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# Hybrid Optimization Algorithms of Firefly with GA and PSO for the Optimal Design of Water Distribution Networks

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**Abstract:** A novel two-hybrid optimization model of particle swarm optimization (FAPSO) and firefly algorithm with genetic algorithm (FAGA) are introduced to improve the performance of the conventional firefly algorithm for the least cost design of water distribution networks. The performance of the models is tested through application to three of the well-known benchmark networks available in the literature and also to the real case study of the El-Mostakbal City network, Ismailia, Egypt. The performance of the different algorithms was determined by evaluating the minimum, maximum, mean and standard deviation of costs, the function evaluation number, the consumed computational time for 1000 evaluations and the success rate calculated using the fuzzy logic concept for different optimal solutions slightly greater than the known optimal solution (by about 1.0% and 2.0%) were utilized for testing the convergence and search capabilities of the models. It was found that the FAGA model is superior to the standard firefly and FAPSO models in exploring the search space, exploiting the promising areas and convergence to the optimal solution and can be considered as a reasonable optimization technique for the management of water distribution networks.

**Keywords:** water distribution networks; hybrid optimization; firefly algorithm; genetic algorithm; particle swarm optimization; performance evaluation

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## 1. Introduction

Water distribution networks are one of the most important necessary infrastructures for the development of countries worldwide. Huge investment is required for the construction of such networks, and a relatively small decrease in this cost leads to a considerable total saving which can be achieved by selecting the pipe diameters from a set of available market sizes to minimize the total construction cost. This process is referred to as the optimal design of water distribution networks. Numerous optimization techniques were and are still being developed for such purpose of searching in a very large non-differentiable, and nonconvex design space. It is a discrete type of problem and very computational demand. However, still, there is no agreement on what optimization method is best for a particular design problem, and considerable research challenges remain essential [1].

Standard optimization techniques early developed include linear programming (LP) applied only to linear objective functions, equations and constraints, nonlinear programming (NLP) to deal with nonlinear problems and dynamic programming (DP) to solve stochastic and nonlinear problems. These techniques get stuck in local optimum solutions and do not guarantee global optimum performance. Besides, limitations when applied to problems with high dimensionality, uncertainty and requires high computational time [2].

In the last few decades, heuristic and metaheuristic techniques have been developed to overcome such limitations. Among these techniques are genetic algorithm, particle swarm optimization and firefly algorithm. The genetic algorithm (GA), first introduced by Holland [3], is the most popular evolutionary population-based algorithm which has been adopted in many complex optimization problems in water resources applications. Many researchers have attempted to improve the computational efficiency of the algorithm. Reca et al. 2017 [4] introduced the bounded genetic algorithm (B-GA) model to reduce the search space by decreasing the number of available pipe sizes that can be used for each of the network pipes. Recently, Sangroula et al. [5] introduced the smart optimization program for water distribution networks (SOP-WDN) which is based on a genetic algorithm linked with the EPANET hydraulic simulation solver. Particle swarm optimization PSO first introduced by Eberhart and Kennedy [6], is inspired by the behavior of a flock of birds and has been widely used in various optimization problems in different fields because of its simplicity and the few parameters that have to be adjusted.

Most of the optimization techniques were introduced in their standard form, then followed by different modifications that were found necessary to improve their reliability, robustness and convergence so that they become more and more effective for the optimization of water distribution networks. Among these modifications is the hybridization of different optimization algorithms, which recently have been widely considered by many researchers for performance improvement. (bacterial foraging, genetic algorithm and ant colony, [7]) (particle swarm and Hooke–Jeeves, [8]), (particle swarm and tabu search, [9]), (particle swarm optimization and cuckoo search, [10]), (grasshopper optimization algorithm and genetic algorithm, [11]). The firefly optimization algorithm has received considerable attention through being hybridized with other algorithms for different applications, as shown in Table 1, which summarizes the different hybrid firefly models.

**Table 1.** Different hybrid firefly optimization algorithms.

| Author's | Hybrid Firefly Model | Case Study |
|---|---|---|
| Zervoudakis et al. (2020) [12] | Firefly and Genetic Algorithm | Product Line Design Problem |
| Abdullah et al. (2012) [13] | Firefly-Differential Evolution (HEFA) | Complex and Nonlinear Problems |
| Tahershamsi et al. (2014) [14] | Firefly-Harmoni Search | Optimization of Water Distribution Systems |
| Gu et al. (2013) [15] | Firefly and Harmony Search | Global Numerical Optimization |
| Kora and Krishna (2016) [16] | Firefly and Particle Swarm Optimization | Detection of Bundle Branch Block |
| Elkhechafi et al. (2017) [17] | Firefly- Genetic Algorithm | Global Optimization |
| Aydilek (2018) [18] | Firefly-Particle Swarm Optimization | Computationally Expensive Numerical Problems |
| Nhu et al. (2020) [19] | Firefly-Particle Swarm Optimization | Rainfall induced Flash Floods |
| Khan et al. (2020) [20] | Firefly-Particle Swarm Optimization | Standard IEEE 30-Bus Test System |
| Yadav et al. (2021) [21] | Firefly and Biogeography-Base Optimization | Software Production Line |
| Wahid and Ghazali (2021) [22] | Firefly and Genetic Algorithm | Minimization and Maximization Functions |
| Bilal and Millie Pant (2020) [23] | Firefly and Particle Swarm Optimization | Optimization of Water Distribution Systems |

## 2. Materials and Methods

### *2.1. Formulation of Pipe Networks Optimization*

The problem of pipe network optimization is to find the best combination of pipe diameters among a set of commercially available diameters as discrete decision variables that provides the least construction cost for the network satisfying prescribed constraints. The problem can be formulated as reported in Ezzeldin and Djebedjian [24] by the minimization of the objective function $f$ satisfying both design constraint and hydraulic constraints, continuity and energy, as follow;

#### 2.1.1. Total Pipe Cost

The total pipe cost of the network can be expressed by:

$$C_T = \sum_{i=1}^{N_{pipes}} c_i(D_i) * L_i \tag{1}$$

where $C_T$ is the total construction cost of the network, $N_{pipes}$ is the number of pipes, $c_i$ $(D_i)$ is the cost of pipe $i$ of discrete diameter $D_i$ per unit length, and $L_i$ is the length of pipe $i$.

#### 2.1.2. Objective Function

The objective function of cost minimization is:

$$Minimize\ f = \left\{ \begin{matrix} C_T\ if\ H_{j,min} - H_j \le 0 \\ C_T + C_p\ else \end{matrix} \right\}, \tag{2}$$

where $C_p$ is the penalty cost = $P_C \sum_{j=1}^{N_{nodes}} \left(H_{j,min} - H_j\right)$, $P_C$ is the penalty cost coefficient taken equal to 10,000, $H_{j,min}$ is the minimum allowable head at node $j$, $H_j$ is the head at node $j$ and $N_{nodes}$ is the number of nodes in the pipe network.

#### 2.1.3. Hydraulic Constraints

Continuity constraint;

$$\sum_{j=1}^{N_{nodes}} Q_j = 0, \tag{3}$$

$Q_j$ is the discharge at node $j$.
Energy constraint;

$$\sum h_f = E_p, \tag{4}$$

$h_f$ is head loss due to friction in pipe calculated using Hazen–Williams formula given by $h_f = \frac{10{,}674 * L_i * Q_i^{1.852}}{C_i^{1.852} * D_i^{4.87}}$; $C_i$ is the Hazen–Williams Coefficient, $Q_i$ is the discharge in pipe $i$ and $E_p$ is the energy supplied by a pump.
Design constraint;

$$D_{min} \le D_i \le D_{max}\ i = 1 \ldots\ldots N_{pipes}, \tag{5}$$

$D_{min}$, $D_{max}$ are the minimum and maximum commercially available pipe diameters, respectively.
Nodal head constraint;

$$H_j \ge H_{j,min}\ j = 1 \ldots\ldots N_{nodes}, \tag{6}$$

A MATLAB code is developed to execute the hydraulic simulation model given by Equations (3) through (6) and an optimization code written in MATLAB is used to solve the optimization model. Both simulation and optimization models are linked to solving the simulation–optimization model.

### *2.2. Benchmark Networks*

Three benchmark networks are considered in this study, namely, two-loop (Alperovits and Shamir [25]) with an optimal solution of 419,000 cost units, Hanoi (Fujiwara and Khang [26]) with an optimal solution of 6081 × 10^6$, and New York water supply system (Schaake and Lai [27]) with optimal solution of 38,637,600$. The first network is the hypothetical two-loop network shown in Figure 1, which consists of 8 pipes of 1000 m constant length and 7 nodes all fed by gravity from a single reservoir 210 m fixed elevation. Each pipe in the network is selected among 14 available discrete pipe diameters of 1, 2, 3, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, and 24 inches with arbitrary unit costs of 2, 5, 8, 11, 16, 23, 32, 50, 60, 90, 130, 170, 300 and 550, respectively. The minimum allowable nodal head is 30 m, and the Hazen–Williams Coefficient is 130 for all pipes—the optimization algorithm searches for the optimal solution in $14^8$ possible solutions for the network design.

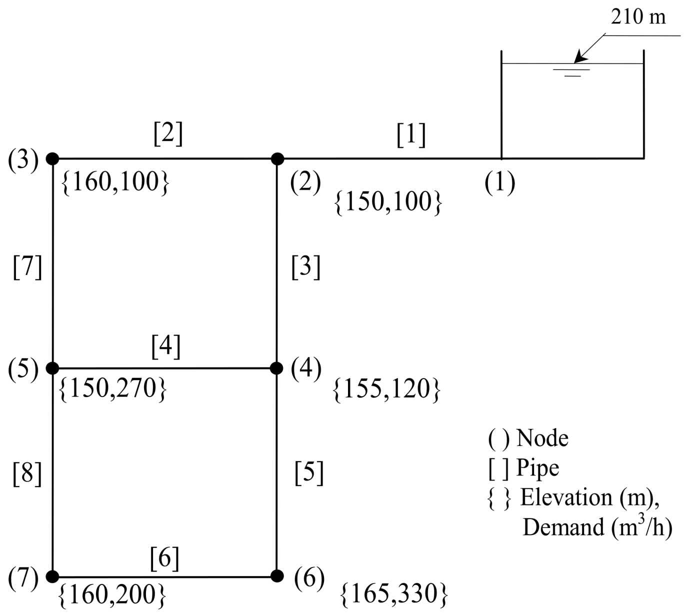


**Figure 1.** Two-loop network.

The second network is the Hanoi City network, Vietnam, shown in Figure 2, which is a three-loop network consisting of 34 links joined at 32 nodes and fed by gravity from a constant head reservoir of 100 m in elevation. The design of the network is restricted to selecting 6 discrete commercially available pipe sizes of 12, 16, 20, 24, 30, and 40 inches of, which cost 45.73, 70.4, 98.38, 129.3, 180.8 and 278.3 \$/m, respectively. The minimum head at each node is required to be greater or equal to 30 m. above ground level, with the Hazen–Williams coefficient being 130 for all pipes. The optimal design of the network is searched among $6^{34}$ possible network designs.

The third network is the New York City water supply system, for which the layout is shown in Figure 3. The network is fed from the single source Hillview Reservoir at a constant level of 300 ft and comprises 21 pipes and 20 nodes arranged in two loops. As a result of the growing demands at certain nodes 16, 17, 18, 19, and 20 in the existing network, it was required that the network be rehabilitated in order to increase the prespecified nodal pressures to meet the new conditions. A parallel expansion has been proposed by constructing new gravity tunnels parallel to the existing ones to increase the heads at nodes 16, 17, and 18 to 260, 272.8, and 272.8 and 255 ft., respectively, while maintaining a head of 255 ft. at other nodes. The solution space has $16^{21}$ possible designs with a Hazen–Williams roughness coefficient of 100. The available pipe diameters are in inches and cost \$/ft. are 0(0), 36(93.5), 48(134), 60(176), 72(221), 84(267), 96(316), 108(365), 120(417), 132(469), 144(522), 156(577), 168(632), 180(689), 192(746), and 204(804).

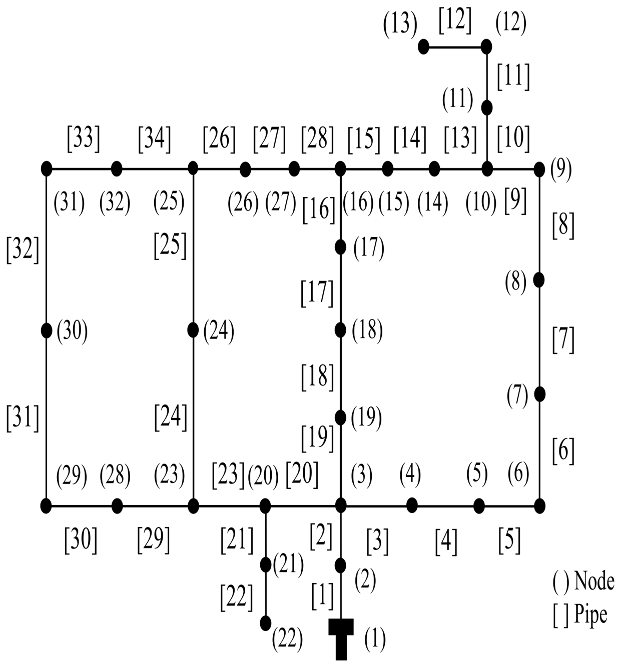


**Figure 2.** Hanoi city network, Vietnam.

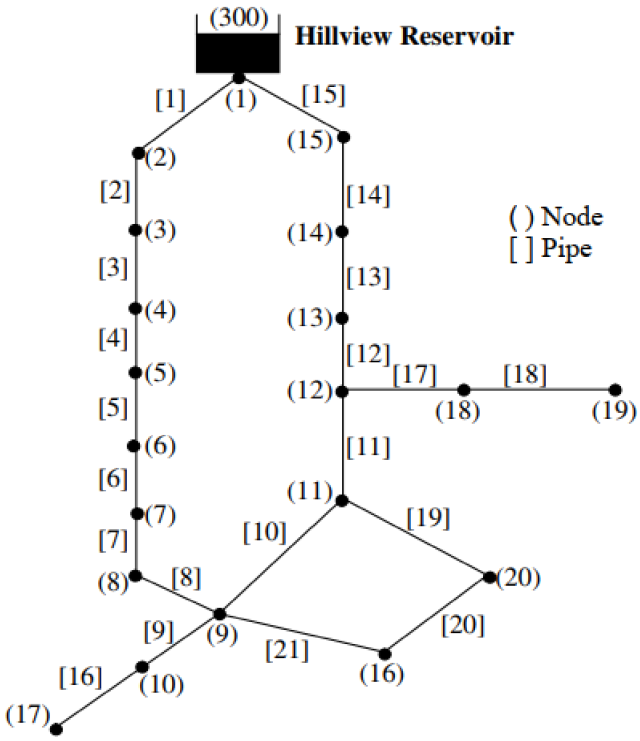


**Figure 3.** New York City water supply system.

For the selected benchmark networks of known optimal solutions obtained by many researchers, the optimization models are tested for reaching the known optimal solutions with better performance and search capabilities in the solution space.

### *2.3. Real Case Study of El-Mostakbal City Network*

The models are also applied to the real case study of the El-Mostakbal city network shown in Figure 4, which has been constructed as an extension to the Ismailia city network in Egypt (Rayan et al. [28]). The network is of the unknown optimal solution for which the models are applied and tested to be able to reach a new optimal solution better than that of 4,926,560.7 LE using the modified Jaya algorithm (Abdel-Gawad [29]). As a real large-scale network that has a huge solution space of $10^{44}$ possible solutions, the El-Mostakbal city network is recommended to be used for testing the search capability and performance of the different optimization algorithms (Abdel-Gawad [29]). The network has 44 pipes and 33 nodes. The available pipe sizes are 0.15, 0.2, 0.25, 0.3, 0.4, 0.5, 0.6, 0.8, 1.0 and 1.2 m. at the cost of 188, 255, 333, 419, 570, 735, 1110, 1485, 2505 and 3220 LE/m., respectively. The network is designed to satisfy a minimum required head of 22 m at all nodes with Hazen–Williams coefficient of 22 m for all pipes. The network has previously been analyzed by many researchers using different optimization techniques, including Rayan et al. [28], El-Ghandour and Elbeltagi [30], Ezzeldin and Djebedjian [24] and Abdel-Gawad [29].

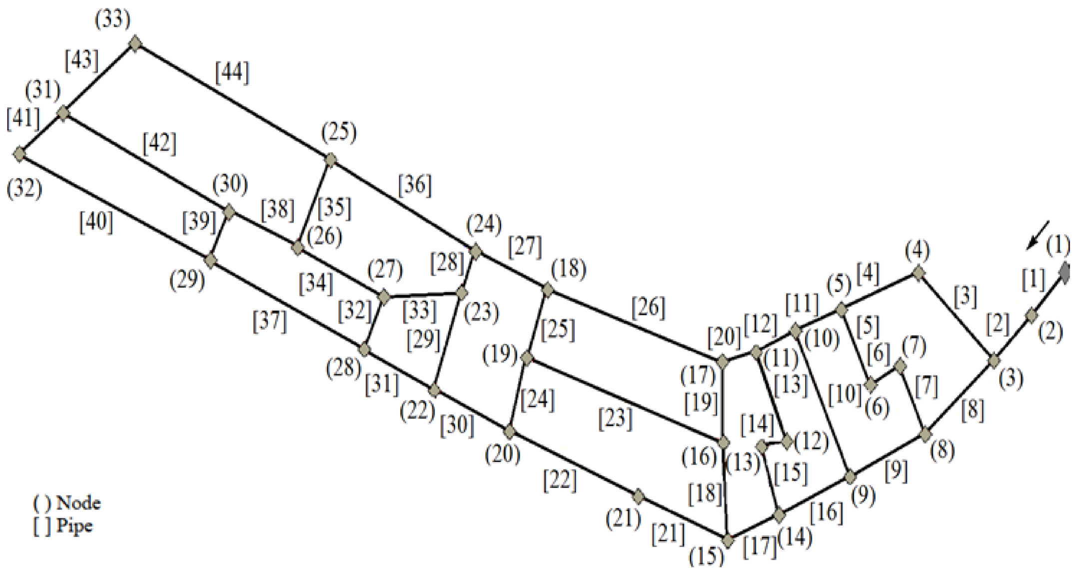


**Figure 4.** El-Mostakbal city network, Ismailia, Egypt [28].

## 3. Firefly Optimization Algorithm

The firefly optimization algorithm first introduced by Yang [31] is a swarm intelligence population-based metaheuristic optimization technique inspired by the natural behavior of fireflies and can be used to solve both continuous and discrete optimization problems [12]. The algorithm proved to be an efficient search method for non-linear optimization problems and, when compared to PSO and GA for finding the global optima of various test functions, showed superiority in terms of efficiency and success rate [31]. However, the algorithm could fall into a locally optimal solution or suffer from low accuracy when solving high-dimensional optimization problems [32]. The algorithm has been applied to many problems related to water resources engineering [33–36]. Up to the knowledge of the authors, the firefly algorithm has not yet been applied for the least cost design of water distribution systems.

### *3.1. Formulation of Firefly Algorithm (FA)*

Firefly algorithm, FA is based on the following three rules, (1) all fireflies are unisex, (2) attractiveness is proportional to their brightness, and (3) the brightness of a firefly is determined by the landscape of the objective function.

A detailed formulation of the firefly algorithm, as found in Yarpiz [37], can be summarized as follow:

1. Initialize the input parameters for *FA*.
2. Generate an initial population of $n_{pop}$ fireflies for the dimension of $N_{pipes.}$
3. The total construction cost of the network, $C_T$ and the corresponding constraint for each firefly is evaluated using the simulation model.

4. The fitness of each firefly, $f_i$, $i$ = 1, 2, 3, ..., $n_{\text{pop}}$ (the summation of total construction cost and penalty due to the constraints violation [Equation (2)]) is computed.
5. Compare the finesses $f_i$ and $f_j$ for each of the two fireflies $i$ and $j$, respectively, ($i$ and $j$ = 1: $n_{pop}$ and $i \neq j$).
6. If $f_i > f_j$, firefly $i$ moves towards firefly $j$. Update the position of firefly $i$, $X_i(t)$ according to Equation (7) and calculate its fitness $f'_i$ at the new position, $X_i(t+1)$.

$$X_i(t+1) = X_i(t) + \beta.r_n\big(X_i(t) - X_j(t)\big) + \alpha_1(t)\Delta R_n \tag{7}$$

where $\beta = \beta_0 e^{-\gamma r_{ij}^2}$ *is* the attractiveness, $\beta_0$ = coefficient base value at $r$ = 0, $\gamma$ = light absorption coefficient and $r_{i,j}$ calculated as:

$$r_{ij} = {}^{d_{ij}}/_{d_{max}} \tag{8}$$

where $d_{ij}$ is the distance between any two fireflies $i$ and $j$ which can be determined by the cartesian distance in the form:

$$d_{ij} = \sqrt{\sum_{k=1}^{N_{pipes}} \left(X_{i,k} - X_{j,k}\right)^2}$$

and

$$d_{max} = \sqrt{\sum_{k=1}^{N_{pipes}} \left(D_{max_k} - D_{min_k}\right)^2}$$

$X_{i,k}$ is the $k^{th}$ component of the spatial coordinate $X_i$ of $i^{th}$ firefly, $D_{min}$ and $D_{max}$ are vectors of the minimum and maximum allowable diameters, $r_n$ is a vector with uniformly distributed random numbers, $\Delta$ = 0.05 ($D_{max} - D_{min}$) is the uniform mutation range, $R_n$ is a vector with continuous uniform distribution with the lower endpoints-1 and upper endpoint 1 and $\alpha_1(t) = \alpha_0 d_r^{t-1}$ is the mutation coefficient at iteration $t$ in which $\alpha_0$ = initial mutation coefficient and $d_r$ is the mutation coefficient damping ratio

7. If $f'_i < f_i$ replace the position of the firefly $i$, $X_i(t)$ with the updated one, $X_i(t+1)$ otherwise keep the old position of the firefly $i$.
8. Repeat Steps 5 to 7 until the maximum number of iterations, $n_{iter}$ is reached.
9. Rank the fireflies and find the current best solution.

### *3.2. Hybrid Firefly-Particle Swarm Optimization (FAPSO) Model*

Aydilec [18] Combined the search ability of firefly FA and particle swarm PSO optimization algorithms through hybrid firefly and particle swarm optimization model which used the ability of PSO to provide fast convergence in exploration (local optima) in the global search while FA is generally used in local search due to its ability in fine-tuning in exploitation (global optima). The model can be summarized in the following steps:

1. Initialize the input parameters of the *FA* and *PSO* algorithms.
2. Generate an initial population of $n_{pop}$ particles with random positions and velocity on $N_{pipes}$ dimensions in the solution space.
3. Calculate the fitness, $f_i$ for each particle, $i$ in the population ($i$ =1, 2. 3. ..., $n_{pop}$)
4. Select the social global best, *gbest* and personal best, *pbest* particles.
5. Compare each particle's fitness $f_i$ value in the population with *gbest* in the last iteration $(t-2)$. If $f_i$ < or equal $gbest(t-2)$ ($t$ > 2, $t$ indicates the iteration number) start local search using *FA* as given in Equations (9) and (10)

$$X_i(t+1) = X_i(t) + \beta(X_i(t) - gbest(t-2)) + \alpha\left(r_n - \frac{1}{2}\right) \tag{9}$$

$$V_i\ (t+1) = X_i(t+1) - X_i(t)$$

$$\text{the attractiveness } \beta = \beta_o e^{-\gamma r_{ij}^2} \text{ in which } r_{ij} = db_{ij}/d_{max} \quad (10)$$

where $db_{ij}$ is calculated as:

$$db_{ij} = \sqrt{\sum_{k=1}^{N_{pipes}} (X_{i,k}(t) - gbest_k(t-2))^2}$$

Otherwise, the particle will be handled by *PSO*. The velocity, $V_i$ and position, $X_i$ of the $i^{th}$ particles are given by Equations (11) and (12) as:

$$V_{i,k}(t+1) = wV_{i,k}(t) + c_1 r_1 (pbest_{i,k}(t) - X_{i,k}(t)) + c_2 r_2 (gbest_k(t) - X_{i,k}(t))$$

$$V_{max} > V_{i,k}(t+1) > V_{min} \quad (11)$$

$$X_i(t+1) = X_i(t) + V_i(t+1) \quad (12)$$

where $V_i(t+1)$ is the particle velocity in iteration $(t+1)$, $w = 0.90 - t(\frac{0.40}{n_{iter}})$ is the inertia weight. $r_1$ and $r_2$ are random numbers in range of [0,1]. $c_1$ and $c_2$ are the acceleration coefficient, and $V_{max}$ is the maximum change of the particle velocity.

6. Compare fitness, $f_i$ for each particle, $i$ in the population with those of *gbest* and *pbest* particles. Update *gbest* for the population and *pbest* of every particle.
7. Repeat steps 5 to 6 until the maximum number of iterations, $n_{iter}$ is reached.

### 3.3. Hybrid Firefly-Genetic Algorithm Model (FAGA)

Zervoudakis et al. [12] proposed a hybrid approach *(FAGA)* based on the firefly algorithm, *FA* and the genetic algorithm, *GA*, to combine the advantages of both algorithms for which the procedure is summarized as follows:

1. Generate a random initial population of $n_{pop}$ fireflies.
2. Calculate the fitness, $f_i$ for each firefly, $i$ in the population ($i$ =1, 2. 3. ... , $n_{pop}$)
3. Compare the fitness $f_i$ and $f_j$ for each of the two fireflies $i$, $j$, respectively, ($i$ and $j$ = 1: $n_{pop}$ and $i \neq j$).
4. Apply genetic crossover for the two fireflies $i$ and $j$ for the case $f_j < f_i$ according to Equations (13) and (14).

$$X_i(t+1) = L * X_i(t) + (1-L) * X_j(t) \quad (13)$$

$$X_j(t+1) = L * X_j(t) + (1-L) * X_i(t) \quad (14)$$

$L$ is a vector with continuous uniform distribution with the lower endpoints 0 and upper endpoint $(1 + r)$ where $r$ is a single uniformly distributed random number in the interval (0,1).

$L * X_i(t)$ multiplies arrays $L$ and $X_i(t)$ by multiplying corresponding elements.

On the contrary, if $f_j > f_i$, apply the genetic mutation in both fireflies as given in Equations (15) and (16).

$$X_{i,k}(t+1) = X_{i,k}(t) + sigma(k) * R \quad (15)$$

$$X_{j,k}(t+1) = X_{j,k}(t) + sigma(k) * R \quad (16)$$

where $sigma = 0.10\ (D_{max} - D_{min})$, $k$ is a vector with $n$ values; $n = (mu * N_{pipes})$ and $mu$ is the mutation coefficient which are sampled uniformly at random without replacement, from the integers 1 to $n_{pipes}$, and $R$ is a vector of random $n$ values drawn from the standard normal distribution.

5. Replace the old solutions for the fireflies $i$ and $j$ with the new ones if they have better finesses.
6. Repeat steps 3 to 5 until reaching the maximum number of iterations, $n_{iter}$.

### 3.4. Models Parameters

Setting the values of the parameters for each algorithm is a crucial issue in the optimization process to reach an optimal solution for each of the four tested networks. Several trial runs were carried out for each algorithm to select the most appropriate values of its related parameters. Table 2 shows the final values of the parameters for each algorithm.

**Table 2.** Values of the parameters for the models *FA*, *FAPSO* and *FAGA*.

| Model | Parameter | Pipe Network | | | |
|---|---|---|---|---|---|
| | | Two-Loop | Hanoi | New York | El-Mostakbal |
| *FA* | $n_{iter}$ | 1000 | 1000 | 1000 | 1000 |
| | $n_{pop}$ | 10 | 40 | 40 | 40 |
| | $\Gamma$ | 1 | 1 | 1 | 1 |
| | $\beta_0$ | 2 | 2 | 2 | 2 |
| | $\alpha_0$ | 0.2 | 0.2 | 0.2 | 0.2 |
| *FAPSO* | $n_{iter}$ | 130 | 150 | 200 | 150 |
| | $n_{pop}$ | 70 | 350 | 200 | 400 |
| | $c_1$ | 1 | 1.49 | 1.49 | 1.49 |
| | $c_2$ | 1.1 | 1.49 | 1.1 | 1.49 |
| | $\Gamma$ | 1 | 1 | 1 | 1 |
| | $\beta_0$ | 2 | 2 | 2 | 2 |
| | $A$ | 0.2 | 0.2 | 0.2 | 0.2 |
| *FAGA* | $n_{iter}$ | 1000 | 1000 | 1000 | 1000 |
| | $n_{pop}$ | 10 | 40 | 40 | 40 |
| | $Mu$ | 0.15 | 0.15 | 0.1 | 0.2 |

## 4. Application and Results

The optimal solutions for the El-Mostakbal city water distribution network obtained using different optimization algorithms available in the literature are listed in Table 3, showing that the hybrid model *FAGA* introduced in the present study succeeded in reaching a new optimal solution of 4,923,731.5 L.E. compared to the last available optimal solution of 4,926,560.7 obtained by Abdel-Gawad [29] using the modified Jaya algorithm. The optimal diameters of the network are shown in Table 4.

**Table 3.** Optimal cost for El-Mostakbal city network using different optimization algorithms.

| Author's | Optimization Technique | Optimal Cost |
|---|---|---|
| Rayan et al. (2003) [28] | SUMT | 6,770,787 |
| El-Ghandour and El-Beltagi (2018) [30] | GA | 5,268,431 |
| | PSO | 4,968,881.5 |
| | ACO | 5,484,596 |
| | MA | 5,055,519 |
| | SFLA | 5,181,846 |
| Ezzeldin and Djebedjian (2020) [24] | WOA | 4,932,467.1 |
| Abdel-Gawad (2021) [29] | FSAJA | 4,926,560.7 |
| Present Study | FA | 5,676,331.79 |
| | FAPSO | 4,932,901 |
| | FAGA | 4,923,731.5 |

Note(s): SUMT (Sequential Unconstrained Minimization Technique), GA (Genetic Algorithm), PSO (Particle Swarm Optimization), ACO (Ant Colony Optimization), MA (Memetic Algorithm), WOA (Whale Optimization Algorithm), FSAJA (Free Sensitivity Analysis Jaya Algorithm), FA (Firefly Algorithm), FAPSO (Hybrid Firefly-Particle Swarm Optimization), FAGA (Hybrid Firefly-Genetic Algorithm).

**Table 4.** Optimal pipe diameters of El-Mostakbal city network.

| Pipe Number (Optimal Diameter, mm.) | | | | | |
|---|---|---|---|---|---|
| 1 (600) | 2 (500) | 3 (500) | 4 (500) | 5 (150) | 6 (150) |
| 7 (150) | 8 (150) | 9 (150) | 10 (150) | 11 (500) | 12 (500) |
| 13 (150) | 14 (150) | 15 (150) | 16 (150) | 17 (150) | 18 (150) |
| 19 (150) | 20 (500) | 21 (150) | 22 (150) | 23 (150) | 24 (150) |
| 25 (150) | 26 (400) | 27 (400) | 28 (250) | 29 (150) | 30 (150) |
| 31 (150) | 32 (150) | 33 (200) | 34 (150) | 35 (250) | 36 (300) |
| 37 (150) | 38 (250) | 39 (250) | 40 (200) | 41 (150) | 42 (150) |
| 43 (150) | 44 (200) | | | | |

*Performance Evaluation*

The performance of the three optimization models applied to the networks considered in the study was assessed according to two stages. The first stage includes a set of different measures, namely, (1) minimum cost, maximum cost, mean and standard deviation columns 1, 2, 3, and 4), respectively; (2) convergence criteria measured by the function evaluation number and the computational time required for performing 1000 evaluations (columns 5 and 6), respectively. The results of the first stage of assessment are shown in Table 5, which clearly illustrates that the FAGA hybrid model has the best values for the minimum and maximum cost, the mean and standard deviation for all networks, which means better search capability in the huge search space ($14^8$, $6^{34}$, $10^{44}$ and $16^{21}$ for the two-loop, Hanoi, El-Mostakbal and New York networks, respectively). Besides, the values of the number of function evaluations and the computing time for 1000 evaluations are the lowest for all networks compared to FA and FAPSO, which means faster convergence of the hybrid FAGA model towards the optimal global solution. The case study of El-Mostakbal city network optimized by firefly algorithm and the two hybrid models, FAPSO and FAGA is shown in Figure 5 which clearly illustrates the faster convergence of the FAGA model in reaching an optimal solution of 4,923,731.5 LE. at a number of function evaluations of 37,440 compared to 5,966,072.39 L.E. and 4,964,187.63 L.E. for FA and FAPSO, respectively at the same number of function evaluations.

**Table 5.** Performance evaluation according to the first stage of assessment.

| Network | Optimization Algorithm | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|---|
| | | Min. Cost | Max. Cost | Mean | Standard. Deviation | F.E.N. | Sec Per 1000 Eval |
| Two-Loop | FA | 419,000 | 441,000 | 425,150 | 8317.86 | 6205 | 88.8 |
| | FAPSO | 419,000 | 453,000 | 435,700 | 11,388.36 | 2596 | 85.3 |
| | FAGA | 419,000 | 420,000 | 419,160 | 370.33 | 2380 | 82 |
| Hanoi | FA | 6,566,082.81 | 8,307,245.89 | 7,402,370.25 | 524,647.62 | 52,249 | 91.8 |
| | FAPSO | 6,195,529.34 | 69,044,904.1 | 6,507,346.32 | 208,328.43 | 102,960 | 88.3 |
| | FAGA | 6,087,729.57 | 6,375,686.7 | 6,252,830.16 | 79,998.3 | 37,410 | 82 |
| New York | FA | 38,637,600 | 62,390,579.7 | 44,093,383.99 | 5,396,845.17 | 22,335 | 91 |
| | FAPSO | 38,637,600 | 61,551,400 | 40,393,718.25 | 5,139,391.02 | 13,916 | 89.2 |
| | FAGA | 38,637,600 | 38,796,300 | 38,662,992 | 58,771.06 | 9120 | 88.1 |
| El-Mostakbal | FA | 5,676,331.79 | 6,263,583.1 | 5,913,233.06 | 170,902.64 | 55,216 | 94.3 |
| | FAPSO | 4,932,901 | 5,214,838 | 5,046,771.6 | 92,426.21 | 58,842 | 90.5 |
| | FAGA | 4,923,731.5 | 5,025,247.3 | 4,949,974.37 | 35,382.66 | 37,440 | 88 |

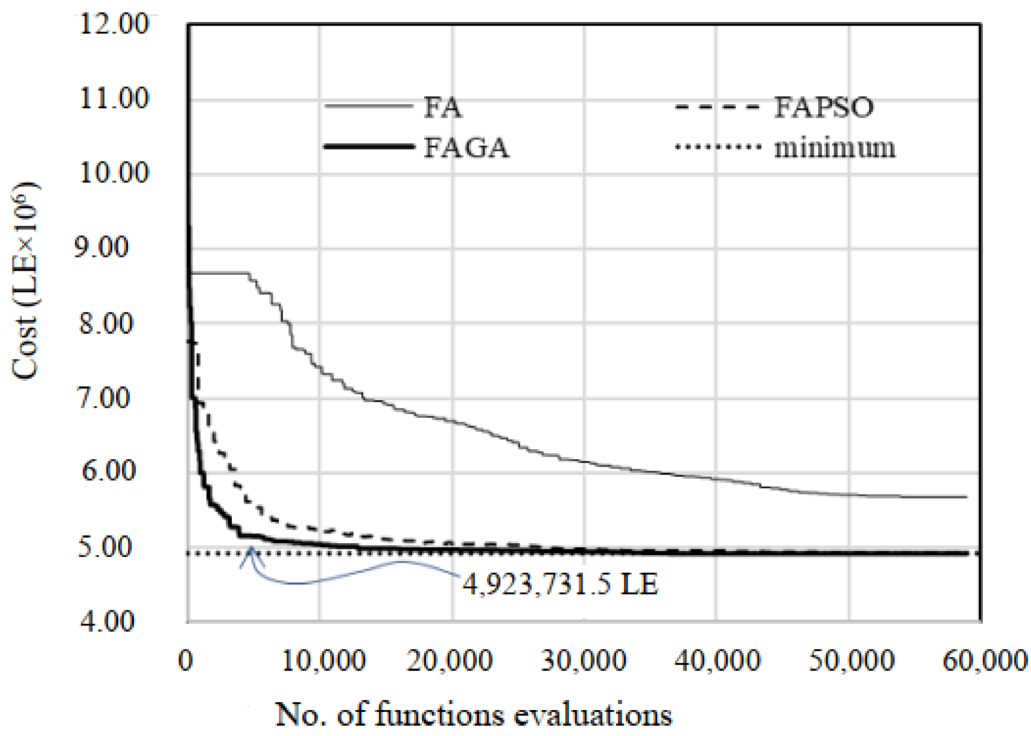


**Figure 5.** Convolution of the cost with the number of functions evaluations for El-Mostakbal city network.

In the second stage of performance evaluation, the performance of the firefly algorithm and the two hybrid models, FAPSO and FAGA, is assessed using the success rate, *Sr*, estimated by the fuzzy logic concept. The success rate has been first introduced by Mora-Melia et al. [38] as a measure of the quality and convergence of an optimization algorithm. Quality refers to the ability of an algorithm to obtain the maximum number of good solutions as a ratio to the total number of simulations performed. In the present study, the success rate is utilized as a measure of an algorithm to obtain the maximum number of good solutions (near-optimal solutions) for the prespecified total number of simulations. An optimization error $C$ is defined as the limit of exceedance of the good solution beyond the optimal solution of the network. Values of $C$ considered in this study were 0, 0.01 and 0.02. The success rate is then evaluated using the fuzzy logic concept first introduced by Cullinane et al. [39] and later used by El-Ghandour et al. [40] to determine the nodal hydraulic availability indices. The following steps describe the procedure of the fuzzy logic method:

1. Determine the known optimal solution $f(x^*)$ for the pipe networks (two-loop, 419,000, Hanoi, $6.081 \times 10^6$ and New York, 38,637,600). If the known optimal solution is not available, $f(x^*)$ is replaced with the best-known optimal solution (EL-Mostakbal, 4,923,731.5 obtained from the present study).
2. The robustness of the optimization algorithm is measured by accepting optimal solutions $f(x)_{max}$ slightly greater than the known optimal solution $f(x^*)$ such that $f(x)_{max} = (1 + C) * f(x^*)$ where $C$ = 0, 0.01 and 0.02.
3. Run each of the three optimization algorithms considered in this study, FA, FAPSO and *FAGA*, 20 times for each of the four networks and denote the objective function at the termination point, $f\left(x_{opt}\right)_i$, $i = 1, 2, 3, \ldots, 20$.
4. Estimate the Acceptance Index $AI_i$ as given in Equation (17) using the principles of fuzzy logic [39]. Values of optimization error, $C$ = 0, 0.01, and 0.02, are assumed to be acceptable. Zero value of $C$ means a tenuous relationship between $AI_i$ and $f\left(x_{opt}\right)_i$. At the same time, the second and third values of $C$ denote continuous function (S-shape fuzzy membership function) to simulate the relationship between the Acceptance index, $AI_i$ and $f\left(x_{opt}\right)_i$. As given in Equation (17), it is clear that

$AI_i$ takes a value equal to 1 if $f(x_{opt})_i = f(x^*)$ and value between 1 and zero if $(1+C) * f(x^*) > f(x_{opt})_i > f(x^*)$ while it takes value of zero if $f(x_{opt})_i$ more than or equal $(1+C) * f(x^*)$.

$$AI_i = \begin{cases} 0\ if\ f(x_{opt})_i > f(x_{max}) \\ 2\left\{\frac{f(x_{opt})_i - (1+C)\times f(x^*)}{C\times f(x^*)}\right\}^2 if\ \left(1+\frac{C}{2}\right)\times f(x^*) < f(x_{opt})_i < (1+C)\times f(x^*) \\ 1-2\left\{\frac{f(x_{opt})_i - f(x^*)}{C\times f(x^*)}\right\}^2 if\ f(x^*) < f(x_{opt})_i < \left(1+\frac{C}{2}\right)\times f(x^*) \\ 1\ if\ f(x_{opt})_i = f(x^*) \end{cases} \quad (17)$$

Estimate the success rate $Sr$ as:

$$Sr = 100 * \sum_{i=1}^{20} AI_i\ /20 \quad (18)$$

The results of the success rate obtained using the proposed algorithms for the different networks are shown in Table 6 which clearly illustrates the effectiveness and quality of the *FAGA* hybrid model as the values of the success rate are remarkably higher than the corresponding values for the traditional FA and hybrid FAPSO models for all networks especially EL-Mostakbal network considering the prespecified limits of the optimization error *C*.

**Table 6.** Success rates using the proposed optimization algorithms.

| Network | Optimization Algorithm | Success Rate (Sr %) | | |
|---|---|---|---|---|
| | | *C* = 0 | *C* = 0.01 | *C* = 0.02 |
| Two-loop | FA | 25 | 56.01 | 60.65 |
| | FAPSO | 15 | 15 | 16.69 |
| | FAGA | 84 | 98.18 | 99.54 |
| Hanoi | FA | 0 | 0 | 0 |
| | FAPSO | 0 | 0 | 0.1314 |
| | FAGA | 2 | 7.5 | 10.73 |
| New York | FA | 5 | 8.31 | 13.03 |
| | FAPSO | 5 | 5 | 5 |
| | FAGA | 84 | 94.6 | 98.65 |
| El-Mostakbal | FA | 0 | 0 | 0 |
| | FAPSO | 0 | 11.39 | 29.31 |
| | FAGA | 40 * | 66.89 | 77.1 |

Estimation of the acceptance index *AI* (Equation (17)) is based mainly on obtaining an optimal solution $f(x_{opt})_i$ greater than the known optimal solution of the network by a value of optimization error *C* = 0, 0.01, and 0.02. For *C* = 0, only the runs providing optimal solutions $f(x^*)$ are accepted and take the value of *AI* = 1, while *C* = 0.01 and 0.02 only runs giving optimal solutions equal or greater than the optimal solution by about C take values of *AI* = (0,1). Figure 6 shows the continuous function (S-shape fuzzy logic) for the relationship between acceptance index and cost ratio $f(x_{opt})_i / f(x^*)$. Table 7 illustrates the procedure of calculating the success rate Sr (Equation (18)) for the El-Mostakbal city network by performing 20 runs and considering the different values of *C*. The values of the acceptance index *AI* of FAGA for the El-Mostakbal city network (twenty runs—different values of optimization error C) are given in Table 7. The values of *Sr* are also calculated and given in the table.

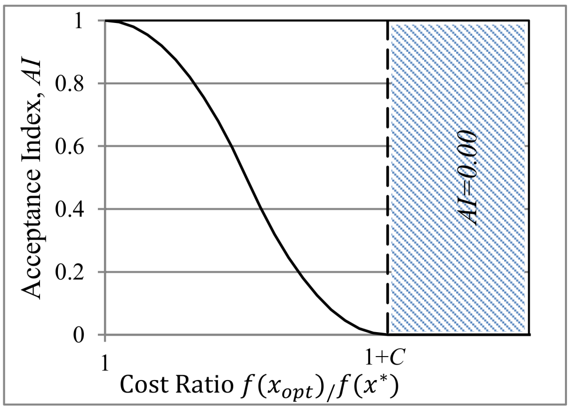


**Figure 6.** S-shape fuzzy logic relationship of acceptance index and cost ratio.

**Table 7.** Estimation of success rate ($S_r$%) of FAGA optimization model for El-Mostakbal City network.

| **Run No.** | **Acceptance Index (AI)** | | | **Run No.** | **Acceptance Index (AI)** | | |
|---|---|---|---|---|---|---|---|
| | ***C* = 0.00** | ***C* = 0.01** | ***C* = 0.02** | | ***C* = 0.00** | ***C* = 0.01** | ***C* = 0.02** |
| 1 | 1 | 1 | 1 | 11 | 0 | 0.687 | 0.922 |
| 2 | 1 | 1 | 1 | 12 | 0 | 0.687 | 0.922 |
| 3 | 1 | 1 | 1 | 13 | 0 | 0.687 | 0.922 |
| 4 | 1 | 1 | 1 | 14 | 0 | 0.687 | 0.922 |
| 5 | 1 | 1 | 1 | 15 | 0 | 0.687 | 0.922 |
| 6 | 1 | 1 | 1 | 16 | 0 | 0.247 | 0.790 |
| 7 | 1 | 1 | 1 | 17 | 0 | 0 | 0.091 |
| 8 | 1 | 1 | 1 | 18 | 0 | 0 | 0.004 |
| 9 | 0 | 0.879 | 0.970 | 19 | 0 | 0 | 0.002 |
| 10 | 0 | 0.819 | 0.955 | 20 | 0 | 0 | 0 |
| * Sr = (13.38/20) × 100 = 40%. | | | | ∑ | 8.00 | 13.38 | 15.42 |
| | | | | Sr % | 40.0 * | 66.90 | 77.10 |

## 5. Conclusions

The present research introduced two hybrid models, firefly-particle swarm optimization (FAPSO) and firefly-genetic algorithm (FAGA), to enhance the performance of the standard firefly algorithm (FA). The proposed models were tested through application to the three well-known benchmark networks of known optimal solutions, namely, two-loop, Hanoi and New York, and also to the real large-scale case study of El-Mostakbal city network, Egypt, of the unknown optimal solution obtained yet. The results revealed that the proposed FAGA model was able to reach the known optimal solutions of 419,000 cost units, $6.081 \times 10^6$ \$ and 38,637,600 \$ for the benchmark networks, respectively. For the real case study of the El-Mostakbal city network, the FAGA model succeeded in reaching a new optimal solution of 4,923,731.5 L.E. compared to the last optimal cost of 4,926,560.7 L.E. available in the literature. Additionally, performance evaluation of the proposed algorithms in terms of function evaluation number, computational time, selected related cost measures, namely, minimum, maximum, mean and standard deviation and finally, success rate, revealed that FAGA, when compared to the standard FA and the hybrid model FAPSO had a better search capability in huge solution spaces, faster convergence towards an optimal solution, balancing between exploration and exploitation phases, the higher capability of finding the optimal solution. Finally, it can be concluded that the FAGA hybrid optimization algorithm is a very promising optimization tool and has an attractive ability to efficiently handle pipe network optimization problems. For future studies, it is recommended that the model be applied to multi-objective pipe network optimization.

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