



Article Takeovers, Freezeouts, and Risk Arbitrage

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Abstract: This paper develops a dynamic model of tender offers in which there is trading on the target's shares during the takeover, and bidders can freeze out target shareholders (compulsorily acquire remaining shares not tendered at the bid price), features that prevail on almost all takeovers. We show that trading allows for the entry of arbitrageurs with large blocks of shares who can hold out a freezeout—a threat that forces the bidder to offer a high preemptive bid. There is also a positive relationship between the takeover premium and arbitrageurs' accumulation of shares before the takeover announcement, and the less liquid the target stock, the stronger this relationship is. Moreover, freezeouts eliminate the free-rider problem, but front-end loaded bids, such as two-tiered and partial offers, do not benefit bidders because arbitrageurs can undo any potential benefit and eliminate the coerciveness of these offers. Similarly, the takeover premium is also largely unrelated to the bidder's ability to dilute the target's shareholders after the acquisition, also due to potential arbitrage activity.

Keywords: takeovers; freezeouts; arbitrage; hold-out power; bargaining; dynamic game

JEL Classification: C78; D82; G34

1. Introduction

Tender offers in the U.S. and the U.K. are often immediately followed by a second-step freezeout merger in which the acquirer ends up with full ownership of the target.¹ In a freezeout merger, untendered shares are compulsorily acquired at the tender offer price, once the minimum fraction of shares required to approve a freezeout merger has been tendered. For example, during 1995–2023, there were 2160 tender offers for U.S. target firms, and 1953 (or 90.4%) of those were followed by a second-step freezeout merger.²

Freezeout tender offers allow bidders to overcome the Grossman and Hart (1980) [1] free-rider problem in takeovers by making an offer conditioned upon shareholders tendering the minimum fraction of shares required for a freezeout. Shareholders cannot free-ride because if the offer is successful, the bidder will automatically own enough shares to acquire the free-rider shares compulsorily.³

In a static setting, using take-it-or-leave-it offers conditioned on a freezeout, a bidder can extract all the surplus from target shareholders, which is undoubtedly contrary to the results of a vast empirical literature (e.g., Bradley et al. (1988) [2]). The shortcoming of the static framework is that, in practice, bidders cannot credibly commit to take-it-or-leave-it offers. This motivates us to study takeovers in a dynamic environment in which offers can be revised and extended over time, and there is trading in the target's shares during the takeover.

What is the outcome of a dynamic tender offer conditioned upon a freezeout when trading is allowed? We show that trading while the offer is open, allows arbitrageurs to accumulate blocks of shares, which gives them the power to hold out the takeover because the bidder is unable to obtain the necessary number of shares for a freezeout if blockholders do not tender their shares. Even though arbitrageurs and large shareholders are extremely interested in the success of a takeover, they can credibly delay their tendering decision until the bidder offers a takeover premium larger than a critical value commensurate with



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Copyright: © 2024 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). arbitrageurs' hold-out power. Shareholders' ability to trade, which allows arbitrageurs to increase their ownership of shares, provides a threat that forces the bidder to offer a high preemptive bid, even though there may be no de facto trading during the tender offer. A concentrated ownership structure allows the target shareholders to leverage their rights and increase their bargaining power vis á vis the bidder, forcing him to pay a high takeover premium despite the bidder's freezeout rights. Large shareholders, therefore, perform a different role in our model than they do in Shleifer and Vishny (1988) [3], where their presence is beneficial because they help solve the free-rider problem.

To be sure, when the offer is announced, there may be no arbitrageur or large shareholder owning target shares. How can arbitrageurs profit from buying blocks that will drive up the premium, given that shareholders might not sell their shares, free-riding on the potential benefits created by arbitrageurs? Noise traders, as in Kyle and Vila (1991) [4] and Kyle (1985) [5], can provide camouflage to enable arbitrageurs to profit by trading in the target shares. Similarly, as in Cornelli and Li (2002) [6], the knowledge of the arbitrageurs' presence gives them an endogenous informational advantage that can lead to trade with other shareholders and traders (however, in our study, the role of arbitrageurs is not to resolve the free-rider problem). We extend the analysis of Kyle and Vila (1991) [4] and Cornelli and Li (2002) [6] to a sequential trading model like Kyle's (1985) [5] in which several arbitrageurs trade blocks of shares during the tender offer.

The economics of tender offers is formalized as a bargaining game among the bidder and arbitrageurs, where the number of players in the bargaining is endogenous and may change due to trading. The bidder and arbitrageurs all have hold-out or veto power and can use this power to determine the offer price. The idea that blocking by arbitrageurs may be relevant for the pricing of takeovers was first developed by Bergstrom et al. (1993) [7], based on a cooperative Nash bargaining game. This paper extends the analysis of Bergstrom et al. to a dynamic setting in which there can be trading during the tender offer and develops several new results and empirical implications.

We start showing that the supply curve of shares is upward sloping. More specifically, the supply curve is the relation between the equilibrium bid price and the number of shares that the bidder needs to acquire in the offer. Intuitively, the supply curve is upward-sloping because the greater the number of shares demanded by the bidder, the larger the number of shareholders who can form hold-out blocks. Moreover, when there is a large number of arbitrageurs with hold-out power, they can credibly demand a larger share of the takeover gains in exchange for tendering their shares, which implies that the supply curve is upward-sloping (see also Stulz (1988) [8], Stulz et al.(1990) [9], and Burkart et al. (1998) [10].

Moreover, there is a positive relationship between the equilibrium takeover premium and arbitrageurs' accumulation of shares before the tender offer. Although arbitrageurs can enter after the announcement of the offer, their entry at this stage is uncertain and happens with a probability of less than one. Therefore, the more arbitrageurs are present at the announcement, the more hold-out power shareholders have to force a higher premium. Furthermore, this relationship is weaker the more liquid the stock is, because it is then more likely that new arbitrage blocks can be formed during the tender offer. Consistent with the implications of the model, Jindra and Walkling (2004) [11] and Jiang et al. (2018) [12] find that there exists a positive and significant relationship between arbitrage activity before the announcement of the offer (proxied by a measure of abnormal trading volume) and the takeover premium.

The model also predicts that there should be a positive relationship between arbitrage activity after the announcement of the offer and revisions in the bid measured by the ratio of the closing and opening bid.⁴ Larcker and Lys (1987) [14] provide evidence that in several transactions where arbitrageurs accumulated over five percent of the shares after the announcement of the offer, the takeover premium increased by more than 9 percent. This evidence is consistent with our interpretation that arbitrageurs use their power to hold out the transaction to force the bidder to raise the takeover premium.

Another contribution of this paper is to develop a comprehensive characterization of the structure of tender offers. Tender offers are composed of a mix of strategic elements that usually appear prominently on the front page of virtually every offer of purchase: the bid price, the maximum number of shares sought in the offer, the acceptance condition, as well as whether there will be a second-step freezeout merger or not. Can the bidder benefit from using front-end loaded bids, such as two-tier or partial offers? What if the bidder can dilute the target post-acquisition of control? The analysis yields a novel characterization of the tender offer structure with several surprising results consistent with existing empirical findings.

We show that front-end loaded bids, such as two-tiered offers and partial offers, do not provide any strategic benefits to bidders in addition to any-or-all offers. Arbitrageurs eliminate, in equilibrium, any element of coerciveness associated with an offer. For example, a coercive two-tiered offer with a low blended price allows arbitrageurs to accumulate large stakes in the open market profitably. As we have argued before, as long as arbitrageurs accumulate large blocks of shares, they can prevent the bidder from freezing out shareholders at a low back-end price, even though dispersed shareholders stampede to tender their shares. Besides, arbitrageurs can use their power to hold out the two-tiered offer to demand from the bidder a blended premium equal to the premium in any-or-all offers. Indeed, Comment and Jarrell (1987) [15] find that the average total premium is insignificantly different in executed two-tiered and any-or-all offers. This model also explains Jarrell and Poulsen's (1987) [16] finding that the adoption of fair-price charter amendments has an insignificant effect on a firm's stock price. Interestingly, Comment and Jarrell (1987) [15] provide indirect evidence that arbitrage activity is more intense during two-tiered than any-or-all offers, which is consistent with the more important role played by arbitrageurs during two-tiered offers.

Also, there is a novel relationship between the equilibrium takeover premium and the fraction of the target's post-acquisition value that the bidder can dilute in private benefits. According to Grossman and Hart (1980) [1], the equilibrium takeover bid equals the post-takeover value with dilution. However, this need not be the case if the bidder can freezeout shareholders. For example, if target shareholders enjoy a good level of protection against dilution by a controlling shareholder, the bidder can acquire all shares with an any-or-all offer conditioned on a second-step freezeout at a price lower than the post-acquisition stock price. Consequently, the bid price is determined not by the level of dilution but rather by the supermajority requirement for a freezeout merger.

More surprisingly, dilution does not determine the takeover premium, even when target shareholders enjoy a very weak level of protection. A bidder can only takeover the target with a bid price greater than the price at which a majority of shareholders are willing to tender. Since corporate charters may require fewer shares for the acquisition of control than for a freezeout merger, the takeover premium may be somewhat lower when the bidder can considerably dilute shareholders post-acquisition because of the somewhat reduced hold-out power of shareholders—the upward-sloping supply curve relation. Interestingly, though, whenever the charter specifies a a similar fraction of shares for a freezeout merger and acquisition of control, which is, for example, common for firms incorporated in Delaware, then the takeover premium should not depend at all on the level of dilution. Therefore, the model provides a novel relationship between the takeover models with dilution in the literature, such as Grossman and Hart (1980) [1] and Burkart et al. (1998) [10].

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 solves for the equilibrium of the dynamic tender offer game with trading and analyzes the role of arbitrageurs and large shareholders in takeovers. Section 4 characterizes the structure of tender offers. Section 5 discusses the empirical implications of the model, and the conclusion follows. The Appendix A contains the proofs of propositions.

2. The Tender Offer Model

We first describe the takeover laws that lay out the rules of the game and motivate the dynamic tender offer game with trading that is proposed next.

2.1. Takeover Laws: The Rules of the Game

In the U.S., takeovers are regulated by the Williams Act, enacted into federal law in July 1968, and also regulated by corporate laws that are under state jurisdiction. The purpose of the Williams Act is to provide target shareholders with full and fair disclosure of information and sufficient time to evaluate and act upon the information. In the U.K., the Takeover Panel, created in March 1968, is the regulatory body that administers the City Code on Takeovers and Mergers. Similar to the Williams Act, the Code was designed to ensure good business standards and fairness to shareholders.⁵

We motivate the theoretical model of tender offers using the rules prevailing in the U.S. and the U.K., although the model is applicable to takeovers in any country as long as the same basic rules apply. The basic rules used in our theoretical model are the following: rules that ensure disclosure of information,⁶ specify the minimum duration for tender offers,⁷ ensure equal treatment of shareholders,⁸ and entitles shareholders to withdrawal and proration rights.⁹

Besides the tender offer rules, the merger provisions are the other key component to understanding takeovers.¹⁰ The merger provisions are relevant for takeovers because usually tender offers are immediately followed by a second-step freezeout merger once the bidder has succeeded in acquiring enough shares to gain board approval and the shareholder votes necessary to approve the merger. In the second-step merger, the target is merged into a corporation controlled by the acquirer (usually a wholly-owned subsidiary or a shell company created for the acquisition). Shareholders who did not tender their shares either accept the freezeout merger price or, if they disagree with the merger, request appraisal rights and receive a value appraised by the courts for their shares.¹¹ We will see that the ability to conduct a second-step freezeout merger is a powerful mechanism for discouraging free-riding and influences the price paid during the tender offer and the response of shareholders to the tender offer.

We will refer to the percentage of votes required to approve the second step merger as the freezeout parameter f throughout the paper. In the U.K. and several other European countries, such as Sweden, the fraction required for a freezeout merger is equal to 90 percent. In the U.S., however, the fraction of shares required for a freezeout varies significantly among states and has undergone several changes. Before 1962, the great majority of states in the U.S. had a 2/3 supermajority requirement. The Model Business Corporate Act of 1962 reduced the percentage required to a simple majority. In 1967, Delaware adopted the simple majority provision, and other major states, such as California, Michigan, and New Jersey, followed suit. However, several large states, such as New York, Ohio, and Massachusetts, still maintain the 2/3 supermajority requirement. Notice also, that several firms incorporated in states with a simple majority requirement, such as Delaware, have amended their charters, adopting supermajority merger provisions. Indeed, 267, or 18 percent, of the 1500 large capitalization companies profiled by the Investor Responsibility Research Center in 1995, had adopted supermajority merger requirements (ranging from 2/3 to 80 percent of the shares).

The other important parameter for our model is the minimum fraction of shares the bidder needs to obtain in the tender offer to gain control of the target—the control acquisition parameter k. In the U.K., all bids must be conditional upon the bidder's acquiring, pursuant to the offer, over 50 percent of the voting share capital.¹² Although in the U.S., there is no such rule, all offers considered in the paper will be conditional upon the bidder acquiring at least 50 percent of the shares. Unconditional offers, though, are not allowed in our framework.¹³ However, the bidder may well choose to condition the tender offers upon a higher fraction than a simple majority being tendered. We allow for

any-or-all, two-tiered, and partial offers, but unless otherwise explicitly noted, we will be considering any-or-all offers.

2.2. The Model

The firm has one infinitely divisible share with a value normalized to v = 0 under the incumbent manager's control. There is one bidder that can increase the value of the target to v = \$1 per share upon gaining control of the target, and the bidder does not have any stake in the target. There are no other competing bidders, or all other competitors can only improve the value of the target significantly less than the natural bidder. All the model's relevant information is common knowledge to all market participants.

The takeover may be either friendly or hostile. Let $\alpha \in [0, 1)$ denote the number of shares that are not going to be tendered into the offer. In a hostile takeover, the shares owned or controlled by the target's insiders are not tendered into the offer, and the board of directors does not recommend the offer. In contrast, the opposite holds in a friendly takeover. Thus, we expect α to be bigger in hostile takeovers. In our model, the only difference between a hostile and friendly takeover is that α is bigger in the former than in the latter.¹⁴

Even in a friendly transaction, some shares are not tendered because they are owned by shareholders who abstain and are not informed of the offer, shareholders that have unreasonable beliefs about share valuations or shareholders who are in a high tax bracket and do not want to realize capital gains. In a hostile acquisition, shares owned by insiders, shares owned by employee stock ownership plans (ESOPs) controlled by insiders, as well as shares owned by shareholders who follow the board's recommendation, are also assumed not tendered into the offer.

Consider that the bidder makes an offer conditioned upon f shares being tendered. A large shareholder, either an individual or a group acting in concert or cooperatively, owning at least a stake of size

$$\beta = 1 - \alpha - f \tag{1}$$

is able to hold out or veto the success of an offer conditioned on f shares by not tendering his shares, even if all the other shareholders decide to tender (excluding the α shares that, by definition, are not tendered). Large shareholders with blocks of size β will be called arbitrageurs, and such blocks can be formed as a result of trading activity either before the tender offer is announced, perhaps based on insider information or during the tender offer, as described next. Jiang et al. (2018) [12] point out that often arbitrageurs acquire a block of shares in announced takeovers. They attempt to block the deal under its current terms and try to persuade other shareholders to do the same unless the acquirer improves the takeover premium.

The tender offer game is an infinite horizon repeated game with three stages at each period. Every period starts with the offering stage, in which an offer is proposed. Shareholders then play the tendering game, in which they choose to tender, simultaneously, a fraction of their shares. The takeover succeeds if the fraction of shares tendered is $T \ge f$ (because the offer was conditioned upon f shares being tendered). If the takeover does not succeed, then there is a delay of Δt units of time, and during this period, a trading session takes place. The same three-stage game is repeated until the takeover succeeds.¹⁵

The extensive form of the game is described next.

Offering stage: Say that at the beginning of the offering stage, there are *n* arbitrageurs (or large shareholders) with a stake of at least $\beta = 1 - f - \alpha$ shares. Each arbitrageur has the power to hold out the takeover if any one of them refuses to tender his block of shares, given that an exogenous fraction of shares α is not tendered. At the beginning of the offering stage, the bidder and the *n* arbitrageurs with veto power are chosen at random with equal probability $\frac{1}{n+1}$ to propose an offer (they are chosen with equal probability because each one of them has equal bargaining power).¹⁶ Let the bidder's offer be $p = p_B$ and the arbitrageurs' offer be $p = p_A$.¹⁷ After the bidder proposes an offer, shareholders play the tendering game, in which they decide whether to accept or reject the bidder's offer.

If shareholders accept the offer in the tendering game, the takeover succeeds, and if they reject, there is a delay of Δt , and during this period, there is a trading session. Similarly, after an arbitrageur proposes an offer, the bidder either accepts or rejects his offer. If the bidder accepts the offer, then this becomes the new offer and the game proceeds with the tendering game, and if the bidder rejects the offer, there is a delay of Δt in which trading takes place.

Tendering game: Shareholders decide simultaneously how many shares they are going to tender. Say that a total of *T* shares has been tendered. The takeover is successful if $T \ge f$. The bidder then obtains control of the target, takes up the tendered shares at the offer price, and has the option to freeze out shareholders who did not tender at the offer price. If the takeover is successful, the game is over, and the players obtain the payoffs specified below. If the takeover is unsuccessful in the current period, T < f, then there is a delay of Δt , after which shareholders play the trading game.

Trading game: Three kinds of traders participate in any trading session: noise traders, arbitrageurs, and value-based investors/shareholders. The bidder and the incumbent manager are not allowed to trade during the tender offer.¹⁸ During any trading session, noise traders demand an exogenous random quantity $\tilde{\epsilon}$ that can be equal to $\beta = 1 - \alpha - f$ with positive probability or otherwise is equal to 0. Trading is costly, and both buyers and sellers of shares expend c > 0 per share traded. The distributions of noise traders are also considered to be independent and identically distributed over time. Both arbitrageurs and value-based investors choose a demand schedule, and trading takes place at the market clearing price.

More specifically, arbitrageurs choose the quantity x_i they want to trade. They do not have any (exogenous) private information and are allowed to take either long or short positions. Investors have a demand schedule (pricing function) P(y). Competition among investors forces the pricing function to be equal (gross of trading costs) to the expected value of shares, P(y) = E[p|y]. The market clearing condition, $\tilde{\epsilon} + \sum x_i + y = 0$, determines the price at which trading takes place. As a result of trading, a new ownership structure for the target arises, and new arbitrageurs with large blocks can be formed (or large blocks can be dismantled). Let *n* be the new number of arbitrageurs with at least β shares after a trading session. After the trading session is over, the same three-stage game is repeated, starting with the new ownership structure that resulted from the previous trading session.

All players in the model are assumed risk-neutral and have a cost of capital r.¹⁹ Therefore, they discount payoffs by $\delta = e^{-r\Delta t} < 1$ after an offer is rejected, and there is a delay of Δt . If the takeover never succeeds, the payoffs of all players are zero (the status quo payoff). If the takeover succeeds at period t with an offer equal to p and a total of T shares is tendered, the payoffs are as follows: If the bidder freezes out shareholders, then his payoff is equal to $\delta^{t-1}(1-p)$; if he does not freeze out, his payoff is equal to $\delta^{t-1}T(1-p)$. The payoff of a shareholder who tenders t shares and keeps (1-t) shares is equal to $\delta^{t-1}p$ if there is a freezeout, or otherwise is equal to $\delta^{t-1}[pt + (1-t)]$.²⁰

Our goal is to characterize the equilibrium of the game. The equilibrium concept used is stationary subgame perfect Nash equilibrium (SPE), where the number of arbitrageurs owning blocks of shares are the states.

3. Tender Offers and Arbitrage

In this section, we start the analysis of any-or-all offers conditioned upon f shares being tendered with an immediate second-step freezeout, which is the most commonly used type of offer. By definition, these offers specify no maximum and accept all shares tendered or none if a minimum of f shares are not tendered. Also, the bidder is committed to obtain all untendered shares in a follow-up freezeout merger promptly, paying shareholders who did not tender the same consideration paid to shareholders who tendered.

Freezeout tender offers are a powerful tool that bidders can use in a takeover. If the bidder could credibly make a take-it-or-leave-it bid conditioned on a freezeout, then such a

bid would not only eliminate the free-rider problem, but also allow the bidder to extract all the surplus from shareholders.

For example, consider the static setup used by Grossman and Hart (1980) [1], where a bidder can commit to a take-it-or-leave-it offer. Assume also that the bidder can commit to a bid conditional on a minimum of f shares being tendered, where f is the freezeout parameter (i.e., the bidder is committed not to accept any shares if the acceptance condition of the offer is not met, even if he wanted to).²¹ Therefore, on one hand, if less than f shares are tendered, the bid fails, and, on the other hand, if more than f shares are tendered, the bidder has the option to buy all the shares that have not been tendered at the offer price. The ability to make a take-it-or-leave-it offer, combined with the ability to condition the bid on a freezeout enables the bidder to extract all the surplus from target shareholders.

Proposition 1. (Static Takeovers with Freezeouts) Suppose that the bidder can commit to making an any-or-all, take-it-or-leave-it offer conditioned upon f shares with a second-step freezeout merger. Then the bidder's optimal strategy is to offer a price $p = \varepsilon > 0$, with ε arbitrarily close but strictly higher than zero. The optimal response of shareholders is to tender all their shares, and the bid succeeds with probability 1.

Conditioning on the freezeout parameter eliminates the free-rider problem of Grossman and Hart (1980) [1] because a shareholder that does not tender in an successful offer will have her shares frozen up or taken up by the bidder at the same price at which shareholders tendered their shares. Thus, by conditioning his offer on f shares, where f is the freezeout parameter, the bidder makes shareholders' payoffs identical regardless of their tendering decision whenever they are not pivotal and do not influence the outcome (success or failure) of the offer. However, since the payoffs of all shareholders are strictly higher when the offer is successful, it is a weakly dominant strategy for all shareholders to tender all their shares because their payoffs are higher whenever they are pivotal.²²

A static model with take-it-or-leave-it offers seems unable to capture the outcome of tender offers because usually the target receives a significant takeover premium. The shortcoming of the static model is that even though bidders may be able to commit to the acceptance condition, but they are not able to credibly commit to take-it-or-leave-it offers. Tender offers are thus more appropriately studied in a dynamic framework where bidders can negotiate with shareholders, and offers can be revised and extended over time until accepted by shareholders.

3.1. The Dynamics of Tender Offers

What is the outcome of the dynamic tender offer game? We will show that in equilibrium, shareholders will trade their shares in order to concentrate the ownership structure in the hands of arbitrageurs with the ability to hold out the tender offer. This more concentrated ownership structure allows target shareholders to leverage their rights and increase their bargaining power vis á vis the bidder, forcing the bidder to raise the takeover premium despite his freezeout rights.

Consider that the bidder makes an offer conditioned upon f shares being tendered. Shareholders who alone (or as a group) own at least a fraction of 1 - f of the shares can veto the takeover when individually (or acting in a cooperative or concerted manner) they strategically do not tender into the offer. We refer to those shareholders as arbitrageurs. Observe that even though arbitrageurs with veto power are highly interested in the takeover's success, they can strategically delay their tendering decision until the bidder gives them a proportional share of the takeover gains. To be sure, when the offer is announced, there might not be any arbitrageur owning target shares, as all shares might be owed by dispersed shareholders or by other passive shareholders (e.g., some types of institutional investors). However, trading while the offer is open, allows the ownership of shares to switch to new shareholders who are active and strategic.

Notice that because some shareholders, for some exogenous reasons, might not tender their shares into the offer, the number of shares that are needed by arbitrageurs to give them hold out power is only $\beta = 1 - \alpha - f$, where α is the fraction of shares that are not tendered. Therefore, the total number of arbitrageurs with veto power can be any integer *n* smaller than or equal to the maximum feasible number of blocks of size β that can be formed, which is equal to

$$n_f = \left[\frac{1-\alpha}{1-\alpha-f}\right],\tag{2}$$

where [x] is the integer part of x. Notice also that because of the "all-holders" and "bestprice" provisions of Rule 14d-10, the payoffs of all shareholders are the same and equal to the final offer price p. This rule thus prevents the bidder from making side payments to arbitrageurs in exchange for their agreement to tender. Of course, the bidder's payoff in a successful offer p conditioned on a freezeout is (1 - p), because the bidder acquires all shares at p.

We will formalize the economics of freezeout tender offers using a structure similar to a bargaining game with several players with veto power, where the number of players in the bargaining is endogenous and changing throughout the game. Aside from the fact that the number of players in the bargain is endogenous, this bargaining game is similar to Gul (1989) [19] and Hart and Mas-Colell (1996) [20].

In any stationary equilibrium of the tender offer game, the strategies of the bidder and shareholders can depend only on payoff-relevant variables, and thus must be independent of any other aspects of the history of moves. Stationarity then implies that the strategies of the bidder and shareholders at any given period can depend only on the number of arbitrageurs *n* with β or more shares at the beginning of the tender offer period. Naturally, we also expect that the equilibrium price that the bidder will have to pay for the target will increase the number of arbitrageurs with holdout power.

We will proceed to find a stationary perfect equilibrium of the tender offer game as follows. Assume that we have found a stationary perfect equilibrium, and say that the equilibrium outcome is such that the stock price is equal to p(n), an increasing function of the number of arbitrageurs n present at the beginning of a new period of the game. We then analyze the conditions imposed by stationarity and subgame perfectness on the equilibrium strategies and derive a system of equations that solve for p(n). We develop the analysis proceeding backward, starting from the trading game.

3.2. The Trading Game

How can arbitrageurs accumulate blocks during the tender offer without any private information? Can profitable arbitrage activity occur if there are trading costs and stock markets are efficient? A model with noise trading, such as Kyle and Vila (1991) [4] and Kyle (1985) [5], can provide camouflage that enables arbitrageurs to profit by trading in the target shares and to accumulate blocks of shares despite the fact that all information about the tender offer is publicly known to all market participants.

Costly arbitrage activity can occur in our setting, even though arbitrageurs have no ex-ante inside information, in the same way, that there can be arbitrage activity in Cornelli and Li (2002) [6]: the knowledge of the arbitrageurs' presence gives them an endogenous informational advantage that can lead to trade with other shareholders (see also Maug (1998) [23]. We extend the analysis of Cornelli and Li (2002) [6] and Kyle and Vila (1991) [4] to a dynamic trading model, such as Kyle (1985) [5], in which there are several arbitrageur trading blocks of shares during the tender offer.

Our next result shows that the following strategy profile is a competitive Nash equilibrium of the trading game: one arbitrageur places an order to buy a block of β shares with probability ϕ equal to

$$\phi = \left(1 - \frac{2c}{\pi[p(n+1) - p(n)]}\right)^+$$
(3)

and with probability $(1 - \phi)$ does not buy any shares—where $x^+ = \max(0, x)$ and

$$\pi = P(\tilde{\varepsilon} = \beta); \tag{4}$$

other arbitrageurs do not trade any shares; investors/shareholders demand for shares is equal to P(y), a function of the order flow y, satisfying: P(0) = p(n) + c, $P(\beta) = \phi p(n+1) + (1-\phi)p(n) + c$ and $P(2\beta) = p(n+1) + c$.

Therefore, if trading costs are small, $2c < \pi[p(n+1) - p(n)]$, the number of arbitrageurs increases to n + 1 with probability $\phi > 0$ and remains equal to n with probability $1 - \phi$. If trading costs are large, $2c \ge \pi[p(n+1) - p(n)]$, then $\phi = 0$, and the number of arbitrageurs remains equal to n with probability 1. Notice also, that the probability of entry ϕ , is non-increasing in the trading costs c and converges to 1 as trading costs approach zero.

The demand schedule of traders is such that shares are priced competitively and efficiently, given the information about the order flow and given their knowledge of the equilibrium strategies used by arbitrageurs. So, for example, when the order flow is y = 0, traders know for sure that no arbitrageurs are buying blocks, and thus, shares are worth p(n), and when the order flow is $y = 2\beta$, traders know for sure that an arbitrageur is buying one block, and thus, shares are worth p(n + 1). However, when the order flow is $y = \beta$, noise traders provide camouflage for arbitrageurs, and shares are worth $\phi p(n + 1) + (1 - \phi)p(n)$. The arbitrageur, however, has private knowledge that shares are worth either p(n + 1), if he is buying a block, or only p(n), if he is not buying.

In equilibrium, competition among arbitrageurs drives their profits to zero. For example, arbitrageurs' expected profits from buying a block with β shares, given the demand schedule of traders, are equal to

$$E[\Pi(\beta)] = \frac{\beta \pi [p(n+1) - (\phi p(n+1) + (1-\phi)p(n) + c)] + \beta (1-\pi) [p(n+1) - (p(n+1) + c)] - \beta c}{\beta c}$$

and $E[\Pi(\beta)] = 0$, if $2c \le \pi[p(n+1) - p(n)]$, and $E[\Pi(\beta)] < 0$, otherwise. These results are proved in the lemma below.

Lemma 1. Consider a trading stage game with n existing arbitrageurs with a stake β , noise traders that demand β shares with positive probability $\pi = P(\tilde{\epsilon} = \beta)$, and where trading costs per share are equal to c. Say that the value of the firm is equal to $p(\cdot)$, an increasing function in the number of arbitrageurs, and let $\phi \in [0, 1]$ be given by

$$\phi = \left(1 - \frac{2c}{\pi[p(n+1) - p(n)]}\right)^+.$$
(5)

The trading game has a competitive Nash equilibrium, as described above, in which the number of arbitrageurs increases to n + 1 *with probability* ϕ *and remains equal to n with probability* 1 - ϕ .

Interestingly, noise traders do not lose any money, despite their liquidity needs and the fact that they are trading with arbitrageurs who have an informational advantage (this is also present in Cornelli and Li (2002) [6]. Arbitrageurs profit from trading, even though they do not have any ex-ante insider information, trading is costly, and markets price shares efficiently. This is so because arbitrageurs perform a valuable service for shareholders—drive up the share price—and their information advantage is endogenous.

3.3. The Stationary Perfect Equilibrium

We will obtain the stationary perfect equilibrium by investigating the conditions imposed by stationarity and subgame perfectness on the equilibrium strategies. What are the conditions that must be satisfied by a stationary perfect equilibrium? The analysis of the trading game revealed that there is a Nash equilibrium of the trading game in which the number of arbitrage blocks increases to n + 1 with probability $\phi(n)$ and remains equal to n with probability $1 - \phi(n)$, where

$$\phi(n) = \left(1 - \frac{2c}{\pi[p(n+1) - p(n)]}\right)^+.$$
(6)

The expected value of shares before the trading session is then $\phi(n)p(n+1) + (1 - \phi(n))p(n)$, because shares are worth p(n+1) if a new arbitrageur enters, and p(n) otherwise.

At the tendering stage, shareholders, including dispersed and large shareholders, tender all their shares, if and only if, the offer is greater than or equal to

$$p_B(n) = \delta[\phi(n)p(n+1) + (1 - \phi(n))p(n)], \tag{7}$$

because if the takeover fails, there is a delay of Δt , where $\delta = e^{-r\Delta t}$, after which there is a trading session in which the expected value of shares is $\phi(n)p(n+1) + (1-\phi(n))p(n)$. Observe that because the offer is conditioned on a freezeout, shareholders do not take into account the possibility of free-riding when deciding whether or not to tender their shares. Similarly, the bidder accepts the arbitrageurs' offer if and only if it is lower than or equal to $p_A(n)$ given by

$$1 - p_A(n) = \delta[\phi(n)(1 - p(n+1)) + (1 - \phi(n))(1 - p(n))],$$
(8)

because if the offer is rejected, there is a delay of Δt , after which the bidder obtains (1 - p(n + 1)) if a new arbitrageur enters or (1 - p(n)) if he does not.

At the offering stage, the bidder's optimal offer is $p_B(n)$ equal to the minimum that shareholders are willing to accept and the arbitrageurs' optimal offer is $p_A(n)$ equal to the maximum that the bidder is willing to accept. The stock price p(n) at the beginning of the offering stage must then satisfy

$$p(n) = \frac{np_A(n) + p_B(n)}{n+1},$$
(9)

because with probability $\frac{n}{n+1}$, arbitrageurs offer $p_A(n)$ —and the bidder and all other shareholders accept this offer—and with probability $\frac{1}{n+1}$, the bidder offers $p_B(n)$ —and shareholders accept this offer.

We prove that there is a unique solution for the system of Equations (6)–(9). We also show that the maximum number of arbitrageurs who can enter during the tender offer is \underline{n} given by:

$$\underline{n} = \min\left(n_f, \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{\pi}{c}}\right),$$
(10)

where n_f is the maximum number of blocks feasible (see Equation (2)). In the case where there is already a large number of arbitrageurs present so that $n \ge \underline{n}$, the entry of new arbitrageurs occurs with probability zero, and the equilibrium bid solution of the system of equations is simply

$$p(n) = v(n),$$
 (11)
 $\phi(n) = 0,$

where v(n) is defined as a measure of the bargaining power of arbitrageurs

$$v(n) = \frac{n}{n+1}.$$
(12)

In the case where $n < \underline{n}$, entry of new arbitrageurs can take place with positive probability. The equilibrium bid, the solution of the system of equations in the range where $n < \underline{n}$, and the following expressions determine the entry probability of new arbitrageurs

$$p(n) = \delta\left(p(n+1) - \frac{2c}{\pi}\right) + (1-\delta)v(n),$$

$$\phi(n) = \frac{p(n) - v(n)}{p(n) - v(n) + \left(\frac{\delta}{1-\delta}\right)\frac{2c}{\pi}}.$$
(13)

Finally, notice that the expressions for the bidders and arbitrageurs' offers, $p_B(n)$ and $p_A(n)$, are given by equations

$$p_B(n) = \delta[\phi(n)p(n+1) + (1 - \phi(n))p(n)],$$
(14)

$$p_A(n) = \delta[\phi(n)p(n+1) + (1 - \phi(n))p(n)] + (1 - \delta).$$

Notice that for $n \ge \underline{n}$, shares are worth p(n) = v(n), and the marginal gain associated with the entry of a new arbitrageur is a decreasing function of the number of arbitrageurs. The value \underline{n} is obtained as the minimum n such that it is not profitable for a new arbitrageur to enter, which is equivalent to $2c = \pi [v(\underline{n}+1) - v(\underline{n})] = \frac{1}{\underline{n}(\underline{n}+1)}$. When the number of arbitrageurs is smaller than \underline{n} , then arbitrageurs can profitably enter. The possibility of arbitrageurs entering drives the stock price to $p(n) = \delta (p(n+1) - \frac{2c}{\pi}) + (1-\delta)v(n)$, which is higher than the price if no new arbitrageur entered (equal to v(n)).

We prove in the following proposition that the following strategy profile is an SPE equilibrium. Consider any subgame starting with n arbitrageurs: (i) At the offering stage, the bidder makes an offer $p_B(n)$ conditioned on f shares when it is his turn to propose, and shareholders with β or more shares offer $p_A(n)$ when it is their turn to propose. (ii) At the tendering stage, shareholders, including dispersed and large shareholders, tender all their shares if the offer is greater than or equal to $p_B(n)$, and do not tender any shares otherwise. Also, the bidder's response to an arbitrageur offers to accept any offer of arbitrageurs lower than or equal to $p_A(n)$ and to reject any offer above $p_A(n)$. (iii) At the trading session, one arbitrageur with no blocks places orders to buy blocks of β shares with probability $\phi(n)$ and does not buy any shares with probability $1 - \phi(n)$; all other arbitrageurs do not trade any shares, and investors' demand schedule is equal to $P_n(y)$.

These results are proved in the following proposition.

Proposition 2. (Takeovers and arbitrage with freezeouts) Let the cost of trading per share be equal to c > 0, and noise traders' demand for shares be $\tilde{\epsilon}$ equal to β shares with positive probability π , and zero otherwise. Let <u>n</u> and v(n) be given by expressions (10) and (12). Then there exists a stationary perfect equilibrium in which an any-or-all offer conditioned upon f shares being tendered with a second-step freezeout succeeds with probability 1. Furthermore, for δ arbitrarily close to 1, the equilibrium bid price depends on the number of arbitrageurs n at the announcement of the offer as follows:

- (*i*) If $n \ge \underline{n}$, the equilibrium bid is p(n) = v(n), where v(n) is increasing in n.
- (ii) If $n < \underline{n}$, the equilibrium bid is $p(n) = p(\underline{n}) + [n \underline{n}]\frac{2c}{\pi} > v(n)$, increasing in the number *n* of arbitrageurs.

Therefore, insider trading activity before the announcement of the tender offer drives up the takeover premium. Moreover, the additional takeover premium attributed to the entry of one more arbitrageur is equal to $\Delta p = \frac{2c}{\pi}$ and is increasing trading costs (and decreasing the liquidity of the stock).

Naturally, arbitrageurs with inside information can easily and profitably accumulate shares before the announcement of the tender offer, and thus, some arbitrage blocks can already be present at the opening of the offer. The proposition shows that there is a positive relationship between arbitrageurs' accumulation of shares before the tender offer and the takeover premium. Although arbitrageurs can enter after the announcement of the offer, their entry at this stage is uncertain and happens with probability less than one. Furthermore, the relationship between the premium and arbitrage activity is stronger when the stock is less liquid because the lower the liquidity of the stock, the more unexpected is the arbitrageurs' entry, and thus, the stock price reacts more when the entry occurs (note, that the liquidity of the stock can be proxied by $\frac{\pi}{c}$ where *c* is the trading cost and $\pi = P(\tilde{\epsilon} = \beta)$ is the probability of noise trading).

In the extreme case where trading costs are arbitrarily small, naturally, there should be no relationship between arbitrage activity before the announcement and the premium. The equilibrium bid converges to $v(n_f)$, and the bid does not depend on the existing number of arbitrageurs at the announcement of the offer: the maximum number of arbitrageurs can be formed with high probability in a few trading sessions after the announcement of the offer.

So far, we concentrated on the relationship between the takeover premium and arbitrage activity before the announcement of the offer. However, we also predict that there should be a positive relationship between arbitrage activity after the announcement and revisions in the bid measured as the ratio of the final and opening bid. The more arbitrageurs successfully accumulate blocks during the offer, the more they are able to force the bidder to increase the premium using their enhanced holdout power. We expect, though, to find that in most cases, few revisions in the bid take place because the bidder, in equilibrium, makes a high initial preemptive bid that is immediately accepted by shareholders (see also Fishman (1988) [24]. The equilibrium is such that all offers succeed with probability 1 in the first period of the game. Thus, there is no de facto entry of arbitrageurs during the tender offer. However, the threat of entry is strong enough to make the bidder pay a high preemptive bid.²³

Consistent with the implications of the model, Jindra and Walkling (2004) [11] and Jiang et al. (2018) [12] find that there exists a positive and significant relationship between arbitrage activity before the announcement of the offer (proxied by a measure of abnormal trading volume) and the takeover premium. Furthermore, Larcker and Lys (1987) [14] have some evidence that seems to support that there is a positive relationship between arbitrage activity after the announcement of the offer and revisions in the bid. They show that arbitrageurs often accumulate shares after the announcement of the offer, and that in transactions where arbitrageurs enter, the takeover premium increases. We postpone a more detailed discussion of the empirical evidence related to takeovers and arbitrage until Section 5.

3.4. The Supply Curve of Shares

We have so far restricted our attention to cases where the acceptance condition is equal to the freezeout parameter. However, the bidder could have chosen to make an offer conditioned on obtaining majority control and not an offer conditioned on a freezeout, which is, in general, a more stringent condition when there is a supermajority merger requirement.

However, we show that whenever the tender offer is conditioned on a fraction lower than the freezeout requirement, shareholders take into account the possibility of free-riding when deciding whether to tender their shares and the bidder can only profit on gains in his toehold. As Harrington and Prokop (1993) [21] showed the free-rider problem of Grossman and Hart (1980) [1] is even more pronounced in the dynamic case than in the static one. In a dynamic setting, it becomes even more challenging to convince shareholders to tender because they know that if the offer fails in the current period, it can be extended for an additional period, and if the offer succeeds, they gain more by not tendering their shares. The results of Harrington and Prokop also hold for the game considered here: if the corporation has a large number of shares traded, the bidder can only profit on gains in his toehold when not using offers conditioned on a freezeout. **Proposition 3** (Supply curve of shares). Consider a corporation with a large number of shares, discount rate δ arbitrarily close to 1, and a cost of trading shares arbitrarily close to zero. The equilibrium bid price is equal to

$$v(f) = v(n_f), \tag{15}$$

and the bid is structured as an any-or-all offer conditioned upon f shares with a second-step freezeout. Therefore, the supply curve of shares is upward sloping in the freezeout parameter. Furthermore, the takeover premium in a hostile acquisition is higher than in a friendly acquisition because the more shares that are certain not to be tendered (such as insiders' shares), the more hold-out power arbitrageurs have to extract a higher premium from the bidder.

The model above yields a supply curve for shares that are endogenously upwardsloping, where the supply curve is the relation between the equilibrium bid price and the fraction f of shares. Intuitively, the supply curve is upward-sloping because the greater the minimum number of shares demanded by the bidder, the larger the number of shareholders who can hold out. Moreover, when there is a large number of arbitrageurs with hold-out power, they can credibly demand a larger share of the takeover gains in exchange for tendering their shares, which implies that the supply curve is upward-sloping. The result that the supply curve is upward-sloping is also a feature of earlier models in the literature, such as Stulz (1988) [8] and Burkart et al. (1998) [10], although for different reasons than those proposed here. In Stulz, the upward-sloping supply curve is obtained based on heterogeneous shareholder valuation of shares. In Burkart et al. [10], it is obtained based on ex-post moral hazard.²⁴ See Section 5 for a discussion of the empirical evidence related to Proposition 3.

In order to illustrate the result of the proposition, we consider some examples of freezeout parameters that prevailed in the U.S. during several periods. For example, say that in a hostile takeover, the incumbent management owns 5 percent of the target and that 5 percent of the shares are owned by shareholders who are non-responsive or uninformed about the takeover and do not tender their shares so that $\alpha = 10\%$. In New York before the introduction of the freezeout laws in 1985, the freezeout parameter was 2/3, and likewise for Delaware before 1967. According to our previous results, shareholders have $\frac{3}{4}$ of the bargaining power. Therefore, the equilibrium bid is $\frac{3}{4}$ of the total takeover gains, and the bidder's profit is $\frac{1}{4}$ of the gains. In Delaware after 1967, a simple majority of the votes was required for a freezeout; there can be two blockholders with veto power, with a stake of 40 percent, the tender offer price is $\frac{2}{3}$, and the bidder's profits are $\frac{1}{3}$ of the takeover gains.

The examples above illustrate that with freezeout parameter values prevailing in the U.S., the model generates a distribution of gains between bidder and target shareholders that is skewed toward the target (see also Bergstrom et al. (1993) [7], despite the fact that we assumed that there is no competition for the target (bidders' profits in contested offers are, obviously, likely to be lower than without competition).²⁵

4. The Structure of Tender Offers

In this section, we analyze the structure of tender offers chosen by bidders. So far, we have restricted our attention to any-or-all bids with an immediate second-step freezeout merger in which the consideration paid to shareholders who are frozen out is equal to the bid price. Can the bidder benefit from using front-end loaded bids such as two-tier or partial offers? What is the outcome of takeovers in which the bidder can dilute target shareholders post-acquisition? What if the bidder is not able to commit to the acceptance condition? Our goal is to understand what determines the joint choice of the most important strategic elements of a tender offer: the bid price, the maximum number of shares sought in the offer, the acceptance condition, as well as the choice of whether or not to undertake a second-step merger.²⁶ The analysis yields a novel characterization of the structure of tender offers with several surprising results that are consistent with existing empirical findings.

Among other things, we show that front-end loaded bids, such as two-tiered offers and partial offers do not provide any strategic benefits to bidders in addition to any-or-all offers. In order words, bidders can maximize profits simply by using any-or-all offers. The intuition for this result is that arbitrageurs eliminate, in equilibrium, the coerciveness associated with two-tier and partial offers. For example, suppose that a two-tiered offer with a low blended price is going to be accepted in equilibrium because shareholders stampede to tender. The stock price should then be very close to the blended price, allowing arbitrageurs to accumulate large stakes in the open market at a low price. As we have argued before, as long as arbitrageurs accumulate large blocks of shares, they can prevent the bidder from freezing out shareholders at the back-end price, even though all other dispersed shareholders are coerced to tender their shares. Besides, arbitrageurs will use their power to hold out the two-tiered offer in order to demand from the bidder, in exchange for tendering their shares, a higher blended price that reflects their proportional share of the takeover gains.

Moreover, we show that there is a novel relationship between the equilibrium bid price and the fraction of the post-acquisition value of the target that the bidder can dilute private benefits. According to Grossman and Hart (1980) [1], the equilibrium takeover bid would simply be equal to the post-takeover value with dilution. Interestingly though, this need not be the equilibrium if the bidder is able to freezeout shareholders. For example, if target shareholders enjoy a good level of protection against dilution by a controlling shareholder, the bidder can acquire all shares with an any-or-all offer conditioned on a second-step freezeout at a bid price lower than the post-acquisition value of shares. Consequently, the bid price is determined not by the level of dilution but rather by the supermajority requirement for a freezeout merger. Even more surprisingly, this result also holds when target shareholders enjoy very weak levels of protection against dilution. A bidder can only take over the target with a bid greater than or equal to the bid necessary to acquire control, which is determined by arbitrageurs who can use their hold-out power to block the bidder from acquiring a majority of the target's shares. Therefore, an interesting new empirical implication of the model is that there should not be a significant relationship between the takeover premium and the level of dilution post-acquisition of the target. This implication of the model is in contrast with other takeover models with dilution in the literature, such as Grossman and Hart (1980) [1] and Burkart et al. (1998) [10].

4.1. Commitment to the Acceptance Condition

So far, we have seen that offers with a minimum tender condition equal to f can be an effective way to address the free-rider problem. We have, though, always assumed that the bidder was able to commit to the acceptance condition. This commitment means that even if slightly less than f shares were tendered in an offer, the bidder would not be allowed to waive the minimum condition and accept the tendered shares for payment. Nevertheless, Holmstrom and Nalebuf (1992) [27] have argued that conditional offers might not be a credible way to solve the free-rider problem if the bidder were not able to credibly commit to the acceptance condition. The equilibrium where all shareholders tender in an any-or-all offer with an acceptance condition equal to f, could unravel if the bidder could not commit to the conditionality of the offer: a dispersed shareholder might not tender if he believes that the bidder would take over anyway, even if less than f shares were tendered because he would be better off keeping his shares in this event, and he would not be worse off if the bidder obtained more than f shares and immediately followed up with a freezeout of shareholders who did not tender at the exact bid price.

Would any-or-all offers conditioned on a freezeout unravel without the ability to commit to the acceptance condition? Possibly taking into account the importance of the acceptance condition, the SEC in the U.S., and the Takeover Panel in the U.K. has created rules that allow bidders to commit to it credibly.²⁷ For example, the SEC interprets the waiver of an acceptance condition near the end of a tender offer as a material change in the terms of the offer that requires further extension of the offer for at least five business days

after the waiver. While the commitment rules of the SEC and the Takeover Panel address the problem raised by Holmstrom and Nalebuf (1992) [27], they open another interesting question. How critical is the rule that allows bidders to commit to the acceptance condition?

Somewhat surprisingly, we show that this rule is not necessary for the success of anyor-all freezeout offers, and the outcome is unchanged with or without the rule. Therefore, the issue raised by Holmstrom and Nalebuf, while relevant within their static takeover framework, is not applicable in a the dynamic framework of tender offers.

Intuitively, an explicit mechanism of commitment is not necessary because it is in the bidder's best interest to take over if and only if he is able to immediately conduct a second-step freezeout after the expiration of the tender offer. The bidder's payoff from extending the offer until he is able to immediately freezeout outweighs the benefits of waiving the acceptance condition and taking over without the supermajority required for a freezeout.

Proposition 4. Let $f < \delta < 1$, and let $p(\cdot)$ be the bid price given by Equation (13). An anyor-all offer with bid price $p(\cdot)$, acceptance condition equal to f, and a second-step freezeout is an equilibrium, even if the bidder is unable to commit not to waive the acceptance condition.

The proof of the result comes from comparing the bidder's profits when he waives the acceptance condition and his profits when he does not waive it. On the one hand, the bidder's profit when waiving the acceptance condition and taking over when less than *f* shares are tendered, is smaller than f(1 - p): after the takeover, the stock price post-acquisition increases to \$1, and shareholders will vote against a second-step freezeout unless the bidder offers \$1, per share as a consideration for the merger. On the other hand, the bidder's profit when less than *f* shares are tendered and he does not waive the acceptance condition, is equal to $\delta(1 - p)$: he extends the offer for another period and shareholders will, in equilibrium, tender more than *f* shares for sure in the next period, allowing the bidder to freezeout shareholders.

Therefore, if $f < \delta$, the bidder is able to credibly commit to the acceptance condition, and thus the any-or-all offer of Proposition 2, conditioned upon *f* shares tendered, is an equilibrium, even if the bidder is unable to commit not to waive the acceptance condition, providing yet another solution to the issue raised by Holmstrom and Nalebuf [17].

4.2. Two-Tiered Offers

Two-tiered offers are another well-known mechanism to resolve the free-rider problem, which has been extensively studied in the literature. In a two-tiered bid, the bidder specifies a maximum amount of shares sought in the first tier of the offer at the front-end price. Also, the bid is conditioned upon a minimum of f shares being tendered, and the tender offer is followed up by a second-step freezeout merger in which the remaining shares are taken up at a lower back-end price.²⁸ The relevant price for shareholders in a two-tier offer is the blended price, which is the weighted average of the price paid in the front-end and the back-end, where the weight is the fraction of shares receiving each price.

The potential strategic benefit of two-tiered offers, inducing shareholders to tender and solving the free-rider problem, is straightforward. However, two-tiered offers are controversial because they not only solve the free-rider problem but also have the potential to coerce shareholders to tender even if they do not want to. Two-tiered offers can create a stampede of dispersed shareholders tendering their shares because they will be concerned about receiving the lower back-end price if they do not tender and the offer is successful. Can a two-tiered offer really coerce shareholders to tender their shares?

This line of reasoning neglects the potential for arbitrageurs to profit from eliminating the coerciveness of the offer. Intuitively, if a two-tiered offer with a low blended price is going to be accepted anyway because shareholders will be forced to tender, then the stock price should reflect that and should, therefore, be very close to the blended price. Arbitrageurs could then buy shares in the open market at the blended price, accumulating large stakes. As we have argued before, as long as arbitrageurs accumulate at least a block β , they can prevent the bidder from freezing out shareholders at the back-end price, even if all other dispersed shareholders are coerced to tender their shares. Arbitrageurs will then use their power to hold out the two-tiered offer in order to demand from the bidder a higher blended price that reflects their fair share of the takeover gains. We show that, regardless of whether the bidder uses a two-tiered or an any-or-all offer the outcome of the takeovers is the same.

Proposition 5 (Two-tiered offers). There is no additional strategic benefit for the bidder in using two-tiered offers rather than using any-or-all offers with freezeouts. The equilibrium blended price when the bidder uses two-tiered offers is the same as the equilibrium bid when the bidder uses any-or-all offers with freezeouts. In equilibrium, arbitrageurs protect shareholders from coercive two-tiered offers. Therefore, fair price charter provisions do not yield any additional benefit to target shareholders.

This result provides a formalization of Demsetz's (1983) [28] insights. He noted that shareholders can protect themselves from a coercive offer through the formation of large blockholdings by takeover specialists and arbitrageurs during the tender offer. We strengthen Demsetz's intuition, showing that coercion of shareholders cannot happen in equilibrium and the outcome of a two-tiered offer is the same as an any-or-all offer because the arbitrage opportunities that it creates.

Indeed, Comment and Jarrell (1987) [15] find that the average total premium is insignificantly different in executed two-tiered and any-or-all offers. This the model also explains why Jarrell and Poulsen (1987) [16] have found that a firm's adoption of fair-price charter amendments has an insignificant effect on its stock prices. Relatedly, Comment and Jarrell (1987) [15] provide indirect evidence indicating that arbitrage activity is more intense during two-tiered than any-or-all offers, consistent with the more important role played by arbitrageurs during two-tiered offers.

4.3. Takeovers with Dilution

Grossman and Hart (1980) [1] have proposed that one mechanism that could solve the free-rider problem is to allow a bidder who acquires control to divert part of the posttakeover value improvements as private benefits. This addresses the free-rider problem because shares are worth less post-takeover to shareholders who keep their shares than to an acquirer who can extract some private benefits from control. Therefore, in equilibrium, a takeover would succeed when the bid price is equal to the post-takeover value with dilution, and bidders would then be able to profit in a takeover.

We now explore the impact of the bidder's ability to dilute minority shareholders of the target in the presence of tender offers with freezeouts. We modify the model proposed in Section 2 as follows, maintaining everything else the same. Assume that the bidder is able to dilute a fraction *d* of the target value post-acquisition. We maintain the assumption that the economic value of the target post-acquisition is equal to \$1, with the improvements introduced by the acquirer. Therefore, the stock price post-acquisition is similar to 1 - d. The acquirer's payoff, given that he purchases a total of *T* shares at a bid price *p*, is T(1 - d - p) + d, equal to the security benefits of *T* shares owned by the bidder, plus his private benefits *d*, subtracted from the cost of acquiring *T* shares.

According to Grossman and Hart (1980) [1], the equilibrium takeover bid would be equal to the post-takeover value with dilution, p = 1 - d, and bidders would be able to profit in a takeover even though they did not own any previous stake in the target. This offer could be structured either as an any-or-all offer or a partial offer conditioned only upon the bidder's obtaining at least a majority *k* of shares. However, this need not be the equilibrium if the bidder is able to freezeout shareholders. Suppose, for example, that target shareholders enjoy a good enough level of protection against dilution, such that d < 1 - p(f), where p(f) is given by Equation (15). As we have seen before, p(f) is the

expression of the bid price at which the bidder could acquire the target with an any-or-all offer conditioned on a second-step freezeout. Then, since p(f) > 1 - d, it is in the bidder's best interest to acquire the target with a freezeout offer rather than a bid 1 - d that is not conditioned upon at least f shares being tendered. Consequently, for low levels of dilution, the bid is determined not by the the precise amount that can be diluted, but rather by the supermajority requirement for a freezeout merger.

Surprisingly, even if target shareholders do not enjoy much protection against dilution by a controlling shareholder, the takeover premium is not reduced beyond a certain lower bound. Suppose, for example, that the bidder can dilute the target shareholders postacquisition by more than d > 1 - p(k), where p(k) is given by Equation (15) with the majority fraction k replacing f. Of course, in this case, the equilibrium bid under Grossman and Hart (1980) [1] is equal to 1 - d < p(k). However, in our model, the bidder would not be able to take over the target with a bid lower than p(k). In equilibrium, the bidder makes an any-or-all bid p(k), conditional only upon the acquisition of control. Intuitively, the bid must be at least equal to p(k) because it would otherwise not receive the necessary minimum number of shares k. Arbitrageurs with a stake of size $1 - k - \alpha$ can block the bidder from acquiring a controlling stake k. As we have seen before, these shareholders have veto power and can extract from the bidder an offer price of at least p(k). Even though the bidder is able to dilute shareholders significantly post-acquisition, blockholders will use their bargaining power to force the bidder to pay more to gain control of target assets.

Notwithstanding, for intermediate levels of dilution, $d \in [1 - p(k), 1 - p(f)]$, the equilibrium bid does coincide with the Grossman and Hart (1980) [1] equilibrium. In equilibrium, the bidder makes an any-or-all bid 1 - d (conditional on k shares), which is accepted by target shareholders. Note, that the bidder would propose an offer conditioned upon k, rather than make an offer conditioned upon f (unless he reserves the right to waive the condition), in order not to give excessive (and unnecessary) bargaining power to shareholders of the target, who could drive the premium to p(f) > 1 - d. Also, even though p(k) < 1 - d, shareholders' free-riding behavior forces the bidder to bid at least 1 - d in order to induce shareholders to tender.

We summarize these results for the equilibrium of tender offers with freezeouts and dilution in the following proposition.

Proposition 6 (Tender offers with freezeouts and dilution). Let the bid price $p(\cdot)$ be defined as in Equation (15). Also, let the freezeout and control share acquisition parameters be equal to f and k, and let the acquirer be able to dilute in private benefits a fraction d of the post-takeover gains. The equilibrium bid depends on the level of dilution as follows:

- (*i*) For low levels of dilution, d < 1 p(f), the equilibrium bid is an any-or-all offer at p(f) conditioned upon f shares with a second-step freezeout.
- (ii) For intermediate levels of dilution, $d \in [1 p(k), 1 p(f)]$, the equilibrium bid is at 1 d, conditional upon k shares, and can either be a partial or any-or-all offer.
- (iii) For high levels of dilution, d > 1 p(k), the equilibrium bid is an any-or-all offer at p(k), conditional upon k shares. The offer can also be structured as a partial offer for k shares, but then the bid must be equal to p > p(k), solution of $p(k) = p \cdot k + (1 d) \cdot (1 k)$.

The proposition indicates that the ability to dilute target shareholders post-acquisition moderately does not increase the bidder's profits as long as the bidder has the option to freeze out shareholders who do not tender their shares. However, the bidder's profits increase as he is able to dilute shareholders by more. Notwithstanding, the most a bidder's profit can increase, even if he is able to completely dilute shareholders upon gaining control, is equal to [p(f) - p(k)], because target shareholders receive a minimum fraction equal to p(k) of the total economic gains generated by the takeover, regardless of their level of protection in the post-acquisition stage. Note, that if both the freezeout and control acquisition parameters are the same (k = f), as is common, for example, for most companies

incorporated in Delaware, then the takeover premium is completely independent of the level of dilution.

We believe that one important empirical implication of our model is that there should not be a very significant relationship between the takeover premium and the level of dilution post-acquisition of the target. This implication is substantially different than other takeover models with dilution in the literature, such as Grossman and Hart (1980) [1], Burkart et al. (1998) [10], and Bebchuk (1989) [29].

The following example illustrates an application of the result. Consider a hostile acquisition of a target with a freezeout parameter equal to 2/3, a control acquisition parameter equal to 50 percent, and an insider owning $\alpha = 10\%$. As we have seen before, $p(f) = \frac{3}{4}$, and $p(k) = \frac{2}{3}$. Thus, if the dilution level is less than 25 percent, then a bidder would not obtain any extra profits from diluting shareholders, and the takeover bid would be at $p(f) = \frac{3}{4}$, conditioned upon f shares with a second-step freezeout. For dilution levels d between 25 and 33 percent, the bidder can obtain extra profit equal to d - 0.25 because of his ability to dilute d. For even higher levels of dilution, the bid price is fixed at the lower bound $p(k) = \frac{2}{3}$, and offers are conditioned upon only k shares, and thus, the most extra profit that the bidder can obtain from his ability to dilute target shareholders is equal to 8 percent. For markets such as the U.S., where the evidence shows that the minority shareholders enjoy significant levels of protection, the ability to dilute is not likely to play a major role.²⁹

Nevertheless, for many other markets around the world, the ability to dilute may play a role in takeovers. In the U.K., for example, even though the freezeout parameter is 90 percent, the ability of bidders to dilute the target post-acquisition after acquiring only 50 percent of the shares may help them succeed with a lower premium. For example, if the freezeout parameter is 90 percent, insiders with $\alpha = 10\%$ can thwart a hostile bidder from freezing out (i.e., $p(f) = \infty$). However, the bidder can succeed with an offer conditioned upon *k* shares (or with an offer conditioned upon 90 percent with the right to waive this minimum condition, as is common in the U.K.) at a price equal to the maximum of $\frac{2}{3}$ and 1 - d. Therefore, if the level of dilution is above $d \ge 25\%$, then the equilibrium bid price is equal to the maximum of $\frac{2}{3}$ and 1 - d. This example shows that, in many cases, the equilibrium bid price may be determined not by the supermajority required for a freezeout but rather by the ability to dilute the target post-acquisition of control.

4.4. Synergistic Takeovers

In this section, we analyze the case in which the gains associated with the takeover arises from synergies between the acquirer and the target. Synergistic takeovers are different than takeovers with dilution because the post-acquisition value of the target is low for both the bidder and the shareholders who keep their shares until the acquirer is able to pursue a second-step freezeout merger. This is the case, for example, in takeovers where the gains come from economies of scale in distribution, production, research and development, or administrative functions. In order to achieve economies of scale, however, it is key to be able to operate the acquirer and the target as a single entity. The problem is that if the target is not a wholly owned subsidiary of the acquirer (shareholders who did not tender remain as minority shareholders of the target), there can be significant legal restrictions in jointly operating the target and acquirer. Legal systems that offer good protection to minority shareholders, require an arms-length parent-subsidiary relationship with an equal division of costs and benefits between them. However, it can be impossible or very costly to develop systems or contracts that have both a seamless operation and an equal division of surplus and thus, in such circumstances, the second-step freezeout is essential to accomplish the takeover gains (see Gilson and Black (1995) [31]).

Synergistic takeovers can easily be addressed within the framework of the paper. Notice that, unlike the previous cases analyzed, where the bidder could take over with offers conditioned on any fraction above simple majority, in the synergistic takeover case, the bidder loses some flexibility and will only make offers that are strictly conditioned upon f shares being tendered.

Proposition 7 (Synergistic Takeovers). Suppose that the gains from the takeover can only be created if there is a merger of the target and bidder, but not if the target is managed as a separate firm. Then the equilibrium bid is at p(f), and the bid is structured as an any-or-all offer conditioned upon f shares being tendered with a second-step freezeout.

This is a case often neglected in studies of takeovers; however, we believe it has empirical relevance. The result also yields some testable cross-sectional implications. For example, whenever the takeover is of a synergistic type, and there is a supermajority merger requirement (such as 90 percent of the shares in the U.K.), we would expect the bidder to pay a higher premium compared to a takeover in which the bidder can dilute the target significantly, and thus, a successful bid requires only a simple majority of the votes.

5. Empirical Implications

Our results on takeovers and freezeouts are consistent with numerous existing empirical findings. We first discuss the empirical implications related to takeovers and arbitrage and then follow with a discussion of the structure of tender offers.

5.1. Takeovers and Arbitrage

Jindra and Walkling (2004) [11] study the relationship between arbitrage activity and the takeover premium. Jindra and Walkling (2004) [11], using a sample of 362 cash tender offers, find that there exists a positive relation between arbitrage and the takeover premium. As a proxy for the presence of arbitrageurs, they use a measure of abnormal volume. They calculate two measures of abnormal volume: one following the methodology proposed by Schwert (1996) [32] and another suggested by Lakonishok and Vermaelen (1990) [33]. Jindra and Walkling estimate a regression of the percentage takeover premium on the abnormal volume, and they find a positive coefficient using both measures. The coefficient is highly significant (t = 4.78) with Lakonishok and Vermaelen's methodology, and the *t*-statistic is only 0.89 with Schwert's methodology.

In addition, Schwert (1996) [32] finds that the runup in the stock price is positively and significantly correlated with the offer premium. According to Meulbrock (1992) [34], almost half of the runup in the month before a merger or tender offer announcement occurs on the days when insiders trade, and thus arbitrageurs' accumulation of blocks in the target is mainly responsible for the runup. The relationship between the runup and the premium suggests that there is also a positive and significant relationship between arbitrage activity and the premium in Schwert (1996) [32].

Overall, the existing empirical evidence seems to be consistent with the result of Proposition 2, that insider trading activity before the announcement of the tender offer drives the takeover premium up. Furthermore, our results also predict that the coefficient in the regression estimated by Jindra and Walkling should be negatively related to a measure of the liquidity of the stock; Proposition 2 obtains an expression for the coefficient equal to $\frac{2c}{\pi}$, which is a measure that is negatively related to the liquidity of the stock.³⁰

Finally, we also predict that there should be a positive relationship between arbitrage activity after the announcement and revisions in the bid measured as the ratio of the final and opening bid. The more arbitrageurs are able to accumulate blocks during the offer, the more they are able to force the bidder to increase the premium. We expect, though, to find that, in most cases, there should be few revisions in the bid because the bidder, in equilibrium, makes a high initial preemptive bid that shareholders immediately accept. For example, Franks and Harris (1987) [13] report that offers are revised in 123 uncontested takeovers in the U.K., approximately 9 percent of the uncontested takeovers in their sample.

The study by Larcker and Lys (1987) [14] allows us to evaluate the role played by arbitrageurs in a takeover. Their sample consists of 111 tender offers and merger proposals

from 1977 to 1983, where an arbitrageur purchases more than five percent of the outstanding shares with the purpose stated as "arbitrage or other business activities" or "to participate in a potential merger or tender offer". Furthermore, their sample is appropriate to understand the role played by arbitrageurs after the announcement of a takeover because in most transactions in their sample, arbitrageurs accumulated shares after the announcement of the acquisition (except for three transactions). Larcker and Lys' results indicate that the takeover premium increased by an average of 9 percent from the date of arbitrageurs' entry to the expiration of the offer, consistent with our interpretation that arbitrageurs use their power to hold out the transaction to force the bidder to pay a higher price.³¹

5.2. The Structure of Tender Offers

In Section 4 of the paper, we develop a characterization of the structure of tender offers. One of the results we derived (Proposition 4) was that the supply curve of shares, defined as the relation between the equilibrium bid price and the minimum number of shares that the bidder needs to acquire in the offer is (endogenously) upward-sloping. The proposition implies that the takeover premium is an increasing function of the supermajority merger requirement, which is a relationship that, to the best of our knowledge, has not yet been tested.

The proposition also implies that the takeover premium is a decreasing function of the fraction of shares owned by shareholders who do not tender into the offer. Therefore, in a hostile acquisition, the takeover premium is higher because the insider-controlled shares are not tendered into the offer, which increases the hold-out power of other shareholders to demand a higher premium from the bidder. Stulz, Walkling, and Song (1990) [9] have evidence showing that, indeed, there is a positive and significant relationship between insider ownership and the takeover premium.

We have also seen that front-end loaded bids, such as two-tiered offers and partial offers do not provide any strategic benefits to bidders in addition to any-or-all offers. Consistent with our results, Comment and Jarrell (1987) [15] find that the average total premium (based on the blended price) received by shareholders differs insignificantly in executed two-tiered and any-or-all offers: the average premium in any-or-all offers is 56.6 percent above the pre-offer price and is 55.9 percent in two-tiered offers. Additionally, Jarrell and Poulsen (1987) [16] have found that fair-price charter amendments have insignificant effects on the stock price of adopting firms.

Interestingly, Comment and Jarrell (1987) [15] provide some indirect evidence which indicates that arbitrage activity is more intense during two-tiered offers than any-or-all offers. They report that the number of shares traded (cumulative average transactions between the beginning and expiration of the offer) is equal to 55.9 percent in two-tiered offers, while it is only 34.4 percent in any-or-all offers, consistent with the more important role played by arbitrageurs during two-tiered offers.

Moreover, we uncovered a novel relationship between the equilibrium takeover premium and the fraction of the post-acquisition value of the target that the bidder can dilute private benefits. If target shareholders enjoy a good level of protection against dilution by a controlling shareholder, the bidder can acquire all shares with an any-or-all offer conditioned on a second-step freezeout at a price that is even lower than the post-acquisition stock price. Consequently, the bid price is determined not by the level of dilution, but rather by the supermajority requirement for a freezeout merger. More surprisingly, even if target shareholders enjoy only a very weak level of protection, the bidder is not able to take over the target with a bid price lower than the one necessary to acquire the majority control, which can reasonably be greater than the target's post-acquisition value with dilution. Since corporate charters may require fewer shares for acquisition of control than for a freezeout merger, the takeover premium may be somewhat lower when the bidder can considerably dilute shareholders post-acquisition, because of the slightly reduced holdout power of shareholders—the upward-sloping supply curve relation. Interestingly, whenever the charter specifies a similar fraction of shares for control acquisition and freezeouts, then the takeover premium should not depend at all on the level of dilution. Therefore, the model provides a novel and empirically testable relationship between the takeover premium and the level of dilution that is in contrast with other takeover models with dilution in the literature, such as Grossman and Hart (1980) [1] and Burkart, Gromb, and Panunzi (1998) [10].

6. Conclusions

Tender offers are usually associated with an immediate follow-up freezeout merger. This paper proposes a dynamic model of takeover and freezeouts with trading, motivated by the takeover laws prevailing in the U.S. and the U.K. We provide a comprehensive characterization of the equilibrium outcome of takeovers, including the takeover premium, as well as the structure of tender offers. The framework is simple and tractable, and the results of the model are consistent with extensive empirical literature and yields new empirical implications that are yet to be tested.

Arbitrageurs play an important role in determining the takeover premium. For example, the supply curve of shares is endogenously upward-sloping: The greater the number of shares needed by the bidder, the higher the number of arbitrageurs who can form hold-out power. Moreover, when there is a large number of arbitrageurs with hold-out power, they can credibly demand a greater takeover premium in exchange for tendering their shares. Likewise, there is a positive relationship between the premium and arbitrageurs' accumulation of shares before and after the announcement of the offer.

We show that bidders do equally well using either any-or-all offers or front-end loaded bids, such as two-tiered and partial offers, and therefore, fair price charter provisions should be innocuous. Furthermore, the ability to moderately dilute target shareholders does not increase the profits of bidders with freezeout rights. The option to dilute shareholders is not valuable in the presence of the freezeout option, and consequently, there should not be any significant relationship between the takeover premium and dilution levels.

Although this paper focused only on takeovers, there are many other corporate events, such as debt reorganizations of firms in financial distress, in which arbitrageurs play an important role in resolving potential market failures due to free-riding by dispersed shareholders (see Kahan and Tuckman (1993) [35]).³² The dynamic model with trading developed in this paper may also be helpful in studying these other corporate events.

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Appendix A

Proof of Proposition 1. Consider that the bidder makes an offer $p = \varepsilon > 0$ conditioned on f shares. Let the total number of shares tendered be denoted by T. The payoff per share of all shareholders is equal to p if $T \ge f$, regardless of the number of shares tendered because whenever $T \ge f$, the bidder will take over the firm and freezeout shareholders that did not tender. Otherwise, if T < f, then the payoff to all shareholders is equal to 0, the value of the company under the incumbent management. It is then a weakly dominant strategy for all shareholders to tender all their shares: first, your payoff is the same regardless of your tendering, given that the bid is successful decision; second, whenever the total number of shares tendered by all other shareholders, excluding your own shares, is $T_- < f$, the bid could be successful if you tender your own shares t, so that $t > f - T_-$, in which case your payoff is higher. Therefore, it is a weakly dominant strategy for all shareholders to tender.

Proof of Lemma 1. Let the demand schedule of investors be given by P(y), a function of the order flow *y*, defined as follows:

$$P(y) = \begin{cases} p & y < 0\\ p(n) + c & y \in [0, \beta)\\ \phi p(n+1) + (1-\phi)p(n) + c & y = \beta\\ p(n+1) + c & y \in (\beta, 2\beta]\\ \overline{p} & y > 2\beta \end{cases}$$
(A1)

We first prove that the proposed strategy profile of an arbitrageur is a best response given the demand schedule P(y), and then prove that the investors' demand schedule is the best response, given the arbitrageurs' strategy profile.

Suppose that an arbitrageur trades *x* shares such that $x \in (0, \beta)$. The arbitrageur's profits are:

$$E[\Pi(x)] = x\{\pi[p(n) - (p(n) + c)] + (1 - \pi)[p(n) - (p(n+1) + c)] - c\}$$

= -2xc + x(1 - \pi)[p(n) - p(n+1)] < 0

so that no arbitrageur trades $x \in (0, \beta)$. Say that the arbitrageur places an order to sell x shares, x < 0, (or taking a short position if he already owns some shares). Then with probability $1 - \pi$, when noise traders do not demand any shares, the order flow will be y = x < 0, and trading will take place at a price equal to p. Of course, there exists a price p smaller than p(n - 1), so selling blocks or taking short positions will always be unprofitable for arbitrageurs. Similarly, if an arbitrageur buys more than β shares, $x > \beta$, then with probability π the order flow will be $y > \beta$, and trading will take place at a price equal to \overline{p} . Of course, there also exists a price \overline{p} bigger than $p(n_f)$, so that buying more than β shares is unprofitable. \Box

Say now that $x = \beta$, so that arbitrageurs profits are:

$$E[\Pi(\beta)] = \beta \left\{ \begin{array}{l} \pi[p(n+1) - (\phi p(n+1) + (1-\phi)p(n) + c)] + \\ (1-\pi)[p(n+1) - (p(n+1) + c)] - c \end{array} \right\}$$

= $\beta \{-2c + (1-\phi)\pi[p(n+1) - p(n)]\}.$

Let $\overline{c} = \frac{1}{2}\pi[p(n+1) - p(n)]$. If $c \leq \overline{c}$, then $\phi = \frac{\overline{c}-c}{\overline{c}}$, and thus $(1-\phi)\pi[p(n+1) - p(n)] = (\frac{c}{\overline{c}})2\overline{c} = 2c$. Therefore, $E[\Pi(\beta)] = 0$, and the arbitrageur is indifferent between entering and buying a block with β shares. Therefore, the strategy of buying a block of β shares with probability $\phi = \frac{\overline{c}-c}{\overline{c}}$ and with probability $(1-\phi)$ staying out of the market is the best response to P(y). Also, if $c > \overline{c}$ then $\phi = 0$, and thus $E[\Pi(\beta)] = \beta\{-2c + \pi[p(n+1) - p(n)]\} = \beta\{-2c + 2\overline{c}\} < 0$. Therefore, the best response for arbitrageurs, given the pricing function P(y), is not to enter for sure.

The strategy of not trading is an optimal response for other arbitrageurs who either already own blocks or do not, given P(y) and one arbitrageur are playing the strategy described above. Say that an arbitrageur trades x < 0. Then, as we have seen above, there is a probability $(1 - \phi)(1 - \pi)$ that the order flow is y = x < 0, in which case the price is $\overline{x} > 0$. Then, there is a probability $\phi\pi$ that the order flow is $y = 2\beta$, in which case the price \overline{p} is high enough that the arbitrageur incurs losses. Similarly, say that an arbitrageur trades price \overline{p} is high enough that the arbitrageur incurs losses, or else $\phi = 0$, and it is also not profitable to trade any shares, as argued in the previous paragraph. This proves the first part of the proposition.

We now prove that the investors' demand schedule P(y) is the best response given the strategy of the arbitrageurs. In other words, the price quoted by traders is equal to the expected value of the shares, given the observation of the order flow, P(y) = E[p|y]. The order flow is equal to either y = 0, β , or 2β with probability 1. For any order flow that is out-of-equilibrium and occurs with probability 0, we are free to choose any value for the price P(y); therefore, we can fix any P(y) = p if y < 0 and $P(y) = \overline{p}$ if $y > 2\beta$. If $y = 2\beta$, then E[p|y] = (p(n+1)+c) - c = p(n+1) is the value of the shares because an arbitrageur is then buying a block of shares for sure. If $y = \beta$, then $E[p|y] = (\phi p(n+1) + (1-\phi)p(n) + c) - c = \phi p(n+1) + (1-\phi)p(n)$ is the value of the shares because an arbitrageur is buying shares with probability ϕ , in which case shares are worth p(n+1), and with probability $(1 - \phi)$, no arbitrageur is entering, in which case shares shares are worth p(n). If y = 0, then E[p|y] = p(n) is the value of the shares. This finalizes the proof of the proposition.

Proof of Proposition 2. We first show that the strategies proposed are a stationary Nash equilibrium of the tender offer game. Note, that investors' demand for shares is given by $P_n(y)$ equal to:

$$P_{n}(y) = \begin{cases} p & y < 0\\ p(n) + c & y \in [0, \beta)\\ \phi(n)p(n+1) + (1 - \phi(n))p(n) + c & y = \beta\\ p(n+1) + c & y \in (\beta, 2\beta]\\ \overline{p} & y > 2\beta \end{cases}$$
(A2)

We use the one-stage-deviation principle (see Fudenberg and Tirole (p. 109, 1991) [38] to prove that the strategies are a subgame perfect equilibrium. The one-stage-deviation principle states that, in order to prove that a strategy profile for a game in extensive form is a subgame perfect Nash equilibrium, it is sufficient to prove that no player can gain by deviating from the prescribed strategy in a single stage and conforming to the strategy after that.

(i) At the offering stage, the bidder makes an offer $p_B(n)$ conditioned on f shares (and profits $\Pi(p_B(n))$): any offer $p < p_B(n)$, is rejected for sure by arbitrageurs, and the bidder would obtain only $\delta\phi(n)\Pi(p(n+1)) + (1-\phi(n))\Pi(p(n)) = \Pi(p_A(n)) < \Pi(p_B(n))$; any offer $p < p_B(n)$ is accepted by shareholders, and the bidder's profits $\Pi(p) < \Pi(p_B(n))$. Also, any offer that is not conditional on f would fail for sure, because then all shareholders hold out and do not tender any shares, and the bidder obtains only $\Pi(p_A(n)) < \Pi(p_B(n))$.

At the offering stage, an arbitrageur proposes $p_A(n)$ conditional on f shares when it is his turn to propose, and since this offer is immediately accepted by the bidder, and subsequently, all shareholders tender their shares; then the takeover succeeds with probability 1 and shares are worth $p_A(n)$: any offer $p > p_A(n)$ is rejected for sure by the bidder. Shareholders then obtain, $\delta[\phi(n)p(n+1) + (1 - \phi(n))p(n)] = p_B(n) < p_A(n)$; any offer $p < p_A(n)$ is accepted by the bidder and shareholders obtain $p < p_A(n)$. Offers that are not conditional on f shares are also rejected by the bidder.

Therefore, neither the bidder nor the arbitrageur wants to deviate from the equilibrium strategies at the offering stage.

(ii) At the tendering stage, shareholders, including dispersed and large shareholders, tender all their shares if the offer is conditional on f shares and the price is $p \ge p_B(n)$, and do not tender any shares otherwise. Suppose that the takeover fails. Shareholders then obtain $\delta[\phi(n)p(n+1) + (1-\phi(n))p(n)] = p_B(n)$. Suppose that the takeover succeeds, then all shareholders obtain p regardless of whether or not they tendered their shares. Therefore, it is a strictly dominant strategy for arbitrageurs to tender, and it is a weakly dominant strategy for dispersed shareholders to tender whenever $p \ge p_B(n)$. Also, the bidder's response to an arbitrageur's offer is to accept any offer of arbitrageurs $p < p_A(n)$ and reject any offer $p \ge p_A(n)$. If the bidder rejects the offer p, then he obtains $\delta\phi(n)\Pi(p(n+1)) + (1-\phi(n))\Pi(p(n)) = \Pi(p_A(n))$, and $\Pi(p) > \Pi(p_A(n))$ if $p < p_A(n)$ and $\Pi(p) \le \Pi(p_A(n))$ if $p \ge p_A(n)$.

Therefore, neither the bidder nor the arbitrageur wants to deviate from the equilibrium strategies at the tendering (accept/reject) stage.

(iii) The strategies prescribed at the trading game are a Nash equilibrium, by Lemma 1, if and only if

$$\phi(n) = \left(1 - \frac{2c}{\pi[p(n+1) - p(n)]}\right)^+,$$
(A3)

for all integers $n, 0 \le n \le n_f$, where p(n) and $\phi(n)$ are given by

$$p(n) = v(n),$$
 (A4)
 $\phi(n) = 0,$

if $n \ge \underline{n}$ and by

$$p(n) = \delta\left(p(n+1) - \frac{2c}{\pi}\right) + (1-\delta)v(n),$$

$$\phi(n) = \frac{p(n) - v(n)}{p(n) - v(n) + \left(\frac{\delta}{1-\delta}\right)\frac{2c}{\pi}},$$
(A5)

if $n < \underline{n}$ where $\underline{n} = \min\left(n_f, \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{\pi}{c}}\right)$. Note that $\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{\pi}{c}}$ is the solution of $v(n+1) - v(n) = \frac{2c}{\pi}$, and thus, if $n \ge \underline{n}$, then $\phi(n) = 0$ and no arbitrageur enters for sure. For $n < \underline{n}$, replacing the expression for p(n+1) (A3) yields the formula for $\phi(n)$.

Therefore, the prescribed strategies are a Nash equilibrium of the trading game, finally proving the claim that the strategy profile is an SPE equilibrium.

At last, it is straightforward to prove that the limit as δ converges to 1 is equal to p(n) = v(n) if $n \ge \underline{n}$ and equal to $p(n) = p(\underline{n}) + [n - \underline{n}]\Delta > v(n)$ if $n < \underline{n}$.Note that $[n - \underline{n}]$ is the integer part of a negative number. \Box

Proof of Proposition 5. Say that the two-tiered offer is structured as follows: the front-end price is $\frac{\hat{p}}{f}$ for a maximum of f shares; the offer is conditional on f shares, and the back-end price is equal to the appraisal value of shares, which, say, for simplicity is equal to the pretender offer price of 0. The blended price of the offer is thus equal to $\hat{p} = \frac{\hat{p}}{f} \cdot (f) + 0 \cdot (1 - f)$ if all shareholders tender their shares.

We allow for a trading session to take place after the tender offer is announced and before the offer expires. After this trading session, the tender offer game proceeds exactly as before. We will prove that in equilibrium, the blended price is equal to $\hat{p}(n) = p(n+1) - \frac{2c}{\pi}$, where *n* is the number of arbitrageurs owning shares more than or equal to $\beta = 1 - f - \alpha$ and p(n+1) is as in Equation (9). Note, that for δ arbitrarily close to 1, $\hat{p}(n) = p(n)$ for all *n*.

We prove that the following strategy profile is an SPE equilibrium. Consider any subgame at the offering stage with *n* arbitrageurs:

(i) At the offering stage, the bidder makes a two-tiered offer in which the blended price is $p_B(n)$, when it is his turn to propose, where $p_B(0) = 0$ and $p_B(n)$ is as in Equation (7) if n > 0. Arbitrageurs, if any are present, propose a two-tiered offer with a blended price equal to $p_A(n)$, when it is their turn to propose, where $p_A(n)$ is as in Equation (8). (ii) At the tendering stage, dispersed shareholders tender all their shares, and arbitrageurs tender if the offer is greater than or equal to $p_B(n)$, and do not tender any shares otherwise. The insider's strategy is to tender α shares if and only if the bid is greater than or equal to $p_B(n)$ (note that even in a hostile takeover, the insider's response to an arbitrageur offers to accept any offer of arbitrageurs with a blended price lower than or equal to $p_A(n)$ and reject any offer above $p_A(n)$. Let p(n) be as in Equation (9) if n > 0 and p(0) = 0. (iii) At the trading session, one arbitrageur with no blocks places orders to buy blocks of β shares with probability $\phi(n)$ and does not buy any shares with probability $1 - \phi(n)$, where

$$\phi(n) = \left(1 - \frac{2c}{\pi[p(n+1) - p(n)]}\right)^+;$$
(A6)

all other arbitrageurs do not trade any shares, and the investors demand schedule is equal to $P_n(y)$, as in Equation (A2).

The proof that the strategy profile above is an SPE equilibrium follows the same line of reasoning as the proof of Proposition 2. In equilibrium, the value of shares when the bidder is allowed to make a two-tiered offer is, for all $n \ge 0$, equal to

$$\widehat{p}(n) = \phi(n)p(n+1) + (1-\phi(n))p(n) = p(n+1) - \frac{2c}{\pi}.$$

The case of more interest is the one where n = 0. Note, that the bidder is unable to coerce shareholders to tender if the blended price is low because in equilibrium, this would allow arbitrageurs to enter with a high probability equal to $\phi(0) = \left(1 - \frac{2c}{\pi[p(1)]}\right)^+$ and drive the price up to p(1). The equilibrium stock price is then equal to $\hat{p}(0) = \phi(0)p(1) + (1 - \phi(0))0 = p(1) - \frac{2c}{\pi}$, which is the same outcome as in the case where the bidder makes an any-or-all offer. \Box

Notes

- ¹ In a tender offer, the acquirer proposes a per-share price to the target shareholders, who then have the choice of whether or not to sell at the offer price. In a merger, the acquirer and the target board of directors agree on a price, and the target shareholders then vote on whether or not to approve the proposal.
- ² Source: Our analysis using the S&P Capital IQ database.
- ³ The freezeout fraction in the U.S. depends on state regulation and corporate charters. Before the Model Business Corporate Act of 1962, most states had a 2/3 supermajority requirement. After the Act's passage, most states, including Delaware, adopted a simple majority requirement. Some states, such as New York, Ohio, and Massachusetts, retained the 2/3 supermajority requirement. In addition, several U.S. firms—around 18 percent of the 1500 large capitalization firms profiled by the Investor Responsibility Research Center in 1995—have amended their charters to include supermajority merger provisions. In the U.K. and several other European countries, such as Sweden, the freezeout fraction equals 90 percent.
- ⁴ Franks and Harris (1989) [13] report that offers are revised in over 9 percent of the uncontested (single-bidder) takeovers in the U.K.
- ⁵ U.S. takeover rules are contained in Section 14(d)(1)–(7) of the Securities Exchange Act of 1934, and Securities and Exchange Commission (SEC) Rules 14D and 14E. U.K. takeover rules are contained in the City Code on Takeover and Mergers (see Johnston (1980) [17]).
- In the U.S., Rule 14d-1 requires that bidders disclose material information about the offer to shareholders at the commencement of the offer. In the U.K., General Principles 3 and 10 and Rule 8 contain similar provisions.
- ⁷ In the U.S., Rule 14e-1 requires that the offer must be held open for a minimum of 20 business days. Any revision in the offer requires that the offer be kept open for at least ten additional business days. In the U.K., Rule 22 provides for a minimum of 21 days after the posting of the offer and a 14-day delay after revisions.
- ⁸ In the U.S., Rule 14d-10 contains the "all-holders" and "best-price rule" provisions. Under these provisions a tender offer must be made to all target shareholders, and each shareholder must be paid the highest consideration paid to any other shareholder during the offer. In the U.K., similar treatment is prescribed by General Principle 8 and Rule 22.
- ⁹ In the U.S., Rule 14d-7 gives the target shareholders withdrawal rights during the life of the offer (extending the withdrawal rights provided by Section 14(d)(5)). In the U.K., Rule 22 specifies that shareholders have withdrawal rights only after the expiry of 21 days from the first closing date of the initial offer.
- ¹⁰ Typically, a merger transaction occurs only after the boards of both companies involved approve the transaction, and a specified percentage of shareholders of both corporations vote in favor of the transaction. Therefore, tender offers are the only form available to conduct a hostile acquisition when the manager disapproves of the transaction.

- ¹¹ This valuation is based on the fair value of the shares, exclusive of the gains in value created by the bidder. Corporations and state corporate laws also commonly have fair price provisions that require the same price be paid to shareholders in both the tender offer and the second-step merger transaction. We will address the effect of such fair price provisions in the paper.
- ¹² Rule 21 of the Code (see Johnston (1980) [17]).
- ¹³ This is without much loss of generality because truly unconditional offers are very uncommon. A bidder can usually extend the offer until a majority of shares are tendered or just withdraw the offer before the expiration if less than a majority of shares are tendered. Therefore, unconditional offers can be seen, in practice, as offers that are conditional upon the acquisition of majority control (see Hirshleifer and Titman (1990) [18]).
- ¹⁴ We assume in this paper that the insiders do not have any bargaining power other than the ability to control the tendering decision of α shares.
- ¹⁵ The ability of acquirers to change the offer over time motivates the use of a dynamic game. Also, provisions that regulate the minimum duration of the offer and its revisions give shareholders time to trade before the expiration of the offer.
- ¹⁶ This is similar to several other bargaining models, such as Gul (1989) [19], Hart and Mas-Colell (1996) [20].
- ¹⁷ The rules that ensure equal treatment of shareholders eliminate the possibility of any price discrimination during the offer; therefore, all offers are extended to all shareholders.
- ¹⁸ In the U.S., Rule 10b-13 prohibits an offeror from buying any shares of the target company in the open market or private negotiations during the tender offer. There are also rules restricting the issuer or other related persons from repurchasing shares in the market during a third-party tender offer (Rule 13e-1 and insider trading rules of Section 16). However, in the U.K., the bidder is allowed to purchase shares at the offering price in the open market while the offer is open.
- ¹⁹ We assume in the model that arbitrageurs and bidders have the same cost of capital. The model can easily be changed to accommodate the case in which arbitrageurs have a higher cost of borrowing than the bidder.
- ²⁰ Note that, unlike Harrington and Prokop (1993) [21], all shares tendered are purchased at the final bid price *p*, including shares that may have been tendered early on, before an increase in the bid price. This is in accordance with Rule 14d-10 in the U.S. and Rule 22 in the U.K.
- See Section 4.1 for a discussion of rules in the U.S. and the U.K. that allow a bidder to commit to the minimum tender condition.

²² Interestingly, Lee and Oh (2022) [22] analyze freeze-out mergers in cases where shareholders have multiple shares and show that a raider's increased ability to freeze out non-tendering shares lowers shareholders' incentives to free ride.

- ²³ The existence of unsuccessful takeovers can be reconciled with a model in which there is some exogenous variable that influences the success of the offer. Also, a model of bargaining with incomplete information about valuations allows for the possibility of delays in the takeover and entry of arbitrageurs in equilibrium.
- ²⁴ See also Stulz et al. (1990) [9] for other reasons for an upward-sloping supply curve.
- ²⁵ Bradley et al. (1988) [2] suggest that bidders' profits are, on average, only 10 percent of total synergy gains. Their estimate includes contested offers, which occur in approximately 29 percent of the cases. Also, Roll (1986) [25] proposes that many acquirers exhibit irrational behavior and overpay or overestimate the value of targets, and many acquirers overpay for acquisitions motivated by empire building (see Jensen (1986) [26]).
- ²⁶ These four features are the ones that usually appear conspicuously on the front page of offers to purchase in the U.S. and U.K.
- ²⁷ See SEC Rel. No. 34-23421, CCH Fed. Sec. L. Rep. ¶84,016. See Johnston (1980) [17] for similar rules in the U.K.
- In order for a two-tiered offer to be effective, in the takeover with no dilution case, it is necessary that the minimum tender condition is equal to f, which in many cases, when k = f, is identical to conditioning upon obtaining a majority of the shares.
- ²⁹ For example, Barclay and Holderness (1989) [30] suggest that large controlling blocks can obtain, on average, only 5% in private benefits.
- ³⁰ We expect that incorporating a measure of liquidity into the regression of Jindra and Walkling should increase their explanatory power.
- ³¹ It is an interesting issue for further research to determine the motives that led the 123 single-bidders in the Franks and Harris (1987) [13] sample to revise their bids.
- ³² Also, it may be interesting to address efficiency and coordination problems in takeovers with freezeouts. Maug (2006) [36] focuses on the trade-off between efficiency and fairness in freezeouts and proposes alternative rules for valuing minority shares in freezeouts; Baek and Oh (2015) [37] focus on tender offers where shareholders face coordination problems during takeovers and show the freezeout amount determines the blockholder's size and the expected amount of abnormal returns.

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