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# Prices of Mexican Wholesale Electricity Market: An Application of Alpha-Stable Regression

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**Abstract:** This paper presents a proposal to estimate prices in the Mexican Wholesale Electric Market, which began operations in February 2016, which is why it moves from a scheme with a single bidder to a competitive market. There are particularities in the case of the Mexican market, the main one being the gradual increase in the number of competitors observed until now and, on the other hand, the geographic and technical characteristics of the electric power generation. The observed prices to date show great fluctuations in the observed data due to diverse aspects; among the stems we can mention the own seasonality of the demand of electrical energy, the availability of fuel, the problems of congestion in the electrical network, as well as other risks such as natural hazards. For the above, it is relevant in a market context to have a price estimation as accurate as possible for the decision-making of supply and demand. This paper proposes a methodology for the generation of electricity price estimation through the application of stable alpha regressions, since the behavior of the electric market has shown the presence of heavy tails in its price distribution.

**Keywords:** electricity markets; alpha-stable distribution; alpha-stable regression; electricity prices

## 1. Introduction

As part of the recent reform in the Mexican electricity sector, the electricity market has been liberalized, culminating with the creation of the Wholesale Electricity Market. Before the reform, the activities of generation, transmission, distribution, and commercialization of electric power were exclusive of the State. A set of problems that affected the electricity sector were identified: (a) high generation costs; (b) low participation of clean energies in the energy matrix; (c) inadequate infrastructure; (d) transmission losses; and, (e) structural problems in the Federal Electricity Commission (State Company). The objective of the reform was to address these problems [1–3].

In a competitive market, it is necessary to have reliable information to be able to make market decisions. The recent reform in the electric sector in Mexico requires the generation of robust models to forecast the prices of electricity. Having reliable information on the electricity market will allow better decisions to be made by both the market operator and the generators and consumer participants [4].

This work presents an application of alpha-stable regression for estimation of forecasting of electricity prices in the Mexican market. There is a set of relevant proposals in the state of the art on electricity price estimation. However, the vast majority presents applications to consolidate electricity markets. The particularity of the Mexican market is that it is of recent creation and with low private participation, maintaining a practically monopolistic market structure [4]. This, together with other factors such as fuel prices, network congestion, as well as disruptive events, have generated high volatility in prices. One of the advantages of addressing the problem of estimating electricity prices through stable regressions lies in the ability to capture the cyclical behavior of demand determined by consumption peaks and, therefore, spikes in electricity prices. The modeling considering heavy tails in the data will allow capturing the behavior of the data with greater precision and taking into account the presence of peaks of demand and prices when making the expected estimates in a future horizon. The stable regression has been applied efficiently in other disciplines such as finance, machine learning, and soft computing [5–7]. Derived from the literature review, no alpha-stable regression applications were identified for the estimation of electricity prices.

The study presents the general framework of the wholesale electricity market in Mexico, as well as information regarding competitors and electricity prices. The usefulness of the use of heavy tailings distributions that allow capturing in a better way the presence of extreme values according to the behavior of the energy demand is addressed. By estimating prices using alpha-stable regressions, we seek to capture this information and generate efficient forecasts for the decision-making of market participants. Data for the Electricity Sector for Mexico of the period 2016–2017 were used. Historical information was considered from the first year of operation of the wholesale electric market for the realization of the model. The results show an efficient adjustment of the observed price trend, despite the fact that there are outliers in the distribution of prices, especially because of the recent implementation of the competitive market. This situation will be dynamic since competitors are entering the market in a constant manner, so there is an immediate impact of entry into the behavior of electricity prices.

The work is structured as follows: section one describes the Mexican Electricity Sector until 2013, before the reform. Subsequently, the main characteristics of the Wholesale Electricity Market are described, the structure of the wholesale electricity market, and the behavior of electricity prices in the first year of operation. In the second section, is analyzed the alpha-stable distributions and alpha-stable regressions framework. The third section presents the main results of the study. The final section addresses conclusions and recommendations.

## 2. The Mexican Electric Sector

The Mexican Electricity Sector operated with a parastatal company as a single supplier (Federal Electricity Commission (FEC)). Other generators could participate through self-generation schemes, independent producers, cogeneration, small producers, export centers, and continuous own uses with a total gross generation of 42,676 GWh (excluding independent producers) that represents 13% of the total gross generation in 2014 (302,806 GWh). The generation of independent producers was usually sold directly to FEC. By 2014, the total net electricity generation in Mexico was 302,806 GWh; the generation was concentrated in the use of fossil fuels. Of the total FEC and independent producers generation, 79% use fossil fuels and coal, 15% hydro, and the rest is clean energy. In 2011, the generators that only used fossil fuels were 73% of the total electricity generation [8]. The use of fossil fuels for power generation has been discussed due to the fact that production of $CO_2$ contributes to the accumulation of greenhouse gases. Therefore, there are two scenarios of capacity expansion for the period 2013–2027. The first scenario contemplates the expansion of the public power generation with a share of 32% with the use of clean technologies in 2027. The second scenario considers the expansion seeking to increase its generation with non-fossil sources to 35% in 2027 [9].

In reference to generation costs, the level cost of the different technologies for Mexico shows that turbo-gas technology with diesel and gas, and those of internal combustion, are the most expensive plants with a maximum of up to $316 per MW (diesel) (Figure 1). The levelized cost of generation is an essential factor in the determination of energy prices since it is necessary to build an energy matrix focused on reducing emissions and an electricity market that seeks to reduce energy prices.

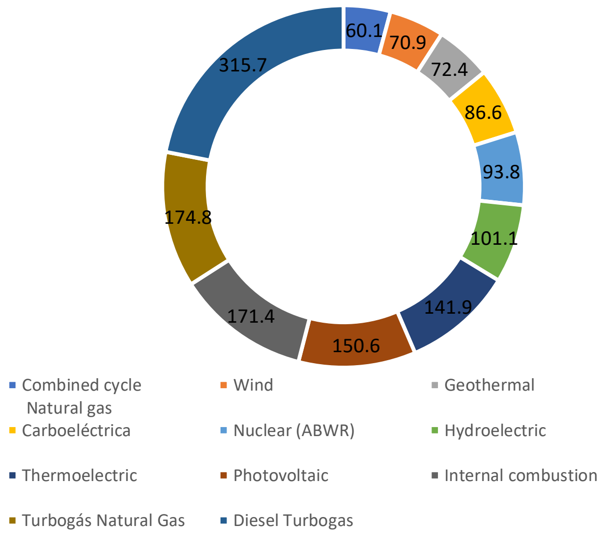


**Figure 1.** Level cost (dollars per net MWh), 2014.

### 2.1. *Mexican Wholesale Electricity Market*

The Wholesale Electric Market (WEM) was implemented in Mexico as a market based on variable costs, whose operation began in February 2016. Currently, there are 62 participants in the market—nine of them are from the Federal Electricity Commission and 53 are from private generators. Of the 62 market participants, 38 are generators, 15 are suppliers of qualified services, seven non-supplier marketers, an intermediation generator, and a basic services provider (Figure 2).

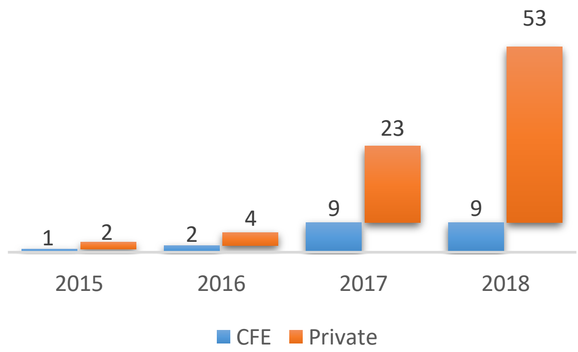


**Figure 2.** Market participants, 2015–2018.

For more details of the market participants, consult the Wholesale Electricity Market Bases published by the Ministry of Energy [10]. The WEM is integrated by a set of market instruments to operate efficiently.

- Short-term Energy Market
- Power Balance
- Clean Energy Certificates Market
- Financial Transmission Rights Market
- Medium and long-term auctions

### 2.2. *The Locational Marginal Prices*

Locational marginal pricing (LMP) reflects the value of the energy at the specific location and time it is delivered. Congestion raises the LMP in the receiving area of the congestion. Operating conditions that limit the delivery capacity of specific transmission lines also can contribute to congestion and result in LMP changes. Locational marginal prices are calculated by the market operator and published on the web. This enables market participants to factor the information into their decision-making. The calculations used to determine LMPs take into account electricity demand, generation costs, and the use of, and limits on, the transmission system. The price tells market participants the cost to serve the next megawatt of load at a specific location. LMPs give price signals that encourage new generation sources to locate in areas where they will receive higher prices [11].

LMP includes: (a) cost of the energy in the delivery node; (b) variable cost of energy generation; (c) cost per congestion; and, (d) cost of the losses. The LMP is the result of an optimization model performed by the market operator, of allocation of units in a day in advance. The model seeks to minimize generation costs subject to the generation capacity of all units and their costs, as well as to satisfy the required demand, taking into account the corresponding required reserves and the restrictions of the transmission network. The algorithm for the dispatch economic system will calculate the marginal price of energy in each node, on a time basis, and it will have three components: marginal energy component, marginal congestion component, and marginal losses component [12].

$$LMP_i = EC_i + LC_i + CC_i \tag{1}$$

where:

$EC_i$ corresponds to the energy component of node $i$
$LC_i$ corresponds to the loss component of node $i$
$CC_i$ corresponds to the congestion component of node $i$

The WEM started operations in February 2016; in the first year of operation, prices show volatility. The main reasons for the volatility of LMP may be due to climatic aspects, fuel availability, congestion, as well as network failures. In Mexico, there are three systems, the National Interconnected System, Baja California Sur, and Baja California Norte. In the present study, only the National Interconnected System will be addressed. The evolution of the LMP in what is carried out by the WEM in its first year of operation shows an average value of $2,144 with a standard deviation of $700 (MXN) (Figure 3).

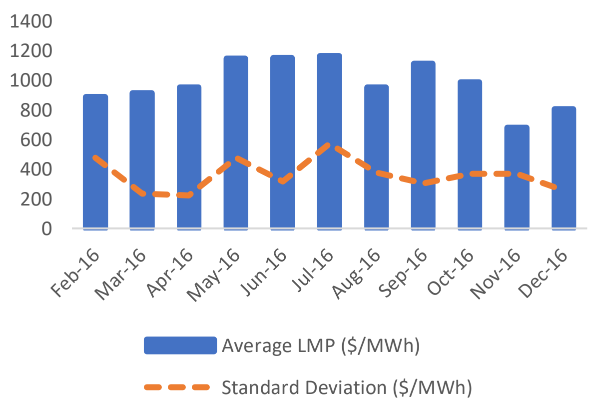


**Figure 3.** Locational marginal pricing (LMP) in the National Interconnected System (NIS).

The prices in the National Interconnected System (NIS) shows high volatility, as can be seen in the dotted line. In general, the high volatility can be attributed to the fact that it is a newly created market with low private participation in the generation and with variations attributable to losses in transmission, congestion in the network, and the cost of fuels (Figure 3).

There is a correlation between energy consumption and the price of energy, so there are peaks of demand according to the time of day and the day of the week. As expected, the weeks with the highest energy demand coincide with higher prices. The highest prices observed were in weeks 25 and 42 (Figure 4).

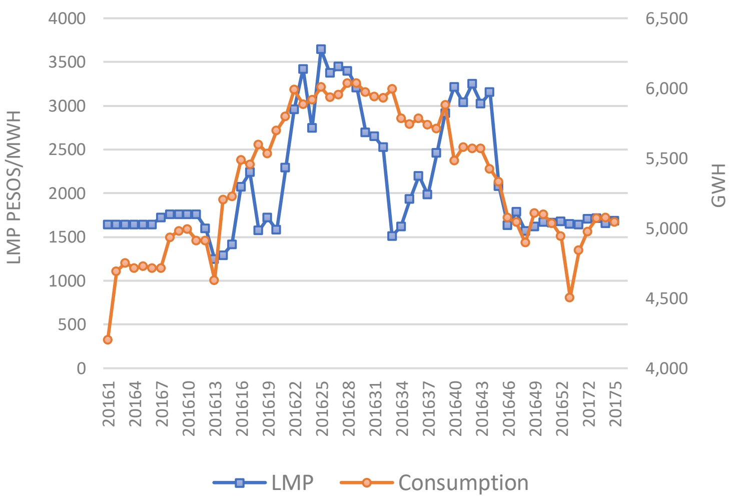


**Figure 4.** LMP vs. consumption by a week in the NIS, January 2016–January 2017.

The relationship between the peaks of demand and the determination of the LMP is a fundamental factor to consider for estimating electricity prices, since many methods may not correctly adjust these peaks of demand that, in turn, reflected in peaks in prices (Figure 4). The objective of the price forecasting model is that, based on the consumption and the expected demand, the price of energy can be estimated taking into account the prevalence of peaks in the prices, for which the use of alpha-stable regression is proposed.

## 3. Alpha-Stable Distributions and Stable Regressions

### *3.1. α-Stable Distributions*

The Gaussian distribution has wide applications in different areas but there are variables that cannot be modeled efficiently with this distribution because of a greater degree of impulsivity, that is, events or events are possible that, described by a Gaussian distribution, would be considered as very unlikely. The $\alpha$-stable distribution has been used in the literature to describe this type of phenomena. The stable distribution has been applied in many areas with efficient results.

In this section, the $\alpha$-stable distribution and its main properties are described, noting that the $\alpha$-stable distribution complies with the Central Limit Theorem and the stability property. On the other hand, it will also denote the complexity of working with this type of distribution, since it lacks general analytical expression. As a consequence, this distribution is not very widespread among the academic community. In spite of this, this difficulty can be overcome by making use of the property of the symmetric $\alpha$-stable distribution that allows expressing a distribution of this type as a Gaussian conditional to a random variable with the $\alpha$-stable distribution.

Definition. $X$ has an $\alpha$-stable distribution if it has the characteristic function [13]:

$$\varphi(w) = \begin{cases} \exp\left\{\left|\gamma w\right|\left[1 + isign(w)\beta\frac{2}{\pi}\log(|w|)\right] + i\mu w\right\}, (\alpha = 1) \\ \exp\left\{-\left|\gamma w\right|^{\alpha}\left[1 - isign(w)\beta tan\left(\frac{\pi\alpha}{2}\right)\right] + i\mu w\right\}, (\ \alpha \neq 1) \end{cases} \tag{2}$$

$\alpha \in (0, 2]$ is the characteristic parameter; this parameter controls impulsivity of the variable $X$, $\beta \in [-1,\ +1]$ is a parameter of symmetry, $\gamma > 0$ is a scaling parameter, and $\mu$ is the position parameter. Derived from the above and by convention, the $\alpha$-stable distributions will be denoted as a function of its four parameters using the following notation.

$$f(\alpha,\ \beta x)(.\left|\gamma, \mu\right) \tag{3}$$

### 3.2. Alpha-Stable Regression

Consider the standard regression model

$$y_i = \sum_{j=1}^{k} x_{ij}\theta_j + \varepsilon_i,\ i = 1,\ldots,N \tag{4}$$

where $y_i$ is an observed dependent variable, the $x_{ij}$ are observed independent variables, are unknown coefficients to be estimated, and are identically and independently distributed. The standard Ordinary Least Squares (OLS) estimator in matrix form:

$$\widehat{\beta_{OLS}} = (X'X)^{-1}X'y \tag{5}$$

It can be expressed as

$$\widehat{\beta_{OLS}} - \beta = (X'X)^{-1}X'\varepsilon \tag{6}$$

Thus, in the simplest case where $X$ is predetermined, $\widehat{\beta_{OLS}} - \beta$ is a linear sum of the elements of $\varepsilon$. With $\varepsilon$ elements, i.i.d. (independent identically distributed) non-normal stable variables, and then $\widehat{\beta_{OLS}}$ has a stable distribution. The variance of $\varepsilon_i$ cannot be estimated. Then, standard Ordinary Least Squares (OLS) inferences are invalid [14,15], proving the following properties of the asymptotic t-statistic.

a. The tails of the distribution function are normal-like at $\pm\infty$.
b. The density has infinite singularities $|1 \mp x|^{-\alpha}$ at $\pm 1$ for $0 < \alpha < 1$ and $\beta \neq \pm 1$, when $1 < \alpha < 2$ the distribution has peaks at $\pm 1$.
c. As $\alpha \to 2$, the density tends to normal and the peaks vanish

Further, the estimation of the parameters at stable distribution have the usual asymptotic properties of a Maximum Likelihood estimator [16], are asymptotically normal, asymptotically unbiased, and have an asymptotic covariance matrix $n^{-1}I(\alpha,\beta,\gamma,\delta)^{-1}$, where $I(\alpha,\ \beta,\ \gamma,\ \delta)$ is Fisher's Information. Assume that $\varepsilon_i = y_i - \sum_{j=1}^{k} x_{ij}\theta_j$ is alpha-stable with parameters $\{\alpha, \beta,\ \gamma,\ 0\}$. Let it be alpha-stable density function $s(x,\alpha,\beta,\gamma,\delta)$, then the density function of $\varepsilon_i$ is [17]:

$$s(\varepsilon_i,\ \alpha,\ \beta,\ \gamma,\ \delta) = \frac{1}{\gamma} s\left(\frac{y_i - \sum_{j=1}^{k} x_{ij}\theta_j}{\gamma},\ \beta,\ 1,\ 0\right) \tag{7}$$

the likelihood function,

$$L(\varepsilon,\ \alpha,\ \beta,\ \gamma,\ \theta_1,\theta_2,\ldots) = \left(\frac{1}{\gamma}\right)^n \prod_{i=1}^{n} s\left(\frac{y_i - \sum_{j=1}^{k} x_{ij}\theta_j}{\gamma},\ \beta,\ 1,\ 0\right) \tag{8}$$

and the loglikelihood,

$$\begin{aligned} l(\varepsilon,\ \alpha,\ \beta,\ \gamma,\ \theta_1,\theta_2,\ldots) &= \sum_{i=1}^{n} -nlog(\gamma) + log\left(s\left(\frac{y_i - \sum_{j=1}^{k} x_{ij}\theta_j}{\gamma},\ \beta,\ 1,\ 0\right)\right) \\ &= \sum_{i=1}^{n} \psi(\hat{\varepsilon_i}) \end{aligned} \tag{9}$$

The maximum likelihood estimators are the solutions of the equations:

$$\frac{\partial l}{\partial \theta_j} = \sum_{i=1}^{n} -\psi'(\hat{\varepsilon}_i)x_{ij} = 0,\ j = 1,2,\ldots,k \tag{10}$$

Thus,

$$\sum_{i=1}^{n} -\frac{\psi'(\hat{\varepsilon}_i)}{\hat{\varepsilon}_i} y_i x_{ij} = \sum_{i=1}^{n} -\frac{\psi'(\hat{\varepsilon}_i)}{\hat{\varepsilon}_i} \sum_{j=1}^{k} x_{ij}\theta_j \tag{11}$$

If $W$ is the diagonal matrix, we can write the model in the matrix format:

$$W = \begin{pmatrix} -\frac{\psi'(\hat{\varepsilon}_1)}{\hat{\varepsilon}_1} & 0 \cdots & 0 \\ 0 & -\frac{\psi'(\hat{\varepsilon}_2)}{\hat{\varepsilon}_2} \cdots & 0 \\ \vdots & \cdots & -\frac{\psi'(\hat{\varepsilon}_n)}{\hat{\varepsilon}_n} \end{pmatrix}$$

Then,

$$X'Wy = (X'WX)\hat{\theta} \tag{12}$$

If $X'WX$ is not singular,

$$\hat{\theta} = (X'WX)^{-1}X'Wy \tag{13}$$

This estimator has the format of a Generalized Least Squares estimator with heteroscedasticity, where the variance of the error term $\varepsilon_i$ is proportional to $\frac{\psi'(\varepsilon_i)}{\varepsilon_i}$. The effect of the Generalized Least Squares adjustment is to give less weight to larger observations. The estimator for stable processes gives higher weights to the center of the distribution and smaller weights to extreme values. This effect increases as $\alpha$ are reduced [17]. This is consistent with what was obtained by [18].

## 4. Estimation of Locational Marginal Prices in National Interconnected System Using Alpha-Stable Regressions

The estimation of electricity prices has been a challenge for all markets. There is a set of methodologies used that consist of models tailored to the market that is studied. That is why, in the Mexican case, the use of alpha-stable regressions is proposed. According to the state of the art, there is a set of methodologies generally used for the estimation of energy price forecasts: multi-agent models, diffusion and Markov models, statistical models, and recently badass models in computational intelligence, as well as the development of hybrid models or ensemble modeling. Temporality is another characteristic aspect in the case of electricity prices since there are different horizons in the market. In addition, according to temporality, it is possible to apply different forecasting methodologies [19]. Figure 5 shows a classification of general methodologies applied in energy price models.

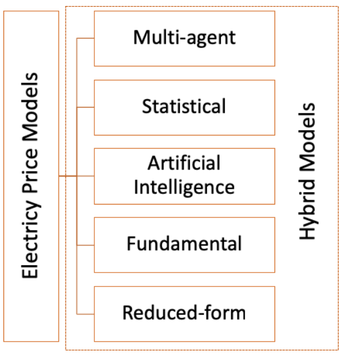


**Figure 5.** Electricity price modeling.

Highlights include works such as [20], where neural networks are applied for the realization of forecasts. Ref. [21] shows a general framework of statistical models for the forecast of prices of a day in advance using models ARIMA, ARMAX, GARCH and discusses the application of stochastic models for the derivation of prices as models of diffusion and Markov jumps. Ref. [22] shows the use of linear time series models and nonlinear ones. Another set of authors focuses on the use of price forecasts modeling the stochastic dynamics of prices, seeking to manage volatility risk and the valuation of derivatives, refs. [23–26]. Another approach is based on the modeling of market behavior through multiagent models such as [27–30] and [31]. However, the limitation of these methods is that they do not allow efficient forecasts to be generated for short periods of time (market in advance time and real-time market). Recently, the application of artificial intelligence models has increased, as in the case of [32–34] and [35], the application of neural network models, fuzzy neural networks are highlighted and committee machines. Refs. [36,37] perform a revision of machine-learning methods such as functional principal component analysis, support vector machine, and time series. On the other hand, ref. [38] highlights the use of time series for short-term forecasts such as ARIMA, seasonal ARIMA, and VAR models for price forecasts in one hour in advance and one day in advance.

A fundamental element in the elaboration of forecasts of electricity prices is the evaluation of its effectiveness; for this, a group of authors has dedicated to studying the efficiency of the forecasts proposing a set of generally accepted methods. It is the case of Mean Absolute Error and Mean Absolute Percentage Error, which are generally accepted methods. [39] performed a comparison of forecast performance measures highlighting the use of mean absolute scaled error. Other authors such as [40,41] use the absolute error or square scaled error. Another alternative is to use standardized error measurements as shown by [42–44], highlighting measures for short-term forecast such as daily or weekly weighted mean absolute errors and root square errors.

The present work proposes the use of alpha-stable regressions, given the presence of heavy tails in the errors, due to the impulsivity of the series. An advantage of using alpha-stable regressions in the presence of errors with heavy tails is that the inference made by OLS estimates before the presence of stable errors has no validity because of the case of infinity variance. In the case of alpha-stable regressions, both the parameters of the model and the parameters of the stable distribution of errors are estimated simultaneously, which makes it possible to generate adequate inference in such cases.

*Estimation of Electricity Price in Mexico*

In the case of the Mexican market that started operations in 2016, information was collected on average locational marginal prices for the National Interconnected System. The information corresponds to weekly data from January 2016 to January 2017, in addition to the real consumption per week and the weekly demand forecast estimated by the National Center for Energy Control (NCEC). The model considers the locational marginal price as a dependent variable and consumption as an independent variable. For the Mexican market, due to its recent start of operations, there is little information available as well as additional variables that could be considered in the model. However, consumption as such is a variable of great importance when determining the expected price of electricity. Based on the summary of the statistics shown in Table 1, the normality tests show that the data do not conform to a normal distribution. Similarly, the parameters of the estimated alpha-stable distribution are shown.

**Table 1.** Summary Statistics.

| | **LMP** | **Consumption** |
|---|---|---|
| | **(Mexican Pesos/MWh)** | **(GW/h)** |
| Mean | 2132.784 | 5340.727 |
| St. dev | 680.7264 | 486.5223 |
| Skewness | 0.8344004 | −0.1500883 |
| Kurtosis | 2.218459 | 1.861669 |
| | Goodness-of-Fit Test for Normal Distribution ($p < 0.05$) | |
| KS test | 0.0176 | 0.0071 |
| SW test | 0.00000 | 0.00416 |

The series used show high variability depending on the peaks of electricity demand according to the time and day of the week. The normality tests show that the data do not adjust to a normal distribution as expected due to the presence of impulsivity in the series. The parameters of the alpha-stable distribution are estimated using the maximum likelihood method and the $S_0$ parameterization (Table 2) [13]. The adjustment of the parameters evidences impulsivity in the series, as well as positive asymmetry.

**Table 2.** Maximum Likelihood Parameters of Stable Distribution.

| | **LMP** | **Consumption** |
|---|---|---|
| | **(Mexican pesos/MWh)** | **(GW/h)** |
| $\alpha$ | 1.5340 | 1.7851 |
| $\beta$ | 0.4121 | 0.2579 |
| $\gamma$ | 477.179 | 341.044 |
| $\delta$ | 2132.70 | 5340.73 |
| Goodness-of-Fit Test for alpha-stable distribution ($p < 0.05$) | | |
| K-S test | 0.82939 | 0.65749 |

Data generated by the electric power demand peaks adjust largely to the behavior of a stable distribution.

The model estimated is:

$$PML = \theta_0 + \theta_1 C + \varepsilon_i,\ i = 1,\ \ldots,\ N \tag{14}$$

The results of the alpha-stable regression are shown in Table 3. The parameters are estimated by two methods, the Maximum Likelihood method (stable model) and the Ordinary Least Squares. As expected, the parameters estimated by the alpha-stable regression have values lower than OLS, due to the penalty to larger observations, whose effect increases as a function of the value of the alpha parameter [45,46].

**Table 3.** Parameters of Stable Regression.

| | $\boldsymbol{\theta_0}$ | $\boldsymbol{\theta_1}$ |
|---|---|---|
| Ordinary Least Squares | −3098.9843 | 0.9795 |
| Maximum Likelihood Stable | −2604.1885 | 0.9510 |
| Stable parameters of $\alpha$ stable residuals (Asymptotic 95% confidence intervals) | | |
| $\alpha$ | 1.34575 (1.19434, 2.49726) | |
| $\beta$ | 0.09980 (−0.14193, 0.34152) | |
| $\gamma$ | 9.56450 (8.68237, 10.44663) | |

Although the series used in the regression are close to the normal case, when calculating the errors, it is observed that they adjust to a stable distribution. Given the properties of the estimation of the stable regression by maximum likelihood, it is a robust method for errors with heavy tails and an efficient method with data that are not strictly stable distributions. Table 4 shows the Fisher information matrix that allows the estimation of the confidence intervals of the parameters of the model, as well as the parameters of the distribution of the errors. The dimension of the matrix is represented by (k + 3), the sum of the parameters in the regression and the coefficients of stable distribution ($\alpha$, $\beta$, and $\gamma$).

**Table 4.** Fisher Information Matrix (k + 3) * (k + 3).

| | | | | |
|---|---|---|---|---|
| 182.4849 | −4.2016 | −8.7818 | −1.0338 | −2201.844 |
| −4.20169 | 79.6711 | −0.5106 | 6.2927 | 13401.96 |
| −8.7818 | −0.5106 | 5.3781 | −0.1778 | −378.819 |
| −1.0338 | 6.2927 | −0.1778 | 2.8481 | 6065.802 |
| −2201.844 | 13401.96 | −378.819 | 6065.802 | 1.46E + 07 |

Figure 6 shows the relationship between an estimated dependent variable and an independent variable by a scatter plot, comparing the adjustment of prices with respect to consumption, showing how, in the case of the adjustment by OLS, it shows greater dispersion with respect to the observed value of the PML, this largely due to the errors that contain heavy tails. In the case of the stable model estimated by maximum likelihood, a better fit to the observed data of the LMP was observed. The alpha-stable adjustment (Maximum Likelihood Stable) is below the OLS values on average, since it weighs the values of the center of the distribution, largely and, to a lesser extent, the extreme values of the tails. This depends on the value of the alpha impulsivity parameter (Figure 6).

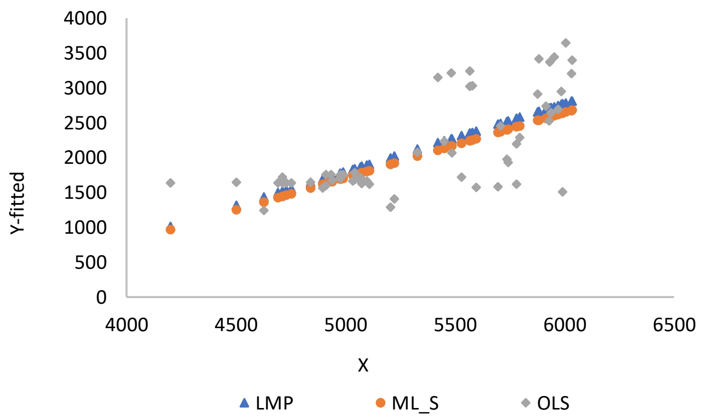


**Figure 6.** OLS and Stable regression.

The adjustment in two steps of the parameters of the model and the parameters of the stable distribution of the errors allows to control the impulsivity of the series and that the inference made with the estimators is robust and consistent. However, it should be noted that, in the case of the LMP for Mexico, the presence of a nonlinear behavior is observed; it is clear that the linear adjustment has limitations (Figure 7). So in another stage of the research, it is recommended to estimate other methodology as Generalized Additive Models (GAM) in combination with the stable adjustment to improve the performance in the presence of heavy tails in errors.

The estimations made by OLS and the alpha-stable regression show that the behavior of electricity prices is nonlinear, so the adjustment with both methodologies can be improved. However, in the case of OLS, it shows poor performance in the adjustment of the data when there are price peaks (peaks in demand) underestimating the values of LMP. In the case of alpha-stable regression, the adjustment to price peaks is better but has limitations in the nonlinear behavior of prices (Figure 7).

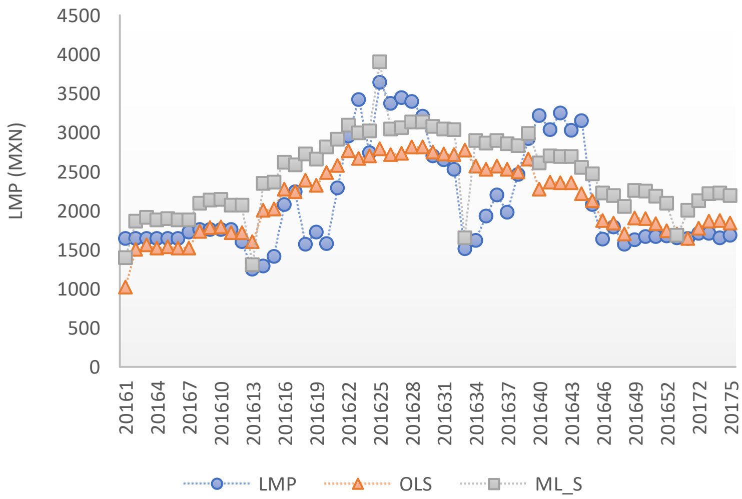


**Figure 7.** Estimations of Electricity Prices.

The residuals of the estimations made by the alpha-stable regression show that, on average, the stable model overestimates the LMP values, even though the adjustment to demand peaks is better. The adjustment of the stable regression (Maximum Likelihood Stable) shows a better performance in the extreme values; this is very low or very high prices of electricity (Figure 8). As inputs for making decisions about offers in the electricity market, it is necessary to have accurate estimates in this case; the Mean Absolute Percentage Error (MAPE) was estimated according to Equation (15).

$$MAPE = \frac{\sum_{t=1}^{N}\left|\frac{E_t}{Y_t}\right|}{N} \tag{15}$$

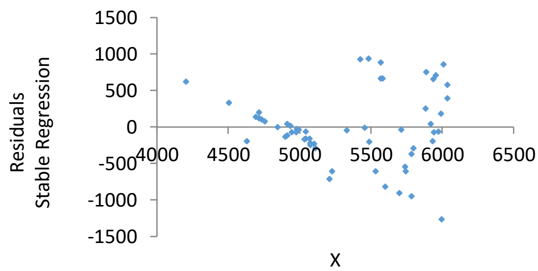


**Figure 8.** Stable regression residuals.

The performance of the estimations comparing both methods shows a MAPE highly influenced by the presence of extreme data that is high prices of the energy, not being the case of the stable regression. In the same way, this behavior is attributed to the nonlinear relationship observed in the variables considered in the model (Figure 9).

The MAPE for the stable estimate (Maximum Likelihood Stable) was 14% and for the OLS case of 25% (Figure 9). It is recommended to explore other adjustment methods to improve the performance of the estimates, such as the combination of the Stable Model and some nonlinear adjustment method such as Generalized Additive Models.

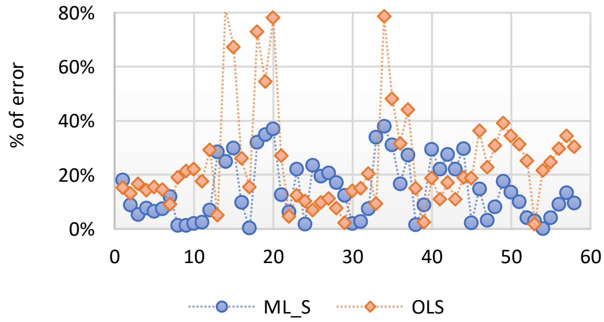


**Figure 9.** MAPE Stable Model and OLS.

## 5. Conclusions and Future Work

The start of operations of the WEM includes the generation of a set of market instruments that foster competition, thus creating the short-term market, the clean energy certificates market, the power market, and the financial transmission rights market. A fundamental element for its optimal development is access to reliable information in a timely manner. In this context, information on energy prices is an essential input for market operations; market participants must have robust estimates that allow them to make decisions for the supply and purchase of energy. The market structure has changed rapidly due to the incorporation of new suppliers in contrast to the previous scheme. The opening of the electricity sector seeks to encourage the entry of more competitors to allow competitive prices to be offered and to encourage the clean energies (through the clean energy certificate market). The behavior of prices observed in the first year of operations shows high volatility in prices, partly attributable to the behavior of fuel costs and the costs of losses.

Taking into consideration the above, this paper proposes a method of estimating energy prices based on alpha-stable regressions, considering the presence of impulsivity in the series of prices and electricity consumption, which result in errors with heavy tails. The main effect of estimating a linear model in these conditions lies in the limitation of the inference power of the model, since there is the possibility that the variance of the errors is infinite. The results of the estimations show the presence of impulsivity in the behavior of the price and consumption of electricity ($\alpha_{LMP} = 1.53$ and $\alpha_C = 1.78$), as expected given the nature of the consumption of electricity with peaks of demand according to the hour, day of the week, and month of consumption. The stable model fits better in the presence of extreme values; however, it does not capture the nonlinear behavior of the relationship between price and electricity consumption identified. MAPE shows a better performance of the stable model compared to OLS, 14%, and 25%, respectively.

Given the short time of operation of the Mexican wholesale electricity market, there is no open availability of more variables that could help improve the specification of the model. It is proposed as future work to integrate additional information to the model as well as to increase the frequency of the data at daily and hourly prices. Similarly, it is recommended to explore the integration of stable models with other nonlinear treatment methodologies. The present work is a first step towards the generation of technical evidence that allows the correct development of the Mexican electric market.

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