Ideal Brayton cycles

For an ideal Brayton cycle (BC cycle), according to Fig.2 (a) the thermal efficiency may be expressed as:

$$\eta_{\mathrm{BC}}=1-\pi^{\frac{1-\gamma}{\gamma}} \tag{E-S1}$$

Specific power may be expressed as:

$$P_{\mathrm{BC}}=c_{\mathrm{p}}T_2\left(\tau-\tau\cdot\pi^{\frac{1-\gamma}{\gamma}}-\pi^{\frac{\gamma-1}{\gamma}}+1\right) \tag{E-S2}$$

The specific thrust of the engine based on the ideal Brayton cycle may be expressed as:

$$F_{\mathrm{s}}=W_{\mathrm{g}}c_{\mathrm{out}}-W_{\mathrm{a}}c_{\mathrm{in}}+(p_{\mathrm{out}}-p_{\mathrm{in}})\cdot A_{\mathrm{out}} \tag{E-S3}$$

In this work, the working fluids completely expand with $p_{\mathrm{out}}=p_{\mathrm{in}}$. The mass flow of fuel is neglected with $W_{\mathrm{g}}=W_{\mathrm{a}}$, which is the unit mass. The intake velocity is zero.

Therefore, the specific thrust can be simplified as:

$$F_{\mathrm{s}}=c_{\mathrm{out}}=\sqrt{2\cdot c_{\mathrm{p}}T_2\left(\tau-\tau\cdot\pi^{\frac{1-\gamma}{\gamma}}-\pi^{\frac{\gamma-1}{\gamma}}+1\right)} \tag{E-S4}$$

The nozzle exit velocity may be expressed as:

$$c_{\mathrm{out}}=\sqrt{2\cdot P_{\mathrm{BC}}} \tag{E-S5}$$

The equation can be simplified to,

$$c_{\mathrm{out}}=\sqrt{2\cdot c_{\mathrm{p}}T_2\left(\tau-\tau\cdot\pi^{\frac{1-\gamma}{\gamma}}-\pi^{\frac{\gamma-1}{\gamma}}+1\right)} \tag{E-S6}$$

The energy fraction for the BC cycle is defined as the ratio of the compression power and the heat addition.

$$\psi_{\mathrm{BC}}=\frac{P_{\mathrm{comp}}}{\Delta H_{3-4}+P_{\mathrm{comp}}}=\frac{\left(\pi^{\frac{\gamma-1}{\gamma}}-1\right)}{\tau-1} \tag{E-S7}$$

Ideal reheated Brayton cycles

For an ideal reheating Brayton cycle (RBC cycle), according to Fig.2 (b), the thermal efficiency can be expressed as :

$$\eta_{\mathrm{RBC}}=\frac{(h_4-h_3)+(h_{4.6}-h_{4.5})-(h_5-h_2)}{(h_4-h_3)+(h_{4.6}-h_{4.5})} \tag{E-S8}$$

It can be acquired by simple derivation:

$$\eta_{\mathrm{RBC}}=1-\frac{\pi_1^{\frac{1-\gamma}{\gamma}}\cdot\tau_2-1}{\tau_1-\pi^{\frac{\gamma-1}{\gamma}}+\tau_2-\pi_2^{\frac{1-\gamma}{\gamma}}\cdot\tau_1} \tag{E-S9}$$

The specific power can be expressed as:

$$P_{\mathrm{RBC}} = (h_4 - h_{4.5}) + (h_{4.6} - h_5) - (h_3 - h_2) \tag{E-S10}$$

In brief,

$$P_{\mathrm{RBC}} = c_p T_2 \left(\tau_1 - {\pi_2}^{\frac{1-\gamma}{\gamma}} \cdot \tau_1 + \tau_2 - {\pi_1}^{\frac{1-\gamma}{\gamma}} \cdot \tau_2 - \pi^{\frac{\gamma-1}{\gamma}} + 1 \right) \tag{E-S11}$$

The nozzle exit velocity can be expressed as:

$$c_{\mathrm{out}} = \sqrt{2 c_p T_2 \left(\tau_1 - {\pi_2}^{\frac{1-\gamma}{\gamma}} \cdot \tau_1 + \tau_2 - {\pi_1}^{\frac{1-\gamma}{\gamma}} \cdot \tau_2 - \pi^{\frac{\gamma-1}{\gamma}} + 1 \right)} \tag{E-S12}$$

The specific thrust can be expressed as:

$$F_{\mathrm{s}} = c_{\mathrm{out}} = \sqrt{2 c_p T_2 \left(\tau_1 - {\pi_2}^{\frac{1-\gamma}{\gamma}} \cdot \tau_1 + \tau_2 - {\pi_1}^{\frac{1-\gamma}{\gamma}} \cdot \tau_2 - \pi^{\frac{\gamma-1}{\gamma}} + 1 \right)} \tag{E-S13}$$

The energy fraction for the RBC cycle is defined as the ratio of the compression power and the heat addition.

$$\psi_{\mathrm{RBC}} = \frac{P_{\mathrm{comp}}}{\Delta H_{3-4} + \Delta H_{4.5-4.6}} = \frac{\left(\pi^{\frac{\gamma-1}{\gamma}} - 1 \right)}{\tau_2 + \tau_1 + \pi^{\frac{\gamma-1}{\gamma}} - \pi_2 \frac{1-\gamma}{\gamma} \tau_1} \tag{E-S14}$$

Ideal reheated Brayton cycles with SOFCs

For an ideal reheated Brayton cycle with SOFCs (RBFC cycle), according to Fig.2 (d) the efficiency is:

$$\eta_{\mathrm{RBFC}} = \frac{(h_4 - h_3) + (h_{4.6} - h_{4.5}) - (h_5 - h_2) + (h_3 - h_2)}{(h_4 - h_3) + (h_{4.6} - h_{4.5}) + (h_3 - h_2)} \tag{E-S15}$$

This can be acquired by simple derivation:

$$\eta_{\mathrm{RBFC}} = 1 - \frac{{\pi_1}^{\frac{1-\gamma}{\gamma}} \cdot \tau_2 - 1}{\tau_1 + {\pi_2}^{\frac{1-\gamma}{\gamma}} \cdot \tau_1 - \tau_2 - 1} \tag{E-S16}$$

For the reheated Brayton cycle with SOFC (RBFC cycle), the specific power is

$$w_{\mathrm{RBFC}} = (h_4 - h_{4.5}) + (h_{4.6} - h_5) \tag{E-S17}$$

In brief,

$$w_{\mathrm{RBFC}} = c_p T_2 \left(\tau_1 - {\pi_2}^{\frac{1-\gamma}{\gamma}} \cdot \tau_1 + \tau_2 - {\pi_1}^{\frac{1-\gamma}{\gamma}} \cdot \tau_2 \right) \tag{E-S18}$$

The nozzle exit velocity can be expressed as:

$$c_{\mathrm{out}} = \sqrt{2 c_p T_2 \left(\tau_1 - {\pi_2}^{\frac{1-\gamma}{\gamma}} \cdot \tau_1 + \tau_2 - {\pi_1}^{\frac{1-\gamma}{\gamma}} \cdot \tau_2 \right)} \tag{E-S19}$$

The specific thrust can be expressed as:

$$F_{\mathrm{s}} = c_{\mathrm{out}} = \sqrt{2 c_p T_2 \left(\tau_1 - {\pi_2}^{\frac{1-\gamma}{\gamma}} \cdot \tau_1 + \tau_2 - {\pi_1}^{\frac{1-\gamma}{\gamma}} \cdot \tau_2 \right)} \tag{E-S20}$$

The energy fraction of the RBFC cycle is defined as the ratio of the compression power and heat addition.

$$\psi_{\mathrm{RBFC}} = \frac{P_{\mathrm{comp}}}{\Delta H_{3-4}+\Delta H_{4.5-4.6}+P_{\mathrm{ele}}} = \frac{\left(\pi^{\frac{\gamma-1}{\gamma}}-1\right)}{\tau_1+\tau_2-\pi_2^{\frac{1-\gamma}{\gamma}}\cdot\tau_1-1} \tag{E-S21}$$

According to Equation (11),

$$c_p \cdot T_2 \cdot \left(\pi^{\frac{\gamma-1}{\gamma}} - 1\right) = c_p \cdot (T_4 - T_3 + T_{4.6} - T_{4.5}) \cdot \left(\frac{1}{\phi\cdot\eta_{\mathrm{fc}}} - 1\right)^{-1} \tag{E-S22}$$

The fuel utilization in the RBFC cycle can be derived by Eq (E-S22) as

$$\phi_{\mathrm{RBFC}} = \left\{\eta_{\mathrm{fc}}\left[\left(\frac{\tau_1-\pi^{\frac{\gamma-1}{\gamma}}+\tau_2-\pi_2^{\frac{1-\gamma}{\gamma}}\cdot\tau_1}{\pi^{\frac{\gamma-1}{\gamma}}-1}\right)+1\right]\right\}^{-1} \tag{E-S23}$$

There is a maximum fuel utilization of the fuel cell in the BFC cycle, which can be expressed as:

$$\phi_{\mathrm{RBFC,max}} = 1 - \frac{c_p\cdot(600+273.15-T_2)}{c_p\cdot(T_4-T_3)+c_p\cdot(T_3-T_2)+c_p\cdot(T_{4.6}-T_{4.5})} \tag{E-S24}$$

In brief,

$$\phi_{\mathrm{RBFC,max}} \frac{\tau_1+\tau_2-\pi_2^{\frac{1-\gamma}{\gamma}}\cdot\tau_1+\frac{873.15}{T_2}}{\tau_1+\tau_2-\pi_2^{\frac{1-\gamma}{\gamma}}\cdot\tau_1-1} \tag{E-S25}$$

In this work, $T_2 = 280$ K. The maximum fuel utilization of the fuel cell in the BFC cycle is:

$$\phi_{\mathrm{RBFC,max}} = \frac{\tau_1+\tau_2-\pi_2^{\frac{1-\gamma}{\gamma}}\cdot\tau_1+3.12}{\tau_1+\tau_2-\pi_2^{\frac{1-\gamma}{\gamma}}\cdot\tau_1-1} \tag{E-S26}$$

Ideal isothermal Brayton cycle with SOFCs

For an ideal isothermal Brayton cycle with SOFC (IBFC cycle), according to Fig.2 (e) the thermal efficiency is

$$\eta_{\mathrm{RBFC}} = \frac{(h_4-h_3)+(h_3-h_2)-(h_5-h_2)+(w_{4-4.5})}{(h_4-h_3)+(h_{4.5}-h_4)+(h_3-h_2)+(w_{4-4.5})} = \frac{(h_4-h_3)+(h_3-h_2)-(h_5-h_2)+R_gT_4ln\pi_1}{(h_4-h_3)+(h_{4.5}-h_4)+(h_3-h_2)+R_gT_4ln\pi_1} \tag{E-S27}$$

In brief,

$$\eta_{\mathrm{RBFC}} = 1 - \frac{\pi_2^{\frac{1-\gamma}{\gamma}}\cdot\tau}{\tau-1} \tag{E-S28}$$

For an ideal isothermal Brayton cycle with SOFC (IBFC cycle), the specific power is

$$w_{\mathrm{RB,cyc}} = w_{4-4.5} + (h_{4.5} - h_5) = -R_gT_4ln\frac{p_{4.5}}{p_4} + (h_{4.5} - h_5) \tag{E-S29}$$

In brief,

$$w_{\mathrm{RB,cyc}} = T_2\left[R_g\tau ln\pi_1 + c_p\left(\tau - -\pi_2^{\frac{1-\gamma}{\gamma}} \cdot \tau\right)\right] \tag{E-S30}$$

The nozzle exit velocity may be expressed as:

$$c_{\text{out}} = \sqrt{T_2\left[R_g\tau ln\pi_1 + c_p\left(\tau - \pi_2^{\frac{1-\gamma}{\gamma}} \cdot \tau\right)\right]} \tag{E-S31}$$

According to Equation (E-S4), the specific thrust can be expressed as:

$$F_{\text{s}} = c_{\text{out}} = \sqrt{T_2\left[R_g\tau ln\pi_1 + c_p\left(\tau - -\pi_2^{\frac{1-\gamma}{\gamma}} \cdot \tau\right)\right]} \tag{E-S32}$$

The energy fraction for the IBFC cycle is defined as the ratio of the compression power and the heat addition.

$$\psi_{\text{IBFC}} = \frac{P_{\text{comp}}}{\Delta H_{3-4}+w_{4-4.5}+P_{\text{ele}}} = \frac{h_3-h_2}{h_4-h_3+R_gT_4ln\frac{p_4}{p_{4.5}}+h_3-h_2} = \frac{\left(\pi^{\frac{\gamma-1}{\gamma}}-1\right)}{\frac{R_g}{c_p}\tau ln\pi_1+\tau-1} \tag{E-S33}$$

According to Equation (11),

$$c_p \cdot T_2 \cdot \left(\pi^{\frac{\gamma-1}{\gamma}} - 1\right) = \left[c_p \cdot (T_4 - T_3) + R_gT_4ln\pi_1\right]\left(\frac{1}{\phi\cdot\eta_{\text{fc}}} - 1\right)^{-1} \tag{E-S34}$$

The fuel utilization in the IBFC cycle can be derived as

$$\phi_{\text{IBFC}} = \left\{\eta_{\text{fc}}\left(\frac{\tau-\pi^{\frac{\gamma-1}{\gamma}}+\frac{R_g}{c_p}\tau ln\pi_1}{\pi^{\frac{\gamma-1}{\gamma}}-1} + 1\right)\right\}^{-1} \tag{E-S35}$$

There is a maximum fuel utilization of the fuel cell in the IBFC cycle, which can be expressed as:

$$\phi_{\text{IBFC,max}} = 1 - \frac{c_p\cdot(600+273.15-T_2)}{c_p\cdot(T_4-T_3)+c_p\cdot(T_3-T_2)+R_gT_4ln\pi_1} \tag{E-S36}$$

In brief,

$$\phi_{\text{IBFC,max}} = \frac{\tau+\frac{R_g}{c_p}\tau ln\pi_1-\frac{873.15}{T_2}}{\tau+\frac{R_g}{c_p}\tau ln\pi_1-1} \tag{E-S37}$$

In this work, $T_2 = 280$ K. The maximum fuel utilization of the fuel cell in the IBFC cycle is:

$$\phi_{\text{IBFC,max}} = \frac{\tau+\frac{R_g}{c_p}\tau ln\pi_1-3.12}{\tau+\frac{R_g}{c_p}\tau ln\pi_1-1} \tag{E-S38}$$