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Network Externalities and Downstream Collusion under Asymmetric Costs: A Note

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Abstract: This paper considers the collusive stability of downstream competition in a vertical market with network externalities and cost asymmetry. A dynamic collusion game is constructed, and backward induction is employed to solve the subgame perfect Nash equilibrium. We show that larger network externalities lead to less collusive incentive for an inefficient firm, while for an efficient firm, this depends on the efficiency gap. An increase in network externalities will destabilize the downstream collusion when the cost asymmetry is large and network externalities are relatively weak.

Keywords: collusion; cost asymmetry; network externalities

JEL Classification: D43; L13; M21

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Citation: Lee, J.-Y.; Fan, C.-C.; Tsai, C.-S. Network Externalities and Downstream Collusion under Asymmetric Costs: A Note. *Games* **2023**, *14*, 29. https://doi.org/10.3390/g14020029

Academic Editors: Randall Calvert, Kjell Hausken and Ulrich Berger

Received: 31 December 2022
Revised: 14 February 2023
Accepted: 15 March 2023
Published: 30 March 2023

1. Introduction

The stability of collusion over time has received significant attention in the literature. It is also widely discussed under different market structures. Friedman [1] first used supergame to discuss the stability of collusion, which is dependent on the discount factor of the player. Deneckere [2] pointed out the impact of product differentiation on collusion. Cost structures also play an important role when it comes to collusion (Collie [3]). In recent years, the effects of network externalities have received extensive attention; Song and Wang [4] first took network externalities into account in the framework of collusion stability with symmetric cost.

In today's industrial structure, vertical supply chains generally exist in the consumer electronics market, which is also characterized by network externalities. Collusion and competition among manufacturers of consumer electronics products is an important research topic. To fill this literature gap, in this paper, we investigate collusion stability in a vertical structure with downstream cost asymmetries in the presence of network externalities, extending the work of Song and Wang [4] and Toshimitsu [5]. They took product substitution into consideration and found that collusion becomes more sustainable for closer substitutes of products under relatively strong network externalities. Toshimitsu [5] demonstrated the conditions under which collusive behavior improves social welfare. In mostly related work, Choi and Lee [6] showed that if the network externality is strong (weak), the collusion of price (quantity) is more stable than quantity (price), which is different from the findings of Collie [3]. In this paper, we aim to take network externalities and cost asymmetry into account to analyze downstream collusive stability in a vertical market. We show that larger network externalities lead to less collusive incentive for an inefficient firm, while for an efficient firm, this depends on the efficiency gap. This is because the input price (cost) can be changed as efficiency changes compared to a one-tier market structure as in Pal and Scrimitore's work [7].

The reminder of the paper is as follows. A literature review is provided in Section 2. The basic model with a linear demand curve and network externalities is presented in Section 3 and solved in Section 4, with an extensive analysis of the results. Section 5 presents a discussion on robustness. Section 6 concludes the paper.

2. Literature Review

Deneckere [2] first showed that when goods are very close substitutes, more tacit collusion is supported under Bertrand duopoly than Cournot duopoly. Rothschild [8] discussed the possibility that firms optimally choose whether to be price or quantity setters in each period is considered, pointing out that asymmetric cartels do not emerge from any of these contributions at the subgame perfect equilibria.

Ross [9] used a supergame theoretic model of collusion to analyze the effects of different levels of product differentiation on cartel stability, and found that greater homogeneity can reduce cartel stability. Lambertini and Sasaki [10] derived the optimal punishments required to sustain collusion under Bertrand and Cournot duopoly with differentiated products. Collie [3] showed that collusion is more sustainable under Cournot duopoly than under Bertrand duopoly with quadratic costs for any degree of product substitutability.

In another literature trend, the issue of vertical collusion is explored, e.g., Nocke and White [11], Barbot [12], Normann [13], Bian et al. [14], Biancini and Ettinger [15], Dingwei et al. [16], Gilo and Yehezkel [17], and Wang and Wang [18]. In particular, Wang and Wang [18] investigated the collusive incentive for far-sighted manufacturers selling via managerial retailers. They showed that revenue delegation can impede upstream collusion in Bertrand competition. Furthermore, the hindering result of managerial delegation is robust if it allows manufacturers to consider partial collusion. Ying et al. [19] found that consumer-oriented CSR's effect on the stability of upstream collusion basically hinges on the downstream competition modes. In particular, for a given degree of CSR and product substitutability, upstream collusion is always less stable under downstream price competition. However, the previous studies did not take the issue of network externalities into consideration.

The effect of network externalities has a non-negligible impact on industrial economic decision making, e.g., see Katz and Shapiro [20–22], Chou and Shy [23], Economides [24], Hoernig [25], Bhattacharjee and Pal [26,27], Pal and Scrimitore [7], Song and Wang [4], and Nakamura [28]. To see how network externalities impact collusive stability, Ruhmer [29] analyzed price collusion between platforms in a two-sided market model, and pointed out that collusion becomes harder to sustain as indirect network externalities become stronger. Unlike the two-sided market model, Pal and Scrimitore [7] highlighted that collusion sustainability under homogenous Cournot game depends on the strength of network externalities in an infinitely repeated game with trigger strategy punishment. Song and Wang [4], in a differentiated oligopoly game, showed that collusion becomes more sustainable for closer substitutes of products under relatively strong network externalities. However, the mentioned papers did not consider the collusive stability of downstream competition in a vertical market with network externalities and cost asymmetry.

3. Basic Model

Consider a market where there is one upstream firm selling an input to two downstream firms for a wholesale price w. Assume that downstream duopoly produces homogenous final products with positive consumption network externalities. For simplicity's sake, the upstream firm's cost is assumed to be normalized to zero and there are no other costs except the input price for downstream firms. Wang and Wang [18], in a vertical structure with many manufacturing firms, investigated the collusive incentive for far-sighted manufacturers selling via managerial retailers. In contrast to the existing literature, they found that revenue delegation can impede upstream collusion in Bertrand models.

Firm 1 produces one unit of products with λ unit of inputs ($0.5 < \lambda < 1$), while firm 2 produces one unit of products with one unit of inputs. The more the λ, the less the cost difference. This way of modeling a firm's cost asymmetry allows us to capture differences in firm capacity and its impact on production efficiency, i.e., firm 1 is more efficient than firm 2. The inverse demand function for product can be expressed as follows (see also Choi and Lee [30]):

$$p = a - q_1 - q_2 + n(y_1 + y_2) \tag{1}$$

where p denotes the final price charged for products, q_i $(i = 1, 2)$ denotes the quantities, and y_i denotes consumers' expectations regarding firm i's total sales, a is the market scale, and $n \in (0, 1)$ measures the network effects. To solve the equilibrium, we impose the "rational expectations" conditions as those set by Katz and Shapiro [20], i.e., $y_1 = q_1$, $y_2 = q_2$.

The profits of firms can be given by

$$\pi_U = \lambda w q_1 + w q_2 \tag{2}$$

$$\pi_1 = (p - \lambda w) q_1 \tag{3}$$

$$\pi_2 = (p - w) q_2 \tag{4}$$

where π_U is the profit for the upstream firm, and π_i $(i = 1, 2)$ is the profit for downstream firm i.

Consider that the firms engage in an infinitely repeated game. We examine the effect of cost asymmetry on the stability of the collusion in a vertical structure with downstream network externalities. Along the punishment path, assume that firms use Friedman's [1] grim trigger strategy.

A two-stage dynamic production game is used to explore the equilibrium. In every following period, the upstream firm decides the input price in the first stage, and each downstream firm simultaneously chooses the outputs in the second stage. We solve the subgame perfect Nash equilibrium (SPNE) through backward induction.

The game structure of the model is as follows (see Figure 1):

Stage 1: upstream firm decides the input price.

Stage 2: downstream firms decide to collude or deviate.

Stage 3: downstream firms simultaneously choose the outputs.

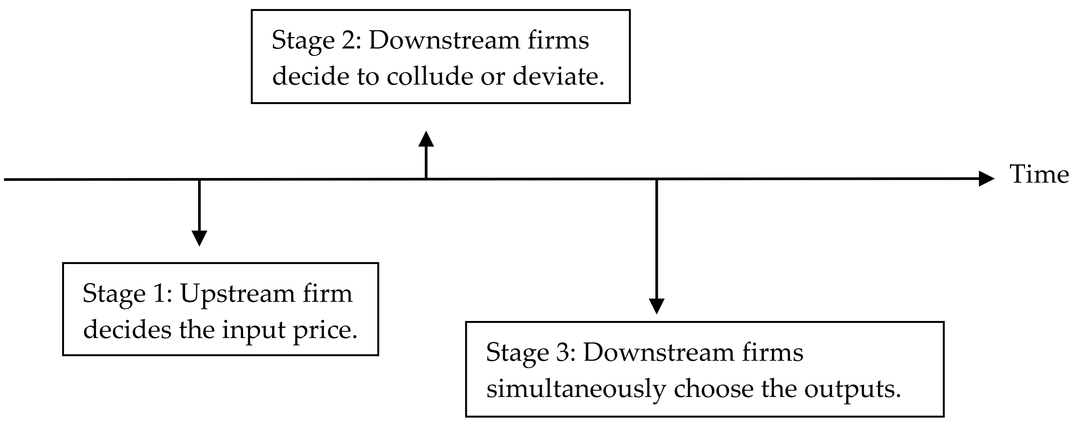

Figure 1. Flowchart of the game.

4. Market Equilibrium and Analyses

4.1. Non-Collusion

Firstly, we consider that each downstream firm chooses its output to maximize its own profit (Equations (3) and (4)) independently. According to the profit maximum problems, the first-order conditions are

$$\frac{\partial \pi_1}{\partial q_1} = a - w\lambda - 2q_1 - q_2 + n(y_1 + y_2) = 0 \tag{5}$$

$$\frac{\partial \pi_2}{\partial q_2} = a - w - q_1 - 2q_2 + n(y_1 + y_2) = 0 \tag{6}$$

Letting $q_1 = y_1$ and $q_2 = y_2$, from Equation (5) to Equation (6), we obtain the response functions of the output as follows:

$$q_1 = \frac{a + w - nw + (-2 + n)w\lambda}{3 - 2n} \tag{7}$$

$$q_2 = \frac{a + w - nw + (-2 + n)w\lambda}{3 - 2n} \tag{8}$$

In the second stage, the response functions of the firms are considered. Substituting the response functions of Equations (7) and (8) into Equation (2), and deciding the input price of the upstream firm, we obtain

$$\frac{\partial \pi_u}{\partial w} = \frac{a(1 + \lambda) + 2w(n(-1 + \lambda)^2 - 2(1 + (-1 + \lambda)\lambda))}{3 - 2n} = 0 \tag{9}$$

Thus, the optimal input price can be derivable as

$$w^N = \frac{(1 + \lambda)a}{2H} \tag{10}$$

where $H \equiv 2 - n(1 - \lambda)^2 - 2(1 - \lambda)\lambda$ and the superscript "N" denotes the non-collusion (competition) under vertical separation with homogenous goods.

$$q_1^N = \frac{1}{2}a\left(\frac{1 - \lambda}{H} - \frac{1}{2n - 3}\right)$$

$$q_2^N = \frac{a\left(2 - n - 5\lambda + 4n\lambda + (5 - 3n)\lambda^2\right)}{2(3 - 2n)H}$$

$$\pi_1^N = \frac{a^2(-5 + n(3 - \lambda)(1 - \lambda) + (5 - 2\lambda)\lambda)^2}{4(3 - 2n)^2 H^2}$$

$$\pi_2^N = \frac{a^2\left(n - 2 + 5\lambda - 4n\lambda - (5 - 3n)\lambda^2\right)^2}{4(3 - 2n)^2 H^2}$$

$$\pi_U^N = \frac{a^2(1 + \lambda)^2}{4(3 - 2n)H}$$

$$SW^N = \frac{a^2(7 - 4n(1 - \lambda)^2 - (10 - 7\lambda)\lambda)(17 + 4n^2(1 - \lambda)^2 - \lambda(14 - 17\lambda) - n(17 - (22 - 17\lambda)\lambda))}{8(3 - 2n)^2 H^2}$$

4.2. Collusion

Secondly, we discuss joint-off maximization. Of note, collusion exists only when the members produce goods with the same costs. In our paper, the strategy is providing the advanced production tech to the inefficient firm 2 to reach the collusion. The profits of firm 2 and upstream firm will then become $\pi_2' = (p - \lambda w)q_2$ and $\pi_U' = \lambda w(q_1 + q_2)$.

We assume a permanent tech transfer occurs, and then after a deviation, the game turns into a Cournot without asymmetry.

Simplifying the bargaining procedure from Verboven [31], we assume the share of the joint profit from firm 1 is α, which comes from the consideration of the ad valorem side payment due to exogenous bargaining power. Hence, it is reasonable to assume $\frac{1}{2} < \alpha < 1$ for the tech efficiency of firm 1. The joint payoff is $\pi_1 + \pi_2'$ and it is maximized when $q_1^C = q_2^C = \frac{a}{8-4n}$ and $w^C = \frac{a}{2\lambda}$. The resulting payoffs of firms are

$$\pi_1^C = \alpha \frac{a^2}{4(2-n)^2},$$

$$\pi_2^C = (1-\alpha)\frac{a^2}{4(2-n)^2},$$

$$\pi_U^C = \frac{a^2}{8-4n},$$

$$SW^C = \frac{a^2(7-3n)}{8(2-n)^2}$$

where the superscript "C" denotes collusion.

We obtain the collusion of the two firms if the profit with collusion for each firm is superior to the Cournot–Nash equilibrium profit. That is, $\pi_2^C > \pi_2^N$, if $\alpha < \hat{\alpha}$ for inefficient firm 2 and $\pi_1^C > \pi_1^N$, if $\alpha > \check{\alpha}$ for efficient firm 1, where $\hat{\alpha} = 1 - \frac{(2-n)^2\left(2-n-5\lambda+4n\lambda+(5-3n)\lambda^2\right)^2}{(3-2n)^2H^2}$ and $\check{\alpha} = \frac{(2-n)^2(5-n(3-\lambda)(1-\lambda)+(2\lambda-5)\lambda)^2}{(3-2n)^2H^2}$. By checking the sensibility of the incentive to the collusion with regard to the degree of network externalities, we obtain Proposition 1.

Proposition 1. *With the larger degree of network externalities, the collusion incentive for the inefficient firm is always smaller, while for the efficient firm, the incentive is smaller if the efficiency gap is large or small enough. If the efficiency gap is moderate, then there is an inverse U-shaped relationship between collusive incentive and network externalities.*

Proof. We have $\frac{\partial\hat{\alpha}}{\partial n} < 0$. If $0 < \lambda < \frac{1}{2}\left(1 - \sqrt{3\left(\sqrt{17}-4\right)}\right)$ or $\frac{1}{2}\left(1 + \sqrt{3\left(\sqrt{17}-4\right)}\right) < \lambda < 1$, then $\frac{\partial\check{\alpha}}{\partial n} > 0$. If $\frac{1}{2}\left(1 - \sqrt{3\left(\sqrt{17}-4\right)}\right) < \lambda < \frac{1}{2}\left(1 + \sqrt{3\left(\sqrt{17}-4\right)}\right)$, and $n^* \equiv \frac{2+\sqrt{2}\sqrt{(1-\lambda)\lambda(1+\lambda)^4}-2\lambda(9-\lambda(10-(3-\lambda)\lambda))}{(1-\lambda)(1-\lambda(11(3-\lambda)\lambda))}$, then (1) $\frac{\partial\check{\alpha}}{\partial n} < 0$, as $0 < n < n^*$. (2) $\frac{\partial\check{\alpha}}{\partial n} > 0$, as $1 > n > n^*$. □

This finding is consistent with Pal and Scrimitore [7], who found that in a network goods oligopoly, there is no incentive to collude unless the network externalities are sufficiently weak. It is well known that the stronger the degree of network externalities, the greater the gain from Cournot competition than that from collusion, as for any given degree of network externalities, there are higher outputs in Cournot competition, and the outward shift of the demand curve is greater than that under collusion.

Nevertheless, there is a crucial difference in the case where the (relative) efficiency is moderate as there is an inverse U-shaped relationship between collusive incentive of the efficient firm and the degree of network externalities. In this case, with moderate efficiency difference and smaller network externalities, there exists a possibility that the collusive incentive for the efficient firm is positively correlated with network externalities; namely, moderate λ indicates a higher input price (When $\lambda < \widetilde{\lambda} \equiv \frac{(n-2)+\sqrt{2}\sqrt{6-7n+2n^2}}{2-n}$, $\frac{\partial w^N}{\partial \lambda} > 0$;

when $\lambda > \widetilde{\lambda}$, $\frac{\partial w^N}{\partial \lambda} < 0$. Hence, when the cost asymmetry is moderate, the input price is higher.) and lower marginal profit due to network externalities, compared to the other two extreme cases.

Via partial differentiation to efficiency difference, we obtain the following Proposition 2.

Proposition 2. *If the efficiency difference between firms is smaller, the collusive incentive of the efficient firm is larger, and the collusive incentive of the inefficient firm is larger until the critical value is reached, and then the relationship will be reversed. Furthermore, the collusion likelihood is smaller.*

Proof. We have $\frac{\partial \check{\alpha}}{\partial \lambda} < 0$ and $\frac{\partial(\hat{\alpha}-\check{\alpha})}{\partial \lambda} > 0$, provided that $\hat{\alpha} > \check{\alpha}$. When $0 < \lambda < \frac{\sqrt{4-2n}+n-2}{n}$, $\frac{\partial \hat{\alpha}}{\partial \lambda} > 0$, and when $\lambda > \frac{\sqrt{4-2n}+n-2}{n}$, $\frac{\partial \hat{\alpha}}{\partial \lambda} < 0$. □

With smaller efficiency asymmetry, apart from underutilized network externalities due to undercut production, there is a lower implicit opportunity cost of free riding (lower market share) for firm 1 under collusion. That is, beating the rival is more important, in spite of the side payment under collusion. The overall effect of competition on the efficient firm will be positive.

For firm 2, the cost-saving effect dominates the comprehensive effect of underutilized network externalities and side payment when the cost asymmetry is large. Hence, in this case, cost asymmetry has a positive effect on the collusion, while the relationship is reversed when the asymmetry is small enough. Taking both firms into account, we find that when the efficiency gap is large, the weakening effect on firm 1 is greater than the temptation for firm 2 to deviate. However, when the efficiency gap is small enough, the likelihood of collusion increases. Our finding is consistent with Ganslandt et al. [32], and Miklós-Thal's [33] who states that collusion is sustainable under cost symmetry, while collusion may be sustainable under cost asymmetry; however, the difficulty for efficient collusion to sustain holds when costs are asymmetric. Miklós-Thal [33] built a model of price competition in a one-tier market structure.

4.3. Deviation and Tacit Collusion

Thirdly, we discuss deviations from tacit collusion. Given the collusive output of the rival, firm 2 (or 1) maximized its profit. If the deviant firm is firm 1, it is reasonable to assume that the deviated firm still holds the efficient technology for the deviant firm and would not let the competitor detect the deviation. If the deviant firm is firm 2, it makes sense that it still has the tech since deviated firm 1 is unaware of the betrayal, so the deviation case is actually the same for the two firms. The upstream firm still makes the same price in the collusion case for the unknown betrayal. The outputs are obtained as follows:

$$q_i^D = \frac{2a\lambda - a}{8\lambda - 4n\lambda}, q_j^D = \frac{a(1-n+2\lambda)}{4(n-2)^2\lambda} \tag{11}$$

where the superscript "*D*" denotes the deviation under vertical separation and the subscript i denotes the deviated firm and j denotes the deviant firm. The profits of firm i and j and social welfare are as listed below:

$$\pi_i^D = \frac{a^2(1-n+2\lambda)(1-2\lambda)}{16(n-2)^3\lambda^2}, \pi_j^D = \frac{a^2(1-n+2\lambda)^2}{16(n-2)^4\lambda^2}, \pi_U^D = \frac{a^2(2(3-n)\lambda-1)}{8(n-2)^2\lambda}, \tag{12}$$

$$SW^D = \frac{a^2(2(3-n)\lambda-1)(1+26\lambda-n(1+6(4-n)\lambda))}{32(2-n)^4\lambda^2} \tag{13}$$

Comparing the social welfare under the three regimes, we see that in the static analysis, $SW^N > SW^D > SW^C$, which is in line with the work of Ciarreta and Gutiérrez-Hita [34]. The latter inequality can be explained for the same shared efficiency tech (the total marginal cost of the industry), but firm 1 has greater production. The former inequality can be explained for the production extension effect dominating the social efficiency effect.

By letting δ denote the discount factor between periods, based on the above analysis, the tacit collusion is sustainable *if and only if* these two conditions are correct for the case of firm 1 or 2 deviation, respectively:

$$\frac{\pi_1^C}{1-\delta_1^*} \geq \pi_1^D + \frac{\delta_1^* \pi_1^N}{1-\delta_1^*}, \quad \frac{\pi_2^C}{1-\delta_2^*} \geq \pi_2^D + \frac{\delta_2^* \pi_2^N}{1-\delta_2^*} \tag{14}$$

Let δ_i^* be the δ satisfying Equation (14) with equality. Given the cost function proposed by Friedman [1], this critical discount factor for the cartel is the lowest one that maintains collusion between the two firms. This means that we should have

$$\delta^* = max\left\{\frac{\pi_1^D - \pi_1^C}{\pi_1^D - \pi_1^N}, \frac{\pi_2^D - \pi_2^C}{\pi_2^D - \pi_2^N}\right\} = max\{\delta_1^*, \delta_2^*\} = \delta_2^*$$

As in Rothschild [9], we can find that firm 2 has the stronger incentive to deviate from the collusion, which is in contrast with Brandão et al. [35]. The explanation is intuitive: with the more efficient production skill, firm 2 gains the most from deviation and loses the least from the punishment, while firm 1 holds on to its shared technology on the other hand.

Considering the infinitely repeated game, again, we make the comparison of social welfare in three cases. With simple calculation, we have $\frac{SW^C}{1-\delta^*} \geq SW^D + \frac{\delta^* SW^N}{1-\delta^*}$, as long as this collusion is sustainable. That is, collusion benefits social welfare, which is in contrast to the conventional literature.

We relax the assumption that $\frac{1}{2} < \alpha < 1$, and derive the profit ratio when the critical discount factor is minimized, $\delta_1^* = \delta_2^* = \delta^{**}(\alpha, a, \lambda, n)$,

$$\frac{\pi_1^D - \pi_1^C}{\pi_1^D - \pi_1^N} = \frac{\pi_2^D - \pi_2^C}{\pi_2^D - \pi_2^N} \tag{15}$$

Solving Equation (15), we have

$$\delta^{**} = \delta(a, \lambda, n)$$

(Due to the complexity of the function, only its implicit function is written here. If the calculation process is required, it can be obtained from the author.) Of note, whether the collusion strategy is via ad valorem or fixed fee (side payment), the discount factor is the same. Comparative static analyses for the influences of network externalities and profit ratio are shown as follows in Propositions 3 and 4.

Proposition 3. *Collusion sustainability is more stable as cost differences decrease.*

Proof. We have $\frac{\partial \delta^{**}}{\partial \lambda} > 0$. □

This result is in line with the finding of Miklós-Thal [33]. There is larger deviation motivation for the sunk cost (firm 1) and a smaller punishment effect for larger Cournot profit (firm 1) and inefficiency (firm 2). Hence, cheating becomes less favorable as λ increases.

Proposition 4. *An increase in network externalities will destabilize the collusion when they are small and will stabilize it when they are large.*

Proof. As shown in Figure 2, if $n > n^{**}$, $\frac{\partial \delta^{**}}{\partial n} > 0$, and if $n < n^{**}$, $\frac{\partial \delta^{**}}{\partial n} < 0$. □

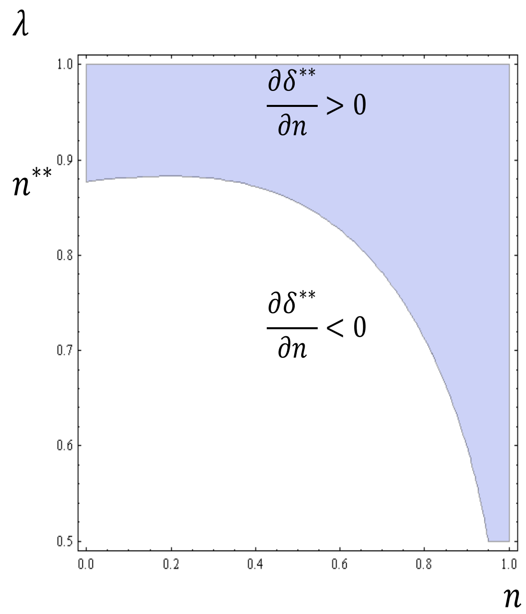

Figure 2. Cost asymmetry and network effects. (**: critical value of variable).

In Figure 2, if $n < n^{**}$ (the superscript "**" denotes the critical value of network externalities), we have $\frac{\partial \delta^{**}}{\partial n} < 0$, and so, when the network externalities are relatively weak, the critical discount factor decreases in the degree of network externalities. When the cost asymmetry is large and network externalities are relatively weak (strong), $n < (>) n^{**}$, collusion becomes less (more) difficult to sustain. There are two opposite effects. With larger network externalities, the gains from output expanding will be unilaterally larger and the deviating motivation is stronger. Additionally, the losses from the punishment will be smaller for a smaller input price (With simple calculation, we find $\frac{\partial w}{\partial n} > 0$.). For firm 2 especially, there is an amplified output reduction effect for the loss of efficient tech. As a result, the deviation effect is dominated by (dominates) the punishment effect for weak (strong) network externalities.

5. Robustness of Our Claims

5.1. Collusion without Technology Transfer

If there is no technology transfer, cost asymmetry always exists in the market. Firm 2′s profit will remain at $\pi_2 = (p - w)q_2$. The joint payoff is $\pi_1 + \pi_2$, and the first-order conditions are the following:

$$\frac{\partial(\pi_1+\pi_2)}{\partial q_1} = a - w\lambda - 2q_1 - 2q_2 + n(y_1 + y_2) = 0$$
$$\frac{\partial(\pi_1+\pi_2)}{\partial q_2} = a - w - 2q_1 - 2q_2 + n(y_1 + y_2) < 0$$

It is maximized when $q_1^{CN} = \frac{2a}{8-4n}$, $q_2^{CN} = 0$ and $w^{CN} = \frac{a}{2\lambda}$. The resulting payoffs of firms are

$$\pi_1^{CN} = \alpha\frac{a^2}{4(2-n)^2}, \pi_2^{CN} = (1-\alpha)\frac{a^2}{4(2-n)^2}, \pi_U^{CN} = \frac{a^2}{(8-4n)\lambda}. \quad (16)$$

where the superscript "*CN*" denotes collusion without technology transfer.

As in Section 4, we examine deviations from tacit collusion, and obtain outputs as follows:

$$q_i^{DN} = \frac{2a\lambda - a}{8\lambda - 4n\lambda}, q_j^{DN} = \frac{a(3-n)(2\lambda-1)}{4(n-2)^2\lambda} \quad (17)$$

$$\pi_i^{DN} = \frac{a^2(2\lambda-1)(n(7-2n(1-\lambda)-6\lambda)+2\lambda-5)}{16(n-2)^3\lambda^2},\ \pi_j^{DN} = \frac{a^2(n-3)^2(1-2\lambda)^2}{16(n-2)^4\lambda^2} \quad (18)$$

where the superscript "DN" denotes the deviation under vertical separation without technology transfer. This is similar to the solution process in Section 4. We have

$$\delta^{***} = \delta(a,\lambda,n)$$

As we pointed out in Section 4, there is larger deviation motivation for the sunk cost (firm 1), and a smaller punishment effect for larger Cournot profit (firm 1) and inefficiency (firm 2). Hence, cheating becomes less favorable as λ increases, and when the cost asymmetry is large and network externalities are relatively weak (strong), collusion becomes less (more) difficult to sustain.

5.2. Deviation of Both Firms and Tacit collusion

If both firms decide to deviate from the collusion, the upstream firm still makes the same price in the collusion case for the unknown betrayal. We obtain the outputs as follows:

$$q_i^{BD} = q_j^{BD} = \frac{a-w\lambda}{3-2n} \quad (19)$$

where the superscript "BD" denotes the deviation under the vertical separation of both the downstream firm and the subscript i and j denote the deviant firm. The profits of firm i and j and social welfare are listed below:

$$\pi_i^{BD} = \pi_j^{BD} = \frac{a^2}{4(3-2n)^2}, \pi_U^{BD} = \frac{a^2}{6-4n}, \quad (20)$$

$$SW^{BD} = \frac{a^2(5-3n)}{2(3-2n)^2} \quad (21)$$

Comparing the profit between one-sided deviation and two-sided deviation, we have

$$\pi_j^D - \pi_j^{BD} = \frac{a^2(n-1-2\lambda)^2}{16(n-2)^4\lambda^2} - \frac{a^2}{4(3-2n)^2} > 0, \text{ for all } n. \quad (22)$$

According to Equation (22), the profit from one-side deviation is larger than that from two-sided deviation. This indicates that two-sided deviation never happens, and our one-sided deviation results are thus robust.

6. Conclusions

This paper extends the tacit collusion literature by combining network externalities and cost asymmetry in a vertical structure. We demonstrate that larger network externalities lead to less collusive incentives for an inefficient firm while for an efficient firm, this depends on the efficiency gap for the changed input price (cost) as efficiency changes. The inefficient firm has more motivation to deviate from collusion if its profit share is small, since it has more to gain from deviation to capture advanced production tech and a lower input price. If the discount factor is minimized, the cost differences are enlarged, and the relationship between network externalities and collusive sustainability depends on the cost differences. Additionally, when the collusion is sustained, the social welfare in this case is also dominant.

Even when there is no technology transfer in the market, the cartel will choose low-cost production technology. Its profit will be the same as in the case where technology transfer is present under the collusion of firms. Due to the cost-saving effect, there will be a "win-win-win" solution among consumers, firms, and society. For the practical implications of this model, if the collusion of firms can effectively reduce production costs, then from the perspective of the antitrust authority, the government can still allow appropriate joint behavior of firms.

The limitation of this study is that there is only one upstream of intermediate goods and the homogenous final goods in Cournot competition. In future studies, the collusion of downstream firms could be explored in the case of two or more upstream firms, and Bertrand competition with differentiated products could be considered.

The advantages of this study lie in the use of a simple linear demand function that yields a clear analytical solution. However, the conclusion does not necessarily hold when the demand function is quadratically differentiable.

Author Contributions: Conceptualization, J.-Y.L., C.-C.F. and C.-S.T.; Methodology, J.-Y.L. and C.-S.T.; Software, C.-C.F.; Formal analysis, J.-Y.L. and C.-S.T.; Writing—original draft, J.-Y.L., C.-C.F. and C.-S.T.; Writing—review and editing, J.-Y.L. and C.-S.T. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

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