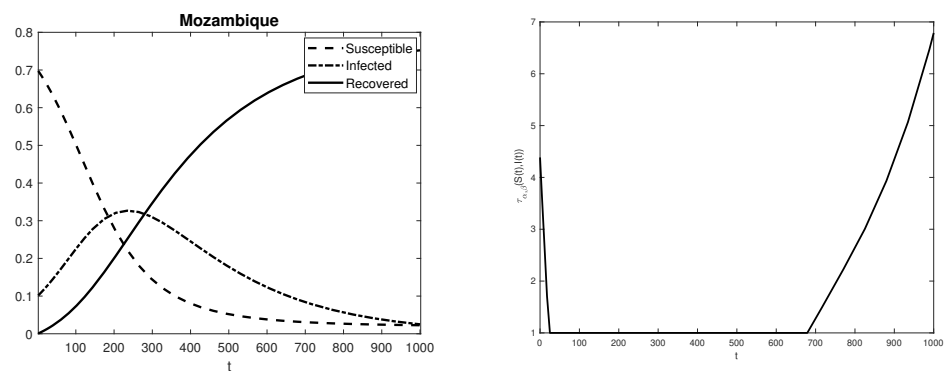
**Figure 4.** Solution to the SIR model (1), with initial value given by the vector (4), for India (left) and corresponding pattern of  $\tau_{\alpha,\beta}(S(t), I(t))$  (right).



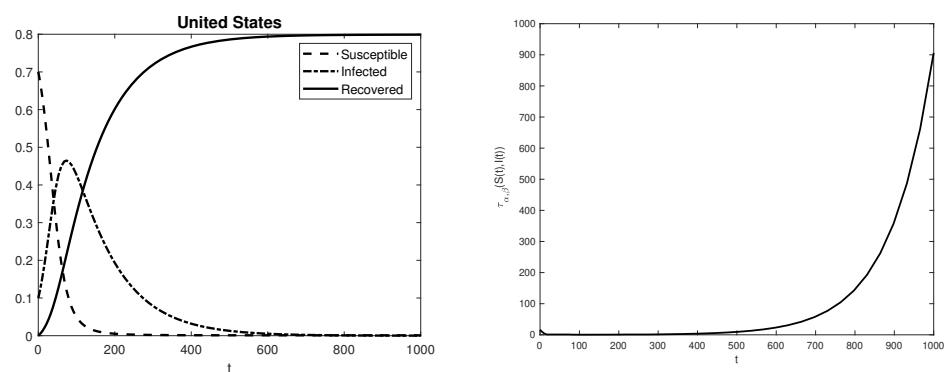
**Figure 5.** Solution to the SIR model (1), with initial value given by the vector (4), for Italy (left) and corresponding pattern of  $\tau_{\alpha,\beta}(S(t), I(t))$  (right).



**Figure 6.** Solution to the SIR model (1), with initial value given by the vector (4), for Mexico (left) and corresponding pattern of  $\tau_{\alpha,\beta}(S(t), I(t))$  (right).



**Figure 7.** Solution to the SIR model (1), with initial value given by the vector (4), for Mozambique (left) and corresponding pattern of  $\tau_{\alpha,\beta}(S(t), I(t))$  (right).



**Figure 8.** Solution to the SIR model (1), with initial value given by the vector (4), for United States (left) and corresponding pattern of  $\tau_{\alpha,\beta}(S(t), I(t))$  (right).

**Table 3.** Number of time units required to reach the maximum of infected in France, India, Italy, Mexico and the United States, referring to 2019.

	Number of Time Units
Australia	66.15
Brazil	77.00
France	61.16
India	145.75
Italy	88.90
Mexico	87.02
Mozambique	234.40
United States	74.05

#### 4. Conclusions

The analysis carried out in this paper is useful to give a measure, suggested by the stiffness ratio, of the speed of re-affirmation of the truth after the spread out of a fake news. In particular, the analysis suggests to use SIR models with high stiffness ratio to describe the diffusion of fake information when the country is exposed to a slower transit of fake news. Less stiff models are particularly suitable when the transit of fake news is slower and its survival time in the exposed population is higher. The employed model is the standard SIR system of differential Equation (1), but certainly more complex deterministic and stochastic models may be used in order to describe the diffusion of fake news as an epidemic phenomenon as, for instance, in [50–58]. Moreover, it would be worth investigating how to detect fake news through sentiment analysis of tweets as suggested by [59,60].

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**Data Availability Statement:** As stated in the treatise, most of the data are publicly available in [45,47].

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