



# Article Electronic Structure and Transport Properties of Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub> Single Crystals

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**Abstract:** The electrical resistivity and the Hall effect of topological insulator  $Bi_2Te_3$  and  $Bi_2Se_3$  single crystals were studied in the temperature range from 4.2 to 300 K and in magnetic fields up to 10 T. Theoretical calculations of the electronic structure of these compounds were carried out in density functional approach, taking into account spin–orbit coupling and crystal structure data for temperatures of 5, 50 and 300 K. A clear correlation was found between the density of electronic states at the Fermi level and the current carrier concentration. In the case of  $Bi_2Te_3$ , the density of states at the Fermi level and the current carrier concentration increase with increasing temperature, from 0.296 states  $eV^{-1}$  cell<sup>-1</sup> (5 K) to 0.307 states  $eV^{-1}$  cell<sup>-1</sup> (300 K) and from  $0.9 \times 10^{19}$  cm<sup>-3</sup> (5 K) to 2.6  $\times 10^{19}$  cm<sup>-3</sup> (300 K), respectively. On the contrary, in the case of  $Bi_2Se_3$ , the density of states decreases with increasing temperature, from 0.201 states  $eV^{-1}$  cell<sup>-1</sup> (5 K) to 0.198 states  $eV^{-1}$  cell<sup>-1</sup> (300 K), and, as a consequence, the charge carrier concentration also decreases from 2.94  $\times 10^{19}$  cm<sup>-3</sup> (300 K).

**Keywords:** 2D materials; topological insulator; Bi<sub>2</sub>Te<sub>3</sub>; Bi<sub>2</sub>Se<sub>3</sub>; electronic structure; DFT; materials informatics; topological resistivity; Hall effect; current carrier concentration

# 1. Introduction

The quantum Hall effect, in which the Hall conductivity of a two-dimensional insulator in a high magnetic field is quantized, is one of the important discoveries in condensed matter physics [1]. Special conducting edge states appear in the material in the quantum Hall effect regime. This effect is shown to have a topological nature, and such edge states can be associated with a topological invariant called the Chern number [2,3]. A nonzero Chern number determines the presence of conducting edge states, and a zero Chern number means an insulating state in the bulk, which is observed in the quantum Hall effect. Thus, topological materials can be considered as a special state of matter at the intersection of real materials and abstract mathematical topology. Such materials include topological insulators and topological semimetals. The quantum Hall effect can be considered the first two-dimensional topological insulator. Then, three-dimensional topological insulators were theoretically predicted [4,5] and experimentally discovered [6,7]. Recently, Dirac and Weyl topological semimetals were discovered [8–12].

A topological insulator is an insulator or semiconductor in bulk, whereas a special quantum state of electrons occurs on its surface, which makes charge carriers "topologically protected" from scattering. Such surface states are analogues of the edge states in the quantum Hall effect, and the spin–orbit coupling plays a role of the magnetic field. The metallic surface states of a topological insulator are called Dirac cones, which can be



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**Copyright:** © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). assigned a nonzero Chern number that determines the nontrivial topology of the band structure [5,8,9].

Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub> are typical representatives of the family of topological insulators [13,14]. With the help of external influences (magnetic field, temperature, pressure, etc.), one can fine-tune their electronic structure and, consequently, purposefully change their physical properties. This, in turn, can be used in various devices. Due to their special surface states, Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub> have great application potential and are successfully used in spintronic [15–18] and thermoelectronic [19–23] devices, biological and chemical sensors [24–26], and photonic and optoelectric applications [27,28]. Therefore, obtaining new information about the features of the electronic structure and electronic transport in such topological materials is of great interest and is relevant from both fundamental and applied points of view.

Despite the qualitatively similar electronic structure of Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub>, there are differences the band gap, the energy position of Dirac points on the surface band spectrum, and the strength of spin–orbit coupling. Taking these into account leads to a decrease/increase in the band gap in Bi<sub>2</sub>Te<sub>3</sub>/Bi<sub>2</sub>Se<sub>3</sub>, respectively; see, for example, [13,16,29–31]. All this inevitably manifests itself in electronic properties.

The density of electronic states at the Fermi level  $N(E_F)$  is one of the most important characteristics, and is closely related to many electronic characteristics, particularly the current carrier concentration n. In [32,33], the Hall effect was experimentally studied in Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub>, and it was shown that the current carrier concentration n varies with temperature in different ways: it increases with temperature in the case of Bi<sub>2</sub>Te<sub>3</sub> [32] and almost does not depend on temperature for Bi<sub>2</sub>Se<sub>3</sub> [33]. One possible reason for this difference in the behavior of n(T) may be the different behavior of  $N(E_F)$  with temperature. This formed the basis of this work.

The aim of this work is to establish a relationship between the density of electronic states  $N(E_F)$  and the current carrier concentration n in Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub> topological insulators. The density of electronic states and band structure were determined in the theoretical calculations using the density functional approach, considering spin–orbit coupling, and the charge carrier concentration was determined from experimental studies of the Hall effect in the temperature range from 4.2 K to 300 K in a magnetic field of 10 T.

## 2. Materials and Methods

Topological insulator  $Bi_2Te_3$  and  $Bi_2Se_3$  single crystals were grown by the Bridgman-Stockbarger method. The Bi, Te, or Se components were taken in the required proportion, that is, 2:3; then these components were ground, mixed, and placed in a quartz ampoule with an elongated sharp tip. The ampoule was evacuated to a residual pressure of  $\sim 10^{-4}$  atm and placed in a furnace with a large temperature gradient of about 50 degrees/cm. Then, the ampoule was heated to a temperature of about 750 °C until the initial components were completely melted. The ampoule was kept for 2 h, and then it descended slowly, at a rate of  $\sim 2-5$  mm/h, into the cold zone of the furnace. The single crystals grown during this process had a cylindrical shape with a sharp tip and dimensions of  $\sim 5-10$  mm in diameter and  $\sim 10-20$  mm in length. The crystal structure and chemical composition of the grown single crystals were studied by X-ray diffraction analysis and scanning electron microscopy at the Collaborative Access Center "Testing Center of Nanotechnology and Advanced Materials" of M.N. Mikheev Institute of Metal Physics of the Ural Branch of the Russian Academy of Sciences (IMP UB RAS).

The theoretical calculations of the electronic and band structures of Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub> were carried out in the Quantum ESPRESSO set of computer programs [34,35]. The experimental crystal structure data were taken from the calculations for bulk unit cells of Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub>. Generalized gradient approximation within the Perdew–Burke–Ernzerhof form, usually abbreviated as PBE, for the exchange–correlation functional [36], was used for the electronic structure calculations. Spin–orbit coupling was taken into account in all calculations to provide correct band structure and band gap values, employing full

relativistic ultrasoft pseudopotentials as set in the standard Quantum Espresso library of pseudopotentials [37]. A kinetic energy cutoff of 70 Ry was taken for wavefunctions, and 700 Ry for charge density and potential. A grid of  $12 \times 12 \times 12 k$ -points was used in the first Brillouin zone for integration using the tetrahedron method. All ions were found to have no magnetic moments in the calculations.

Figure 1 shows the X-ray diffraction patterns of the Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub> single crystal. Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub> single crystals were found to have a rhombohedral structure (space group *R3m*). Figure A1a shows an image of the crystal structure of Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub>. They belong to a group of compounds that crystallize into a layered structure, the layers in which are perpendicular to the threefold symmetry axis. Using X-ray data, the lattice parameters of both single crystals were determined. The lattice parameters are *a* = 4.389 Å, *c* = 30.483 Å and *a* = 4.134 Å, *c* = 28.68 Å for Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub>, respectively (Table 1). The obtained parameters are in good agreement with the available literature data (see, for example, [38]).



**Figure 1.** A fragment of the diffraction pattern of the  $Bi_2Te_3$  (**a**) and  $Bi_2Se_3$  (**b**) ground single crystals. The red dashes are the positions of the Bragg peaks.

Compound	Crystal Structure	Lattice Parameters	Chemical Composition
Bi <sub>2</sub> Te <sub>3</sub>	Rhombohedral (space group <i>R3m</i> )	a = 4.389 Å c = 30.483 Å	Bi <sub>2.02</sub> Te <sub>2.98</sub>
Bi <sub>2</sub> Se <sub>3</sub>	Rhombohedral (space group <i>R</i> 3 <i>m</i> )	a = 4.134  Å c = 28.68  Å	Bi <sub>2.01</sub> Se <sub>2.99</sub>

Table 1. Type of crystal structure and lattice parameters of Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub>.

The chemical composition of the single crystals was studied using a Tescan Mira scanning electron microscope (SEM) equipped with Oxford Instruments (Tescan Brno s.r.o., Czech Republic) INCA x-act EDS spectroscope and electron backscatter diffraction. According to the studies, the real chemical composition of the single crystals is in good agreement with the nominal one (Table 1). Figure A2 shows SEM images of the surface microstructure of Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub>, which indicate the high quality of the grown crystals and are comparable with the data presented in [39,40].

The electrical resistivity and the Hall effect were measured by the four-, and fivecontact method (see, for example, [41,42]) in magnetic fields up to 10 T in the temperature range from 4.2 to 300 K using an Oxford Instruments system at the Collaborative Access Center of IMP UB RAS.

### 3. Results and Discussion

## 3.1. Band and Electronic Structures

The electronic and band structures of  $Bi_2Te_3$  and  $Bi_2Se_3$  were calculated theoretically in DFT-GGA approach, taking into account spin–orbit coupling which is essential to obtain the insulating band and electronic structure.

The insulating state in both compounds is a result of the band inversion near highsymmetry point  $\Gamma$ , which implies the presence of the surface states at the Fermi energy. One can also notice another topological feature in the band structure of Bi<sub>2</sub>Te<sub>3</sub>, Figure 2a, which is a point of band degeneration just below the Fermi level right at the high-symmetry point  $\Gamma$  with surrounding linear dispersion. In the band structure of Bi<sub>2</sub>Te<sub>3</sub> the bandgap was calculated as 0.48 eV; see Figure 2a. For the second compound, Bi<sub>2</sub>Se<sub>3</sub>, the bandgap in the band structure was obtained as 0.41 eV; see Figure 2b. The energy gap values and insulator state are in agreement with the previous calculations [38].



**Figure 2.** Band structure of  $Bi_2Te_3$  (**a**) and  $Bi_2Se_3$  (**b**). The Fermi energy is shown at zero as a horizontal dashed line.

From Figure 2, one can see that both Bi compounds are calculated as insulators in the band structure. However, for the plotted electronic structure shown in Figure 3, the bandgap is reproduced as a pseudogap due to the smearing procedure of the density of states (DOS) plot. The main contributions to DOS near the Fermi energy are caused by the p Bi and p Te/Se electronic states (Figure 3b,c,e,f) with the other electronic states being less represented in this energy range. For Bi<sub>2</sub>Se<sub>3</sub>, the total density of states at the Fermi energy, which is located at zero energy, was found to be equal to 0.198 states  $eV^{-1}$  cell<sup>-1</sup>. For Bi<sub>2</sub>Te<sub>3</sub>, the total density of states at the Fermi energy, was found to be equal to 0.307 states  $eV^{-1}$  cell<sup>-1</sup>. One can notice that the bandwidth of the electronic states in Bi<sub>2</sub>Te<sub>3</sub> is wider, and peaks are more intense than those in Bi<sub>2</sub>Se<sub>3</sub>. Similar calculations were made for the crystal structure data for low temperatures (5 and 50 K) from [38]; the results are very similar to those plotted in Figures 2 and 3, for this reason are not shown, however, see Figure A3 for the DOS near the Fermi level  $N(E_F)$  at temperatures of 5,



50 and 300 K deviate, with small differences. Below, we analyze these results in comparison with the experimental data.

**Figure 3.** Electronic structure of  $Bi_2Te_3$  (**a**–**c**) and  $Bi_2Se_3$  (**d**–**f**). The Fermi energy is shown at zero as a vertical dashed line.

#### 3.2. Electronic Transport Properties

Figure 4 shows the temperature dependences of the electrical resistivity  $\rho(T)$  of Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub> single crystals. The dependence  $\rho(T)$  is shown to have a metallic character for both samples. The residual resistivity  $\rho_0$  is  $3.8 \times 10^{-5} \Omega \cdot \text{cm}$  and  $5.2 \times 10^{-5} \Omega \cdot \text{cm}$  for Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub>, respectively. Note that the residual resistivity ratio (RRR) of the Bi<sub>2</sub>Te<sub>3</sub> single crystal ( $\rho_{300 \text{ K}}/\rho_{4.2 \text{ K}} = 26$ ) exceeds the RRR of Bi<sub>2</sub>Se<sub>3</sub> ( $\rho_{300 \text{ K}}/\rho_{4.2 \text{ K}} = 5.4$ ), which indicates a higher "electrical purity" of the Bi<sub>2</sub>Te<sub>3</sub> single crystal.



Figure 4. Temperature dependences of electrical resistivity of Bi<sub>2</sub>Te<sub>3</sub> (a) and Bi<sub>2</sub>Se<sub>3</sub> (b).

Figures 5 and 6 show the temperature dependences of the Hall coefficient  $R_{\rm H}$  and the current carrier concentration n of the Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub> single crystals in a magnetic field B = 10 T, obtained from data on the Hall resistivity  $\rho_{xy}$  in the framework of a single-band model using the following equations:

$$R_H = \frac{\rho_{xy}}{B},\tag{1}$$

$$n = \frac{1}{e \cdot R_H} \tag{2}$$

where *e* is the electron charge. Since the Hall coefficient is negative for  $Bi_2Te_3$  and  $Bi_2Se_3$  (Figure 5), the majority charge carriers are electrons. For  $Bi_2Se_3$ , one can note a slight change in the value of the current carrier concentration with temperature (Figure 6), which is consistent with previous studies [33].



Figure 5. Temperature dependences of the Hall coefficient of Bi<sub>2</sub>Te<sub>3</sub> (a) and Bi<sub>2</sub>Se<sub>3</sub> (b).



Figure 6. Temperature dependences of current carrier concentration in Bi<sub>2</sub>Te<sub>3</sub> (a) and Bi<sub>2</sub>Se<sub>3</sub> (b).

Using the data obtained for the electrical resistivity  $\rho$  and the Hall coefficient  $R_{\rm H}$ , the charge carrier mobilities were determined as  $\mu = R_{\rm H}/\rho$  for Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub> (Figure 7). The mobility is seen to decrease with increasing temperature for both Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub>, which is associated with an increase in the efficiency of current carrier scattering. The mobility is  $18.9 \times 10^3 \text{ cm}^2/(\text{V} \cdot \text{s})$  and  $4.1 \times 10^3 \text{ cm}^2/(\text{V} \cdot \text{s})$  at T = 4.2 K for Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub>, respectively. The higher value of  $\mu$  for Bi<sub>2</sub>Te<sub>3</sub> at low temperatures is due to the higher RRR for this single crystal compared to Bi<sub>2</sub>Se<sub>3</sub>.

A comparison of the electronic transport characteristics of bulk  $Bi_2Te_3$  and  $Bi_2Se_3$  single crystals obtained in this study with previously reported data for bulk crystals and thin films of  $Bi_2Te_3$  and  $Bi_2Se_3$  grown by other methods is given in Table 2.



Figure 7. Temperature dependences of the mobility of  $Bi_2Te_3$  (a) and  $Bi_2Se_3$  (b).

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Bulk Bis Teo Bulk	Growth Method	RRR	$ ho_0, \Omega \cdot cm$	<i>n</i> , cm <sup>-3</sup> *	$\mu$ , cm <sup>2</sup> /(V·s) *	Reference
Bia Tea Bulk	Bridgman–Stockbarger method	26	$3.8 imes10^{-5}$	$8.70 imes10^{18}$	$18.9 imes10^3$	This study
	Spark plasma sintering	-	-	$\sim 2.2  imes 10^{19}$	$\sim 10^{3}$	[32]
Bulk Film	Self-flux method	-	$\sim 0.1  imes 10^{-3}$	-	-	[43]
	Metal organic chemical vapor deposition	~2.55	$\sim 1.35 \times 10^{-3}$	$\sim 6 \times 10^{18}$	~800	[44]
Bulk Bulk Bi <sub>2</sub> Se <sub>3</sub> Bulk Bulk Film	Bridgman-Stockbarger method	5.4	$5.2 \times 10^{-5}$	$2.94 imes10^{19}$	$4.1  imes 10^3$	This study
	Heating stoichiometric mixtures of pure elements	-	-	~2 × 10 <sup>19</sup> at 100 K	~10 <sup>3</sup> at 100 K	[33]
	Self-flux method	-	$\sim 0.1  imes 10^{-3}$	-	-	[40]
	-	~2	$\sim 0.22 \times 10^{-3}$	$\sim \!\! 4.5  imes 10^{19}$	~680	[45]
	Vapor phase epitaxy	~2.17	$0.608  imes 10^{-3}$	$\sim \! 1.07 \times 10^{19}$	954	[46]
Nanoplate	Vapor-liquid-solid mechanism	-	-	$5.2 imes10^{18}$	$8.8 imes10^3$	[47]

\* Data are given at T = 4.2 K (2 K), unless otherwise indicated.

#### 3.3. Current Carrier Concentration Analysis

Figure 8 shows the calculated values of the density of states at the Fermi level  $N(E_F)$  at temperatures of 5 K, 50 K, and 300 K, as well as the charge carrier concentrations n determined from the experimental data at the same temperatures. As can be seen from Figure 8, there is a good correlation between the behavior of  $N(E_F)$  and n with temperature for both Bi<sub>2</sub>Te<sub>3</sub> and Bi<sub>2</sub>Se<sub>3</sub>. In the case of Bi<sub>2</sub>Te<sub>3</sub>,  $N(E_F)$  and n increase with temperature, whereas in the case of Bi<sub>2</sub>Se<sub>3</sub>,  $N(E_F)$  and n decrease with increasing temperature.



**Figure 8.** Density of states at the Fermi level  $N(E_F)$  and current carrier concentration *n* of Bi<sub>2</sub>Te<sub>3</sub> (**a**) and Bi<sub>2</sub>Se<sub>3</sub> (**b**) determined at temperatures of 5 K, 50 K and 300 K. Filled circles represent the carrier concentration, open circles represent the density of states at the Fermi level.

## 4. Conclusions

The concentrations and mobility of current carriers in topological insulator  $Bi_2Te_3$ and  $Bi_2Se_3$  single crystals are estimated using a single-band model -. The calculations of the band and electronic structures of  $Bi_2Te_3$  and  $Bi_2Se_3$  made using the density functional approach confirmed the bandgap in both compounds. It is shown that the charge carrier concentration in  $Bi_2Te_3$  increases with increasing temperature, whereas the charge carrier concentration in  $Bi_2Se_3$ , on the contrary, slightly decreases with temperature, which is consistent with the previously reported experimental results. A good correlation has been established between the behavior of the values calculated for the density of states at the Fermi level and the charge carrier concentration determined on the basis of experimental data with temperature.

**Author Contributions:** Conceptualization, V.V.M. and A.V.L.; methodology, V.V.M., A.V.L. and S.V.N.; software, S.T.B. and A.V.L.; validation, V.V.M. and A.V.L.; formal analysis, V.V.M. and A.V.L.; investigation, V.V.M., A.V.L., S.T.B., E.B.M., A.N.P. and B.M.F.; resources, V.V.M., A.V.L., S.V.N. and E.B.M.; data curation, V.V.M. and A.V.L.; writing—original draft preparation, V.V.M., A.V.L., S.V.N., A.N.P. and B.M.F.; writing—review and editing, V.V.M. and A.V.L.; visualization, V.V.M., A.V.L., S.T.B., S.T.B., A.N.P. and B.M.F.; supervision, V.V.M. and A.V.L.; project administration, A.V.L.; funding acquisition, A.V.L. All authors have read and agreed to the published version of the manuscript.

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**Data Availability Statement:** The data presented in this study are available on request from the corresponding author.

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Appendix A



**Figure A1.** Crystal structure of  $Bi_2Te_3/Bi_2Se_3$  (**a**) plotted in Vesta [48] with the red balls represent bismuth atoms and the blue balls represent tellurium/selenium atoms. The Brillouin zone (**b**) with high symmetry points is shown as per [49].



Figure A2. SEM images of the surface microstructure of Bi<sub>2</sub>Te<sub>3</sub> (a) and Bi<sub>2</sub>Se<sub>3</sub> (b).



**Figure A3.** Total density of states near the Fermi level of  $Bi_2Te_3$  (**a**) and  $Bi_2Se_3$  (**b**) calculated for crystal structure data of 5, 50 and 300 K. The Fermi energy is shown at zero as a vertical dashed line.

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