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Total Variation-Based Metrics for Assessing Complementarity in Energy Resources Time Series

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Abstract: The growing share of intermittent renewable energy sources raised complementarity to a central concept in the electricity supply industry. The straightforward case of two sources suggests that to guarantee supply, the time series of both sources should be negatively correlated. Extrapolation made Pearson's correlation coefficient (ρ) the most widely used metric to quantify complementarity. This article shows several theoretical and practical drawbacks of correlation coefficients to measure complementarity. Consequently, it proposes three new alternative metrics robust to those drawbacks based on the natural interpretation of the concept: the Total Variation Complementarity Index (ϕ), the Variance Complementarity Index (ϕ'), and the Standard Deviation Complementarity Index (ϕ_s). We illustrate the use of the three indices by presenting one theoretical and three real case studies: (a) two first-order autoregressive processes, (b) one wind and one hydropower energy time series in Colombia at the daily time resolution, (c) monthly water inflows to two hydropower reservoirs of Colombia with different hydrologic regimes, and (d) monthly water inflows of the 15 largest hydropower reservoirs in Colombia. The conclusion is that ϕ outperforms the use of ρ to quantify complementarity because (i) ϕ takes into account scale, whereas ρ is insensitive to scale; (ii) ρ does not work for more than two sources; (iii) ρ overestimates complementarity; and (iv) ϕ takes into account other characteristics of the series. ϕ' corrects the scale insensitivity of ρ. Moreover, it works with more than two sources. However, it corrects neither the overestimation nor the importance of other characteristics. ϕ_s improves ϕ' concerning the overestimation, but it lets out other series characteristics. Therefore, we recommend total variation complementarity as an integral way of quantifying complementarity.

Keywords: electricity supply; natural resources; climatology; hydrology; electricity markets; reliability; flexibility

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1. Introduction

The proliferation of energy sources made possible by technological advances and consequent cost reduction introduced the term complementarity into the energy jargon. Most of those new sources have a large natural variability.

The concept of complementarity is of particular interest when the resource's availability or costs are variable in time. Despite the lack of a formal definition, intuition suggests that complementarity refers to the possibility of using different resources according to their availability to meet a given demand. However, each one separately accounts for only a fraction of the demand.

Correlation coefficients are by far the most widely used metric for assessing complementarity between energy sources' time series. This approach has been used, for example, in Poland [1], Portugal [2,3], Spain [4], Italy [5,6], USA [7], Germany [8], Australia [9], Brazil [10–13], China [14–16], Sweden [17], Mexico [18], Canada [19], Lesotho [20], Finland [21], Argentina [22], Britain [23], Saudi Arabia [24], and Chile [25]. It has also been used at the international level (e.g., Latin America [26], Iberian Peninsula [27]), and even at the intercontinental level (e.g., Canada vs. Noruega [19] and Europe vs. Africa [28]).

In the case of Colombia, Pearson's correlation coefficient has been the most used metric [29–36]. Paredes and Ramírez [30] concluded that the highest negative mean seasonal R coefficients were at the wind and solar sites on the Oriental Plains and solar sites in the Antioquia region. They also found seasonal complementarity between wind sites in the north and east and northern rivers, between wind sites in the south and center and the rivers there, and between north and east solar sites and northern rivers. Henao et al. [34] found that solar and wind resources in the Caribbean Coast and the central Andes regions can complement the country's hydropower sector during the dry seasons of the annual climatological cycle and ENSO events. Peña Gallardo et al. [32] concluded that there is good energetic complementarity in the north and northeastern regions of the country throughout the year. Parra et al. [33] concluded that the Colombian power system evidenced an overall low complementarity behavior between its renewable power plants. This low complementarity indicates that most hydro-based power plants present similar temporal and amplitude characteristics, making the power system vulnerable to droughts. Wind and solar power plants have a high potential to complement Colombia's hydroelectric system, suggesting that more power capacity is needed to enhance such complementarity.

Under this approach, there is a high complementarity between two resources when their time series have a high negative correlation coefficient, sometimes called *anticorrelation* (e.g., [7,16,23,27,28]).

However, a complementarity metric should measure the regularity of the sum of all available resources, and correlation coefficients fail to accomplish that. The use of correlation coefficients also has the following issues, also presented by us in Cantor et al. [36]:

1. Negative correlation is not complementarity. Although complementarity entails negative correlation, negative correlation does not entail complementarity. For example, the correlation between speed and height in a pendulum is negative, but they are not complementary.
2. Dimensions matter. In the above example, speed and height have different units. Obviously, for two or more variables to complement, they must have the same dimension. If they have different dimensions, one cannot sum them, and their complementarity has no sense. In the pendulum example above, the complementary variables are kinetic energy and potential energy.
3. The scale of the variables does matter. For example, streamflows from a large river and a small creek could exhibit a sizable negative correlation, but the large one dominates their sum. Hence, the complementarity value could be small.
4. Linearity of the relation is also an issue. Even if correlation coefficients were suitable for assessing complementarity, selecting Pearson's, Kendall's, Spearman's, or any other coefficient needs a physical or mathematical justification.
5. It is natural to consider the complementarity of more than two resources, but a satisfactory correlation analysis is limited to two series.

Besides correlation, among the other metrics used to evaluate complementarity, one can mention the one proposed by Beluco et al. [37] as the product of three partial indices: (i) partial time complementarity index, which evaluates the time interval between the minimum values of two sources; (ii) partial energy complementarity index, which evaluates the relationship between the average values of two sources; (iii) partial amplitude-complementarity index, which assesses the differences between maximum and minimum values of the two energy sources. This idea works well for simple annual sinusoidal series but fails for more complicated series. Borba and Brito [38] proposed the dimensionless ratio between the actual generation discarding excess power and the average generation. This index is more a measure of volatility than complementarity. Han et al. [39] presented two indices of the complementary rate of fluctuation (CROF) and complementary rate of the ramp (CROR) to analyze the complementary degree of sequences with different fluctuation characteristics. Their rate of fluctuation seems to follow the concept of total variation that we present. Neto et al. [40] used the concept of Daily Physical Guarantee (DPG), similar to firm power, to evaluate different renewable energy sources' complementarity. The general idea is interesting but depends on the system's characteristics and does not provide a direct complementarity metric. A method was introduced in [29,41] that assesses temporal complementarity between three variable energy sources, using a combination of correlation coefficients, Euclidean vectors, compromise programming, and normalization. The method's basis is correlation analysis, and therefore their proposal shares the criticism we made above. For a complete review of different methods to evaluate complementarity, see [42]. In our opinion, none of these metrics are satisfactory enough to assess complementarity.

This paper proposes three novel indices based on the mathematical concept of Total Variation to solve these issues (see Section 2). Conceptually, the total variation of a time series is the sum of all the changes in the series. A constant series has a total variation of zero. Therefore, a total variation of zero serves as the standard of regularity. Measuring the deviation of a sum from that standard is the motivation for defining complementarity. The other two indices use the same idea but with variance or standard deviation instead of the total variation. To illustrate the use and properties of the new indices, we apply them to four cases of study (Section 3): (1) two autoregressive processes; (2) wind vs. hydro in the Colombian electricity market; (3) two hydropower sources in the Colombian electricity market; (4) fifteen hydropower sources in the Colombian electricity market.

2. Materials and Methods

In ordinary language, two things complement each other when their combination is complete or perfect. To complement is to add to something in a way that enhances or improves it. In math, two numbers complement each other when their sum is constant, usually 1 or 100. A measure of complementarity between two functions should assess the perfection of their sum. For perfection, here, one takes the regularity of the sum. Therefore, the measure of complementarity is related to the measure of regularity. A convenient way of measuring the regularity of a function is through its total variation, which allows for simple discontinuities. It has more satisfactory properties than the variance [43] (p. 328) [44] (p. 140).

Section 2.1 reviews the concept and main mathematical properties of Total Variation. Then, Sections 2.2–2.4 propose indices, ϕ, ϕ', and ϕ_s as novel metrics of complementarity. Afterward, Sections 3.1–3.3 present one theoretical and three real case studies to evaluate the suitability of ϕ and ϕ' to quantify complementarity: (1) two AR(1) processes, (2) daily wind and hydropower generation time series in Colombia, (3) the daily inflows to two hydropower reservoirs in Colombia, and (4) monthly water inflows of the 15 largest hydropower reservoirs in Colombia. Results are compared between the indices. Section 3.4 presents the application of the ϕ index for the evaluation of the complementarity of multiple time series.

2.1. Total Variation

According to [43,44], the total variation of a function f defined on an interval $[a, b]$ is

$$\bigvee_a^b f = \sup \sum_{i=1}^{n} |f(t_i) - f(t_{i-1})|, \tag{1}$$

where the supremum is taken over all possible finite partitions $a = t_1 < \ldots < t_n = b$ of $[a, b]$.

If the total variation is bounded (finite), f is said to be of *bounded variation* on $[a, b]$. Functions of bounded variations may have only removable discontinuities (jumps). Some important properties of the total variation are worth recalling [43]:

1. The total variation of a constant function is zero, and conversely, if $\bigvee_a^b f = 0$ then f is constant on $[a, b]$.

2. The total variation of a monotonic function f on $[a,b]$ is $|f(b)-f(a)|$.
3. $\bigvee_a^b(\alpha f) = |\alpha| \bigvee_a^b(f)$ for any constant α.
4. If f_1 and f_2 are functions of bounded variation, then so is f_1+f_2, and

$$\bigvee_a^b(f_1+f_2) \le \bigvee_a^b(f_1) + \bigvee_a^b(f_2) \tag{2}$$

5. If $a<b<c$ then

$$\bigvee_a^c(f) = \bigvee_a^b(f) + \bigvee_b^c(f). \tag{3}$$

From the above properties, it is clear that the total variation of a function f with values $f(t_k)$ at the points $a=t_0<t_1<\ldots<t_n=b$, and linear interpolation in between the given points is

$$\bigvee_a^b(f) = \sum_{k=0}^{n-1} |f(t_{k+1}) - f(t_k)|. \tag{4}$$

This property is useful to compute the total variation of a time series.

To illustrate, consider the following sequence of functions in $[0,1]$: $g_n(t) = \sin(2\pi n t)$, $n \ge 1$. Clearly, all the functions have zero mean and their variance is $1/2$, but $\bigvee_0^1(g_n) = 4n$ [44].

Using the definition of Total Variation, it is natural to define the following complementarity index.

2.2. Total Variation Complementarity Index

Given two functions $f_1(t)$ and $f_2(t)$, not both constant, their complementarity index $\phi(f_1,f_2)$ over the time interval $[a,b]$ is defined as

$$\phi(f_1,f_2) = 1 - \frac{\bigvee_a^b(f_1+f_2)}{\bigvee_a^b(f_1)+\bigvee_a^b(f_2)}. \tag{5}$$

Evidently, $0 \le \phi(f_1,f_2) \le 1$, and when $\phi(f_1,f_2)=1$ a perfect complementarity exists between f_1 and f_2, and that occurs only if f_1+f_2 is a constant. The generalization to any number of functions is direct.

As a special case, consider the Total Variation Complementarity index between the series of anomalies. In weather and climate, an anomaly is the difference of a series compared with its normal or average value. Clearly, the total variation complementarity ϕ between the series $f_1, f_2, \ldots, f_n$, is equal to the complementarity between the series of anomalies with respect to a constant mean for each function

$$\phi(f_1,f_2,\ldots,f_n) = \phi(f_1-\mu_1, f_2-\mu_2, \ldots, f_n-\mu_n), \tag{6}$$

because

$$\frac{\bigvee_a^b(f_1+\ldots+f_n)}{\bigvee_a^b(f_1)+\ldots+\bigvee_a^b(f_n)} = \frac{\bigvee_a^b[(f_1-\mu_1)+\ldots+(f_n-\mu_n)]}{\bigvee_a^b(f_1-\mu_1)+\ldots+\bigvee_a^b(f_n-\mu_n)}. \tag{7}$$

Summary of the ϕ index characteristics:

1. If f_1 is the vertical reflection of f_2, then f_1+f_2 is a constant, therefore, $\bigvee_{t_a}^{t_b}(f_1+f_2)=0$ and $\phi=1$.
2. $0 \le \phi \le 1$. For the extreme cases, $\phi=1$ means that there is perfect complementarity, and for $\phi=0$ there is no complementarity.
3. ϕ is symmetric, this is, $\phi(f_1,f_2)=\phi(f_2,f_1)$. Similarly, the symmetry holds for any permutation of the arguments in the case of more than two functions.
4. The ϕ index presents the same result when evaluated on anomaly series compared with its corresponding pure series (see Equation (6)).

5. The ϕ index could be applied to two or more series.

2.3. Variance Complementarity Index

If one wants to stay within the correlation concept, it is possible to correct the use of ρ as a metric for complementarity to consider the variables' scale. For that, considering Equation (5), one can replace total variation by variance. This is, given two functions $f_1(t)$ and $f_2(t)$, not both constant, their variance complementarity index $\phi_v(f_1, f_2)$ is

$$\phi_v(f_1, f_2) = 1 - \frac{\mathrm{Var}[f_1 + f_2]}{\mathrm{Var}[f_1] + \mathrm{Var}[f_2]}, \tag{8}$$

which simplifies to

$$\phi_v(f_1, f_2) = -\frac{2\,\mathrm{Cov}[f_1, f_2]}{\mathrm{Var}[f_1] + \mathrm{Var}[f_2]} = -\rho_{1,2}\frac{2\sqrt{\mathrm{Var}[f_1]\,\mathrm{Var}[f_2]}}{\mathrm{Var}[f_1] + \mathrm{Var}[f_2]}. \tag{9}$$

Notice that ϕ_v so defined is the negative of the correlation coefficient times the ratio of the geometric and arithmetic means of the variances. This last factor is always less or equal to one, with equality when the two variances are equal. Therefore, $\phi_v(f_1, f_2)$ varies between -1 and 1. It is convenient to re-scale it to the range zero to one by means of

$$\phi'(f_1, f_2) = (\phi_v + 1)/2. \tag{10}$$

To illustrate the effect of the scale differences on the complementarity, consider as an example two series with cross-correlation $\rho_{1,2} = -0.8$, and the ratio of their standard deviations $\sigma_1/\sigma_2 = 5$. Without the correction due to the difference of scales, one overestimates their complementarity, either to be 0.8 in the range $(-1, 1)$ or 0.9 in the range $(0, 1)$. However, considering Equation (9), the complementarities are $\phi_v = 0.31$, or $\phi' = 0.65$. Notice that differences between the means do not play any role. The ratio of the variances is the one that measures the relative scale of the series.

Summary of the ϕ' index characteristics:

1. If f_1 is the vertical reflection of f_2, $\mathrm{Var}[f_1] = \mathrm{Var}[f_2]$, then $\rho_{1,2} = -1$, $\phi_v = 1$ and $\phi' = 1$.
2. $0 \le \phi' \le 1$. For the extreme cases, $\phi' = 1$ means that there is perfect complementarity, and for $\phi' = 0$ there is no complementarity.
3. ϕ' is symmetric.
4. The ϕ' index presents the same result when evaluated on anomaly series compared with its corresponding pure series.
5. The ϕ' index could be applied just to two series.

2.4. Standard Deviation Complementarity Index

Using similar reasoning as in Section 2.3, the Standard Deviation Complementarity Index, instead of the variance, measures the variability of the sum with the standard deviation. This way, one stays in the same dimension of the original variables instead of the squares. If σ denotes the standard deviation, one can define the index as

$$\phi_s(f_1, f_2) = 1 - \frac{\sigma[f_1 + f_2]}{\sigma[f_1] + \sigma[f_2]} = 1 - \sqrt{1 - \frac{2\sigma[f_1]\sigma[f_2](1 - \rho_{1,2})}{(\sigma[f_1] + \sigma[f_2])^2}}. \tag{11}$$

Recognizing the square of the ratio of the geometric to the arithmetic mean, one can see that ϕ_s is in the range $(0, 1)$, so there is no need for a re-scaling similar to Equation (10) in the variance index. Moreover, if $\sigma[f_1] = \sigma[f_2]$, the equation can be simplified to

$$\phi_s(f_1, f_2) = 1 - \sqrt{\frac{1 + \rho_{1,2}}{2}}. \tag{12}$$

The standard deviation complementarity index shares all the corresponding characteristics of ϕ' listed in the summary above, except number 5.

3. Results

3.1. Case Study 1: Two First-Order Autoregressive Processes

Autoregressive processes are a well-known time series method widely used for all sorts of applications [45]. This section shows an expression for the total variance complementarity index between two cross-correlated autoregressive processes of order one. The purpose is to clarify the two ways of measuring complementarity, namely the total variation and the variance complementarity index.

Let $\mathbf{z}_t$ be a vector time series with two components, x_t and y_t, both with zero mean and unit variance. The multivariate autoregressive process of order one is

$$\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \mathbf{B}\mathbf{v}_{t+1}, \tag{13}$$

where the matrices $\mathbf{A}$ and $\mathbf{B}$ are constant parameters, and $\mathbf{v}_t$ is a vector time series of uncorrelated random variables with zero mean and unit variance. This is

$$\mathrm{E}[\mathbf{v}_t] = \mathbf{0},\ \mathrm{Var}[\mathbf{v}_t] = \mathrm{E}[\mathbf{v}_t\mathbf{v}_t^T] = \mathbf{I},\ \mathrm{Cov}[\mathbf{v}_t, \mathbf{v}_{t+j}] = \mathrm{E}[\mathbf{v}_t\mathbf{v}_{t+j}^T] = \mathbf{0},$$

for all $j \neq 0$, and $\mathbf{I}$ the identity matrix. For $j > 0$, the vector $\mathbf{v}_{t+j}$ is uncorrelated with $\mathbf{z}_t$,

$$\mathrm{Cov}[\mathbf{z}_t, \mathbf{v}_{t+j}] = \mathrm{E}[\mathbf{z}_t\mathbf{v}_{t+j}^T] = \mathbf{0} \text{ for } j > 0.$$

The second order correlation structure of the process $\mathbf{z}_t$ depends on the matrices $\mathbf{M}_0 = \mathrm{Var}[\mathbf{z}_t] = \mathrm{E}[\mathbf{z}_t\mathbf{z}_t^T]$ and $\mathbf{M}_1 = \mathrm{Cov}[\mathbf{z}_{t+1}, \mathbf{z}_t] = \mathrm{E}[\mathbf{z}_{t+1}\mathbf{z}_t^T]$, with components

$$\mathbf{M}_0 = \begin{pmatrix} \mathrm{Var}[x_t] & \mathrm{Cov}[x_t, y_t] \\ \mathrm{Cov}[y_t, x_t] & \mathrm{Var}[y_t] \end{pmatrix} = \begin{pmatrix} 1 & \rho_{xy} \\ \rho_{xy} & 1 \end{pmatrix},$$

that is symmetric and t-independent, whereas

$$\mathbf{M}_1 = \begin{pmatrix} \mathrm{Cov}[x_{t+1}, x_t] & \mathrm{Cov}[x_{t+1}, y_t] \\ \mathrm{Cov}[y_{t+1}, x_t] & \mathrm{Cov}[y_{t+1}, y_t] \end{pmatrix} = \begin{pmatrix} \rho_x & \rho_{x_{t+1}, y_t} \\ \rho_{y_{t+1}, x_t} & \rho_y \end{pmatrix},$$

is not symmetric.

To find expressions for the parametric matrices postmultiply Equation (13) by $\mathbf{z}_t$ and take the expected value

$$\mathbf{M_1} = \mathbf{A}\mathbf{M_0},$$

therefore

$$\mathbf{A} = \mathbf{M_1}\mathbf{M_0^{-1}}. \tag{14}$$

Similarly, postmultiply Equation (13) by $\mathbf{z}_{t+1}$ and take expected values to obtain

$$\mathbf{M_0} = \mathbf{M_1}\mathbf{M_0^{-1}}\mathbf{M_1^T} + \mathbf{B}\mathbf{B^T}. \tag{15}$$

One can then obtain $\mathbf{B}$ from Equation (15) assuming it is triangular. A simple way is to assume that $\rho_{x_{t+1}, y_t} = \rho_{xy}\rho_x$ and $\rho_{y_{t+1}, x_t} = \rho_{xy}\rho_y$ [46]. In that case, $\mathbf{M_1} = \mathbf{D}\mathbf{M_0}$, with $\mathbf{D}$ a diagonal matrix

$$\mathbf{D} = \begin{pmatrix} \rho_x & 0 \\ 0 & \rho_y \end{pmatrix}.$$

Therefore, Equation (14) is now

$$\mathbf{A} = \mathbf{M_1}\mathbf{M_0^{-1}} = \mathbf{D}. \tag{16}$$

Similarly, Equation (15) simplifies to

$$\mathbf{BB^T} = \mathbf{M_0} - \mathbf{DM_0D} = \begin{pmatrix} 1-\rho_x^2 & \rho_{xy}(1-\rho_x\rho_y) \\ \rho_{xy}(1-\rho_x\rho_y) & 1-\rho_y^2 \end{pmatrix}, \tag{17}$$

that one can solve for $\mathbf{B}$

$$\mathbf{B} = \begin{pmatrix} \sqrt{1-\rho_x^2} & 0 \\ \dfrac{\rho_{xy}(1-\rho_x\rho_y)}{\sqrt{1-\rho_x^2}} & \sqrt{1-\rho_y^2 - \dfrac{\rho_{xy}^2(1-\rho_x\rho_y)^2}{1-\rho_x^2}} \end{pmatrix}. \tag{18}$$

The expected value of the total variation of a univariate autoregressive process (the first component of Equation (13)) is

$$\mathrm{E}\left[\bigvee_1^n x_t\right] = (n-1) \times \frac{2}{\sqrt{\pi}}\sqrt{1-\rho_x}. \tag{19}$$

The Figure 1 shows the expected value as a function of the lag-one autocorrelation coefficient ρ_x.

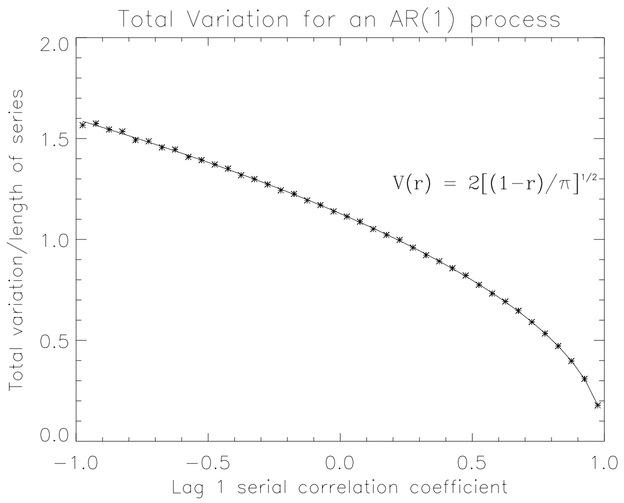

Figure 1. Expected value of the total variation of a univariate autoregressive process as a function of the lag-one autocorrelation coefficient $\rho_x = r$. The points are the average of 100 simulations. The continuous line is Equation (19).

The result in Equation (19) comes realizing that

$$|x_{t+1} - x_t| = |\sqrt{1-\rho^2}v_{t+1} - (1-\rho)x_t|,$$

where both x_t and v_t are independent random variables, with zero mean and unit variance. In addition, one usually assumes that their distribution is Gaussian. Therefore, the expected value of $\bigvee_1^n(x_t)$ is

$$\mathrm{E}\left[\bigvee_1^n x_t\right] = \mathrm{E}\left[\sum_{k=0}^{n-2}|x_{t+1}-x_t|\right] = \sum_{k=0}^{n-2}\mathrm{E}\left[|\sqrt{1-\rho^2}v_{t+1} - (1-\rho)x_t|\right]. \tag{20}$$

A simple calculation of the expected value in Equation (20) gives Equation (19).

Similar arguments apply to the calculation of the expected value of the total variation of the sum $x_t + y_t$

$$\mathrm{E}\left[\bigvee_1^n (x_t + y_t)\right] = \sum_{k=0}^{n-2} \mathrm{E}|(b_{1,1} + b_{2,1})v_{t+1} + b_{2,2}w_{t+1} - (1 - a_{1,1})x_t - (1 - a_{2,2})y_t|, \quad (21)$$

where $a_{i,j}$ and $b_{i,j}$ are, respectively, the components of the matrices **A** and **B** in Equations (16) and (18). All the random variables in Equation (21) are Gaussian, with zero mean and unit variance. v_{t+1} and w_{t+1} are mutually independent and independent of x_t, y_t. The covariance between x_t and y_t is ρ_{xy}. Therefore, using properties of the Gaussian distribution, the absolute value in Equation (21) corresponds to a Gaussian distribution with zero mean and variance equal to

$$(b_{1,1} + b_{2,1})^2 + b_{2,2}^2 + (1 - a_{1,1})^2 + (1 - a_{2,2})^2 + 2(1 - a_{1,1})(1 - a_{2,2})\rho_{xy} = 2(1 + \rho_{xy})(2 - \rho_x - \rho_y).$$

Finally, the expected value of the total variation of the sum is

$$\mathrm{E}\left[\bigvee_1^n (x_t + y_t)\right] = (n-1) \times \frac{2}{\sqrt{\pi}} \sqrt{(1 + \rho_{xy})(2 - \rho_x - \rho_y)}. \quad (22)$$

Figure 2 illustrates this result.

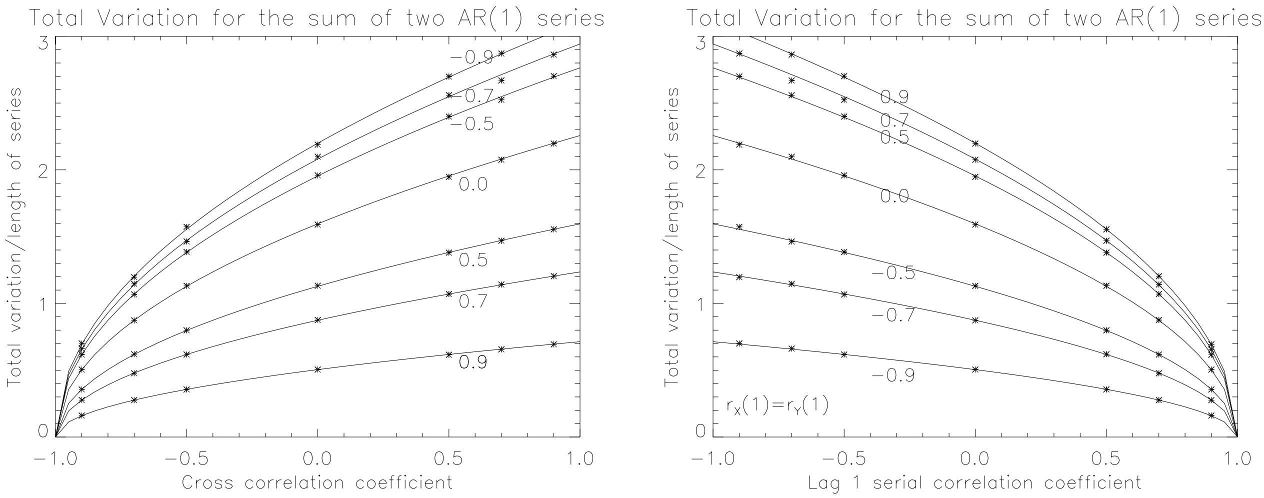

Figure 2. Left panel: Expected value of the total variation of the sum of two AR(1) series as a function of the cross-correlation coefficient ρ_{xy} for different values of the serial autocorrelation coefficient $\rho_y = \rho_x$, curves are labeled with the respective values of ρ_y. **Right panel**: same as **left panel** but as a function of $\rho_y = \rho_x$ for different vales of ρ_{xy}, curves are labeled with the respective values of ρ_{xy}. Points are average of 100 simulations and the continuous line is computed with Equation (22).

Using the above results, Equations (19) and (22), the total variation complementarity index between two autoregressive time series with lag-one correlation coefficients ρ_x and ρ_y, and cross correlation ρ_{xy}, is

$$\phi(x_t, y_t) = 1 - \frac{\mathrm{V}_1^n(x_t + y_t)}{\mathrm{V}_1^n(x_t) + \mathrm{V}_1^n(y_t)} = 1 - \frac{\sqrt{(1 + \rho_{xy})(2 - \rho_x - \rho_y)}}{\sqrt{1 - \rho_x} + \sqrt{1 - \rho_y}}. \quad (23)$$

Figure 3 illustrates some particular cases of this result. We are interested in the comparison of ϕ with ϕ' and ϕ_s. Recall that the unit variance of x_t and y_t implies that $\phi'(x_t, y_t) = (1 - \rho_{xy})/2$ and that $\phi_s = 1 - \sqrt{(1 + \rho_{xy})/2}$.

A first observation is that for $\rho_{xy} = -1$, all three indices ϕ, ϕ', and ϕ_s are equal to unity, perfect complementarity regardless of ρ_x and of ρ_y.

Next, if $\rho_x = \rho_y$, the total variation complementarity is independent of ρ_x, with $\phi = 1 - \sqrt{(1+\rho_{xy})/2} = \phi_s \le \phi' = (1-\rho_{xy})/2$ (left panel of Figure 3). Interestingly, as functions of ρ_{xy}, ϕ and ϕ_s depend on the square root, whereas ϕ' is linear. The explanation is that both the definition of the total variance (see Equation (13)) and of the standard deviation are linear on the values of the series, but the cross-correlation is a second-order moment.

Still within the equality of the lag-one correlation coefficients, the case $\rho_{xy} = 0$, and therefore $\phi = \phi_s = 1 - \sqrt{2}/2 \approx 0.29$, is interesting in comparison with $\phi' = 0.5$. That high-variance complementarity index in the absence of cross-correlation comes from how Equation (10) transforms the range of the index between zero and one. Positive cross-correlations give $0 < \phi' < 1/2$, something that seems high. An alternative to Equation (10) could be to take $-\rho_{xy}$ for negative correlations and zero for positive ones. However, both are arbitrary. Indeed, one does not need re-scaling for ϕ nor ϕ_s. For two series with null cross-correlation, approximately 30% of the total variation of their sum cancels out just by chance. In contrast, one of the alternatives of normalizing the variance complementarity to the range $(0,1)$ is too high (50%), and the other is too low (0%). The comment in the previous paragraph has a bearing on this comparison, as the correspondence between ϕ and ϕ_s shows.

For constant $\rho_{xy} = -0.7$ in the center panel of Figure 3, the total variation complementarity does depend on both ρ_x and ρ_y, although ϕ' has the constant value of 0.85. ϕ' is between 28% and 47% larger than ϕ, depending on the values of the autocorrelation coefficients. On the other hand, the comparison between ϕ and ϕ_s is better for various values of both ρ_x and ρ_y.

For the case $\rho_x = -\rho_y$ (right panel of Figure 3), ϕ depends on ρ_x, except for ϕ' close to one. In all cases, ϕ' is larger than ϕ. With respect to the comparison between ϕ_s and ϕ, the first observation is that $\phi_s = \phi$ for $\rho_x = 0$. Second, for larger values of the autocorrelation, ϕ_s over-estimates ϕ, showing the importance of taking into account autocorrelation.

In general, it is worth noticing the contrast between ϕ' and ϕ_s being independent of ρ_x and ρ_y, whereas ϕ is significantly dependent on both.

Recall that all the cases considered in this section use unit variance for the two series to make a fair comparison. Therefore, the possible scale differences do not play a role; only the effect of the serial autocorrelation does.

We conclude that the total variation complementarity index has more information than the variance complementarity or standard deviation indices. It considers the effect of the autocorrelations of the series. In addition, the index based on cross-correlation, ϕ', significantly overestimates the complementarity compared with ϕ. In contrast, for ϕ_s, the overestimation is less of an issue for a range of autocorrelation values.

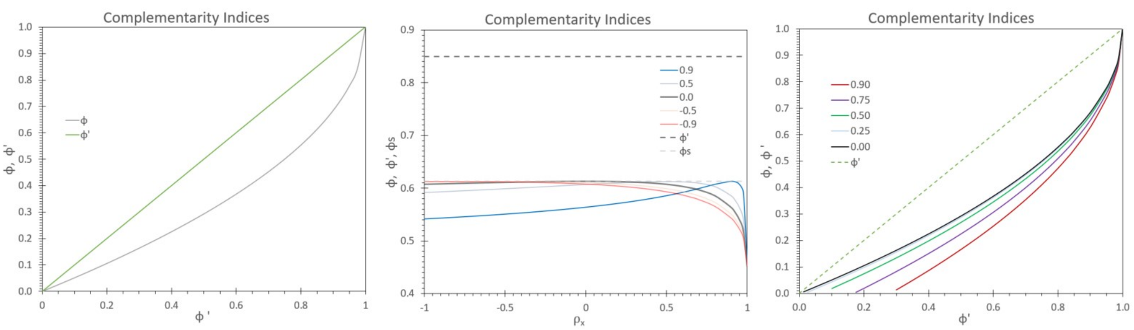

Figure 3. Comparison of total variation complementarity indices, ϕ, ϕ', and ϕ_s as function of ρ_x, ρ_y and $\rho_{x,y}$ for the sum of two AR(1) series: **left panel**, when $\rho_y = \rho_x$, in this case $\phi = \phi_s$; **center panel**, when $\rho_{xy} = -0.7$, and for the annotated values of ρ_y, in this case $\phi' = 0.85$ and $\phi_s = 0.613$; and **right panel**, for the annotated values of $\rho_y = -\rho_x$ as a function of ϕ', in this case $\phi_s = \phi$ for $\rho_x = 0$.

3.2. Case Study 2: Wind vs. Hydro in the Colombian Electricity Market

The total installed capacity for electricity generation in Colombia is approximately 17.5 GW. A total of 68.3% of it comes from hydro sources, while the only wind plant accounts for near 0.1%. This high difference in the scale of both sources has an important impact in the assessment of their complementarity.

To quantify the complementarity between wind and hydro-resources in the Colombian electricity system, we use the daily time series provided by XM, the electric system administrator, and operator in Colombia. For hydro-resources, we analyzed natural inflows (in units of energy) to the main 15 hydropower reservoirs, with a total effective installed capacity of 7668 MW (≈70% of total hydro-capacity) and records from 2000 to 2021. These rivers are San Lorenzo, Nare, Guatapé, San Carlos, Tenche, Guadalupe, Grande, Porce II, Miel I, Urrá, Betania, Batá, Guavio, Alto Anchicayá, and Salvajina. For wind resources, we analyzed daily wind power generation from the Jepirachi plant, with 19.5 MW of effective installed capacity and records for 2004–2021. Notice that the dimensions of both series are the same (energy). The reader can see more details of the hydropower system of Colombia at the XM website (see http://paratec.xm.com.co/paratec/SitePages/Default.aspx, accessed on 30 April 2022).

Figure 4 shows the estimation of ϕ, ϕ', and ϕ_s indices to the daily series and the annual cycles of hydro-inflows and wind generation with moving averages of 31 days (top panel). Considering the large-scale difference between the accumulated hydro-inflows and the Jepirachi plant, the scaled series's complementarity was also studied (bottom panel), with a scale factor of 1/100 for the hydro-inflow series and 1/10 for the wind generation series. Table 1 presents the results for all analyzed cases.

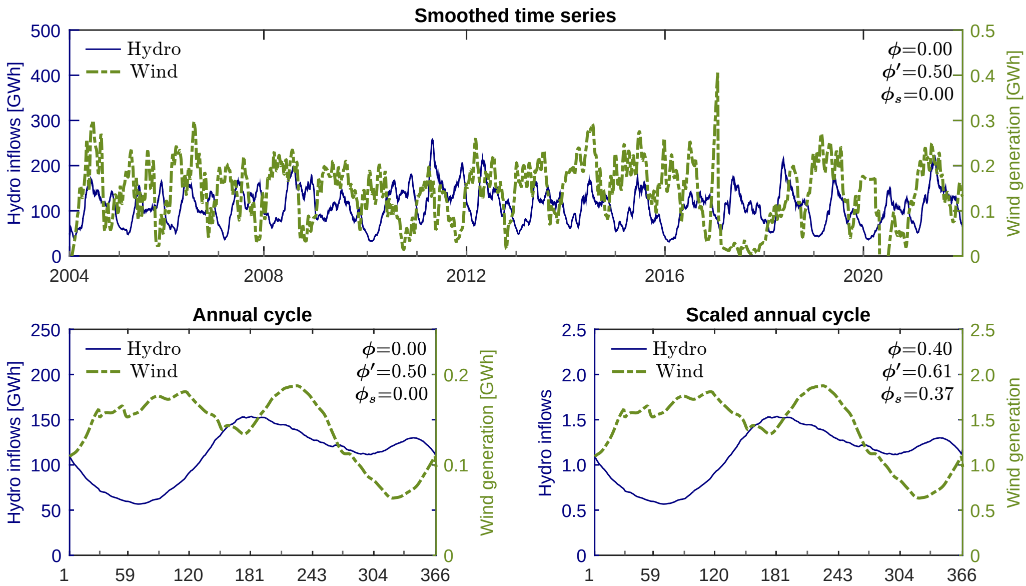

Figure 4. Complementarity indices ϕ, ϕ' and ϕ_s between wind and hydraulic resources in the Colombia electricity market (time series were smoothed by a 31-day moving average filter). The scale factor in the case of scaled annual cycle is 1/100 for hydro-inflow series and 1/10 for wind-power generation series.

Table 1. Complementarity between daily wind and hydro-resources in Colombia. Both time series were smoothed by a 31-day moving average filter. ρ_1 represents the exponential value of the slope of the linear regression for the autocorrelation function of both resources.

Group	σ_H^2 $[GW^2h^2]$	ρ_{1H} [-]	σ_W^2 $[GW^2h^2]$	ρ_{1W} [-]	$\rho_{H,W}$ [-]	ϕ [-]	ϕ' [-]	ϕ_s [-]
Smoothed time series	1508	0.98	0.01	0.98	−0.14	0.00	0.50	0.00
Annual cycle	878	-	0.00	-	−0.22	0.00	0.50	0.00
Scaled series	0.09	-	0.13	-	−0.22	0.40	0.61	0.37

As expected, because of the two series' scale differences, the indices ϕ, ϕ_v (not shown), and ϕ_s do not show complementarity. In contrast, the ϕ' index indicates a complementarity of 0.50 for the daily series and the annual cycle. The reason for this huge overestimation is the way one re-scales the index ϕ_v to obtain ϕ' (Equation (10)).

When using the scaled series, the ϕ index increases to 0.40, and ϕ_s to 0.37. We consider that this scaled analysis evaluates the resources' potential complementarity if the resources' scales were comparable. There is still a considerable difference in the result of ϕ'. This tendency of ϕ' to always exceed ϕ, overestimating the complementarity quantification, comes from Equation (10).

3.3. Case Study 3: Two Hydropower Sources in the Colombian Electricity Market

Nare and Guavio are two important rivers in the hydropower reservoir system of Colombia. According to XM, these rivers belong to different hydrological regions (see details in the XM website: http://portalbissrs.xm.com.co/hdrlg/Paginas/Aportes/apoeneyporreg.aspx, accessed on 30 April 2022). The Andes mountain range separates the two basins, causing significant differences in their hydroclimatic patterns. Seasonality of rainfall is unimodal in the Guavio basin and bimodal in the Nare basin [47]. Streamflows show these regimes, too (Figure 5).

We computed the complementarity indices between the daily natural inflows (in energy units) of the Nare and Guavio rivers for 2000–2021. Figure 5 shows the results, where the left panel shows the 31-day moving average daily inflows and the right panel the annual cycle at daily time-step for both rivers. Table 2 lists the results for all analyzed cases.

The scale of both resources is not as different as in the previous case. Consequently, the ϕ index is similar for the daily and scaled series with values close to 0.20. Using the equivalent to Equation (23) that considers the values of variance (not shown), with the parameters contained in Table 2, one gets $\phi = 0.20$. The same value that one obtains using the time series and Equations (4) and (5). This correspondence suggests that the autoregressive model is good for estimating complementarity. Again, ϕ' shows a higher result than ϕ and ϕ_s for all cases, overestimating the complementarity to around 0.38. The pattern is that the variance complementarity index tends to overestimate complementarity.

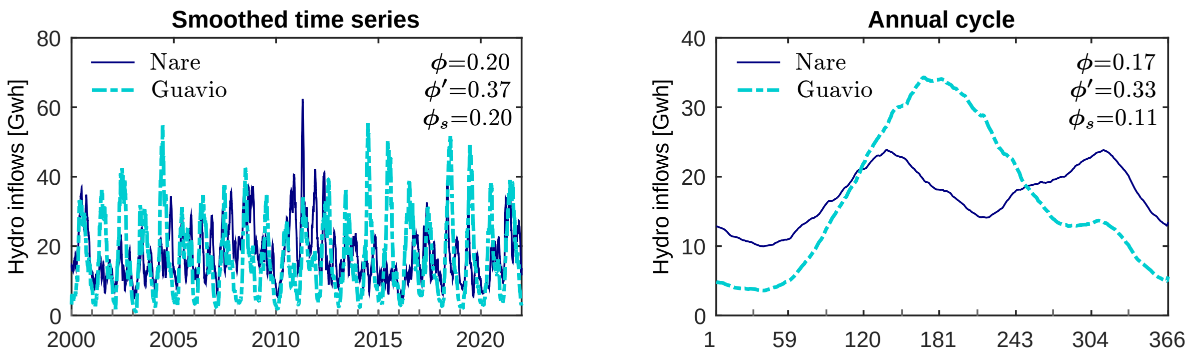

Figure 5. Assessment of complementarity by the ϕ, ϕ', and ϕ_s indices between the Nare and Guavio rivers. **Left panel**: 31-day moving average daily inflows. **Right panel**: annual cycle at daily time-step.

Table 2. Complementarity between 31-day moving average daily streamflows of Nare and Guavio rivers. ρ_1 represents the linear regression slope for the autocorrelation function of both resources.

Group	σ^2_{Nare} $[GW^2h^2]$	ρ_{1Nare} [-]	σ^2_{Guavio} $[GW^2h^2]$	$\rho_{1Guavio}$ [-]	$\rho_{Nare,Guavio}$ [-]	$\boldsymbol{\phi}$ [-]	$\boldsymbol{\phi'}$	$\boldsymbol{\phi_s}$ [-]
Smoothed time series	62	0.98	129	0.98	0.27	0.20	0.37	0.20
Annual cycle	17	-	101	-	0.50	0.17	0.33	0.11
Scaled series	0.46	-	1.45	-	0.50	0.18	0.29	0.12

3.4. Case Study 4: Hydropower-Integrated Sources in the Colombian Electricity Market

One of the main advantages of the ϕ index is that one can evaluate the complementarity between any number of series. To illustrate this capability, we quantify the complementarity between the monthly energy inflows to the 15 largest reservoirs of the Colombian hydropower system (described in Section 3.2). This analysis is worthwhile since these reservoirs are in different country regions with different geographic and climatic characteristics.

Figure 6 presents the daily series and annual cycle of the integrated hydro system with moving averages of 31 days. The ϕ index shows a complementarity of 0.34 for the smoothed series and 0.30 for the annual cycles. The complementarity is somewhat higher for the unsmoothed series (not shown), reaching 0.38 and 0.46, respectively.

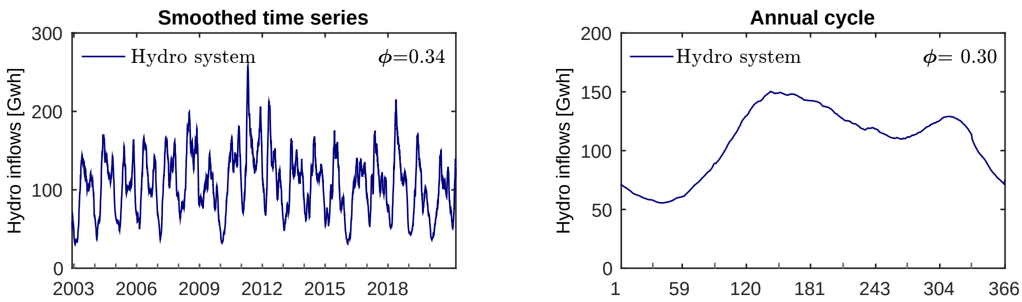

Figure 6. Quantification of the complementarity with ϕ index between the energy inflows to the main 15 hydropower reservoirs of the Colombian electricity system, with smoothed moving averages of 31 days.

4. Discussion

This article proposes a novel method based on the concept of Total Variation. The new index (ϕ) is a consistent and suitable metric for complementarity between any number of series. The index ϕ measures the regularity of the sum of the variables; this is one minus a measure of irregularity. The new index is not affected by the issues pointed out regarding correlation coefficients. Some essential properties of ϕ are: (1) ϕ is sensitive to the scale of the variables; (2) ϕ works only with dimensionally homogeneous variables; (3) ϕ applies to any number of variables; (4) ϕ also depends on the persistence (autocorrelation) of the series, not only on their cross-correlation.

Given the intermittency of renewable energy sources, consistent evaluation of complementarity is a critical issue. Our proposed index fulfills this need, substantially contributing to energy planning.

The definition of ϕ suggested using the variance instead of total variation to measure irregularity. The result uses the correlation coefficient to assess complementarity. This modification corrects the drawback of differences in the scales of the series discussed above. We call this modification the variance complementarity index, ϕ'. Nevertheless, the correction produces overestimations of complementarity. This overestimation comes from the correlation being a second-order moment. This explanation leads to using the standard deviation instead of the variance to measure the irregularity of the sum. This third index, the Standard Deviation Complementarity Index, corrects the overestimation.

The applications to the Colombian power system show that the variance complementarity overestimates complementarity. In comparison, the Standard Deviation Complementarity Index is closer to the total variation index. However, the analytical results for the expected value of two cross-correlated autoregressive series demonstrate that ϕ_s does not capture all the complementarity's dependencies. In particular, the example shows that serial autocorrelation does affect complementarity.

Finally, we used the proposed indices to quantify the Colombian electricity system's complementarity. This assessment is important due to its hydrological and climatic regional differences. The total variation complementarity index between the daily series of the hydro-resources is 0.30. This number estimates the complementarity between the 15 main rivers feeding the Colombian power system, which has never been achieved before. The total variation complementarity index, ϕ, between hydro and wind resources is almost null due to the wind component's negligible contribution. When one uses scaled variables, it is 0.39, representing the potential complementarity between these resources and could stimulate a higher level of investment in wind capacity development. The proposed index put a solid basis and a concrete number on those intuitive ideas.

5. Conclusions

There is no agreement in the electricity industry and the scientific community on how to define and assess the complementarity of energy sources. The most frequently used approach is to associate complementarity with negative correlation. This article showed that correlation indices have several intrinsic shortcomings when used for that intention and proposes the uniformity of the sum of the resources as a more physically based concept of complementarity. As a consequence, dimensional homogeneity is a necessary condition for any set of variables before complementarity can be assessed.

There are other drawbacks to the use of correlation coefficients to assess complementarity. (1) If the series scales are very different, correlation coefficients may mislead complementarity. A negative correlation is a necessary but not sufficient condition for complementarity. (2) Correlation coefficients can only be used with pairs of variables, while the applied cases worldwide usually encompass multiple energy sources.

In this article, we propose a new method (ϕ) based on the concept of Total Variation, which considers as a starting concept to assess complementarity the regularity measure of the sum of the variables. This method is sensitive to the scale of the variables, works only with dimensionally homogeneous variables, depends on the persistence of the series (not only on their cross-correlation), and can be applied to more than two series.

We also present two more indices based on variance (ϕ') and standard deviation (ϕ_s). We found that the (ϕ') index produces complementarity overestimation, on the other hand, although (ϕ_s) corrects the overestimation, this index does not capture all the complementarity's dependencies between the time series.

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