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# Competitive–Cooperative Strategy Based on Altruistic Behavior for Dual-Channel Supply Chains

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**Abstract:** As market competition becomes increasingly fierce, it becomes more and more important for members of the supply chain to maximize market sales and improve the economic benefits of all parties through altruistic cooperation. Considering the complex relationship between online and offline retail channels, this paper proposes a competitive–cooperative strategy based on altruistic behavior for the dual-channel supply chain, by applying the theory of the co-competition game. First, we introduce the problem with respect to the relationship between online and offline retail channels, and establish the competitive–cooperative strategy model based on altruistic behavior. Then, we prove the equilibrium strategy for existence and stability of the proposed model through mathematical deduction. Next, a multi-object optimal model is excluded by applying the Pareto principle, and the NSGA-II-based algorithm is obtained to acquire the Nash equilibrium point. Finally, we present the case testing results, which indicate that the proposed model is robust and can improve the channel efficiency of the supply chain.

**Keywords:** competition and cooperation; altruism; the Nash equilibrium point; supply chain

## 1. Introduction

With the rapid development of e-commerce and Internet technology, the online retail industry, characterized by high dynamism and demand volatility, is becoming more and more interesting for researchers in supply chain management. According to the latest findings of the China Internet Network Information Center from December 2017, the number of Internet users in China has reached 772 million, and Internet shopping users has reached 533 million. At the same time, the utilization rate of network shopping has increased to 69.1%. This development has prompted many manufacturers (HP, Apple, Lenovo) to try to open electronic channels on the basis of traditional retail channels. Establishing a way in which the network marketing channel and traditional retail channel can coexist as a dual-channel supply chain has become a trend. The integration and cooperation of all actors in a supply network constitute the key elements for improving the performance of the entire value chain, and are becoming the focus of academic concern [1–3].

As the competition between online and offline retailers becomes fiercer and fiercer, the malignant price war has left offline enterprises on the verge of closure, and online businesses have had to reduce product quality to guarantee profits. Consumers, on the other hand, have taken to the idea of “offline experience, online purchase”. Bell believes that the experience of offline channels can increase the sales of online channels [4]. For example, when customers buy clothes, they may go to the real stores to discover what they cost and try them on, but will then purchase them via a cheaper online channel. In this case, the efforts of offline retailers (the store display, hiring specialist salespeople, and experiencing trials of products) do not generate any revenue, but give their competitors, i.e., online retailers, a free ride. Similarly, Gallion discusses the shift of some customers from the online to the brick-and-mortar channel (the channel shift effect) [5]. When a consumer is ready to purchase a mobile

phone, he or she may search for relevant information online, such as the reference price, performance parameters, online customer service, and user evaluations, etc., and will then go to nearby stores to buy one. Consumers tend to be conservative and less willing to take a risk, so they prefer to go to a store to buy the product. In fact, online retailers can reap benefits from offline retailers when consumers have experiences in real stores. These two kinds of consumption choices have inevitably led to studies concerning supply chain integration and collaboration issues [6,7]. However, the same conflict caused by the dual channel will lead to more intense competition [8]. Considering this kind of retailer-led, dual-channel conflict scenario, this paper proposes a co-occurrence method of altruistic behavior to resolve this conflict and improve the channel performance of the supply chain. It also discusses the existence and stability of the Nash equilibrium in this situation.

The remainder this paper will run as follows. Relevant literature is reviewed in Section 2. In Section 3, a model of the system is introduced. In Section 4, an equilibrium strategy relating to the competing and cooperating model is proposed, and the equilibrium and stability are analyzed. In Section 5, a simulation model is established, and the effectiveness of the strategy is verified. Finally, in Section 6, a summary is provided.

## 2. Literature Review

According to several reports [9–11], Apple sets up retail stores in the United States to improve their online sales; IBM assigns their online orders to offline retailers; HP provides offline retailers with online order commissions to mitigate the conflict. Most of the literature is based on pricing and marketing efforts. According to Chiang [1], the manufacturers who adopt game theory analysis while analyzing pricing restrict the pricing behavior of offline-channel retailers, but the decline in wholesale prices does not necessarily have negative effects on offline-channel retailers. Cattani [2] deeply researched the problem concerning the competition and conflict between online and traditional channels; ascertaining how to determine a price becomes the key. Considering the degree to which e-commerce implementation is used, a two-stage game model was established by Chenyun and Wang Huanzhi [12] to study the pricing behavior of dual-channel retailers. Gangshu Cai's research shows that a consistent pricing scheme between manufacturers and retailers can reduce the dual-channel conflict and increase retailers' profits [13]. The importance of price competition under the dual channels was discussed by Grewal and Dhruv [14]. Yan Wei [15] discussed the traditional marketing strategies that influence platform sales. By establishing the dual-channel model analysis, it was concluded that combining the Pareto optimum of the traditional marketing strategy with the e-commerce platform can benefit all members. From an effort–cost analysis, Chatterjee concluded that customers' shopping tendencies are related to the effort, cost, and latency of shopping [16]. Dan Bin [17] discussed the coordination of the dual-channel supply chain in the context of electronics businesses. Brunner [18] suggested that the two-way free-riding of two retailers without price competition should be affected by the cost of the sales effort. Taleizadeh [19,20] found the the impact of marketing efforts on the decision profit of supply chain members, as well as the optimal decisions concerning the price, quality, and effort level in the supply chain of game theory. By studying the impact of marketing efforts and the collection rate on decision variables, Zerang [21] compared and analyzed the optimal decision in different scenarios. Li Jianbin et al. [22] studied the impact of pricing and sales efforts on decision-making resulting from the free-rider behavior of online retailers and physical retailers.

The literature in this paper assumes that the majority of decisions are self-serving, and it does not take into account the social preferences of policymakers. A large number of experiments on economic behavior and empirical studies show that in real life, people pay attention to the interests of others while paying attention to their own—also, social preference has an important influence on people's decision-making. Cui [23] was the first to introduce one of these social attributes into the two-stage supply chain model. It made decision-makers think about the strategic behavior of a fair preference. On the basis of Cui's research, Ozgun [24] considered nonlinear demand functions, which are easy to coordinate with the supply chain model of the exponential demand function. Liu et al. [25] proposed a

revision of pricing, using the principle of fairness to solve the inevitable free-rider behavior of online and offline channels of different levels of service, under the dual-channel supply chain structure. Their results have significant theoretical significance, although there are many parameters involved in the utility function of fair preference that affect the practicality of relevant models in this literature.

Now, with respect to the two channels of supply chain management, both the study of social preferences and the impact of social preference on supply chain decisions have largely focused on fair preference; there is a considerable lack of interest in altruistic preferences. In reality, there are behaviors stemming from altruistic preferences, and decision-makers with altruistic preferences will not only pay attention to their own interests, but also the interests of others. The interests of parts suppliers have even helped companies to upgrade their technology to maintain long-term relationships, e.g., Toyota [26]. Fehr and Gächter [27] put forward the reciprocal behavior and positive reciprocal cooperation method of feedback. Negative reciprocity means that revenge is not selfish. A related study found that whether different subjects can achieve cooperation depends on their altruistic preferences and the associated prestige, rather than their benefits [28]. Ge Zehui [29] built an evolution decision model and applied it to the analysis of altruistic attributes of the supply network. The paper points out that considering the altruism attribute is good for suppliers and the supply chain system. Thus, taking altruistic preference into account in the supply chain has a certain practical significance.

In the above literature, consideration is given to the pricing influence of dual-channel price competition strategy coordination, fairness, altruism, and free-riding behavior. It does not consider the relationship between altruism, marketing efforts, and pricing, or their impact on the performance of the whole supply chain. Yan Zhang [30] found that the positive altruistic behaviors of suppliers and retailers can help to alleviate the dual marginalization effect and improve the performance of the supply chain. However, they do not consider that the market demand is non-linear, and the altruistic behaviors of online and offline retailers affect the performance of the supply chain. Our study is based on an online retailer and offline retailers within a double-channel system. Setting up an online sales channel of suppliers and retailers can be regarded as an online retail activity, and such a channel can therefore be considered a follower in the Stackelberg game. In our model, we assume that the market demand function is non-linear, considering the impact of marketing efforts and pricing strategies on the supply chain performance, in the case of altruism. The multi-objective optimization problem of the second stage is solved by using Non-dominated Sorting Genetic Algorithm-II (NSGA-II).

This article contributes to existing literature by introducing the altruism attribute into the dual-channel supply chain model, using this property as a behavioral characteristic of online retailers and complex competition relationships between offline retailers in order to explore the altruism attribute of the decision variables of the subject of the supply chain and the impact of income.

### **3. Competitive–Cooperative Strategy Model Based on Altruistic Behavior**

#### *3.1. Problem Description*

Considering two independent corporate entities, one an online retailer and the other an offline retailer, which have separate online sales platforms and offline entity stores, this paper assumes that online retailers can conduct online marketing as well as offline brand advertising and promotion. An improvement in online marketing ability can enhance the purchase desire of customers, and an improvement in offline brand advertising and promotion ability can help customers to remember the brands of the products in the offline entity stores, and thereby attract more consumers to participate in the stores' promotional activities. Similarly, offline retailers can engage in offline marketing, online product experience, and on-site delivery. An improvement in offline marketing ability can strengthen the desire of consumers to purchase, and an improvement in online product experience and on-site delivery can enhance consumers' experience with the online e-business platform, make them purchase goods online, and prompt them to opt for the nearest delivery to reduce logistics costs.

In summation, considering the long-term competition and cooperation of the dual-channel supply chain, two key factors can affect its competition and cooperation strategy:

1. The self-preference coefficient  $\tau$  of the online and offline retailer represents the idea that the greater the self-interest of the enterprise, the more selfish the enterprise will be, and the higher the self-concern will be. On the contrary, the greater the preference for others of the enterprise, the lower the degree of self-concern will be. However, if cooperation is needed, the enterprise must have concern for others ( $1 - \tau$ ), which is a key factor in the cooperation between two competing retailers concerning how to define degrees of benefits for themselves and for others.
2. The level of effort—cooperation between online retailers and offline retailers requires that online retailers determine their level of effort in their own business ( $e_{uo}$ ) as well as in brand advertising and promotion for other companies ( $e_{ud}$ ). Additionally, offline retailers need to determine their level of effort in their own business ( $e_{do}$ ), as well as in product experience and on-site delivery for other companies ( $e_{du}$ ). Four key elements are involved in choosing levels of effort, which can make the whole competing and cooperating strategy achieve the Pareto optimum.

### 3.2. Model Building

The variables are defined as follows:

$w$ : the wholesale price of the retailer;

$p_u$ : the online sale price of goods;

$p_d$ : the offline sale price of goods;

$e_{uo}$ : the online retailers' level of effort in marketing;

$e_{ud}$ : the online retailers' level of effort in brand advertising and promotion for offline retailers;

$e_{do}$ : the offline retailers' level of effort in marketing;

$e_{du}$ : the offline retailers' level of effort in developing product experience and on-site delivery procedures for online retailers;

$c_u$ : online retailers' cost of sales;

$c_d$ : offline retailers' cost of sales;

$q_u$ : order quantities of online retailers;

$q_d$ : order quantities of offline retailers;

$D_u, D_d$ : consumers' demand for products from online retailers and offline retailers, respectively;

$T$ : inventory factor;

$g(\cdot)$ : consumers' demand function;

$f(\cdot), F(\cdot)$ : the probability density function and the distribution of the probability function of consumers' random demand.

The consumer demand for online and offline retailers is set, respectively, as:

$$D_u = g(p_u)e_{uo}^\alpha e_{du}^\beta + \varepsilon, \quad D_d = g(p_d)e_{do}^\gamma e_{du}^\delta + \varepsilon$$

in which  $0 \leq \alpha \leq 1, 0 \leq \beta \leq 1, \alpha + \beta = 1, 0 \leq \gamma \leq 1, 0 \leq \delta \leq 1, \gamma + \delta = 1$ , indicating the validity of efforts for oneself as well as altruistic efforts in the sales process [31]. The formula  $\varepsilon \in [0, +\infty]$  refers to marketing efforts to create a random demand from consumers. The equation  $T_u = q_u - g(p_u)e_{uo}^\alpha e_{du}^\beta, T_d = q_d - g(p_d)e_{do}^\gamma e_{du}^\delta$  is set as an inventory factor, standing for the difference between the amount of ordered products and the determined amount of demand. If the random demand is  $\varepsilon > T_u, \varepsilon > T_d$  then the products will be out of stock—otherwise, the inventory backlog will appear [32]. It is assumed that the demand function is quadratic and satisfies  $g(\cdot) > 0, g'(\cdot) < 0, g''(\cdot) < 0$  [33].

The expected value of the number of products sold by the online retailer to consumers is:

$$E[\min\{q_u, D_u\}] = q_u - \int_0^{T_u} F(x) dx \quad (1)$$

The expected value of the number of products sold by the offline retailer to consumers is:

$$E[\min\{q_d, D_d\}] = q_d - \int_0^{T_d} F(x) dx \quad (2)$$

The expected profit of online retailers is:

$$\begin{aligned} U_u(e_{uo}, e_{ud}, e_{du}, p_u, q_u) &= p_u E[\min\{q_u, D_u\}] - (w + c_u)q_u - e_{uo} - e_{ud} \\ &= (p_u - w - c_u)q_u - p_u \int_0^{T_u} F(x) dx - e_{uo} - e_{ud} \end{aligned} \quad (3)$$

The expected profit of offline retailers is:

$$\begin{aligned} U_d(e_{do}, e_{du}, e_{ud}, p_d, q_d) &= p_d E[\min\{q_d, D_d\}] - (w + c_d)q_d - e_{do} - e_{du} \\ &= (p_d - w - c_d)q_d - p_d \int_0^{T_d} F(x) dx - e_{do} - e_{du} \end{aligned} \quad (4)$$

When both online retailers and offline retailers have altruistic preferences, the two sides will not only consider their own profits, but also the profits of the other, when making decisions. We introduced the variables  $\tau_u, \tau_d$ , representing the online and offline retailers' altruistic coefficients, respectively, so that the utility functions of both sides can be expressed, respectively, as

$$\text{Max } \pi_u = \tau_u U_u + (1 - \tau_u) U_d \quad (5)$$

$$\text{Max } \pi_d = \tau_d U_d + (1 - \tau_d) U_u \quad (6)$$

As an enterprise is always pursuing the maximum of its own interests,  $0.5 \leq \tau_u \leq 1, 0.5 \leq \tau_d \leq 1$  is consistent with the real situation.  $w, c_u, c_d$  are constant.

#### 4. Equilibrium Strategy Existence and Stability Analysis of the Model

##### 4.1. The Existence of Equilibrium Strategy in Independent Decision-Making

Considering the fact that most offline retailers in the real market make their own strategies as forerunners for determining the price of a commodity, then as followers, online retailers always make their strategy and then decide their own commodity prices afterwards. This can be considered as the Stackelberg game, and can be solved using the reverse induction method. Therefore, we can first make an independent strategy considering both retailers. In the case of a random market demand, without taking altruism and the efforts made for others into consideration, we find that  $D_u = g(p_u), D_d = g(p_d)$ . When the quantity of the online retailer's order quantity is  $q_u > D_u$ , the expected profit is:

$$\begin{aligned} U_u(p_u, q_u) &= p_u D_u - (w + c_u)q_u = (p_u - w - c_u)g(p_u) - (w + c_u)[(q_u - g(p_u))] \\ &< (p_u - w - c_u)g(p_u) = U_u(p_u, g(p_u), w) \end{aligned} \quad (7)$$

When the quantity of the online retailer's order quantity ( $q_u < D_u$ ), we can definitely find that  $U_u(p_u, q_u, w) < U_u(p_u, g(p_u), w)$ . Therefore, only when the order quantity is  $q_u = D_u$ , can we obtain the optimal choice. In this case, the optimal profit that the online retailer can obtain is:

$$\varphi_w(p_u) = U_u(p_u, g(p_u)) = g(p_u)(p_u - w - c_u), p_u \geq w - c_u \quad (8)$$

Similarly, the best profit of the offline retailer is:

$$\varphi_w(p_d) = U_d(p_u, g(p_d)) = g(p_d)(p_d - w - c_d), p_d \geq w - c_d \quad (9)$$

**Theorem 1.** Let  $\overline{p_u}, \overline{p_d}$  be the upper bounds of the online and offline sales prices, respectively. If  $p_u \in [w + c_u, \overline{p_u}]$ ,  $p_d \in [w + c_d, \overline{p_d}]$ , then  $\varphi_w(p_u)$  and  $\varphi_w(p_d)$ , and the maximum point must be  $p_u^*$  and  $p_d^*$ .

**Proof.** As  $\varphi_w(p_u)$  is continuous in  $p_u \in [w + c_u, \overline{p_u}]$ , there must be an extreme value of  $p_u^*$ . Therefore,  $\varphi''_w(p_u) = (p_u - w - c_u)g''(p_u) + 2g'(p_u)$ . Moreover, as  $g'(p_u) < 0, g''(p_u) < 0$ , we can see that  $\varphi''_w(p_u) < 0$ . Thus, we can say that  $\varphi_w(p_u)$  is a unique maximum point  $p_u^*$  of the concave function, and, in the same way, it is clear that  $\varphi_w(p_d)$  is the only maximum point  $p_d^*$  of the concave function.

Under the above basic conditions, assuming that the price of the commodity product  $p_d$  set by the offline retailer is known first, the first-order optimal conditions of the online retailer are also known, considering the degree of effort and the self-interest factor:

$$\frac{\partial \pi_u}{\partial p_u} = \tau_u \left( q_u + p_u g'(p_u) e_{uo}^\alpha e_{du}^\beta F(T_u) - \int_0^{T_u} F(x) dx \right) = 0 \quad (10)$$

$$\frac{\partial \pi_u}{\partial q_u} = \tau_u (p_u (1 - F(T_u)) - w - c_u) = 0 \quad (11)$$

$$\frac{\partial \pi_u}{\partial e_{uo}} = \tau_u \left[ \alpha p_u g(p_u) e_{uo}^{\alpha-1} e_{du}^\beta F(T_u) - 1 \right] = 0 \quad (12)$$

$$\frac{\partial \pi_u}{\partial e_{ud}} = (1 - \tau_u) \delta p_d g(p_d) e_{do}^\gamma e_{ud}^{\delta-1} F(T_d) - \tau_u = 0 \quad (13)$$

From Equation (11), we can obtain

$$p_u - p_u F(T_u) - w - c_u = 0, T_u = F^{-1} \left( \frac{p_u - w - c_u}{p_u} \right) \quad (14)$$

From Equations (8) and (12), we can obtain

$$e_{uo}^{1-\alpha} = \alpha p_u e_{du}^\beta g(p_u) (p_u - w - c_u) = \alpha p_u e_{du}^\beta \varphi_w(p_u) \quad (15)$$

From Equations (9) and (13), we can obtain

$$e_{ud}^{1-\delta} = \frac{(1 - \tau_u) \delta p_d g(p_d) e_{do}^\gamma F(T_d)}{\tau_u} = \frac{(1 - \tau_u) \delta p_d e_{do}^\gamma \varphi_w(p_d)}{\tau_u} \quad (16)$$

$$q_u = T_u + g(p_u) e_{uo}^\alpha e_{du}^\beta = \alpha^{\frac{\alpha}{1-\alpha}} e_{du}^{\frac{\beta}{1-\alpha}} \varphi_w(p_u)^{\frac{\alpha}{1-\alpha}} + F^{-1} \left( \frac{p_u - w - c_u}{p_u} \right) \quad (17)$$

If Equations (15) and (17) are inserted into (10), we can obtain

$$\begin{aligned} & \alpha^{\frac{\alpha}{1-\alpha}} e_{du}^{\frac{\beta}{1-\alpha}} \varphi_w(p_u)^{\frac{\alpha}{1-\alpha}} [g(p_u) + (p_u - w - c_u)g'(p_u)] + \int_0^{F^{-1}(\frac{p_u - w - c_u}{p_u})} (1 - F(x)) dx \\ & = \alpha^{\frac{\alpha}{1-\alpha}} e_{du}^{\frac{\beta}{1-\alpha}} \varphi_w(p_u)^{\frac{\alpha}{1-\alpha}} \varphi_w'(p_u) + \int_0^{F^{-1}(\frac{p_u - w - c_u}{p_u})} (1 - F(x)) dx = M_{ud}(p_u) \end{aligned} \quad (18)$$

We call  $M_{ud}(p_u)$  an augmented marginal profit. At this time,  $\pi_u(p_u) = (p_u, e_{uo}(p_u), e_{ud}(p_d), q_u(p_u))$  can be obtained by solving the unary equation,  $M_{ud}(p_u) = 0$ .  $\square$

**Theorem 2.** Let  $p_u^*$  be the maximum point for  $\varphi_w(p_u)$ , and  $p_u^{**}$  be the minimum point for  $\varphi_w(p_u)$ . In the process of competition and cooperation between online and offline retailers, for any given  $e_{du} > 0$

and  $e_{do} > 0$ , there must exist the optimal response strategy of online retailers,  $p_u(e_{du}), e_{ud}(e_{do}, e_{du}, p_d), q_u(e_{du})$  and  $e_{uo}(e_{du})$ . Thus, we can see that:

- (1) If  $M_{ud}(p_u^{**}) < 0$  then there must be an internal solution,  $p_u^c \in [p_u^*, p_u^{**}]$ , to meet

$$M_{ud}(p_u^c) = 0, M_{ud}'(p_u^c) \leq 0 \tag{19}$$

At the same time, for anyone that can meet the above equations,  $p_u^c \in [w + c_u, \bar{p}]$   $\pi_u(p_u^c)$  must be an optimal local response of an online retailer.

- (2) If  $M_{ud}(p_u^{**}) > 0$ , then the optimal response of an online retailer is  $\pi_u(\bar{p}_u)$ .
- (3) If  $M_{ud}(p_u^{**}) = 0$ , then the optimal response of an online retailer is  $\pi_u(\bar{p}_u)$  or  $\pi_u(p_u^{**})$ .

**Proof.** (1) The objective function of online retailers is:

$$\pi_u(e_{uo}, e_{ud}, p_u, q_u) = \tau_u \left[ (p_u - w - c_u)q_u - p_u \int_0^{T_u} F(x)dx - e_{uo} - e_{ud} \right] + (1 - \tau_u) \left[ (p_d - w - c_d)q_d - p_d \int_0^{T_d} F(x)dx - e_{do} - e_{du} \right]$$

According to Theorem 1, we can see that the profit function of the online retailer  $\varphi_w(p_u)$  is a concave function, so it is monotonically increasing in  $p_u \in [w + c_u, p_u^*]$ . Therefore,  $M_{ud}(p_u^*) > 0$ . In addition, since  $M_{ud}(p_u^{**}) < 0$ , and  $M_{ud}(p_u)$  is continuously differentiable, there must exist a price  $p_u^c$  in the interval  $(p_u^*, p_u^{**})$  to satisfy Equation (19), denoted by  $e_{uo}^c = e_{uo}(p_u^c), e_{ud}^c = e_{ud}(p_u^c), q_u^c = q_u(p_u^c)$ . From  $M_{ud}(p_u^c) = 0$  and Equations (16) and (17),  $(e_{uo}^c, e_{ud}^c, q_u^c, p_u^c)$  satisfies the first order optimal condition (Equations 10–13). In the following section, we discuss the second-order optimality condition.

$$\pi_u(e_{uo}, p_u, q_u) = \tau_u \left[ (p_u - w - c_u)q_u - p_u \int_0^{T_u} F(x)dx - e_{uo} - \frac{1-\lambda}{\lambda}e_{uo} \right] + (1 - \tau_u) \left[ (p_d - w - c_d)q_d - p_d \int_0^{T_d} F(x)dx - e_{do} - e_{du} \right] \tag{20}$$

Online retailers are limited to struggling with themselves and offline retailers to meet such a relationship as  $e_{uo} = \lambda e_u, e_{ud} = (1 - \lambda)e_u$ ; we can see that  $e_{ud} = \frac{1-\lambda}{\lambda}e_{uo}$ . If we insert this into  $\pi_u$ , we can obtain:

$$\begin{aligned} \frac{\partial \pi_u}{\partial p_u} &= \tau_u \left( q_u - p_u g'(p_u) e_{uo}^\alpha e_{du}^\beta F(T_u) - \int_0^{T_u} F(x)dx \right) = 0 \\ \frac{\partial \pi_u}{\partial q_u} &= \tau_u (p_u (1 - F(T_u)) - w - c_u) = 0 \\ \frac{\partial \pi_u}{\partial e_{uo}} &= \tau_u \left[ \alpha p_u g(p_u) e_{uo}^{\alpha-1} e_{du}^\beta F(T_u) - 1 \right] = 0 \end{aligned} \tag{21}$$

Therefore,  $(e_{uo}^c, q_u^c, p_u^c)$  satisfies the following first-order optimality condition:

If we want to discover the partial guide of  $p_u, q_u, e_{uo}$  from  $\frac{\partial \pi_u}{\partial p_u}, \frac{\partial \pi_u}{\partial p_u}, \frac{\partial \pi_u}{\partial p_u}$ , we can obtain the second-order partial derivative:

$$\frac{\partial^2 \pi_u}{\partial e_{uo}^2} \Big|_{(e_{uo}^c, q_u^c, p_u^c)} < 0 \tag{22}$$

$$\begin{aligned} &\begin{vmatrix} \frac{\partial^2 \pi_u}{\partial e_{uo}^2} & \frac{\partial^2 \pi_u}{\partial e_{uo} \partial q_u} \\ \frac{\partial^2 \pi_u}{\partial q_u \partial e_{uo}} & \frac{\partial^2 \pi_u}{\partial q_u^2} \end{vmatrix} \Big|_{(e_{uo}^c, q_u^c, p_u^c)} \\ &= \alpha p_u^2 g(p_u) e_{du}^\beta e_{uo}^{\alpha-2} (1 - \alpha) F(T_u) f(T_u) = \frac{(1-\alpha)p_u f(T_u)}{e_{uo}} > 0 \end{aligned} \tag{23}$$

$$\begin{aligned}
 & \begin{vmatrix} \frac{\partial^2 \pi_u}{\partial e_{uo}^2} & \frac{\partial^2 \pi_u}{\partial e_{uo} \partial q_u} & \frac{\partial^2 \pi_u}{\partial e_{uo} \partial p_u} \\ \frac{\partial^2 \pi_u}{\partial q_u \partial e_{uo}} & \frac{\partial^2 \pi_u}{\partial q_u^2} & \frac{\partial^2 \pi_u}{\partial q_u \partial p_u} \\ \frac{\partial^2 \pi_u}{\partial p_u \partial e_{uo}} & \frac{\partial^2 \pi_u}{\partial p_u \partial q_u} & \frac{\partial^2 \pi_u}{\partial p_u^2} \end{vmatrix}_{(e_{uo}^c, q_u^c, p_u^c)} \\
 &= (1 - \alpha)[1 - F(T_u)]^2 e_{uo}^{-1} + \alpha e_{du}^{2\beta} e_{uo}^{2\alpha-2} p_u f(T_u) \left\{ \alpha [g(p_u) + F(T_u)g'(p_u)p_u]^2 \right. \\
 & \quad \left. + (1 - \alpha)F(T_u)g(p_u)p_u [2g'(p_u) - F(T_u)g''(p_u)p_u] \right\} \\
 &= (1 - \alpha)[1 - F(T_u)]^2 e_{uo}^{-1} + \alpha e_{du}^{2\beta} e_{uo}^{2\alpha-2} p_u f(T_u) [\alpha \varphi'(p_u)^2 + (1 - \alpha)\varphi(p_u)\varphi''(p_u)] \\
 &= (1 - \alpha)e_{uo}^{-1} p_u f(T_u) \left\{ \frac{(w+c_u)^2}{p_u^3 f(T_u)} + \alpha^{\frac{\alpha}{1-\alpha}} e_{du}^\beta \left[ \frac{\alpha}{1-\alpha} \varphi(p_u)^{\frac{2\alpha-1}{1-\alpha}} \varphi'(p_u)^2 + \varphi(p_u)^{\frac{\alpha}{1-\alpha}} \varphi''(p_u) \right] \right\} \\
 &= (1 - \alpha)e_{uo}^{-1} p_u f(T_u) M_{ud}'(p_u) \leq 0
 \end{aligned}$$

According to the order method of the principal minor determinant, from Equations (22)–(24), we can judge that the Hessian matrix of the function  $\pi_u(e_{uo}, p_u, q_u)$  in  $(e_{uo}^c, q_u^c, p_u^c)$  is negative. Therefore, it satisfies the second-order optimality condition—that is,  $(e_{uo}^c, e_{ud}^c, q_u^c, p_u^c)$  is the local maximum point.

- (2) According to the discussion above, we find that the internal optimal solution can meet  $M_{ud}(p_u^c) = 0$  and  $M_{ud}'(p_u^c) \leq 0$ . When  $M_{ud}(p_u^{**}) > 0$ , the maximum point must not be internal, but only at the border. Moreover, according to the fact that  $p_u$  satisfies the margin of profit increase, the optimal price is  $\bar{p}_u$  and  $\pi_u(\bar{p}_u)$  is the optimal reaction solution.
- (3)  $M_{ud}(p_u^{**}) = 0$ ,  $\pi_u(p_u^{**})$  is optimal to meet the first-order optimal conditions. According to the previous proof, we can also ascertain that  $\pi_u(\bar{p}_u)$  is the optimal reaction solution.  $\square$

From the second stage of the optimal solution of the online retailer, we can ascertain that when the change of  $e_{do}, e_{du}$  occurs, the change of  $e_{uo}, e_{ud}, p_u, q_u$  will also occur. At the same time,  $p_u(e_{du}), e_{ud}(e_{do}, e_{du}, p_d), q_u(e_{du})$ , and  $e_{uo}(e_{du})$  can be obtained. The second stage of the profit function of the offline retailer is inserted into the first stage:

$$\pi_d = \tau_d U_d(e_{do}, e_{du}, p_d, q_d) + (1 - \tau_d) U_u(e_{uo}, e_{ud}, p_u, q_u)$$

Among them:

$$\begin{aligned}
 U_d &= (p_d - w - c_d)q_d - p_d \int_0^{T_d} F(x)dx - e_{do} - e_{du} \\
 U_u &= [p_u(e_{du}) - w - c_u]q_u(e_{du}) - p_u(e_{du}) \int_0^{T_u} F(x)dx - e_{uo}(e_{du}) - e_{ud}(e_{do}, e_{du}, p_d)
 \end{aligned}$$

From the first-order optimality condition of the offline retailer at the first stage, we can obtain:

$$\left\{ \begin{aligned}
 & \frac{\partial \pi_d}{\partial p_d} = \tau_d [q_d + p_d g'(p_d) e_{do}^\gamma \left[ \frac{(1-\tau_u)\delta p_d e_{do}^\gamma \varphi_w(p_d)}{\tau_u} \right]^{\frac{\delta}{1-\delta}} F(T_d)] \\
 & + \varphi_w(p_d) \delta e_{do}^\gamma \left[ \frac{(1-\tau_u)\delta p_d e_{do}^\gamma \varphi_w(p_d)}{\tau_u} \right]^{\frac{2\delta-1}{1-\delta}} \left[ \frac{(1-\tau_u)\delta e_{do}^\gamma [\varphi_w(p_d) + p_d \varphi'_w(p_d)]}{\tau_u(1-\delta)} \right] \\
 & - \int_0^{T_d} F(x)dx - (1 - \tau_d) \left[ \frac{(1-\tau_u)\delta p_d e_{do}^\gamma \varphi_w(p_d)}{\tau_u} \right]^{\frac{\delta}{1-\delta}} \left[ \frac{(1-\tau_u)\delta e_{do}^\gamma [\varphi_w(p_d) + p_d \varphi'_w(p_d)]}{\tau_u(1-\delta)} \right] = 0 \\
 & \frac{\partial \pi_d}{\partial q_d} = \tau_d \{ p_d [1 - F(T_d)] - w - c_d \} = 0 \\
 & \frac{\partial \pi_d}{\partial e_{du}} = \tau_d + (1 - \tau_d) \beta g(p_u) [\alpha p_u \varphi(p_u)]^{\frac{\alpha}{1-\alpha}} (p_u - w - c_u) = 0 \\
 & \frac{\partial \pi_d}{\partial e_{do}} = \tau_d \left\{ p_d [\gamma g(p_d) e_{du}^{\gamma-1} e_{du}^\sigma F(T_d) + \frac{\tau_u}{(1-\tau_u)} \frac{(1-\tau_u)\delta p_d \gamma e_{do}^{\gamma-1} \varphi(p_d)}{\tau_u(1-\delta)} \left[ \frac{(1-\tau_u)\delta p_d e_{do}^\gamma \varphi_w(p_d)}{\tau_u} \right]^{\frac{\delta}{1-\delta}} \right\} \\
 & + (1 - \tau_d) \frac{(1-\tau_u)\delta p_d \gamma e_{do}^{\gamma-1} \varphi(p_d)}{\tau_u(1-\delta)} \left[ \frac{(1-\tau_u)\delta p_d e_{do}^\gamma \varphi_w(p_d)}{\tau_u} \right]^{\frac{\delta}{1-\delta}} = 0
 \end{aligned} \right.$$

When the function expression of  $g(x), F(x)$  is given, we find the optimal solution of  $p_u$ , according to the Equation (18); then, by substituting that solution into the above non-linear equation, we can find the optimal strategy solution of the offline retailer.

#### 4.2. The Stability of Equilibrium Strategy

We have discussed the existence of equilibrium points, but the stability of equilibrium points is the guarantee of the reliability of policy choices for the decision-maker. Thus, we discuss the problem of stability below. Let us give the lemma first:

**Lemma 1.** Let it be the game  $\Gamma = \{X_i, f_i\}_{i \in N}$ ,  $\forall i \in N$ , let  $X_i$  be a nonempty convex and compact set of a Hausdorff local convex space  $E_i$ , let  $f_i : X \rightarrow R$  be continuous, and let  $\forall x_i \in X_i$ ,  $u_i \rightarrow f_i(u_i, x_i)$  be quasi-concave on  $X_i$ . There must then exist a Nash equilibrium in the game  $\Gamma$  [34].

$\pi_i(i = u, d)$  is linear on  $X$  and  $\pi_i$  is concave on  $X$ .  $N = \{u, d\}$  where  $u$  is the online retailer and  $d$  is the offline retailer.  $\forall i \in N$ ,  $X_i$  is the strategy set of play  $i$ ,  $X \in R^6$ ,  $X = X_u \times X_d$ .  $\pi_i : X \rightarrow R$  is set as the payoff function of play  $i$ .  $\Lambda = \{\varphi = (\pi_u, \pi_d) : X \rightarrow R$  is continuous on  $X$ .

Define the distance:

$$\rho(\psi^1, \psi^2) = \sup_{x \in X} \|\psi^1(x) - \psi^2(x)\| = \sup_{x \in X} \left( \left| \pi_u^1 - \pi_u^2 \right| + \left| \pi_d^1 - \pi_d^2 \right| \right)$$

Then,  $(\Lambda, \rho)$  is a complete metric space.  $G(\psi)$  denotes the set of all the Nash equilibrium points of  $\psi$  for any  $\psi \in \Lambda$ . According to Lemma 1,  $G(\psi) \neq \emptyset$ . Therefore,  $\psi \rightarrow G(\psi)$  defines a set-valued mapping,  $G : \Lambda \rightarrow P_0(X)$ .

**Definition 1.** Let  $X, Y$  be two Hausdorff topological spaces. A correspondence  $F : Y \rightarrow P_0(X)$  is said to be the upper space (respectively, lower), semi-continuous at  $y \in Y$  if for each open set  $V$  in  $X$  with  $F(y) \subset V$  and  $F(y) \cap V \neq \emptyset$ , respectively, there exists an open neighborhood  $O(y)$  of  $y$  in  $Y$ , such that  $F(y') \subset V$  and  $F(y') \cap V \neq \emptyset$ , respectively, for each  $y' \in O(y)$  [35].

The following Lemma is due to Theorem 2 of [36].

**Lemma 2.** Let  $X$  be a metric space,  $Y$  be a Baire space, and  $F : Y \rightarrow P_0(X)$  be an upper-semi-continuous and compact set-valued (USCO) mapping. Then, there exists a dense residual subset  $Q$  of  $Y$ , such that  $F$  is lower semi-continuous at every  $y \in Q$  [35].

**Lemma 3.** Let  $X$  and  $Y$  be two Hausdorff topological spaces. Let  $\{A_a\}_{a \in \Gamma}$  be a net in  $K(X)$ ,  $\{y^a\}_{a \in \Gamma}$  be a sequence of  $Y$ , and  $\{f^a(x, y)\}_{a \in \Gamma}$  be a sequence of real-valued continuous functions defined by  $X \times Y$ . If  $A_a \rightarrow A \in K(X)$ ,  $y^a \rightarrow y \in Y$  and  $\sup_{(x, y) \in X \times Y} |f^a(x, y) - f(x, y)| \rightarrow 0$ , where  $f$  is a real-valued continuous function defined by  $X \times Y$ , then  $\max_{u \in A_a} f^a(u, y^a) \rightarrow \max_{u \in A} f(u, y)$  [35].

**Theorem 3.** The set-valued mapping  $G$  is an USCO mapping.

**Proof.** Since  $X$  is compact, we only need to prove that the graph of the set-valued mapping  $G$  is closed. To achieve this, we must prove  $\psi^n \in \Lambda$ ,  $\psi^n \rightarrow \psi$ ,  $\forall x^n \in G(\psi^n)$ ,  $x^n \rightarrow x$ , then  $x \in G(\psi)$ . Since  $x^n \in G(\psi^n)$ , then  $\forall i \in N$ . From there,  $\pi_i^n(x_i^n, x_i^n) = \max_{y_i \in X_i} \pi_i(x_i^n, y_i)$ , where  $x_i = X - \{x_i\}$ . As  $\psi^n \rightarrow \psi$ ,  $x^n \rightarrow x$ . Since  $\psi$  is continuous at  $x$ , it holds that

$$\begin{aligned} \left| \pi_i^n(x_i^n, x_i^n) - \pi_i(x_i, x_i) \right| &\leq \left| \pi_i^n(x_i^n, x_i^n) - \pi_i(x_i^n, x_i^n) + \pi_i(x_i^n, x_i^n) - \pi_i(x_i, x_i) \right| \\ &\leq \left| \pi_i^n(x_i^n, x_i^n) - \pi_i(x_i^n, x_i^n) \right| + \left| \pi_i(x_i^n, x_i^n) - \pi_i(x_i, x_i) \right| \\ &\leq \rho(\psi^n, \psi) + \left| \pi_i(x_i^n, x_i^n) - \pi_i(x_i, x_i) \right| \rightarrow 0, (n \rightarrow \infty) \end{aligned}$$

Thus,  $\forall i \in N, \pi_i^n(x_i^n, x_i^n) \rightarrow \pi_i(x_i, x_i), \forall i \in N$ . As  $\pi_i^n, \pi_i$  is continuous, and  $\pi_i^n \rightarrow \pi_i, x_i^n \rightarrow x_i$ , according to Lemma 4, we can obtain  $\max_{y_i \in x_i} \pi_i^n(y_i, x_i^n) \rightarrow \max_{y_i \in x_i} \pi_i(y_i, x_i)$ . Therefore,  $\forall i \in N$ , and we can obtain  $\pi_i(x_i, x_i) = \max_{y_i \in x_i} \pi_i(y_i, x_i)$ —that is,  $x \in G(\psi)$ .  $\square$

**Definition 2.** Let  $M$  be a non-empty and closed subset of  $\Lambda$  and  $y \in M$ .  $x \in G(y)$  is said to be an essential equilibrium of the game  $y$ , relative to  $M$ , provided that for any open neighborhood  $N(x)$  of  $x$  in  $X$  there is an open neighborhood  $O(y)$  of  $y$  in  $\Lambda$ , such that, for any  $y' \in M$  with  $y' \in O(y)$ , there exists  $x' \in G(y')$  with  $x' \in N(x)$ . The game  $y$  is said to be essential relative to  $M$ , if all its equilibria are essentially relative to  $M$  [35].

It is easy to prove the following result, whose proof is omitted.

**Lemma 4.** The game  $y \in M$  is essentially relative to  $M$  if and only if the correspondence  $G : M \rightarrow P_0(X)$  is lower semi-continuously at  $y$  [34].

**Theorem 4.** The Nash equilibrium for the game problem  $\psi$  in the competition and cooperation between online and offline retailers is stable in the space  $\Lambda$ .

**Proof.** From the above, we find that  $(\Lambda, \rho)$  is a complete metric space. By Theorem 3, we know that  $G : \Lambda \rightarrow P_0(X)$  is an USCO mapping. According to the Fort theorem, we can see that there is a dense residual set  $Q$  in  $\Lambda$ , such that  $G$  is continuous at  $\psi \in Q$ . By Lemma 4, the game  $\psi$  is essential. Therefore, the Nash equilibrium point in the game problem  $\psi$  is stable in the space  $\Lambda$ .  $\square$

According to the above proof, the equilibrium points of the competitive and cooperative game do not cause a large fluctuation under the slight perturbation of the payment function. This shows that the equilibrium points are stable in our model.

## 5. NSGA-II-Based Algorithm to Obtain the Nash Equilibrium Point and Testing

### 5.1. Multi-Object Optimal Model for Obtaining the Nash Equilibrium Point

The solution to the problem related to the Nash equilibrium point, regarding the competition and cooperation between online and offline retailers mentioned above, can be transformed into a multi-objective optimization problem. The multi-objective problem is expressed as follows:

$$\begin{aligned} \max \pi(x_{cu}, x_{cd}, x_s) &= \max\{\pi_u(x_{cu}, x_{cd}, x_s), \pi_d(x_{cu}, x_{cd}, x_s)\} \\ \text{s.t. } h_i(x_{cu}, x_{cd}, x_s) &\leq 0, i = 1, 2, 3, 4, 5, 6; \end{aligned}$$

where  $\pi_u(x_{cu}, x_{cd}, x_s)$  is the utility function of the online retailer, and  $\pi_d(x_{cu}, x_{cd}, x_s)$  is the utility function of the offline retailer.  $h_i(x_{cu}, x_{cd}, x_s)$  is the inequality constraint function,  $X_{cu} = \{p_u, e_{uo}, e_{ud}, q_u, \tau_u\}$  is the decision variable of the online retailer, and  $X_{du} = \{p_d, e_{do}, e_{du}, q_d, \tau_d\}$  is the decision variable of the offline retailer.  $x_s = \{\alpha, \beta, \gamma, \delta\}$  is the state variable. For the constraint condition, two aspects should be taken into consideration:

- (1) The effectiveness of online retailers' efforts ( $e_{uo}^\alpha, e_{ud}^\beta$ ) and offline retailers' efforts ( $e_{do}^\gamma, e_{du}^\delta$ ) is controlled by  $(\alpha, \beta, \gamma, \delta)$ . Therefore, it should meet the conditions of  $\alpha > 0, \beta > 0, \gamma > 0, \delta > 0, \alpha + \beta < 1, \gamma + \delta < 1$ .
- (2) In view of the cost factor, it is clear that the costs of online retailers are lower than those of offline retailers. When retailers price their products, they will certainly consider the online prices  $\bar{p}_u, \bar{p}_d$ . The cost of the efforts ( $e_{uo}^\alpha, e_{ud}^\beta, e_{do}^\gamma, e_{du}^\delta$ ) made by retailers can be regarded as the amount of investment in the unit of their goods. Therefore, it should meet the conditions of  $e_{do}^\alpha + e_{du}^\beta < \bar{p}_u * q_u, e_{do}^\gamma + e_{du}^\delta < \bar{p}_d * q_d$ .

### 5.2. NSGA-II-Based Algorithm to Obtain the Nash Equilibrium Point

The algorithm of Non-dominated Sorting Genetic Algorithm-II(NSGA-II) is a multi-objective non-dominated sorting genetic algorithm based on an elite strategy. Its advantage is that it does not need the weight coefficient of each target in the man-made model. It also has a faster convergence rate and robustness [37–39]. Based on the NSGA-II algorithm, we can solve the multi-objective model in the state and decision variables. The calculation steps are as follows:

- (1) Initialize the operating parameters and decision variables, as well as the value of the range of state variables of the multi-objective model. Set the population size as  $N = 30$ , the maximum number of iterations as  $G = 2000$ , and initialization as  $t = 0$ ;
- (2) Randomly generate the individual  $m$ , and individuals  $x_i, x = 1, 2, \dots, m$ . Initialize the population  $P_i$ ;
- (3) Decode the chromosome, according to the constraint conditions, and calculate the objective function values  $\pi_u(x_{cu}, x_{cd})$  and  $\pi_d(x_{cu}, x_{cd})$ ;
- (4) Carry out the non-dominant sorting of the population  $P_i$ , and calculate the individual crowding density on the same non-inferior level;
- (5) Using the tournament method, randomly take two individual compositions to produce  $m/2$  and to conduct a cross-mutation operation, to generate the new group  $Q_i$ ;
- (6) Recalculate the value of the objective function  $\pi_u(x_{cu}, x_{cd}, x_s)$  and  $\pi_d(x_{cu}, x_{cd}, x_s)$  of the offspring population  $Q_i$ ;
- (7) Calculate  $S_i = P_i \cup Q_i$ , conduct a non-dominated sorting of the population  $S_i$ , calculate the individual crowding density on the same non-inferior level, and select  $m$  individuals from  $S_i$  to produce the new species  $P_{i+1}, t = t + 1$  according to the results;
- (8) If the termination condition is met, then the output will be  $P_{i+1}$ ; otherwise, go back to step (5).

### 5.3. Case Testing Results and Analysis

#### 5.3.1. Parameters Setting

Let the market demand function be  $g(p) = (a - p^2)/b$ . The stochastic demand  $\varepsilon$  will obey the normal distribution, which is:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

According to the online sales data and offline sales data of the goods of one e-business and offline enterprise, the parameters  $\mu_u, \mu_d$  and the standard deviation  $\sigma_u, \sigma_d$  are obtained as samples. The parameters of the demand function  $g(p)$  are also obtained by data fitting. The specific operating parameters are presented in Table 1.

Table 1. Fitting parameters.

Operating Parameters	$a$	$b$	$\bar{\mu}_u$	$\sigma_u$	$\bar{\mu}_d$	$\sigma_d$
Parameter Value	538207	0.993361	1498.533	490.7321	1328.566	381.1306

The minimum and maximum values of the decision variables are set based on the sales data of the two companies shown in Table 2:

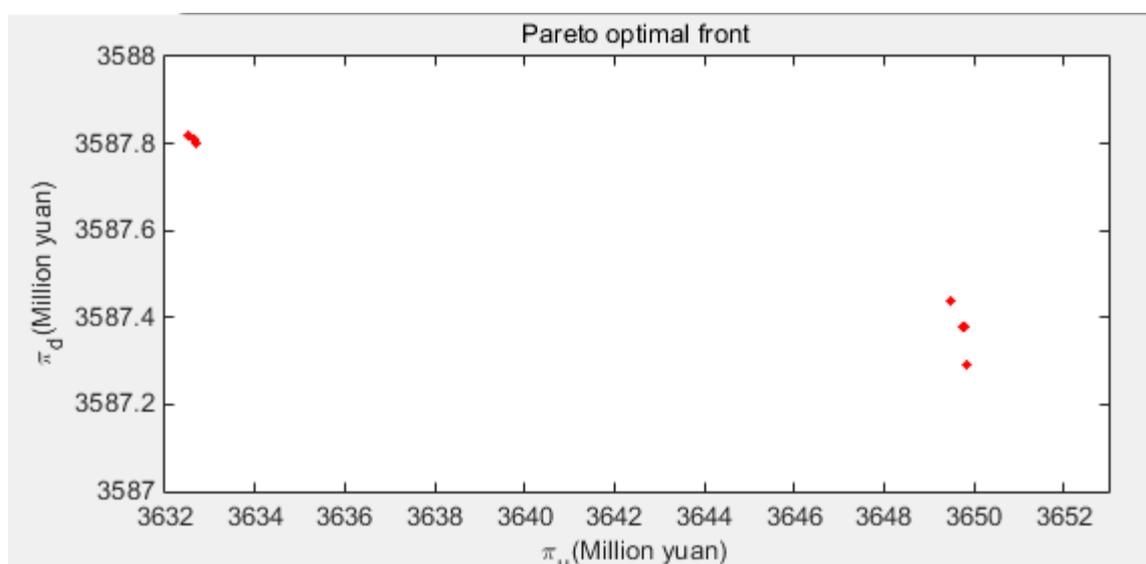
**Table 2.** The minimum and maximum values of the decision variables.

Decision Variables	Minimum Values	Maximum Values	Decision Variables	Minimum Values	Maximum Values
$\alpha$	0	1	$e_{ud}$	0	$8.90 \times 10^6$
$\beta$	0	1	$p_d$	130	498
$\gamma$	0	1	$p_u$	100	498
$\delta$	0	1	$q_u$	3000	$1.70 \times 10^5$
$e_{uo}$	0	$1.09 \times 10^7$	$q_d$	3000	$1.70 \times 10^5$
$e_{du}$	0	$8.90 \times 10^6$	$\tau_u$	0.5	1
$e_{do}$	0	$1.09 \times 10^7$	$\tau_d$	0.5	1

Sales data 2016 from Company A and Company B.

### 5.3.2. Results Analysis

According to Tables 1 and 2, the initialization parameters are inserted into the above algorithm by writing in the MATLAB simulation program. The results obtained are shown in Figure 1.



**Figure 1.** Obtained Pareto optimal front.

From Figure 1, it can be seen that a set of Nash equilibrium points can be obtained by using this model, which verifies the existence and stability of the Nash equilibrium solution of the model, and also shows the feasibility of the competition and cooperation between the online retailer and the offline retailer. From Table 3 and Figure 1, we can find that, with the change of  $\alpha, \beta, \gamma, \delta$ , when the altruistic coefficient of the online retailer is  $\tau_u = 0.82$ , the expected utilities of the Nash equilibrium strategy of the game are  $\pi_d \in [3633, 3634], \pi_u \in [3587, 3588]$ ; however, if the altruistic coefficient of the online retailer is  $\tau_u \in \{0.94, 0.95\}$ , then the expected utilities of the Nash equilibrium strategy of the game are  $\pi_d \in [3633, 3634], \pi_u \in [3587, 3588]$ .

Table 3. Nash equilibrium strategies.

Decision Variables	Number of Obtained Candidate Optimal Solutions of Decision Variables						
	1	2	3	4	5	6	7
A	0.473	0.48	0.48	0.48	0.496	0.496	0.496
B	0.206	0.207	0.207	0.207	0.159	0.16	0.16
$\Gamma$	0.094	0.094	0.093	0.093	0.032	0.032	0.031
$\Delta$	0.235	0.238	0.238	0.238	0.203	0.203	0.202
$e_{uo}$	1048.6	1048.6	1048.6	1048.6	1048.6	1048.6	1048.6
$e_{du}$	247.46	247.46	247.46	247.46	247.46	247.46	247.46
$e_{do}$	68.97	68.97	68.97	68.97	68.97	68.97	68.97
$e_{ud}$	598.19	598.19	598.19	598.19	598.19	598.19	598.19
$p_d$	461	461	461	461	461	461	461
$p_u$	461	461	461	461	461	461	461
$q_u$	147,297	147,297	147,297	147,297	147,297	147,297	147,297
$q_d$	116,140	116,140	116,140	116,140	116,140	116,140	116,140
$\tau_u$	0.82	0.82	0.82	0.82	0.94	0.95	0.95
$\tau_d$	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\pi_u$	3632.52	3632.63	3632.7	3632.71	3649.5	3649.74	3649.8
$\pi_d$	3587.82	3587.81	3587.8	3587.8	3587.44	3587.38	3587.38

According to the data presented in Table 3, the following phenomena can be found:

- At all the Nash equilibrium points, the decision variables  $p_u^*$ ,  $p_d^*$  in the balanced strategy set are equal. This means that if online and offline retailers want to obtain optimal results in competition and cooperation, in order to maximize the profit of the entire system, the online pricing and offline pricing must be consistent, which is consistent with dealings in real situations. For example, the online retail price and offline retail price of Maotai is the same, and the online retail price and offline retail price of Huawei mobile phones is also the same. That means the model is valid.
- The altruistic coefficient of the online retailer is in the range of [0.82, 0.95], and the altruistic coefficient of the offline retailer is 0.5, which indicates that the altruistic intention of the online retailer is small, while that of the offline retailer is large. The slight altruistic behavior of online retailers is due to the higher cost of marketing efforts and the favorable decision-making advantage of the follower. The offline retailers are completely altruistic because the marketing effort is less costly, and as leaders, it shows that they have more sincerity with regard to cooperation.
- Using the same pricing, the online retailer's marketing effort costs, marketing effect, and order quantity are much higher than that of offline retailers. Altruistic behavior can better reduce the profit difference and mediate the conflict between the two channels.

## 6. Conclusions

In the new retail market under the Internet+ environment, the consumer can buy products online after offline checking, and can purchase the products offline based on their online experience. This paper puts forward a conditional collaborative sharing and cooperation method to solve the conflict between online and offline retail channels caused by market sharing. At the same time, considering the altruism mode based on (1) the system of online and offline retailers, (2) the use of altruistic behavior and competition game research methods, and (3) the basis of independent decision-making, this paper establishes conditions for the use of altruism and the mutual effort of the competition model. This paper studies how to determine the optimal sales strategy (effort effectiveness, altruism, and pricing), set under the condition of sharing sales channels, to improve the performance of the whole supply chain. Through the model of the Nash equilibrium, and the existence and stability of the digital simulation analysis, this paper discusses this altruistic relationship, and found that online retailers have slightly altruistic behavior, while offline retailers have entirely altruistic behavior. From this finding, the profit difference between online and offline retailers can be drastically reduced,

and the conflict between them can be mediated. The conflict boundary between online and offline retailers can be studied on the basis of information asymmetry in the future, which is closer to reality, and at the same time can provide policy research recommendations for when the government should coordinate the coexistence of online and offline retailers.

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