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Multi-Criterion Two-Sided Matching of Public–Private Partnership Infrastructure Projects: Criteria and Methods

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Abstract: Two kinds of evaluative criteria are associated with Public–Private Partnership (PPP) infrastructure projects, i.e., private evaluative criteria and public evaluative criteria. These evaluative criteria are inversely related, that is, the higher the public benefits; the lower the private surplus. To balance evaluative criteria in the Two-Sided Matching (TSM) decision, this paper develops a quantitative matching decision model to select an optimal matching scheme for PPP infrastructure projects based on the Hesitant Fuzzy Set (HFS) under unknown evaluative criterion weights. In the model, HFS is introduced to describe values of the evaluative criteria and multi-criterion information is fully considered given by groups. The optimal model is built and solved by maximizing the whole deviation of each criterion so that the evaluative criterion weights are determined objectively. Then, the match-degree of the two sides is calculated and a multi-objective optimization model is introduced to select an optimal matching scheme via a min-max approach. The results provide new insights and implications of the influence on evaluative criteria in the TSM decision.

Keywords: Infrastructure projects; Public–Private Partnership; Two-Sided Matching; Hesitant Fuzzy Set

1. Introduction

With the ever-increasing pace of urbanization in China, limited financial resources of the government have been unable to keep up [1–4]. To relieve infrastructure funding pressure and raise the funds rate, an increasing number of local governments in China began to encourage the private sector to participate in public investment projects. Therefore, Public–Private Partnership (PPP) has been explored and becomes a preferred solution for contemporary infrastructure projects in the public sector [5,6]. To be more specific, PPP is a long-term cooperation between the public and private sectors for joint construction of urban infrastructure projects, or offering public goods and service based on the concession agreement [7]. In PPP infrastructure projects, the formation of partnerships is Two-Side Matching (TSM) between the public and private sectors. "*Sharing interests and responsibilities and long-term cooperation*" are the most critical principles of PPP infrastructure projects [8]. During the construction and operation process, the cooperative partners need to share interests and responsibilities within the public and private sectors. Thus, appropriate selection of cooperative partners in a PPP project is the basis of the project success [6,9].

However, it is often difficult or impossible to give decision-making subjects (i.e., administration organizations) precise evaluation on matching objects where there are several possible evaluative values [10]. Moreover, group decision making is used most in modern decision making techniques to try to avoid mistake decision caused by cognitive bias, and thus there might exist disagreement among the decision makers (DMs) who cannot persuade each other [11]. Taking buying an established house as an example, suppose that four members in the family of the buyer are asked to provide the degrees to which a house is superior to another, and the members prefer to use the values between 0 and 1 to express their preferences. One member provides 0.65, one provides 0.75, one provides 0.75, and the other one provides 0.80. To describe the original evaluative information, the obfuscation tools, such as fuzzy set, intuitive fuzzy set, have their own limitations. As a generalized form of fuzzy set, Hesitant Fuzzy Set (HFS) [12] whose membership degree is allowed to be several evaluative values, provides new ideas for solving hesitation and disagreement of DMs. In the above example, the degrees to which a house is superior to another can be represented by a Hesitant Fuzzy Element (HFE) {0.65,0.75,0.75,0.80}. In addition, determining the evaluative criterion weights is also important since it affects the accuracy of the matching decision significantly [10].

In order to overcome the above limitations, this paper introduces HFS into TSM and considers multi-criterion decision of PPP infrastructure projects, in which the information of evaluative criterion weights is completely unknown. Specifically, the objectives of this study are (1) to determine evaluative criterion weights objectively through maximizing deviation between evaluative criteria and constructing an optimization model; and (2) to obtain an optimal matching scheme by calculating compatibility of the two parties via Hesitant Fuzzy Element Ordered Weighted Averaging (HFEOWA), which is further used to construct an optimal matching model. In addition, the method proposed in this paper can be further expanded to interval the hesitant fuzzy value.

2. Literature Review

The starting point of this paper was a literature review. A literature review was performed, followed by PPP infrastructure projects, TSM satisfaction evaluative criteria, and selection approaches of PPP infrastructure projects according to the current project management literature.

2.1. PPP Infrastructure Projects

PPP, as an important part of a financing pattern, has been reviewed [13] and gained increasing attention within the academic construction literature [14]. Concerning the decision process itself, the focus has mainly been on two aspects.

Firstly, an important topic is risk identification and its allocation between the public and private sectors. Several empirical [15,16] and, to a lesser extent, theoretical studies [14] focus on risk identification and allocation for the delivery of PPP infrastructure projects. Some studies have been conducted to identify and assess the risks for procuring PPP infrastructure projects and to address their proper allocation. For example, many specific issues on PPP infrastructure projects have been proposed, such as the top three risk factors (i.e., government intervention, government corruption, and poor public decision making process) [17], and the opportunities and impact of political risks under PPP schemes [1].

The second aspect involves the pricing of governmental support interventions to guarantee a minimum revenue in the course of the operation [18,19], and the impact of governmental capacity regulations [20]. A minimum traffic guarantee real options model was presented and applied to the projected 1000-mile-long BR-163 toll road that will link the Brazilian Midwest to the Amazon River [21]. With different government support mechanisms, such as design-bid-build, design-build, design-build-finance, and design-build-finance-operate, an option-pricing framework has been developed that enables financial assessment of different types of road projects [18].

2.2. TSM Satisfaction Evaluative Criteria of PPP Infrastructure Projects

The PPP is generally considered a win-win mechanism that represents a collection of resources and efforts for both the public and private sectors [6,7,13]. TSM of PPP infrastructure projects involves both evaluation of the enterprises to the government and the evaluation of the government to the enterprises, which results in a long-term and stable partnership, thereby ultimately maximizing satisfaction of both sides [22,23]. Therefore, the matching satisfaction degree of both the government and the enterprises should be fully considered in the TSM decision process of PPP infrastructure projects [1,24].

Selecting the private partners is a key issue for PPP infrastructure projects. Zhang [25] identified evaluative criteria that are classified into: financial, technical, safety, health, and environmental, and managerial for PPP projects in general [26]. Ng et al. [9] indicated that it is also useful to consider the special evaluative criteria associated with the PPP procurement process such as separate financing, design, construction, and operational responsibilities. Also further, four evaluative criterion categories of the public and private sectors for PPP infrastructure projects were established by Zhang et al. [27] by using a structured questionnaire survey, which included finance, technology, safety and health, and environment. In general, evaluative criteria of the public and private sectors can be classified into several categories:

- *Financial strength*: Many scholars have reported various kinds of financial strength for TSM of PPP infrastructure projects, such as capital strength [22–24,28], financing capacity [17,24,28,29], and financial guarantee [30]. This criterion is used to measure financial credibility of private sectors to deal with capital crises [31].
- *Technical ability*: This criterion category is the ability to establish project parameters and in-house expertise [32]. The private sectors have to show their technical ability including advanced technology [23–25,29], project implementation plans [22], personnel reserve [15,23,30] and critical equipment [15,22], to carry out all construction activities required for a specific PPP infrastructure project.
- *Management ability*: One of the most critical factors for the successful completion of PPP infrastructure projects is top management ability [28]. The private sectors have to show their management capacity to carry out all construction activities required for a specific PPP infrastructure project, including management formalization [1,17], project management capacity [22], ability of communication and cooperation [23], concession duration [15,30], risk management ability [22,29], and maintenance ability [24].
- *Relevant experience*: As a central part of TSM decision process of PPP infrastructure projects, relevant experience has received increasing attention in the past two decades, just as financing experience, constructing experience, and operating experience also influence TSM decision [28–30].
- *Credit level:* Credit hazards can easily lead to serious cost increases and schedule delays for PPP infrastructure projects [1]. Thus, Credit level is crucial for TSM decision [1,17,22,28,30].
- *Governmental insurance*: Currently there are no special laws and regulations for PPP infrastructure projects to define the responsibility division between participants, project assessment, contract performance and change, and conflict-solving styles [15,17,27]. However, appropriate laws and regulations are the foundation for the private sector to participate in PPP infrastructure projects and the premise of TSM [7] because of a longer construction cycle and increasing risk of operating project construction.
- *Governmental guarantee:* The recoverable rate of investment is key for the private sector involved in PPP infrastructure projects, thus the public sector should offer the recoverable rate of investment to businesses via offering a government-backed rate [6,22,28], price [17,29], purchase, and compensation in the scope of given benefit level [1,6].

- *Governmental credibility:* The private sector involved in PPP infrastructure projects must be built on project understanding but the key information about the PPP project is disclosed by the public sector [1,15]. Many previous cases have demonstrated that unclear project information leads to a lack of trust in the public sector [6,27]. Moreover, the private sector is in a weak position when playing games with the public sector; and thus, the legitimate rights and interests of the private sector are hard to guarantee [22].
- *Governmental risk-sharing:* There are a series of unpredictable risks due to PPP infrastructure projects characteristically lasting a long time, and it would expose the private sector to serious damage if these risks occurred [1,9,33]. Therefore, risk-sharing of the government affects the quantity and quality of the private sector involved in PPP infrastructure projects [1,5,6,24,25,29]

2.3. TSM Selection Approaches

To effectively balance the main parts of PPP infrastructure projects, researchers proposed approaches for TSM selection from economic and non-economic perspectives [5,14,22]. Considering the uncertainty within PPP infrastructure projects, Carbonara et al. [5] established the statistical distributions of the random input variable and Monte Carlo simulation technique to calculate a 'win-win' solution for both project promoter and the host government in the concession phase; to allow for fair risk sharing between the two parties to satisfy both private and the government through guaranteeing minimum interests of each party; and finally, to fairly allocate risks between parties. To evaluate and compare different PPP alternatives, Xie and Ng [34] first imitated human reasoning and conducted multi-objective decision making through Bayesian network techniques. Then, they proposed a decision making approach through connecting the decision making items, evaluative criteria, and the ultimate objectives from the government, investor, and community perspective, respectively. A weighted score approach was used to combine the objectives of the three main stakeholders into a single value. Wang et al. [10] first constructed a bilateral matching satisfaction index system for the PPP project at both the government and enterprise levels, and established the matching satisfaction judgment matrices of the two sides via intuitionistic fuzzy numbers. Then, they used the Choquet integral to match the satisfaction evaluation vectors of the two sides, and obtained the final matching results by constructing a multi-objective decision model. Ng et al. proposed a simulation model to assist a public partner to identify the concession period based on the expected investment and tariff regime. Then, they presented needs for establishing different scenarios to represent the risks and uncertainties involved and introduced a fuzzy multi-objective decision model to trade-off the associated three concession items.

Overall, the methods mentioned above are based on general evaluating criteria and qualitative evaluating criteria weights which are pre-assigned in a subjective way. However, the quality of the decision results cannot be only evaluated by the superiority of the evaluating criteria and subjective judgment of DMs, but also by the objective judgment of evaluating criteria weights [2]. Thus, how to develop an objective way to determine the weights associated with the evaluative criteria is no doubt important and interesting. Meanwhile, if there are a few different values in a membership set, traditional methods usually replace these discrete values by continuous values within an interval which do not reflect the differences of these values properly. How to develop a proper method to handle this issue is also challenging.

3. TSM Decision Making Model of PPP Infrastructure Projects Based on HFS

3.1. Description and Assumption

In the past several decades, the Chinese government has initiated extensive infrastructure projects to improve quality of life in China. Initially, these projects have been fully invested by the government. To facilitate building more projects, the government is now inviting more and more private companies for co-investment and building to ensure financial sustainability. For some special projects, each private company can bid one project, due to either heavy financial commitment or limit of capability and/or capacity. At the initial stage, the government would like to have an assessment in order to allocate the right projects to the right enterprises (i.e., different industries) considering their background and expertise for maximizing benefit to all the involved parties. The aim of this paper is to develop a unified framework for PPP projects optimal matching.

During the TSM decision making process, the government and the enterprises are supposed to be evaluated through a group of subjects. Without loss of generality, the number of the subjects in the government is supposed to not exceed the number of the subjects in the enterprise. For each subject, further evaluation will be through a group of criteria. The criteria used to evaluate the subjects in the government and the subjects in the enterprise are referred to as private and public evaluative criteria, respectively. This mutual relationship is described in Figure 1. From this figure, the evaluated subjects in the government are $A_1, A_2, \ldots, A_m$ and the evaluated subjects in the enterprise are $B_1, B_2, \ldots, B_n$ are clear The directed dotted lines between subjects in the government and subjects in the enterprise denote government satisfaction of subjects in the enterprise and enterprise satisfaction of subjects in the government, respectively. Undirected solid lines denote the matches between the subjects in the government and the subjects in the enterprise. The lines joining the subjects in the government with the evaluative criteria denote the evaluative criteria which are considered by the subjects in the enterprise for matching. Similarly, the lines joining the subjects in the enterprise with the evaluative criteria denote the evaluative criteria which are considered by the subjects in the government for matching. For each subject in the government, we assume that the evaluative criteria are assumed to be the same as in the enterprise. The same case is assumed for each subject in the enterprise subject. However, the evaluative criteria considered for the subjects in the government might be different from that considered for the subjects in the enterprise. Under this framework, the subjects in the government will measure enterprise satisfaction through the left evaluative criteria $c_1, c_2, \ldots, c_g$ and the subjects in the enterprise will measure government satisfaction through the right evaluative criteria $d_1, d_2, \ldots, d_l$.

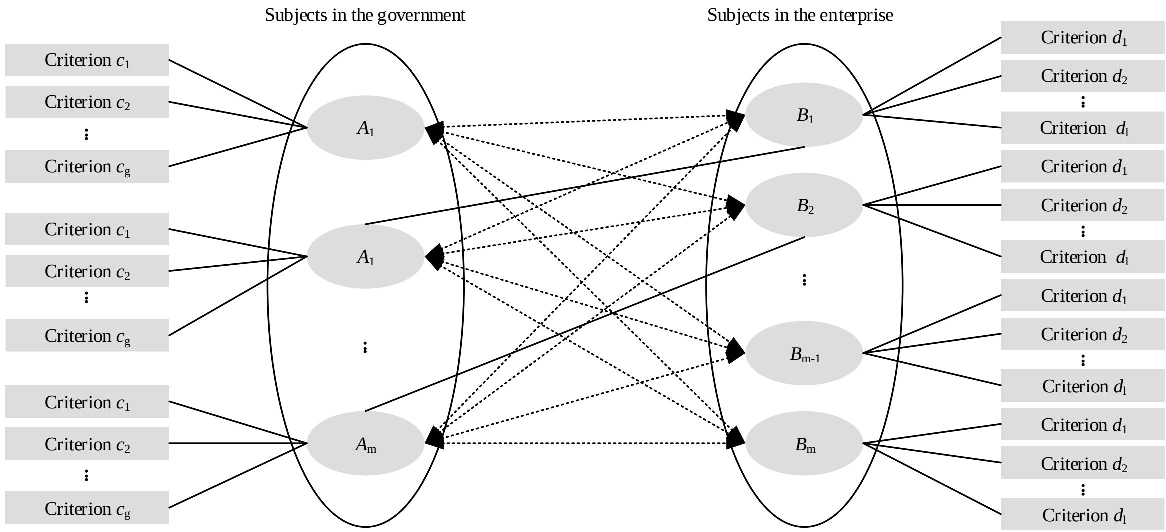

Figure 1. The illustration of Two-Sided Matching (TSM) based on different evaluative criteria.

3.2. TSM Decision Making Model Development

The research framework shown in Figure 2 presents processes and associated techniques used in this study. Major steps include: (a) structuring an evaluative criterion system of PPP infrastructure projects, (b) determining the evaluative criterion weights, (c) calculating the compatibility of both sides (i.e., the subjects in government and the subjects in the enterprise), (d) establishing the matching decision making model, (e) solving the matching decision making model, and (f) comprehensive evaluation.

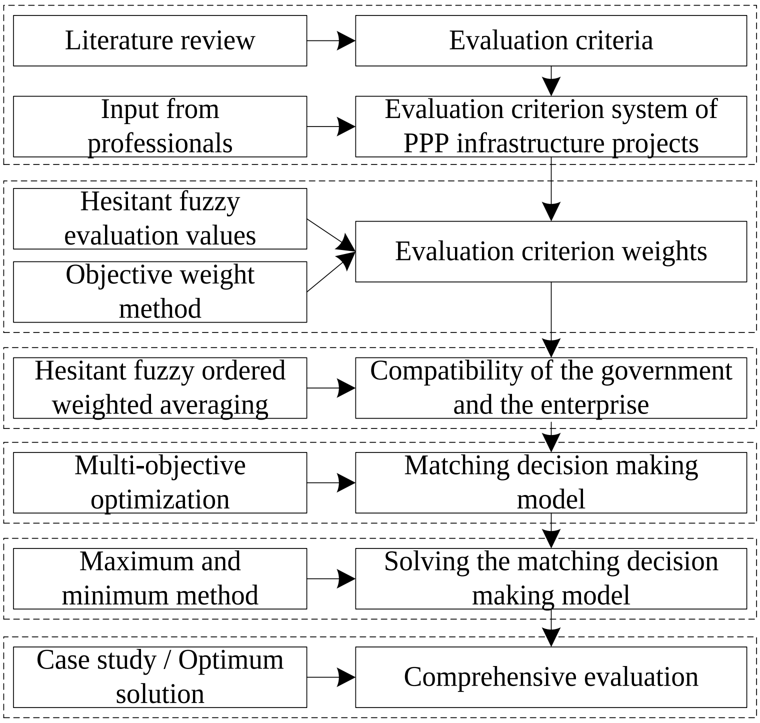

Figure 2. Research framework of TSM based on Hesitant Fuzzy Element (HFE).

Structuring an evaluative criterion system. Through interview and literature review, 29 evaluative criteria are classified into nine categories, including financial strength, technical ability, management ability, relevant experience, credit level, governmental insurance, governmental guarantee, governmental credibility, and governmental risk-sharing for PPP infrastructure projects. Evaluative criteria of matching on both the public and private sectors under the PPP model are shown in Table 1. The private evaluative criteria and the public evaluation criteria are shown in Tables 2 and 3, respectively. All the sub-criterion will be used for the following matching decision.

Determining the evaluative criterion weights. Using subjective and objective criteria in the weight assignment method is essential for coordination and compromise of different evaluative criterion weights because the evaluative criterion weights often are completely unknown [35]. The lower difference between evaluative criterion weights means a closer preference difference between the objects and the matching objects, which is not conducive to optimal matching. Moreover, determining evaluative criterion weights effectively needs to take into account distribution characters of the evaluative values. If there is no difference in values of a certain evaluative criterion for matching objects, then the evaluative criterion has little effect on decision ranking, and thus, its weight should be given a smaller value accordingly. Otherwise, the evaluative criterion has a significant effect on decision ranking. Consequently, its weight should be given a bigger evaluative value accordingly.

Table 1. Evaluative criteria of matching on both the public and private sectors under Public–Private Partnership (PPP) model.

Criteria	Sub-Criteria	Belassi and Tukel [28]	Zhang [25]	Wang et al. [29]	Aziz [30]	Kwak et al. [24]	Chan et al. [15]	Chan et al. [17]	Chou et al. [22]	Ismail [23]	Zhang et al. [27]	Wibowo and Alfen [6]	Liu et al. [1]	Total Number of Hits of a Certain Criterion
Financial strength	Capital strength	√		√	√	√				√				5
	Financing capacity	√	√		√	√	√	√					√	7
	Financial guarantee				√									1
Technical ability	Advanced technology		√	√		√	√		√	√				6
	Project implementation plans			√			√	√	√				√	5
	Personnel reserve				√		√			√				3
	Critical equipment						√		√					2
Management ability	Management formalization	√		√				√					√	4
	Project management capacity	√	√	√				√	√				√	6
	Ability of communication and cooperation	√		√		√	√	√		√			√	7
	Concession duration		√		√	√	√							4
	Risk management ability	√	√	√					√					4
	Maintenance ability		√			√			√					3
Relevant experience	Financing experience		√	√										2
	Constructing experience	√		√	√									3
	Operating experience			√	√									2
Credit level	Enterprise qualification	√												1
	Honor on projects												√	2
	Social reputation				√			√	√				√	4
Government guarantee	Legal guarantee	√	√	√	√		√	√	√	√	√	√	√	11
	Approval guarantee		√				√		√	√		√		5
Government support	Political support	√		√	√	√		√	√	√		√	√	9
	Interest rate guarantee	√	√	√		√	√		√			√	√	8
	Price guarantee			√			√		√			√	√	5
Government credibility	———		√	√								√		3
Government risk-sharing	Restriction of competition			√		√	√		√	√		√	√	7
	Information transparency				√			√				√	√	4
	Contract spirits		√	√	√	√	√				√	√		7
	Risk-taking		√	√		√		√	√			√	√	7
		11	13	18	12	11	13	10	14	8	2	10	14	–

Table 2. Basic decomposition of evaluative criteria on the subjects in the enterprise by the subjects in the government.

Sub-Criterion	Description
1. Capital strength	Total funds, quantity and scale invested for completed construction projects
2. Financing capacity	The scale of financing possible
3. Financial guarantee ability	Ability of obtaining financing guarantee from financial institutions for the construction project
4. Advanced technology	The leading technology in the world
5. Project implementation plans	Plan formulation of activities or process of hard work from start to finish
6. Personnel reserve	Human resources to meet the demand of long-term development goal for the enterprise
7. Critical equipment	Key equipment to ensure continuous, stable, efficient and economic operation of the entire machinery
8. Management formalization	Degree of standardization on enterprise strategic management, marketing management, human resource management, financial and tax management, material management, etc.
9. Project management capacity	Ability of plan, organization, coordination, implementation and control of project managers
10. Ability of communication and cooperation	Skills of coordination and communication with the government
11. Concession duration	Specified period of recycling project costs on investment, operation and maintenance, and obtaining a reasonable return
12. Risk management ability	Ability of minimizing the risk in a certain risky environment
13. Maintenance Ability	Ability of operation and maintenance for infrastructure projects
14. Financing experience	Experiences of financing the similar infrastructure projects
15. Constructing experience	Experiences of constructing the similar infrastructure projects
16. Operating experience	Experiences of operating the similar infrastructure projects
17. Enterprise qualifications	The qualification and level of quality standard adapted to the qualification
18. Honor on projects	National or provincial honor gained by the enterprises for their completed projects
19. Social reputation	Credit evaluation given by partners cooperating in the enterprise

Table 3. Basic decomposition of evaluative criteria on the subjects in the government by the subjects in the enterprise.

Sub-Criterion	Description
1. Legal guarantee	Laws define the responsibility division between the parties, project assessment, contract performance and change, and the way conflicts, etc.
2. Approval guarantee	The government need to provide the provision of legal guarantee to businesses involved in infrastructure projects
3. Police guarantee	The government needs to set up corresponding approval for security and incentives to encourage more businesses to participate in PPP infrastructure projects
4. Interest rate guarantee	The government should offer the recoverable rate of investment to businesses via offering government-backed rate in the scope of given benefit level
5. Price guarantee	The government should offer the recoverable rate of investment to businesses via offering government-backed price in the scope of given benefit level
6. Purchase guarantee	The government should offer the recoverable rate of investment to businesses via offering government-backed purchase in the scope of given benefit level
7. Restriction of competition	Similar projects will not build at any given time
8. Information transparency	Key information about the PPP Project disclosed by the government
9. Contract spirits	The government's contract spirits to guarantee the legitimate rights and interests of businesses
10. Contract spirits	The government ought to initially bear the risks of land policy changes, legal policy changes and exchange rate changes

Based on the above analysis, a nonlinear optimization model is established to determine evaluative criterion weights through maximizing deviation between the values of the evaluative criteria for all matching objects as below:

$$\begin{array}{l} \max\{d(B)\} = \max \sum\limits_{q=1}^{n} d(B_q) = \sum\limits_{q=1}^{n} \sum\limits_{j=1}^{t} \sum\limits_{k \neq j}^{t} \sqrt{\frac{1}{l} \sum\limits_{\lambda=1}^{l} \left(\omega_j h_{qj}^{\sigma(\lambda)} - \omega_k h_{qk}^{\sigma(\lambda)} \right)^2} \\ \text{s.t.} \sum\limits_{j=1}^{n} \omega_j = 1,\ 0 \leq \omega_j \leq 1,\ j = 1, 2, \cdots, t \end{array} \tag{1}$$

where ω_j, ω_k $(j, k = 1, 2, \cdots, t)$ denote evaluative criterion weights of the enterprise $B_q(q = 1, 2, \cdots, n)$ by the government $A_p(p = 1, 2, \cdots, m)$. Here, $d(B)$ denotes the sum of the deviation on HFE evaluative values of all the subjects in the enterprise by the government $A_p(p = 1, 2, \cdots, m)$. $h_{qj}^{\sigma(\lambda)}$ and $h_{qk}^{\sigma(\lambda)}$ are the λth elements in h_{qj} and h_{qk} with an ascending sort order, respectively.

Calculating the compatibility on both sides. In the TSM decision making process, the government and enterprise satisfaction (i.e., compatibility) should be taken into full account and calculated to obtain an optimal matching scheme. In classic multi-attribute decision making theory, evaluative values and evaluative criterion weights of schemes are weighted and summed to gain utility values and the schemes are ranked based on the utility values. Similarly, through weighted aggregating the evaluative value and the evaluative criterion weight of one subject in the enterprise, the utility value of the subject in the enterprise is gained. In general, the utility values and the compatibility of all the subjects in the government and the subjects in the enterprise have the same monotonicity. Based on the above analysis, the utility values and the compatibility of all the subjects in the government and the subjects in the enterprise can be calculated [12,36]. Consequently, the compatibility matrix of all the subjects in the government and the subjects in the enterprise are obtained.

Establishing the matching decision making model. After obtaining the compatibility matrices for all the subjects in the government and the subjects in the enterprise, 0–1 decision making variables $x_{ij}(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$ are introduced to establish the matching decision model. If the value of the 0–1 decision variable is equal to 1, it denotes a matching between the subjects in the government and the subjects in the enterprise; while if the value is equal to 0, it denotes non-matching between the subjects in the government and the subjects in the enterprise. During the TSM decision process, both the subjects in the government and the subjects in the enterprise are supposed to hope to have greater satisfaction. Then, a TSM decision model is established with the goal of maximizing the sum of compatibility for all the subjects in the government and the subjects in the enterprise as following:

$$\max Z_1 = \sum_{p=1}^{m} \sum_{q=1}^{n} \mu_{A_p,B_q} x_{pq} \tag{2}$$

$$\max Z_2 = \sum_{p=1}^{m} \sum_{q=1}^{n} \mu'_{B_q,A_p} x_{pq} \tag{3}$$

$$\text{s.t.} \sum_{q=1}^{n} x_{pq} = 1, x_{pq} \in \{0,1\}, p = 1, 2, \cdots, m \tag{4}$$

$$\sum_{p=1}^{m} x_{pq} \leq 1, x_{pq} \in \{0,1\}, q = 1, 2, \cdots, n \tag{5}$$

In the model above, the compatibility matrix of all the subjects in the government given by subjects in the enterprise and the compatibility matrix of all the subjects in the enterprise given by subjects in the government are $U_{A,B}\left(\mu_{A_p,B_q}\right)_{m \times n}$ and $U'_{B,A}\left(\mu'_{B_q,A_p}\right)_{m \times n}$, respectively. Formulas (2)–(3) are the objective functions of the model to maximize the compatibility on each pair between the subjects in the government and the subjects in the enterprise. Formula (4) is the constraint of the model.

Since the number of the subjects in the government does not exceed the number of the subjects in the enterprise, Formula (4) is an equality constraint, which implicates that one government subject only matches one potential subject in the government and the subjects in the enterprise; while Formula (5) is an inequality constraint, which implicates that one subject in the enterprise matches a potential subject in the government at most.

Solving the matching decision model. The matching decision model (2)–(5) is a multi-objective optimization problem which can be solved by many different algorithms, such as the firefly algorithms [37,38]. This study used the method in Li and Wan [39] to solve this problem. Let the maximum values of the above model $Z_k^{\max}(k = 1, 2)$ be the objective function for considering a single objective function separately. Then, the corresponding optimal solutions can be denoted as:

$$\begin{aligned} & Z_k^{\max} = \max\left(\frac{1}{2}\left(\min\{\beta_k(x)|k = 1, 2\} + \frac{1}{2}\sum_{k=1}^{2}\beta_k(x)\right)\right) \\ & \text{s.t.} \sum_{q=1}^{n} x_{pq} = 1, \sum_{p=1}^{m} x_{pq} \le 1, x_{pq} \in \{0, 1\}, p = 1, 2, \cdots, m; q = 1, 2, \cdots, n \end{aligned} \tag{6}$$

The Model (6) is equivalent to a single-objective optimization as below:

$$\begin{aligned} & \max\left(\frac{1}{2}\left(\min\{\beta_k(x)|k = 1, 2\} + \frac{1}{2}\sum_{k=1}^{2}\beta_k(x)\right)\right) \\ & \text{s.t. } 2\beta_k + \sum_{k=1}^{2}\beta_k = 4\beta, x_{pq} \in \{0, 1\}, k = 1, 2; p = 1, 2, \cdots, m; q = 1, 2, \cdots, n \end{aligned} \tag{7}$$

where β_k is the membership of the objective function $Z_k(k = 1, 2)$ [39]. Finally, using the linear weighted method [40] to calculate Model (7), the optimal solutions of the matching optimization model (2)–(5) are obtained.

4. Application of Proposed Model: Calculation Example

4.1. Data Sources

This selection supposes that there are three government PPP projects (A_1(3), A_2(4), A_3(3)), and six filtered bidding subjects in the enterprise (B_1(3), B_2(4), B_3(3), B_4(3), B_5(4), B_6(2)). To improve the matching efficiency and success rate of the subjects in the government and the subjects in the enterprise in PPP projects and to maximize the benefits from cooperation and reduce the risk of cooperation, the subjects in the government and the subjects in the enterprise are used to evaluate each other on the basis of five evaluative criteria (i.e., c_1: *financial strength*, c_2: *technical ability*, c_3: *management ability*, c_4: *relevant experience* and c_5: *credit level*) of the private sectors, and four evaluative criteria (i.e., d_1: *governmental insurance*, d_2: *governmental guarantee*, d_3: *governmental credibility*, and d_4: *governmental risk-sharing*) of the public sectors developed in the Section *Structuring an evaluative criterion system.* In addition, due to heterogeneity on the preference of the chief executives, HFS is applied to describe evaluative values given by various DMs to maximize the limit retaining the initial evaluative information. The decision making level of each subject in the government and each subject in the enterprise is composed of several senior executives in the same position, i.e., DMs, who are the experts in the field with at least 30 years of working experience in infrastructure projects. Numbers in the brackets are the numbers of the members. Various DMs give their evaluative values on the basis of the evaluative criterion system as shown in Tables 4 and 5.

Table 4. Initial evaluative values of each subject in the enterprise given by the subjects in the government.

Subjects in the Government	Criteria	Initial Evaluative Values					
		B_1	B_2	B_3	B_4	B_5	B_6
A_1	c_1	(0.78,0.75,0.83)	(0.91,0.87,0.90)	(0.55,0.64,0.56)	(0.65,0.72,0.50)	(0.48,0.55,0.65)	(0.47,0.57,0.49)
	c_2	(0.75,0.87,0.67)	(0.85,0.73,0.83)	(0.53,0.49,0.41)	(0.86,0.83,0.78)	(0.68,0.72,0.56)	(0.39,0.47,0.56)
	c_3	(0.85,0.70,0.69)	(0.67,0.53,0.49)	(0.85,0.63,0.71)	(0.48,0.53,0.62)	(0.74,0.55,0.60)	(0.74,0.79,0.83)
	c_4	(0.53,0.49,0.41)	(0.68,0.70,0.83)	(0.57,0.47,0.49)	(0.89,0.92,0.80)	(0.72,0.68,0.48)	(0.55,0.56,0.64)
	c_5	(0.58,0.40,0.59)	(0.63,0.56,0.58)	(0.71,0.47,0.63)	(0.73,0.82,0.69)	(0.78,0.84,0.80)	(0.56,0.58,0.63)
A_2	c_1	(0.69,0.71,0.82,0.75)	(0.83,0.80,0.86,0.71)	(0.47,0.82,0.63,0.55)	(0.63,0.72,0.48,0.52)	(0.58,0.68,0.55,0.62)	(0.53,0.49,0.50,0.49)
	c_2	(0.92,0.71,0.68,0.73)	(0.79,0.87,0.63,0.85)	(0.51,0.81,0.38,0.71)	(0.78,0.84,0.80,0.91)	(0.42,0.38,0.60,0.48)	(0.55,0.72,0.65,0.61)
	c_3	(0.74,0.65,0.72,0.80)	(0.66,0.72,0.83,0.76)	(0.74,0.79,0.68,0.83)	(0.48,0.38,0.50,0.62)	(0.66,0.58,0.42,0.55)	(0.77,0.69,0.82,0.86)
	c_4	(0.65,0.42,0.56,0.50)	(0.84,0.86,0.91,0.85)	(0.65,0.56,0.58,0.49)	(0.84,0.86,0.91,0.89)	(0.72,0.68,0.58,0.72)	(0.52,0.63,0.47,0.55)
	c_5	(0.55,0.48,0.35,0.42)	(0.72,0.56,0.64,0.72)	(0.55,0.48,0.63,0.38)	(0.78,0.60,0.58,0.74)	(0.78,0.69,0.78,0.84)	(0.35,0.55,0.48,0.65)
A_3	c_1	(0.49,0.52,0.64)	(0.74,0.83,0.79)	(0.47,0.49,0.56)	(0.62,0.58,0.72)	(0.42,0.63,0.58)	(0.56,0.75,0.55)
	c_2	(0.55,0.40,0.39)	(0.82,0.79,0.85)	(0.55,0.71,0.60)	(0.84,0.79,0.87)	(0.59,0.72,0.55)	(0.60,0.48,0.49)
	c_3	(0.65,0.73,0.92)	(0.74,0.48,0.79)	(0.83,0.91,0.79)	(0.48,0.32,0.48)	(0.64,0.56,0.55)	(0.79,0.87,0.85)
	c_4	(0.39,0.40,0.41)	(0.68,0.84,0.79)	(0.62,0.71,0.59)	(0.76,0.84,0.80)	(0.63,0.58,0.56)	(0.56,0.65,0.41)
	c_5	(0.50,0.48,0.72)	(0.56,0.55,0.75)	(0.72,0.58,0.63)	(0.56,0.48,0.60)	(0.86,0.83,0.80)	(0.49,0.62,0.47)

Table 5. Initial evaluative values of each subject in the government given by the subjects in the enterprise.

Subjects in the Enterprise	Criteria	Initial Evaluative Values		
		A_1	A_2	A_3
B_1	d_1	(0.58,0.55,0.36)	(0.68,0.72,0.73)	(0.63,0.58,0.60)
	d_2	(0.59,0.40,0.60)	(0.71,0.78,0.82)	(0.74,0.68,0.50)
	d_3	(0.50,0.58,0.38)	(0.65,0.79,0.68)	(0.54,0.60,0.46)
	d_4	(0.92,0.90,0.82)	(0.64,0.49,0.53)	(0.69,0.58,0.38)
B_2	d_1	(0.50,0.69,0.63,0.48)	(0.79,0.81,0.83,0.69)	(0.59,0.60,0.58,0.40)
	d_2	(0.52,0.40,0.49,0.60)	(0.71,0.86,0.75,0.63)	(0.49,0.60,0.38,0.55)
	d_3	(0.56,0.48,0.65,0.71)	(0.63,0.74,0.87,0.72)	(0.53,0.62,0.48,0.39)
	d_4	(0.89,0.84,0.78,0.80)	(0.54,0.63,0.48,0.67)	(0.54,0.57,0.48,0.62)
B_3	d_1	(0.49,0.63,0.58)	(0.68,0.46,0.71)	(0.57,0.85,0.77)
	d_2	(0.62,0.38,0.58)	(0.56,0.67,0.87)	(0.64,0.50,0.55)
	d_3	(0.39,0.42,0.62)	(0.68,0.73,0.86)	(0.63,0.46,0.50)
	d_4	(0.78,0.86,0.82)	(0.57,0.69,0.48)	(0.63,0.58,0.58)
B_4	d_1	(0.55,0.72,0.62)	(0.72,0.68,0.63)	(0.79,0.81,0.63)
	d_2	(0.42,0.58,0.62)	(0.86,0.87,0.65)	(0.55,0.36,0.63)
	d_3	(0.55,0.48,0.71)	(0.86,0.78,0.78)	(0.55,0.66,0.50)
	d_4	(0.79,0.86,0.78)	(0.64,0.49,0.56)	(0.64,0.44,0.56)
B_5	d_1	(0.62,0.73,0.58,0.60)	(0.72,0.46,0.68,0.75)	(0.69,0.58,0.59,0.68)
	d_2	(0.51,0.63,0.60,0.42)	(0.92,0.73,0.82,0.80)	(0.56,0.65,0.48,0.53)
	d_3	(0.36,0.42,0.48,0.56)	(0.86,0.73,0.77,0.75)	(0.58,0.49,0.50,0.62)
	d_4	(0.89,0.80,0.79,0.88)	(0.57,0.63,0.46,0.39)	(0.63,0.57,0.48,0.39)
B_6	d_1	(0.34,0.55)	(0.81,0.78)	(0.59,0.58)
	d_2	(0.58,0.55)	(0.82,0.86)	(0.50,0.68)
	d_3	(0.43,0.69)	(0.68,0.73)	(0.59,0.63)
	d_4	(0.89,0.76)	(0.63,0.49)	(0.78,0.57)

4.2. Model Results

To solve the TSM problem of PPP infrastructure projects, the calculation process of the proposed method is explained in detail in Appendix A.

According to Model (7), the results can be obtained as $Z_1^{\max} = 1.7521$, $Z_1^{\min} = 1.5589$; $Z_2^{\max} = 1.7877$, $Z_2^{\min} = 1.2945$; $\beta_1 = \frac{Z_1 - 1.5589}{1.7521 - 1.5589}$, $\beta_2 = \frac{Z_2 - 1.2945}{1.7877 - 1.2945}$. Then, a single-objective linear programming model (SLPM) for the problem is presented as:

$$\begin{aligned} & \max \beta \\ & \text{s.t. } 2\beta_1 + \beta_1 + \beta_2 \geq 4\beta,\ 2\beta_2 + \beta_1 + \beta_2 \geq 4\beta,\ x_{pq} = 0 \text{ or } 1;\ p = 1,2,3; q = 1,2,\cdots,6 \end{aligned}$$

It is easy to obtain the objective function values as Z_1 = 1.6804, Z_2 = 2.0432. An optimal solution is X* = {(1,1),(2,2),(3,6)}. According to the gained optimal solution, the best matching scheme is {$A_1 \leftrightarrow B_2$, $A_2 \leftrightarrow B_2$, $A_3 \leftrightarrow B_6$}.

4.3. Discussion and Findings

For a more precise discussion, the following three different scenarios are considered.

Case 1: In this case, the match-degree of *the subjects in the government* in the three-dimensional space. For the subject A_1 in the government discussed, there are six possible matchings, i.e., A_1 to B_1, B_2, B_3, B_4, B_5 and B_6, respectively. After A_1 matching is determined, A_2 can be matched with one of the five subjects left in the enterprise. Then A_3 can be matched with one of the four left. The corresponding results are shown in Figure 3. In this figure, different colors are used to show different matching. For example, the blue plots represent that the subject A_1 matches with the subject B_1 in the enterprise; the red plots represent that the subject A_1 matches with the subject B_2 in the enterprise. The X-axis is the sum of the sequence numbers of the subject A_2 and its matched subjects in the enterprise. Taking one of the blue plots for example, the value of X-axis is 6, which indicates that the subject B_4 in the enterprise matches with the subject A_2 as 4 + 2 = 6. Similarly, the Y-axis is the sum of sequence numbers of the subject A_3 and its matched subjects in the enterprise. Again, one of the blue plots is taken as an example. The value of Y-axis is 9, which indicates that the subject B_6 in the enterprise matches with the subject A_3 as 6 + 3 = 9. The Z-axis is the match-degree of subjects in the government. There are two different lines (i.e., blue line and red line) to present different match-degree of the subjects in the government. The blue line and the red line are the maximum and minimum match-degree for all the subjects in the government, respectively. As shown in Figure 3, the maximum match-degree of the subjects in the government is 1.7344; the corresponding matching scheme is $\{A_1 \leftrightarrow B_4, A_2 \leftrightarrow B_1, A_3 \leftrightarrow B_2\}$. The minimum match-degree of the subjects in the government is 1.5589; the corresponding matching scheme is $\{A_1 \leftrightarrow B_3, A_2 \leftrightarrow B_6, A_3 \leftrightarrow B_1\}$. The results show that the change of the match-degree of the subjects in the government is not significant as the matching scheme is changed.

Case 2: In this case, the match-degree of *the subjects in the enterprise* in the three-dimensional space is discussed. For the subject A_1 in the government, there are six possible matching, i.e., A_1 to B_1, B_2, B_3, B_4, B_5 and B_6, respectively. After A_1 matching is determined, A_2 can be matched with one of the five subjects left in the enterprise. Then A_3 can be matched with one of the four left. The corresponding results are shown in Figure 4. In this figure, different colors are used to show different matchings for A_1. For example, the blue plots represent that the subject A_1 matches with the subject B_4 in the enterprise; the red plots represent that the subject A_1 matches with the subject B_2 in the enterprise. The X-axis is the sum of the sequence numbers of the subject A_2 and its matched subjects in the enterprise. Similarly, the Y-axis is the sum of sequence numbers of the subject A_3 and its matched subjects in the enterprise. The Z-axis is the match-degree of subjects in the enterprise. There are two different lines (i.e., the blue line and red line) to present the different match-degrees of the subjects in the enterprise. The blue line and the red line are the maximum and minimum match-degrees for all the subjects in the enterprise, respectively. As shown in Figure 4, the maximum match-degree of the subjects in the enterprise is 2.0432; the corresponding matching scheme is $\{A_1 \leftrightarrow B_6, A_2 \leftrightarrow B_3, A_3 \leftrightarrow B_4\}$. The minimum match-degree of the subjects in the enterprise is 1.4132; the corresponding matching scheme is $\{A_1 \leftrightarrow B_1, A_2 \leftrightarrow B_3, A_3 \leftrightarrow B_6\}$.

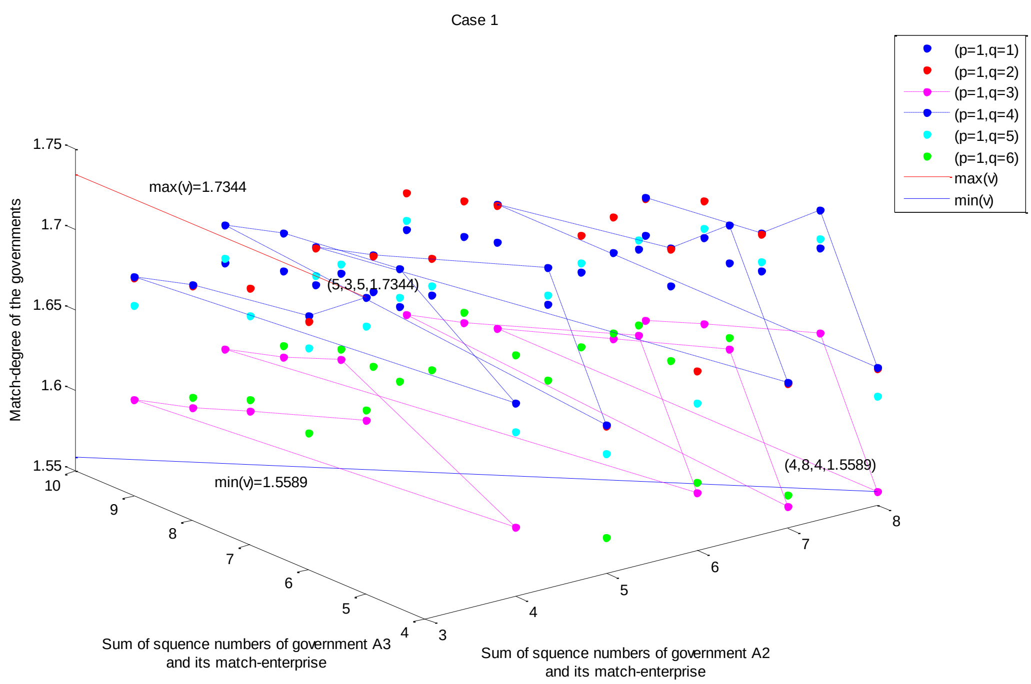

Figure 3. Match-degree of subjects in the government in the three-dimensional space.

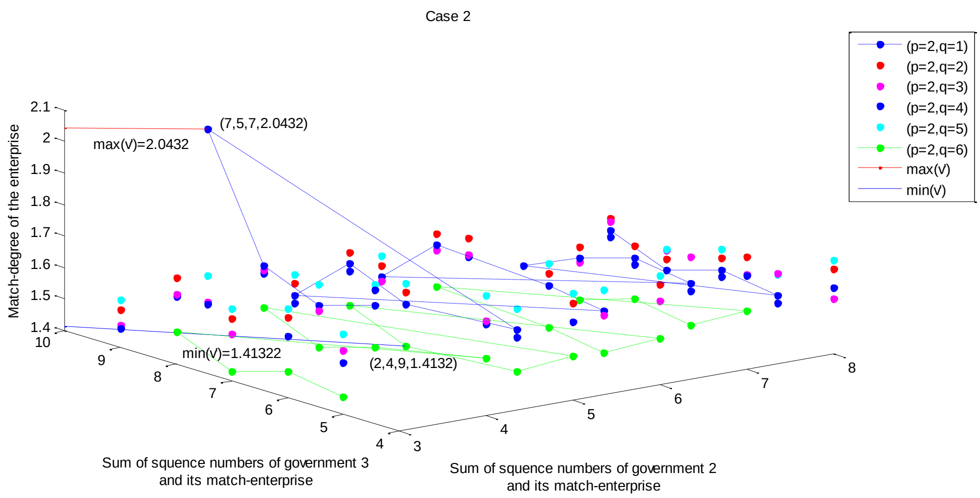

Figure 4. Match-degree of subjects in the enterprise in the three-dimensional space.

Case 3: In this case, the match-degree of *the subjects in the two sides* in the three-dimensional space is discussed. For the subject A_1 in the government, there are six possible matchings, i.e., A_1 to B_1, B_2, B_3, B_4, B_5 and B_6, respectively. After A_1 matching is determined, A_2 can be matched with one of the five subjects left in the enterprise. Then A_3 can be matched with one of the four left. The corresponding results are shown in Figure 5. In this figure, different colors are used to show different matchings for A_1. For example, the blue plots represent that the subject A_1 matches with the subject B_4 in the enterprise; the red plots represent that the subject A_1 matches with the subject B_2 in the enterprise. The X-axis is the sum of sequence numbers of the subject A_2 and its matched subjects in the enterprise. Similarly, the Y-axis is the sum of sequence numbers of the subject A_3 and its matched subjects in the enterprise. The Z-axis is the match-degree of the subjects in the two sides. There are two different lines (i.e., blue line and red line) to present the different match-degrees of the two sides. The blue line and the red line are the maximum and minimum match-degrees for the subjects in the two parts, respectively. As shown in Figure 5, the maximum match-degree of the subjects in the two parts is 3.7236; the corresponding matching scheme is $\{A_1 \leftrightarrow B_1, A_2 \leftrightarrow B_3, A_3 \leftrightarrow B_6\}$. The minimum match-degree of the subjects in the two sides is 3.0438; the corresponding matching scheme is $\{A_1 \leftrightarrow B_6, A_2 \leftrightarrow B_3, A_3 \leftrightarrow B_4\}$.

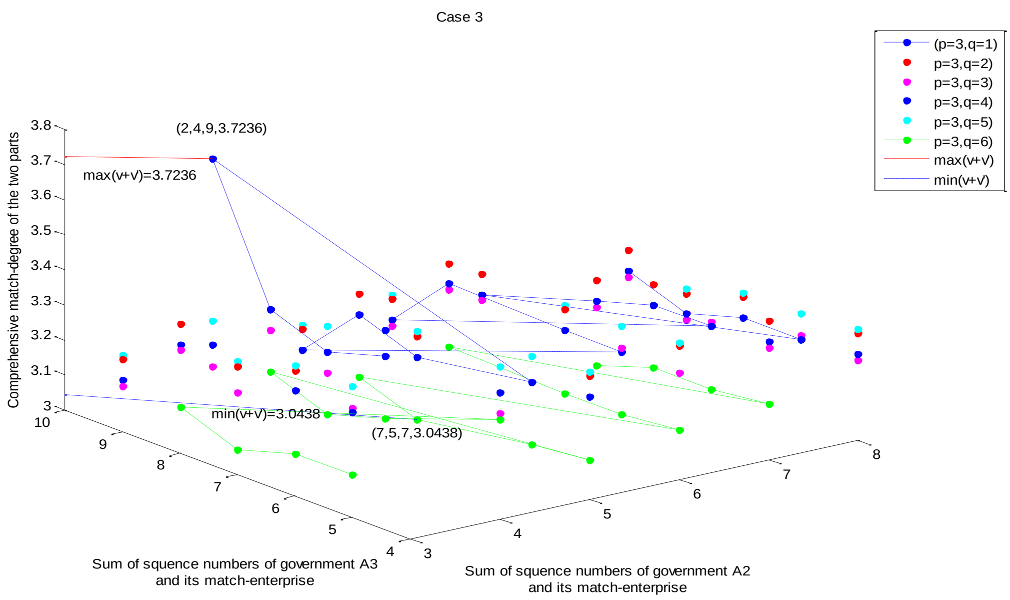

Figure 5. Comprehensive match-degree of the two parts in the three-dimensional space.

From Case 1, Case 2, and Case 3, the best matching scheme in Case 3 is neither the best matching scheme in Case 1, nor the best matching scheme in Case 2. Similarly, the worst matching scheme in Case 3 is neither the worst matching scheme in Case 1, nor the worst matching scheme in Case 2. Moreover, multiple match-degrees of the subjects in the government are close to the best match-degree of the subjects in the government in Case 1. That is, multiple matching schemes are comparatively better for the subjects in the government [14,41]. The United Nations Economic Commission for Europe (UNECE) (2007) claimed that the inefficiency in governance led to the failure in the implementation of PPP in many countries. Thus, a good subject in the government was ranked first as a necessary factor to ensure the success of PPP projects [23]. Accordingly, the best matching schemes in Case 2 and Case 3 fully play to their advantages, because their match-degree in Case 2 and Case 3 become more volatile compared with the others, respectively. However, as shown in Figure 6, taking all matching schemes for the subject A_1 in the government, for instance, the average match-degree of the subjects in the government is superior to the average match-degree of the subjects in the enterprise [15,42,43].

Figure 7 depicts criterion weights on both parties involved. Specifically, the left side displays the values of the evaluative criterion weights for each subject in the government by the subjects in the enterprise; and the right side displays the values of the evaluative criterion weights for each subject in the enterprise by the subjects in the government. The results shown that the evaluative criterion weights of the three subjects in the government are very near and lie on 0.2000; while the evaluative criterion weights of the six subjects in the enterprise remained basically stable, despite a large increase in the third evaluative criterion of the subject B_6 in the enterprise and a large decrease in the first evaluative criterion of the subject B_2 in the enterprise. Also, the average value of evaluative criterion weights on the right side is higher than the average value of evaluative criterion weights on the left side. It reveals that government intervention and corruption may be the major obstacles to the success of PPP projects in China [15].

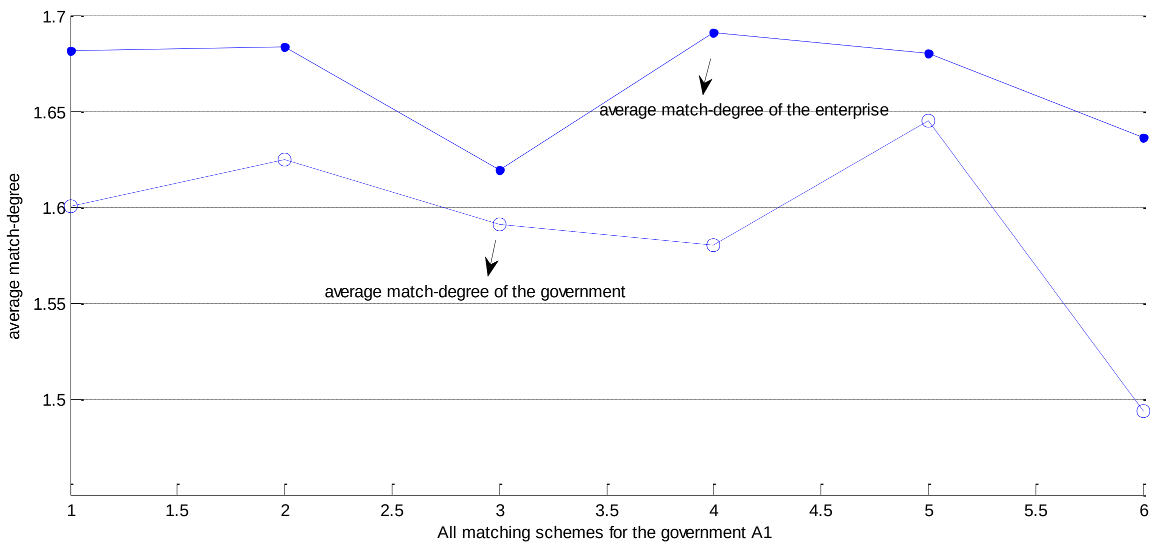

Figure 6. Average match-degree for all matching schemes for the government A_1.

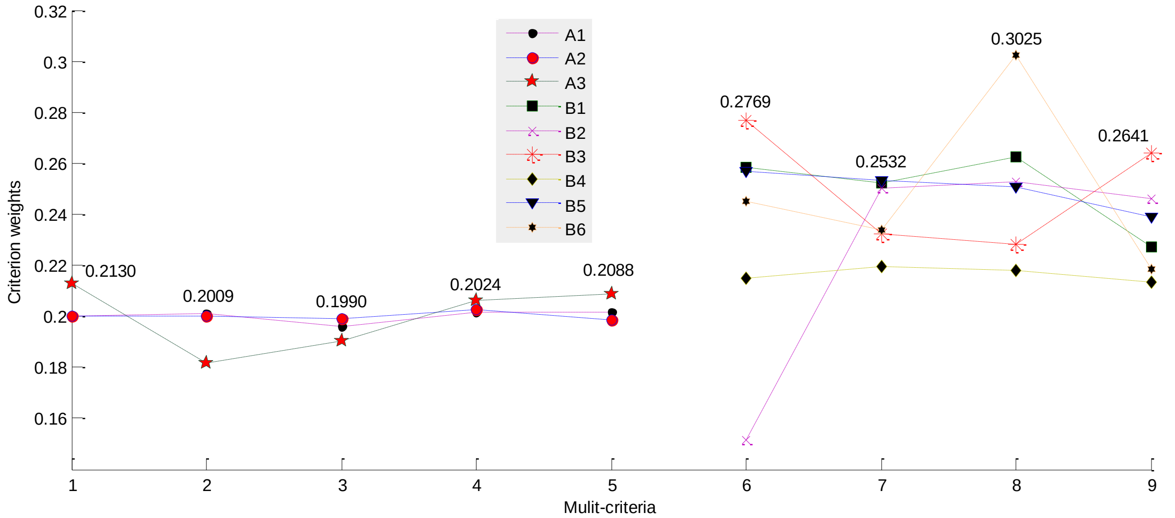

Figure 7. Change of evaluative criterion weights on two parts.

4.4. Practical Case

The Beijing Metro Line 4, officially opened for transportation use on September 28, 2009, is a south–north rail line that runs through urban districts in Beijing, with an overall length of 29 km and 24 stations. With an estimated investment of 153 billion Chinese Yuan (equivalent to 22.22 billion US dollars), this construction project B of Beijing Metro Line 4 invested 46 billion Chinese Yuan (equivalent to 6.68 billion US dollars). Aiming to assure the successful construction of project B, Beijing Infrastructure Investment Co., LTD (BII (3)), as a representative of the subject in the government; Siemens Traffic Technology Group (STTG (3)), Corporation Limited (MTR (4)), Beijing First Group Co., Ltd. (BFG (4)), China Railway Construction Corporation (CRCC (3)) and Beijing Mass Transit Railway Operation Corporation Limited (BMTRO (3)), as representatives of the subjects in the enterprise, expressed their willingness to cooperate. An authoritative tender agency is responsible for the bidding work of the project representative (i.e., BII) and the enterprises in the way of public bidding. Through qualification examination, five filtered bidding subjects in the enterprise (i.e., STTG, MTR, BFG, CRCC, and BMTRO) are eligible to comply with the requirements of subjects in the government. The tender agency receives relevant information from the subject in the government and the five subjects in the enterprise. Various DMs give their evaluative values on the basis of the evaluative criterion system as shown in Tables 6 and 7.

The values of criterion weights on BII are calculated as, in order, 0.1926, 0.1901, 0.1986, 0.2027, and 0.2159; Meanwhile, the values of criterion weights on STTG, MTR, BFG, CRCC, and BMTRO are calculated as, in order, 0.1849, 0.2205, 0.2221 then 0.3725; 0.3952, 0.3111, 0.2552 then 0.3878; 0.2081, 0.3451, 0.2438 then 0.2030; 0.2344, 0.2083, 0.2730 then 0.2843; 0.3450, 0.2732, 0.2000 and 0.1823, respectively. Then, the objective function values of BII matching STTG, MTR, BFG, CRCC and BMTRO are obtained as 0.5046, 0.5291, 0.5548, 0.4953, and 0.5425, respectively. Similarly, objective function values of STTG, MTR, BFG, CRCC and BMTRO matching BII as 0.5013, 0.5933, 0.5508, 0.5565, and 0.4920, respectively. Consequently, the two optimal enterprises are MTR and BFG, which coincides with the actual decision. The calculation process of the proposed method is shown in the Appendix.

Table 6. Initial evaluative values on BII given by Siemens Traffic Technology Group (STTG), Corporation Limited (MTR), Beijing First Group Co., Ltd. (BFG), China Railway Construction Corporation (CRCC), and Beijing Mass Transit Railway Operation Corporation Limited (BMTRO).

Criteria	Initial Evaluative Values				
	STTG	MTR	BFG	CRCC	BMTRO
c_1	(0.63,0.68,0.52)	(0.83,0.75,0.58)	(0.57,0.74,0.71)	(0.72,0.49,0.54)	(0.79,0.55,0.49)
c_2	(0.72,0.55,0.48)	(0.68,0.59,0.47)	(0.63,0.48,0.79)	(0.65,0.39,0.77)	(0.92,0.68,0.78)
c_3	(0.56,0.64,0.39)	(0.74,0.48,0.79)	(0.68,0.63,0.91)	(0.49,0.58,0.87)	(0.44,0.57,0.63)
c_4	(0.58,0.65,0.63)	(0.82,0.41,0.69)	(0.55,0.63,0.75)	(0.57,0.49,0.75)	(0.55,0.58,0.73)
c_5	(0.67,0.69,0.55)	(0.49,0.38,0.54)	(0.62,0.81,0.56)	(0.44,0.36,0.39)	(0.53,0.60,0.75)

Table 7. Initial evaluative values on STTG, MTR, BFG, CRCC, and BMTRO given by BII.

Subjects in the Enterprise	Criteria	Initial Evaluative Values	Subjects in the Enterprise	Criteria	Initial Evaluative Values
STTG	d_1	(0.78,0.89,0.62)	CRCC	d_1	(0.53,0.82,0.78)
	d_2	(0.69,0.64,0.61)		d_2	(0.80,0.72,0.91)
	d_3	(0.55,0.68,0.69)		d_3	(0.64,0.64,0.57)
	d_4	(0.40,0.39,0.36)		d_4	(0.53,0.49,0.73)
MTR	d_1	(0.49,0.50,0.47,0.63)	BMTRO	d_1	(0.36,0.48,0.36)
	d_2	(0.63,0.69,0.55,0.78)		d_2	(0.64,0.52,0.30)
	d_3	(0.77,0.80,0.85,0.83)		d_3	(0.76,0.69,0.63)
	d_4	(0.49,0.50,0.50,0.64)		d_4	(0.81,0.59,0.86)
BFG	d_1	(0.71,0.82,0.60,0.53)			
	d_2	(0.42,0.39,0.47,0.34)			
	d_3	(0.58,0.63,0.54,0.55)			
	d_4	(0.76,0.69,0.61,0.70)			

5. Implications

This section discusses theoretical and practical implications from the analysis of the developed model with the case study. The first theoretical implication is the values of private evaluative criteria and public evaluative criteria; whose values have been hardly accurate in previous studies. To evaluate these two kinds of evaluative criteria, this paper introduces HFS to describe the values of the evaluative criteria, where it fully considers multi-criterion information given by group. The method reveals a new insight into the evaluative approaches on both private and public evaluative criteria of PPP infrastructure projects. Some characteristics of PPP infrastructure projects still need to be considered further with the evaluative values, especially for the matching decision cases which are to satisfy the match-degree of participants, decrease matching risks, and produce an optimal matching scheme [44].

The second insight is the selection of an optimal matching method by balancing private evaluative criteria and public evaluative criteria. Based on the values of private evaluative criteria and public evaluative criteria, the research uncovered the match-degree of the government and the enterprise within different evaluative criterion weights for PPP infrastructure projects. It reduces the best matching method by applying the min-max approach. The developed model provides a new perspective for selecting a best matching method for PPP infrastructure projects.

The practical implication of the research is to assist DMs in selecting the most suitable matching method dynamically and objectively. The subjects in the government can improve their sincerity level, enthusiasm for taking large risks, and react to the dynamics of prices to dispel misgivings of the subjects in the enterprise and to ensure success of matching. Moreover, applying the proposed model to the example demonstrated that a best matching scheme of PPP infrastructure projects is not only to optimize matching partly from the perspective of subjects in the government, but also to consider matching satisfaction of the subjects in the enterprise. It helps the DMs to make the best decision.

Therefore, this research has a strong practical value in guiding and deciding optimal matching during the decision process.

6. Conclusions

This paper developed a quantitative matching decision model of TSM decision in PPP infrastructure projects based on HFS by balancing the match-degree of the public and private sectors. The major contributions of the article are summarized as follows. First, to measure the match-degree of the two sides, this model, in which information on evaluative criterion weights is completely unknown, utilized HFS to describe multi-criterion information given by a group. Second, the best decision method was established by maximizing the whole deviation of each criterion to determine the evaluative criterion weights objectively. The proposed matching decision model and its application are given by balancing the evaluative criteria of the two parts. Third, discussion and implication reveal that risks of land policy changes, legal policy changes, and exchange rate changes of the subjects in the government can also affect the selection of the best matching decision method. As a result, the proposed matching decision approach is the expansion of single value in existing TSM evaluative values, which is closer to hesitant fuzzy and uncertain information in real matching decisions. Certain limitations need to be further studied in future. The research still needs to be investigated and explored more extensively for the best matching plan, especially from the perspectives of the associated relationship between the evaluative criteria.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix. The Calculation Process of the Proposed Method

Firstly, based on the evaluative criteria of matching on both the public and private sectors shown in Table 1, and the private evaluative criteria and the public evaluation criteria shown in Tables 2 and 3, the HFS evaluative values shown in Tables 6 and 7 for each criterion given by two sides involved from lowest to highest are sorted. In Model (1), $d(\beta)$ denotes the sum of the deviation on HFS evaluative values of all the subjects in the enterprise by the pth subject in the government. The below formula is obtained as:

$$R = \begin{bmatrix} (t-1)\sum\limits_{q=1}^{n}\left(\frac{1}{l}\sum\limits_{\lambda=1}^{l}\left(h_{q1}^{\sigma(\lambda)}\right)^2\right) & -\sum\limits_{q=1}^{n}\left(\frac{1}{l}\sum\limits_{\lambda=1}^{l}\left(h_{q1}^{\sigma(\lambda)}h_{q2}^{\sigma(\lambda)}\right)\right) & \cdots & -\sum\limits_{q=1}^{n}\left(\frac{1}{l}\sum\limits_{\lambda=1}^{l}\left(h_{q1}^{\sigma(\lambda)}h_{qt}^{\sigma(\lambda)}\right)\right) \\ (t-1)\sum\limits_{q=1}^{n}\left(\frac{1}{l}\sum\limits_{\lambda=1}^{l}\left(h_{q2}^{\sigma(\lambda)}h_{q1}^{\sigma(\lambda)}\right)\right) & (t-1)\sum\limits_{q=1}^{n}\left(\frac{1}{l}\sum\limits_{\lambda=1}^{l}\left(h_{q2}^{\sigma(\lambda)}\right)^2\right) & \cdots & -\sum\limits_{q=1}^{n}\left(\frac{1}{l}\sum\limits_{\lambda=1}^{l}\left(h_{q2}^{\sigma(\lambda)}h_{qt}^{\sigma(\lambda)}\right)\right) \\ \vdots & \vdots & \ddots & \vdots \\ (t-1)\sum\limits_{q=1}^{n}\left(\frac{1}{l}\sum\limits_{\lambda=1}^{l}\left(h_{qt}^{\sigma(\lambda)}h_{q1}^{\sigma(\lambda)}\right)\right) & (t-1)\sum\limits_{q=1}^{n}\left(\frac{1}{l}\sum\limits_{\lambda=1}^{l}\left(h_{qt}^{\sigma(\lambda)}h_{q2}^{\sigma(\lambda)}\right)\right) & \cdots & (t-1)\sum\limits_{q=1}^{n}\left(\frac{1}{l}\sum\limits_{\lambda=1}^{l}\left(h_{q2}^{\sigma(\lambda)}\right)^2\right) \end{bmatrix} \quad \text{(A1)}$$

Then, the Model (1) can be solved as below:

$$W^* = (\omega_1^*, \omega_2^*, \cdots, \omega_t^*) = \frac{R^{-1}e}{eR^{-1}e} \quad \text{(A2)}$$

According to Formula (A1), evaluative matrices of each part are obtained, followed by:

$$R_{A_1} = \begin{bmatrix} 13.6878 & -8.1966 & -7.8840 & -7.6008 & -7.5671 \\ -8.1966 & 14.0327 & -7.8716 & -7.8435 & -7.7984 \\ -7.8840 & -7.8716 & 13.7473 & -7.4643 & -7.6869 \\ -7.6008 & -7.8435 & -7.4643 & 12.6597 & -7.5038 \\ -7.5671 & -7.7984 & -7.6869 & -7.5038 & 12.8260 \end{bmatrix}$$

$$R_{A_2} = \begin{bmatrix} 12.8667 & -10.7813 & -10.5978 & -10.6170 & -9.2973 \\ -10.7813 & 14.6496 & -11.1821 & -11.3702 & -9.8052 \\ -10.5978 & -11.1821 & 14.2664 & -11.1359 & -9.5717 \\ -10.6170 & -11.3702 & -11.1359 & 14.4533 & -10.0331 \\ -9.2973 & -9.8052 & -9.5717 & -10.0331 & 10.4475 \end{bmatrix}$$

$$R_{A_3} = \begin{bmatrix} 12.2705 & -7.2597 & -7.5208 & -7.0014 & -7.6351 \\ -7.2579 & 16.1477 & -7.7726 & -7.6054 & -7.2808 \\ -7.5208 & -7.7726 & 15.7955 & -7.5559 & -7.7394 \\ -7.0014 & -7.6054 & -7.5559 & 12.6093 & -7.0197 \\ -7.6351 & -7.2808 & -7.7394 & -7.0197 & 12.9015 \end{bmatrix}$$

$$R_{B_1} = \begin{bmatrix} 4.5401 & -3.6401 & -3.2411 & -3.5217 \\ -3.6401 & 5.2173 & -3.4759 & -3.7921 \\ -3.2411 & -3.4759 & 4.1373 & -3.3817 \\ -3.5217 & -3.7921 & -3.3817 & 5.6217 \end{bmatrix} R_{B_2} = \begin{bmatrix} 5.0031 & -4.5562 & -4.8539 & -4.9554 \\ -4.5562 & 4.1706 & -4.4266 & -4.4798 \\ -4.8539 & -4.4266 & 4.7382 & -4.8735 \\ -4.9554 & -4.4798 & -4.8733 & 5.3452 \end{bmatrix}$$

$$R_{B_3} = \begin{bmatrix} 5.0557 & -3.4966 & -3.4278 & -5.4310 \\ -3.4966 & 4.4623 & -3.3025 & -3.5684 \\ -3.4278 & -3.3025 & 4.4084 & -3.4714 \\ -5.4310 & -3.5684 & -3.4714 & 5.4953 \end{bmatrix} R_{B_4} = \begin{bmatrix} 5.6810 & -3.8293 & -4.0413 & -3.9263 \\ -3.8293 & 4.8603 & -3.7851 & -3.5455 \\ -4.0413 & -3.7851 & 5.3087 & -3.7393 \\ -3.9263 & -3.5455 & -3.7393 & 5.1363 \end{bmatrix}$$

$$R_{B_5} = \begin{bmatrix} 4.9916 & -4.9621 & -4.6138 & -4.8403 \\ -4.9621 & 5.1325 & -4.8377 & -4.6985 \\ -4.6138 & -4.8377 & 4.4888 & -4.3039 \\ -4.8403 & -4.6985 & -4.3039 & 5.0204 \end{bmatrix} R_{B_6} = \begin{bmatrix} 4.7342 & -2.5334 & -2.3613 & -2.4312 \\ -2.5334 & 5.5266 & -2.5455 & -2.6932 \\ -2.3613 & -2.5455 & 4.8026 & -2.5617 \\ -2.4312 & -2.6932 & -2.5617 & 5.8800 \end{bmatrix}$$

Calculated values of evaluative criterion weights on matches by the subjects in the government and the subjects in the enterprise are shown in Table A1 according to Formula (A2).

The utility values and the compatibility of the qth subject in the enterprise by the pth subject in the government are [12,36]:

$$v_{A_p,B_q} = 1 - \prod_{j=1}^{t}\left(1 - \left(h_{qj}^{p}\right)^{\sigma(\lambda)}\right)^{\omega_{qj}} \quad \text{(A3)}$$

$$\mu_{A_p,B_q} = \alpha\left(v_{A_p,B_q}\right)^{e} \quad \text{(A4)}$$

where e is a natural exponent; α is a parametric parameter satisfied $0 < \alpha < 1$. The greater the value of α, the more sensitive the government is to a higher (or lower) than expected degree of satisfaction (or disappointment) on the matching results; $\left(h_{qj}^{p}\right)^{\sigma(\lambda)}$ is the rth element in with an ascending sort order.

In a similar way, the utility values and the compatibility of the pth subject in the government by the qth subject in the enterprise can be calculated by the following formulas [12,36]:

$$v'_{A_p,B_q} = 1 - \prod_{i=1}^{s}\left(1 - \left(h'^{q}_{pi}\right)^{\sigma(\psi)}\right)^{\omega_{pi}} \quad \text{(A5)}$$

$$\mu'_{B_q,A_p} = \alpha \left(v'_{H_p,G_q} \right)^e \tag{A6}$$

where $\left(h'^{q}_{pi}\right)^{\sigma(\psi)}$ is the ψth element in h'^{q}_{pi} in ascending sort order.

Calculate compatibility matrices on both sides involved for the given α = 0.6 and Formulas (A3)–(A6) as:

$$U_{A,B} = \begin{bmatrix} 0.5536 & 0.5763 & 0.5003 & 0.5593 & 0.5071 \\ 0.5822 & 0.5999 & 0.5727 & 0.5706 & 0.5660 \\ 0.4926 & 0.5752 & 0.5458 & 0.5373 & 0.5269 \end{bmatrix}$$

$$U'_{B,A} = \begin{bmatrix} 0.5327 & 0.5692 & 0.5178 & 0.5998 & 0.4001 \\ 0.5576 & 0.5842 & 0.5406 & 0.5890 & 0.5008 \\ 0.5989 & 0.4988 & 0.5272 & 0.5468 & 0.3936 \end{bmatrix}$$

Based on the above compatibility matrices on the two sides, an optimal matching model is established according to Formulas (2)–(5) as following:

$$\begin{aligned} &\max\{Z_1\} = \sum_{p=1}^{3} \sum_{q=1}^{6} \mu_{A_p,B_q} x_{pq} \\ &\max\{Z_2\} = \sum_{p=1}^{m} \sum_{q=1}^{n} \mu'_{B_q,A_p} x_{pq} \\ &\text{s.t. } \sum_{q=1}^{6} x_{pq} = 1, \sum_{p=1}^{3} x_{pq} \le 1, x_{pq} \in \{0,1\}, p = 1,2,3; q = 1,2,\cdots,6 \end{aligned}$$

The above model can be transformed and solved by utilizing the maximize method [39].

Table A1. Evaluative criterion weights on both sides involved.

Subjects in the Government	Criterion Weights					Subjects in the Enterprise	Criterion Weights			
	c_1	c_2	c_3	c_4	c_5		d_1	d_2	d_3	d_4
A_1	0.1999	0.2009	0.1960	0.2016	0.2016	B_1	0.2582	0.2523	0.2624	0.2271
A_2	0.2001	0.2001	0.1990	0.2024	0.1984	B_2	0.1514	0.2500	0.2528	0.2461
A_3	0.2130	0.1818	0.1904	0.2060	0.2088	B_3	0.2769	0.2325	0.2282	0.2641
						B_4	0.2150	0.2194	0.2180	0.2136
						B_5	0.2567	0.2532	0.2509	0.2392
						B_6	0.2450	0.2338	0.3025	0.2187

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