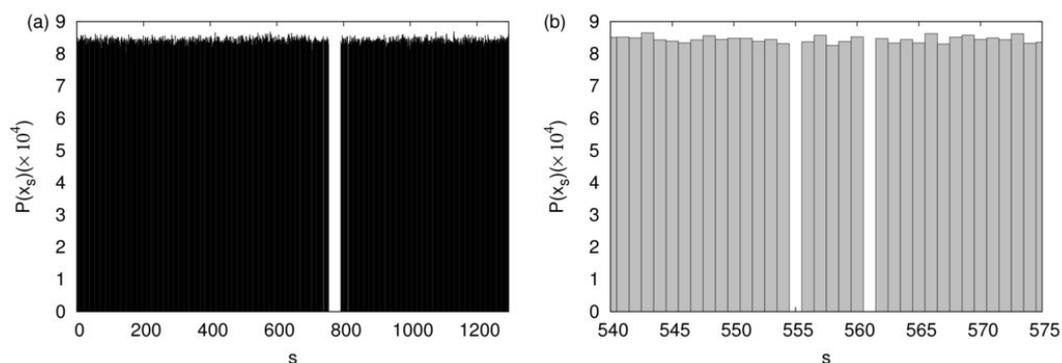
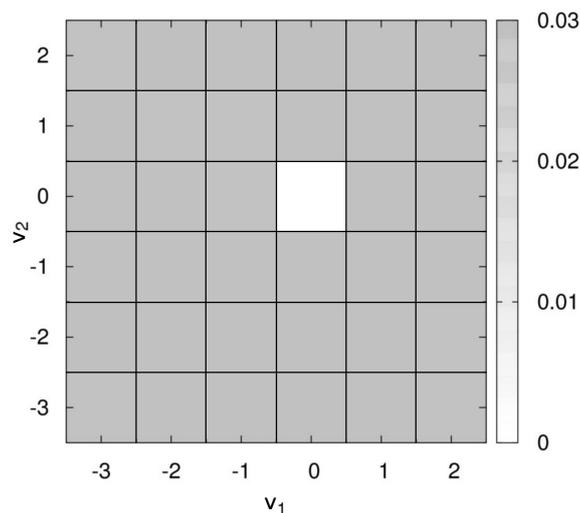


# Supplementary Materials: THREADING DYNAMICS OF RING POLYMER MELTS REVEALED BY DYNAMICALLY CONSTRAINED LATTICE MODEL

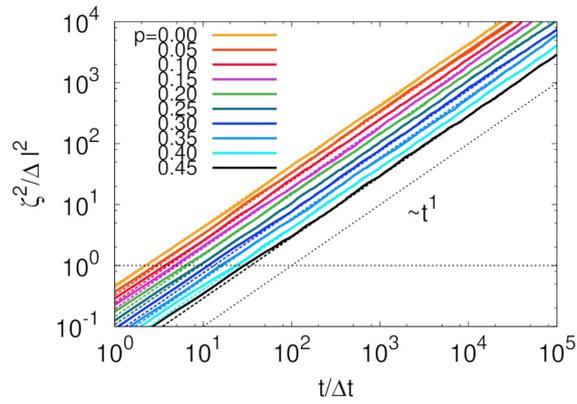
Eunsang Lee<sup>†</sup> and YounJoon Jung<sup>\*†</sup>



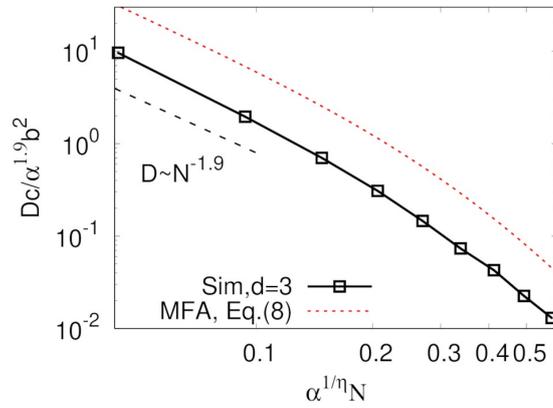
**Figure S1.** Probability of microstates  $\mathbf{x}_s = (v_1, v_2, w_1, w_2)$  in equilibrium for two-dimensional three-particle system with  $L = 6$ , where  $\vec{x}_{12} = (v_1, v_2)$  and  $\vec{x}_{13} = (w_1, w_2)$ . The state index  $s$  is defined by  $s = (v_1 + L_h)L^3 + (v_2 + L_h)L^2 + (w_1 + L_h)L + w_2 + L_h$  where  $L_h = L/2$ . (a) shows the probability of whole states, and (b) shows only for the states with  $\vec{v}_{12} = (-1, 0)$ . In (a),  $756 \leq s \leq 791$  where  $P(\mathbf{x}_s) = 0$  represents the state of  $\vec{v}_{12} = (0, 0)$ , which is not allowed by the particle exclusion. In (b) the states of the probability zero in this graph are for  $\vec{w} = (-1, 0)$  and  $\vec{w} = (0, 0)$ . Except for the case of the particle exclusion, probabilities of all states are almost the same which demonstrates the validity of the detailed balance.



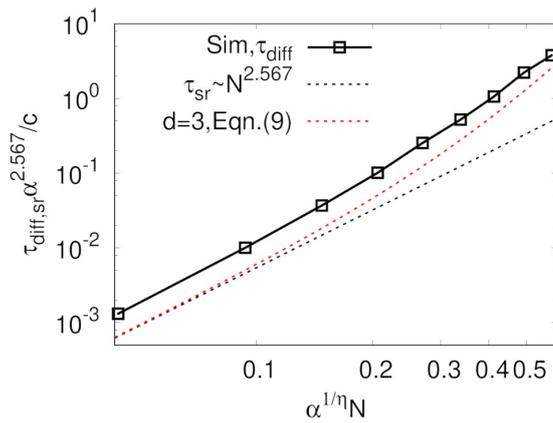
**Figure S2.** Contour plot of probability of microstates  $\mathbf{x}_{12} = (v_1, v_2)$  in equilibrium for two-dimensional two-particle system with  $L = 6$ . Every state except  $\vec{x}_{12} = (0, 0)$  is equally probable.



(a)



(b)



(c)

**Figure S3.** (a) Mean square displacement versus  $t$ , (b) diffusion coefficients and (c) diffusion times as a function of degree of polymerization,  $N$  for the system of  $d = 3$ ,  $f = 0.5$ . In Figs.(b) and (c), red dotted lines represent Eqns. (8) and (9) in the main text, respectively. Black dashed lines indicate the approximate scaling relations in small  $\alpha N$  regime, such that  $D \sim N^{-1.9}$  and  $\tau_{diff} \sim N^{2.567}$ , respectively. In both graphs,  $\alpha$ ,  $b$ , and  $c$  are numerical constants to scale the threading probability unit time and unit length on  $N$ , respectively.