Comparing Fuzzy Sets and Random Sets to Model the Uncertainty of Fuzzy Shorelines

List of Supplementary Materials

- Figure S1. The estimation results of *c* and *m* for FCM classification.
- Table S1. The cluster validity index showing the compactness and the separateness among all clusters (applied using m = 1.6).
- Figure S2. The estimation of threshold interval for c = 2 and various *m* values for FCM classification.
- Table S2. The results of McNemar's test showing the significance of the different accuracies given by *Pleiades* and *Pleiades* + *DTM* ($\alpha = 0.05$) in FCM classification with thresholding.
- Figure S3. The shoreline as the transition zone between water and non-water (Columns 1 and 5); confusion index images (Columns 2 and 6); zooming into the white-dashed rectangle sites (Columns 3 and 7); and shoreline images with fuzziness represented by the confusion index (Columns 4 and 8).
- Table S3. The optimal *n* selected for each threshold interval and the related κ values for generation of random set .
- Figure S4. The curve of differences between two successive standardized core sets d_i .
- Figure S5. Samples of the random set with various extents and their covering functions.
- Figure S6. The set-theoretic variance and the contour of random sets.
- Figure S7. An example of random sets; the core set Γ_1 and its contour (Columns 1 and 5); the support set Γ_0 and its contour (Columns 2 and 6); the transition zone between water and non-water represented by the set-theoretic variance (Columns 3 and 7); and zooming into the yellow rectangle sites (Columns 4 and 8).
- Table S4. The results of McNemar's test showing the significance of the different accuracies given by *Pleiades* and *Pleiades* + *DTM* ($\alpha = 0.05$) in random sets.
- Table S5. The results of McNemar's test showing the significance of the difference given by fuzzy sets and random sets ($\alpha = 0.05$) by using *Pleiades*.
- Table S6. The results of McNemar's test showing the significance of the difference given by fuzzy sets and random sets ($\alpha = 0.05$) using *Pleiades* + *DTM* data.







Figure S1. The estimation results of *c* and *m* for FCM classification. $S_{a\cdot b}$: the name of subsets, *a* is the group number (a = 1, ..., 4) and *b* is the subset number (b = 1, ..., 13).

6.1	C	luster validity index (XB)	
Subset —	<i>c</i> = 2	<i>c</i> = 3	<i>c</i> = 4
<i>S</i> ₁₋₁	0.013	0.023	0.020
<i>S</i> ₁₋₂	0.019	0.030	0.044
<i>S</i> ₁₋₆	0.016	0.021	0.033
<i>S</i> ₁₋₇	0.016	0.027	0.026
<i>S</i> ₁₋₉	0.008	0.034	0.027
<i>S</i> ₁₋₁₀	0.015	0.039	0.033
<i>S</i> ₂₋₃	0.017	0.028	0.027
S ₂₋₁₂	0.035	0.068	0.037
<i>S</i> ₂₋₁₃	0.023	0.027	0.031
S ₃₋₅	0.019	0.038	0.052
<i>S</i> ₃₋₁₁	0.010	0.019	0.023
<i>S</i> ₄₋₄	0.010	0.011	0.084
S ₄₋₈	0.013	0.039	0.040

Table S1. The cluster validity index showing the compactness and the separateness among all clusters (applied using m = 1.6). S_{a-b} : the name of subsets, a is the group number (a = 1, ..., 4) and b is the subset number (b = 1, ..., 13).

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Figure S2. The estimation of threshold interval for c = 2 and various *m* values for FCM classification (see notations in Figure S1 for the name of subsets).

Subset	f11	f12	f21	<i>f</i> 22	Chi-squared (X ²)	p – value
<i>S</i> ₁₋₁	10	9	0	156	7.1	0.01
<i>S</i> ₁₋₂	3	24	7	109	8.3	0.00
<i>S</i> ₁₋₆	6	23	4	119	12.0	0.00
<i>S</i> ₁₋₇	4	34	8	101	14.9	0.00
<i>S</i> ₁₋₉	3	23	9	78	5.3	0.02
<i>S</i> ₁₋₁₀	3	13	4	101	3.8	0.05
S ₂₋₃	7	15	2	128	8.5	0.00
S ₂₋₁₂	1	15	5	100	4.1	0.04
<i>S</i> ₂₋₁₃	7	4	0	107	2.3	0.13
S ₃₋₅	2	38	8	116	18.3	0.00
S ₃₋₁₁	3	11	3	104	3.5	0.06
S ₄₋₄	4	23	5	116	10.3	0.00
<i>S</i> ₄₋₈	1	19	10	91	2.2	0.14

Table S2. The results of McNemar's test showing the significance of the different accuracies given by *Pleiades* and *Pleiades* + *DTM* ($\alpha = 0.05$) in FCM classification with thresholding (see notations in Table S1 for the name of subsets).

*f*11 : number of cases with incorrect classification using both images;

*f*22 : number of cases with correct classification using both images;

f12 : number of cases that are incorrectly classified by Pleiades but correctly classified by Pleiades + DTM; and

*f*21 : number of cases that are correctly classified by *Pleiades* but incorrectly classified by *Pleiades* + *DTM*.



Figure S3. The shoreline as the transition zone between *water* and *non-water* (Columns 1 and 5); confusion index images (Columns 2 and 6); zooming into the white-dashed rectangle sites (Columns 3 and 7); and shoreline images with fuzziness represented by the confusion index (Columns 4 and 8) (see notations in Figure S1 for the name of subsets).

Subset	Number o each	of focal eleme	ents (<i>n</i>) for	к value for each threshold interval			
Subset	0.2 - 0.6	0.3 – 0.7	0.4 - 0.8	0.2 - 0.6	0.3 - 0.7	0.4 - 0.8	
<i>S</i> ₁₋₁	40	80	130	0.84	0.89	0.84	
S ₁₋₂	40	20	80	0.87	0.86	0.86	
<i>S</i> ₁₋₆	40	90	30	0.81	0.88	0.82	
S ₁₋₇	40	60	70	0.70	0.84	0.81	
S ₁₋₉	40	140	20	0.85	0.87	0.87	
S ₁₋₁₀	60	60	30	0.85	0.88	0.87	
S ₂₋₃	70	80	100	0.79	0.87	0.88	
S ₂₋₁₂	20	50	40	0.79	0.87	0.80	
S ₂₋₁₃	180	30	60	0.78	0.81	0.83	
S ₃₋₅	30	60	100	0.83	0.90	0.80	
S ₃₋₁₁	70	60	80	0.85	0.90	0.85	
S ₄₋₄	140	120	30	0.87	0.88	0.85	
S_{4-8}	150	40	20	0.75	0.81	0.75	

Table S3. The optimal *n* selected for each threshold interval and the related κ values for generation of random sets (see notations in Table S1 for the name of subsets).



Figure S4. The curve of differences between two successive standardized core sets d_i (see notations in Figure S1 for the name of subsets).



Figure S5. Samples of the random sets with various extents and their covering functions (see notations in Figure S1 for the name of subsets).



Figure S6. The set-theoretic variance and the contour of random sets (see notations in Figure S1 for the name of subsets).



Figure S7. An example of random sets; the core set Γ_1 and its contour (Columns 1 and 5); the support set Γ_0 and its contour (Columns 2 and 6); the transition zone between *water* and *non-water* represented by the set-theoretic variance (Columns 3 and 7); and zooming into the yellow rectangle sites (Columns 4 and 8) (see notations in Figure S1 for the name of subsets).

		McNemar's Test				
Subset	f11	f12	f21	f22	Chi-squared (X ²)	p – value
<i>S</i> ₁₋₁	13	8	0	154	6.1	0.013
S ₁₋₂	2	26	7	108	9.8	0.002
S ₁₋₆	0	17	8	127	2.6	0.110
S ₁₋₇	4	34	8	101	14.9	0.000
S ₁₋₉	4	22	3	84	13.0	0.000
<i>S</i> ₁₋₁₀	1	12	2	82	5.8	0.016
<i>S</i> ₂₋₃	7	11	4	130	2.4	0.121
<i>S</i> ₂₋₁₂	1	15	5	100	4.05	0.044
<i>S</i> ₂₋₁₃	13	3	3	100	0.0	1.000
S ₃₋₅	2	37	6	119	20.9	0.000
<i>S</i> ₃₋₁₁	3	11	3	104	3.5	0.061
S ₄₋₄	4	22	4	121	11.1	0.001
S ₄₋₈	2	18	10	91	1.8	0.186

Table S4. The results of McNemar's test showing the significance of the different accuracies given by *Pleiades* and *Pleiades* + *DTM* ($\alpha = 0.05$) in random sets (see notations in Table S2 for *f11*, *f12*, *f21* and *f22*, and notations in Table S1 for the name of subsets).

	McNemar Test						
Subset	f11	f12	f21	f22	Chi-squared (X ²)	p – value	
<i>S</i> ₁₋₁	18	1	2	154	0.0	1.000	
<i>S</i> ₁₋₂	25	2	5	111	0.6	0.450	
<i>S</i> ₁₋₆	11	18	6	117	5.0	0.025	
<i>S</i> ₁₋₇	37	0	1	109	0.0	1.000	
<i>S</i> ₁₋₉	26	1	0	86	0.0	1.000	
<i>S</i> ₁₋₁₀	15	0	1	105	0.0	1.000	
<i>S</i> ₂₋₃	19	3	0	130	1.3	0.248	
S ₂₋₁₂	16	1	0	104	0.0	1.000	
S ₂₋₁₃	12	1	0	106	0.0	1.000	
S ₃₋₅	39	1	0	124	0.0	1.000	
S ₃₋₁₁	13	0	1	107	0.0	1.000	
S ₄₋₄	26	1	0	124	0.5	0.480	
S ₄₋₈	19	0	1	101	0.0	1.000	

Table S5. The results of McNemar's test showing the significance of the difference given by fuzzy sets and random sets ($\alpha = 0.05$) by using *Pleiades* (see notations in Table S1 for the name of subsets).

*f*11 : number of cases with incorrect classification using both methods;

*f*22 : number of cases with correct classification using both methods;

f12: number of cases that are incorrectly classified by fuzzy sets but correctly classified by random sets; and

*f*21 : number of cases that are correctly classified by fuzzy sets but incorrectly classified by random sets.

	McNemar Test						
Subset	f11	<i>f</i> 12	f21	f22	Chi-squared (X ²)	p – value	
<i>S</i> ₁₋₁	11	0	2	162	0.5	0.480	
<i>S</i> ₁₋₂	9	0	1	133	0.0	1.000	
S ₁₋₆	4	6	4	138	0.1	0.752	
<i>S</i> ₁₋₇	12	0	1	134	0.0	1.000	
<i>S</i> ₁₋₉	5	7	3	98	0.9	0.343	
<i>S</i> ₁₋₁₀	7	1	0	113	0.0	1.000	
<i>S</i> ₂₋₃	8	2	2	140	0.0	1.000	
<i>S</i> ₂₋₁₂	8	0	1	112	0.0	1.000	
S ₂₋₁₃	8	0	3	108	1.3	0.248	
S ₃₋₅	6	2	1	155	0.0	1.000	
<i>S</i> ₃₋₁₁	5	0	1	115	0.0	1.000	
<i>S</i> ₄₋₄	9	0	1	141	0.0	1.000	
S ₄₋₈	11	1	0	109	0.0	1.000	

Table S6. The results of McNemar's test showing the significance of the difference given by fuzzy sets and random sets ($\alpha = 0.05$) using *Pleiades* + *DTM* data (see notations in Table S5 for *f11*, *f12*, *f21* and *f22*, and notations in Table S1 for the name of subsets).