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Failure Analysis of a New Irrigation Water Allocation Mode Based on Copula Approaches in the Zhanghe Irrigation District, China

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Abstract: The risk analysis of an irrigation water allocation strategy based on physical mechanisms is critically important in practice. Conventional risk analysis only considers the role of the channel system and ignores the factors related to on-farm ponds. This paper proposes a channel-pond joint water supply mode (CPJM) based on copula approaches. Two copulas, the Plackett copula and No.16 copula, are chosen and two types of analyses are carried out with the proposed mode: (1) a risk assessment of CPJM with joint probability and conditional probability; and (2) determination of the water supply strategy given the pond water supply frequency. With a case study of the second channel in the Zhanghe Irrigation District (ZID), Southern China, nine combinations of channel water supply frequency (CWSF) and pond water supply frequency (PWSF) are studied. The results reveal that the failure probabilities of the joint distribution and the conditional distribution of the CPJM are 0.02%–16.54% and 0.45%–33.08%, respectively, with corresponding return period of 42–5000 and 10–222 years. Nevertheless, a previous study has shown that the real probability is 33.3%, which means that the return period is equals to three years. Therefore, the objective failure evaluation of the irrigation water-use strategy is useful for water saving in this channel system. Moreover, the irrigation water allocation strategy can be determined and the failure charts relating the CWSF and PWSF can be obtained for a predetermined PWSF. Thus, the channel-pond joint water supply mode provides a more reasonable estimate of the irrigation water allocation strategy reliability.

Keywords: water resources; channel-pond joint water supply mode; copula; risk; risk analysis; return period

1. Introduction

In recent years, the mismatch between supply and demand of irrigation water resources has become increasingly serious [1,2]. Simultaneously, seasonal drought is becoming increasingly common and agricultural irrigation is increasingly reliant on ponds [3]. Irrigation ponds are a type of water-supply structure that is small in size, and are also referred to as on-farm irrigation tanks or on-farm reservoirs. These structures are widely distributed in Southern China, Southern India, and Sri Lanka [4–7]. The ponds play an important role in decreasing peak flow in main irrigation channels, reallocating water in irrigation districts, enhancing water-efficient irrigation, and supplying water for crop production [2,4,8–13]. Ponds in Southern China not only capture surplus water from the irrigation system and agricultural return water from upstream fields, but also supply water to downstream fields [9,10,14]. The water provided by ponds for irrigation accounts for over 60% in the Zhanghe Irrigation District (ZID) [3].

For irrigation water allocation, controversial and conflict-laden water-allocation issues have raised increasing concerns in recent decades [15–18]. Many models and methods have been developed for agricultural water management and planning. Specifically, Du *et al.*, Zhou *et al.*, and Shangguan *et al.*, used models of deficit irrigation and uncertainty to determine the optimal allocation in a multi-source and multi-crop irrigation area [19–21]. Li *et al.*, and Huang *et al.*, constructed two-stage water allocation models for irrigation water sustainability and planning [22,23]. Zhang proposed a multi-objectives allocation model with Particle Swarm Optimization (PSO) algorithm to develop an irrigation schedule for a winter wheat-summer maize rotation system [24]. Zhang *et al.*, used a large system level transfer model to optimize the regional agricultural water allocation [25]. Sarwar and Eggers used a conjunctive model to evaluate alternative management options for surface and groundwater resources [26]. Other models based on simulation techniques and genetic algorithms with computer modeling have been widely applied in irrigation water resources allocation [27,28]. Remote sensing technology is gradually being incorporated as an assisting tool for irrigation water management by estimating crop water use, evapotranspiration, and soil moisture conditions [29–31].The above models and methods mostly focus on the goals of maximizing the economic benefits, optimizing the multiple-objectives of the irrigation system, and minimizing the ratio of relative water deficiency, but they do not take ponds into consideration or regard ponds as just supporting resources. This causes distorted estimates of the irrigation water allocation strategy failure by ignoring the important role of ponds in the water supply.

Therefore, this study proposes a new irrigation mode, the channel-pond joint water supply mode (CPJM), which is a water resource allocation mode that combines channel water supply (CWS) and pond water supply (PWS) to determine the irrigation water supply strategy. Then, the corresponding risk analysis of the mode was obtained. Since the failure of the mode is caused by the interaction of multiple factors, a single variable-based analysis is unable to reflect the relationships among the multiple factors. Consequently, the results derived from a one-dimensional analysis tend to distort the physical mechanisms involved in the irrigation system. To describe the relationships among multiple variables, many methods have been introduced in hydrologic studies, e.g., multivariate normal distribution function [32–35], exponential distribution [36], Gamma distribution [37], two-dimensional normal distribution, and two-dimensional Gumbel distribution [32,38]. However, all of these multivariate distribution functions assume certain requirements in terms of the marginal distribution of variables; thus, the partial distortion of information is inevitable [39]. Moreover, new ideas and methods have played an important role in water allocation risk analysis. For example, Zhang proposed a risk analysis model involving two phases of environmental and public demand during the program period [40]. Gu *et al.*, performed a multi-objective risk assessment of the optimal deployment of water resources based on stochastic simulations [41]. Additionally, Guo *et al.*, demonstrated the risk of water resource allocation using indices based on the entropy principle [42].However, the literature summary of the risk analysis of water resource allocation lacks quantitative descriptions and the research focuses more on the effects of configuration schemes than on the risk ratio calculation.

Recently, copula theory has been widely applied in constructing joint probability distributions of multivariate data. It provides a general and flexible way to model nonlinear dependences among multivariate data in isolation from their marginal distribution functions [43]. A copula-connected function in which the marginal distribution is stochastic offers us an available solution to describe the relationships between different factors [44,45]. This technique has been widely applied in multivariate hydrological calculations and analyses [46–61]. On the agricultural side, Cisty *et al.*, and Mirabbasi *et al.*, introduced a bivariate copula methodology to describe the characteristics and probability of drought [62,63]. Shiau and Modarres used a copula-based distribution function for drought severity, duration and frequency analysis [64]. Wang *et al.*, constructed a trivariate copula-based model to analyze annual extreme rainfall events in Connecticut [65]. Rauf and Zeephongsekul used non-parametric copulas and marginal distributions to describe rainfall severity and duration in water resource management [66,67]. However, the risk analysis of the irrigation water allocation strategy is rarely conducted using copula theory.

Therefore, the objective of this paper is to propose a new irrigation water-use mode, the CPJM, and estimate the failure probability of the mode with copula-based approaches. In Section 2, the methodology and procedures are explained and illustrated in detail. In Section 3, the CPJM is applied to the second channel in the ZID as a case study. Section 4 describes the results and provides a corresponding discussion. Finally, the conclusions are summarized in Section 5. The CPJM can satisfy the irrigation demand and offer technical support for pond engineering construction and water management.

2. Methodology

2.1. Copula Theory

2.1.1. Description of Copulas

Copulas are functions that couple a multivariate distribution to its one-dimensional marginal distributions. Alternatively, copulas are multivariate distribution functions whose one-dimensional marginal distributions are uniform on the interval of [0,1]. According to Sklar's theorem, there are many copulas, and each copula has its own dependence structure [45]. For modeling bivariate distribution, many copulas in the literatures, e.g., the Gaussian copula, Plackett copula, Frank copula, No.16 copula, Gumbel copula, and Clayton copula, are commonly adopted. They provide an easy way to estimate multivariate joint distribution functions with known marginal distribution functions. A bivariate distribution of X_1 and X_2, *i.e.*, $F(x_1, x_2)$, can be expressed in terms of the copula function $C(u_1, u_2; \theta)$ and the marginal distributions $u_1 = F_1(x_1)$ and $u_2 = F_2(x_2)$:

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2); \theta) = C(u_1, u_2; \theta) \quad (1)$$

where θ is the parameter of the chosen copula function, describing the dependence between X_1 and X_2. In this paper, the two random variables refers to channel water supply (CWS) and pond water supply (PWS). Then, the probability density function,$f(x_1, x_2)$, can be described using Equation (1) as follows:

$$f(x_1, x_2) = f_1(x_1)f(x_2)c(F_1(x_1), F_2(x_2); \theta) = \partial C^2(u_1, u_2; \theta)/\partial u_1 \partial u_2 \quad (2)$$

where $f_1(x_1)$ and $f(x_2)$ are the marginal density functions of X_1 and X_2, respectively, and $c(F_1(x_1), F_2(x_2); \theta)$ is a copula density function, which is given by $C(u_1, u_2; \theta)$, as Equation (2) shows.

The copula parameter $\theta$is given by the correlation coefficient τ between X_1 and X_2. The following integral relationship between τ and θ can be obtained based on the definition of Pearson correlation coefficient:

$$\tau = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{x_1 - \mu_1}{\sigma_1} \frac{x_2 - \mu_2}{\sigma_2} f(x_1, x_2) dx_1 dx_2 \quad (3)$$

where μ_1 and μ_2 are the mean of X_1 and X_2, respectively, and σ_1 and σ_2 are the standard deviation of X_1 and X_2, respectively.

It is evident that the corresponding copula parameter θ can be solved iteratively by the integral equation above using a known marginal distribution and the correlation coefficient τ.

With the aid of SPSS (SPSS Inc., Chicago, IL, USA) software, the correlation coefficient τ between CWS and PWS is −0.305 in a 0.05 two-sided test, which indicates that there is a significant negative correlation between the two variables. The copulas for describing negative correlation coefficients are selected to characterize the dependence between the two variables. Four copulas, Gaussian copula, Frank copula, Plackett copula and No.16 copula are adopted to describe the dependence structure between CWS and PWS, based on a review of the literature. These four copulas are commonly-used multi-dimensional copula functions. The Gaussian copula is an elliptical copula, the Plackett copula is a member of the Plackett copula family, and the Frank and No.16 copulas are members of the

Archimedean copula family. The characteristics of these four copulas and the θ parameter are shown in Table 1.

Table 1. Summary of the adopted copulas and their characteristics.

Copula	Copula Function $C(u_1, u_2; \theta)$	Range of θ
Gaussian	$\int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\theta^2}} \exp[-\frac{x_1{}^2 - 2\theta x_1 x_2 + x_2{}^2}{2(1-\theta^2)}] dx_1 dx_2$	[−1,1]
Plackett	$\frac{S - \sqrt{S^2 - 4u_1 u_2 \theta(\theta-1)}}{2(\theta-1)}$ $S = 1 + (\theta - 1)(u_1 + u_2)$	$(0, \infty) \backslash \{1\}$
Frank	$-\frac{1}{\theta}\ln[1 + \frac{(e^{-\theta u_1}-1)(e^{-\theta u_2}-1)}{e^{-\theta}-1}]$	$(-\infty, \infty) \backslash \{0\}$
No.16	$\frac{1}{2}(S + \sqrt{S^2 + 4\theta})$ $S = u_1 + u_2 - 1 - \theta(\frac{1}{u_1} + \frac{1}{u_2} - 1)$	$[0, \infty)$

Note: Φ^{-1} denotes the inverse of the standard normal distribution function.

2.1.2. Empirical Marginal Distribution

Empirical probabilities are calculated with the Gringorten plotting-position formula [68]:

$$P(K \leqslant k) = \frac{k - 0.04}{N + 0.12} \tag{4}$$

where k is the rank of the k-th smallest observation in the dataset arranged in ascending order; and N is the sample size (number of observations).

2.1.3. Empirical Joint Distribution of Channel Water Supply and Pond Water Supply

Using the same technique as for univariate data, empirical joint distributions of two variables are calculated based on ordered values, as follows:

$$H(y_{1i}, y_{2i}) = P(Y_{1i} \leqslant y_{1i}, Y_{2i} \leqslant y_{2i}) = \frac{z_i - 0.04}{N + 0.12} \tag{5}$$

where N is sample size; z_i is the number of (y_{1i}, y_{2i}) satisfying $(y_{1j} \leqslant y_{1i}, y_{2j} \leqslant y_{2i})$ with managing the data combinations (y_{1i}, y_{2i}) by ranking y_{1i} in ascending order, $i, j = 1, \cdots, N$.

2.1.4. Selection of Marginal Distribution and Copula

Method 1

The process of constructing a joint distribution through copulas can be decomposed into two parts: the marginal distribution and the dependence structure. The quality of the selected marginal distributions and copulas can be evaluated by the root-mean-square error (RMSE), the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) [69]. The marginal distribution corresponding to the minimum RMSE and AIC and the copula corresponding to the minimum AIC and BIC are each obtained.

The RMSE can be expressed as follows:

$$\text{RMSE} = \sqrt{\frac{1}{\text{m} - 1} \sum_{i=1}^{m} (Pe_i - P_i)^2} \tag{6}$$

where m is the number of value pairs; Pe_i and P_i are assumed to be an empirical value and a theoretical value, respectively.

The AIC and BIC can be expressed as:

$$\text{AIC} = -2\ln(MLE) + 2r \tag{7}$$

$$\text{BIC} = -2\ln(MLE) + r \cdot \ln m \tag{8}$$

where r is the number of parameters in the chosen distribution, and MLE is the maximum likelihood estimation value.

The Kolmogorov-Smirnov test is another goodness-of-fit statistic and is applied to determine if the random variable could have the hypothesized, continuous, cumulative distribution function. If the examined calculated value is less than a critical value at a given level of significance (usually 5% in the statistical test), the selected model may be suitable to model the variables.

Method 2

As discussed in the previous sections, the best fit copula can also be determined by comparing the AIC and BIC values when the probabilities of the CPJM failure produced by different copulas differ greatly. In consideration of the fluctuation range of the CPJM failure probability, smaller fluctuation ranges may result in higher precision of the probability. Within the set of copulas, let Ω = {Gaussian, Plackett, Frank and No.16 copulas}, then an index of fluctuation range, ε, can be defined as [44]:

$$\varepsilon = \max\left\{\frac{p_{f\max}}{p_f(C)}, \frac{p_f(C)}{p_{f\min}}\right\} \tag{9}$$

where $p_f(C)$ is the probability of CPJM failure associated with a specific copula; $p_{f\max} = \max\left\{p_f(C), C \in \Omega\right\}$ and $p_{f\min} = \min\left\{p_f(C), C \in \Omega\right\}$.

Hence, among the different values of ε for the chosen copulas, the copula corresponding to the minimum value is the most suitable one for modeling the dependence structure between the parameters for CPJM failure analysis.

2.2. Copula-Based Approaches for Evaluating the Probability of Channel-Pond Joint Water Supply Mode Failure

System risk refers to the probability of failure to perform as expected in a specific service period. This can be generalized as the relationship between the load effect and the bearing capacity. System failure occurs when the load effect is greater than the bearing capacity. Since the load effect and bearing capacity are generally independent from each other when the project is put into operation and the bearing capacity is a constant in a specific research region, the uncertainty in the bearing capacity is usually ignored in the analysis and the probability of system failure is equal to the probability of the load effect.

Irrigation water supply risk is affected by many parameters, including precipitation, temperature, and the source of the irrigation water. The irrigation water source, consisting of channels and ponds in South China, is known to be an important parameter for irrigation water supply analysis. As mentioned previously, the probability information on CPJM is determined by the water supplied by channels and ponds, and the CWS and PWS can be seen as the load effect for the irrigation water source. According to the construction requirement of irrigation reliability [70,71], two cases are taken into consideration: (1) the joint probability of CWS and PWS exceeding a prescribed value, which is used to estimate the actual risk of CPJM failure with statistical data, and (2) the conditional probability of CPJM with a given PWS in a research region, where water is utilized with the priority of pond first, and channel next. These considerations are used to determine the water allocation strategy and predict the water supply rules for ponds and channels. These two types of probability are defined in Equations (10) and (11), respectively.

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$$P(X_1 > x_1, X_2 > x_2) = 1 - F_{X_1}(x_1) - F_{X_2}(x_2) + C(F_{X_1}(x_1), F_{X_2}(x_2)) \tag{10}$$

$$P(X_1 \geqslant x_1 | X_2 \geqslant x_2) = \frac{1 - F_{X_1}(x_1) - F_{X_2}(x_2) + C(F_{X_1}(x_1), F_{X_2}(x_2))}{1 - F_{X_2}(x_2)} \tag{11}$$

where $F_{X_1}(x_1)$ and $F_{X_2}(x_2)$ are the probability distribution functions of CWS and PWS, respectively, and are also the marginal distribution functions of the copula in this paper; $C(F_{X_1}(x_1), F_{X_2}(x_2))$ is the copula function; $P(X_1 > x_1, X_2 > x_2)$ is the JP of CWS and PWS exceeding a prescribed value; $P(X_1 \geqslant x_1 | X_2 \geqslant x_2)$ is the CP of CPJM under conditions in which the PWS exceeds a critical value.

The corresponding return period (RP) of the two cases above is calculated using Equations (12) and (13):

$$T_{(X_1 > x_1, X_2 > x_2)} = \frac{1}{P(X_1 > x_1, X_2 > x_2)} = \frac{1}{1 - F_{X_1}(x_1) - F_{X_2}(x_2) + C(F_{X_1}(x_1), F_{X_2}(x_2))} \tag{12}$$

$$\begin{aligned} T_{(X_1 \geqslant x_1 | X_2 \geqslant x_2)} &= \frac{1}{P(X_1 \geqslant x_1 | X_2 \geqslant x_2)} = \frac{P(X_2 \geqslant x_2)}{P(X_1 \geqslant x_1, X_2 \geqslant x_2)} \\ &= \frac{1 - F_2(x_2)}{1 - F_1(x_1) - F_2(x_2) + C(F_{X_1}(x_1), F_{X_2}(x_2))} \end{aligned} \tag{13}$$

3. Case Study

3.1.Study Area Background

The second channel in the ZID was chosen as the study object (Figure 1). The ZID in the Yangtze River basin, China, has drawn international attention for its success in sustaining rice production despite the dramatic reallocation of water for domestic, industrial, and hydropower uses. Considerable research has also been conducted to study the performance of irrigation tanks as supplemental irrigation sources and to study agricultural non-point source pollutants [5,9]. This irrigation district is located in a subtropical monsoon region with an average annual rainfall of 905 mm. The controlled irrigation area is 504 hm^2, and the majority of the water consumption is for paddy rice. Early rice, semi-late rice, and late rice are cultivated in this district and cover areas of 142 hm^2, 310 hm^2, and 176 hm^2, respectively. This study is based on data observed during 1981–2010 by the Tuanlin Climate Observation Station and CWS data measured during 1981–2010.

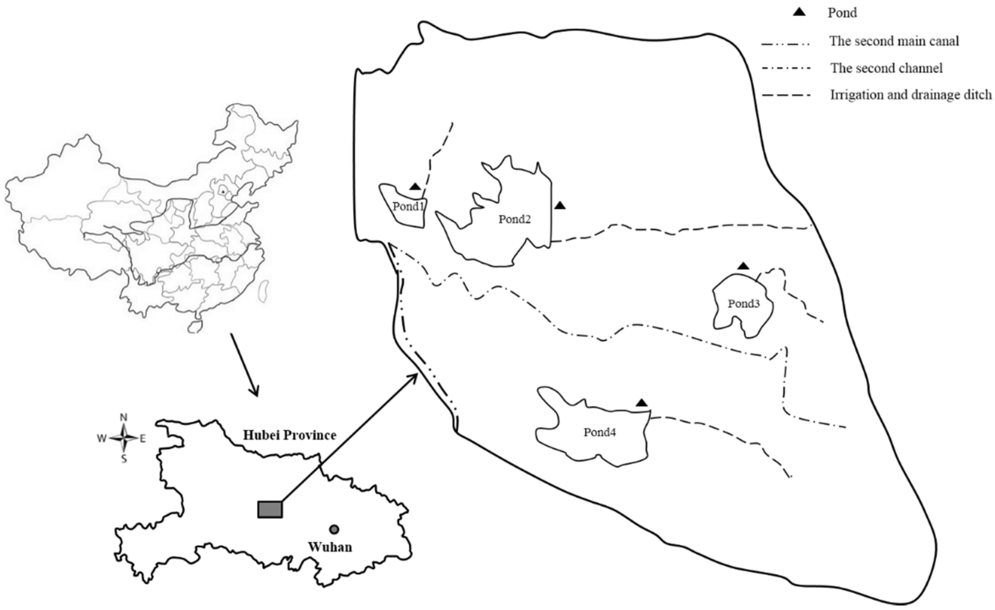


Figure 1. Layout of research region.

The observed rainfall data from 1981–2010 are ranked in descending order, and the empirical probability can be estimated with the Weibull formula. Hydrological years with frequencies of 50%, 75%, and 95% (namely, the years 2006, 1990, and 2010, respectively) are selected to represent three types of typical situations, *i.e.*, normal, dry, and extremely dry years, by assuming that the historical hydro-climatic conditions would occur again in the future. The pond water supply frequency (PWSF) in the selected typical years is 17.86%, 67.86%, and 28.57%, respectively, and the corresponding channel water supply frequency (CWSF) is 42.86%, 89.29%, and 53.57%, respectively. The results suggest inconsistency in the water supply frequency and rainfall frequency. Thus, an irrigation water strategy failure analysis based on different water supply frequencies is essential.

3.1. Determination of Pond Water Supply

Since the research region is located in an irrigation district in Southern China, the water transformation mechanism is complex. Though ponds are an important irrigation water resource, measured PWS data is scarce. The storage period of a pond is defined as the period between the second day of the last irrigation of the year and the day before irrigating the field in the following year, and the water storage of a pond can be determined by the water balance equation, which is assumed to be the initial supply capacity.

Combining the irrigation schedule of paddy rice with the climate data, the PWS can be calculated day by day, thereby determining the annual PWS during 1981–2010. The pond water storage can be calculated according to the following water balance as follows:

$$V_{t+1} = V_t + W_t - D_t - E_t - S_t \tag{14}$$

where V_{t+1} and V_t are the pond water storage of day *t*+1 and *t* in m^3, respectively; W_t is the incoming water entering the pond on day *t* in m^3; D_t is the wasted water of the pond on day t in m^3; E_t is the water evaporated from the pond on day t in m^3, which is obtained by multiplying the measured data an E-601 evaporation pan by the water evaporation conversion coefficient of Yichang Station; and S_t is the leakage amount of the pond in m^3, which is assumed to be constant in this paper, as it is difficult to monitor.

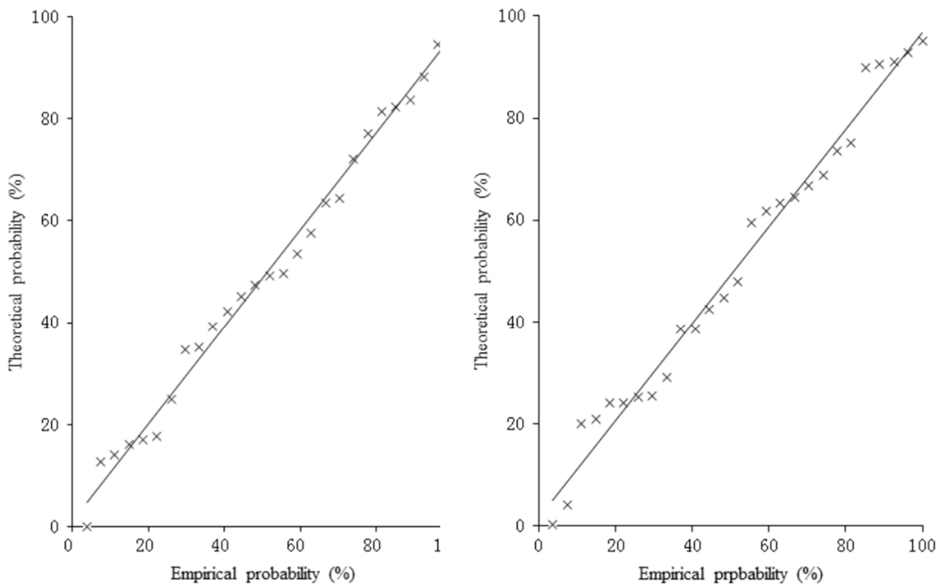
The water quantity in pond is defined as WP_t on day t, the rainfall is P_t, the irrigation amount is W_t, and the total seepage amount of the farmland isS'_t, which is assumed to be constant. The variables herein are all in m^3. Therefore, the water supply constraints of ponds can be described as follows:

If the water quantity of the pond and rainfall is greater than the quantity of the irrigation water demand and seepage, the relational expression can be described as $WP_t + P_t - W_t - S'_t \geqslant 0$. In other words, if the water supply by pond and rainfall can meet the water demand, then PWS is $(W_t + S'_t)$. In other words, if $WP_t + P_t - W_t - S'_t < 0$, PWS is $(WP_t + P_t)$,based on the water use strategy described previously.

The PWS of early, semi-late, and late rice during the whole growth period can be calculated based on the cumulative daily water supply, and the annual PWS during 1981–2010 can thereby be obtained.

3.2. Optimization of Marginal Distribution and Copula

For the variables, several types of candidate probability density functions are tested, including a Pearson-III distribution, an exponential distribution, a normal distribution, and a log-normal distribution, to fit the observation data. According to the results of the Kolmogorov-Smirnov test, the CWS follows a Pearson-III distribution, normal distribution, and log-normal distribution, while the PWS follows a normal distribution and log-normal distribution. Based on the results of RMSE and AIC calculated using Equations (6) and (7) shown in Table 2,the Pearson-III distribution and normal distribution are selected as the marginal distribution functions for CWS and PWS, respectively. The fitting of CWS and PWS is illustrated in Figure 2.



a. the channel water supply by Pearson-III distribution b. the pond water supply by normal distribution

Figure 2. Comparison of best fitting univariate distributions and empirical distributions.

Table 2. Marginal distribution functions of channel and pond water supply.

	Marginal Distribution	RMSE	AIC
Channel	Pearson-III	0.035	−84.81
	Lognormal	0.062	−70.96
	Normal	0.069	−68.17
Pond	Lognormal	0.062	−71.14
	Normal	0.040	−83.86

Using Equations (2) and (3), the parameter θ of the four copulas can be obtained. Then, the AIC, BIC, and ε of the copulas can be calculated using Equations (7)–(9). The values are shown in Table 3. The best fitting copula function is the Plackett copula and the No.16 copula using the previously proposed method 1 and method 2, respectively. The fitting effect of the Frank copula function is demonstrated in Figure 3.

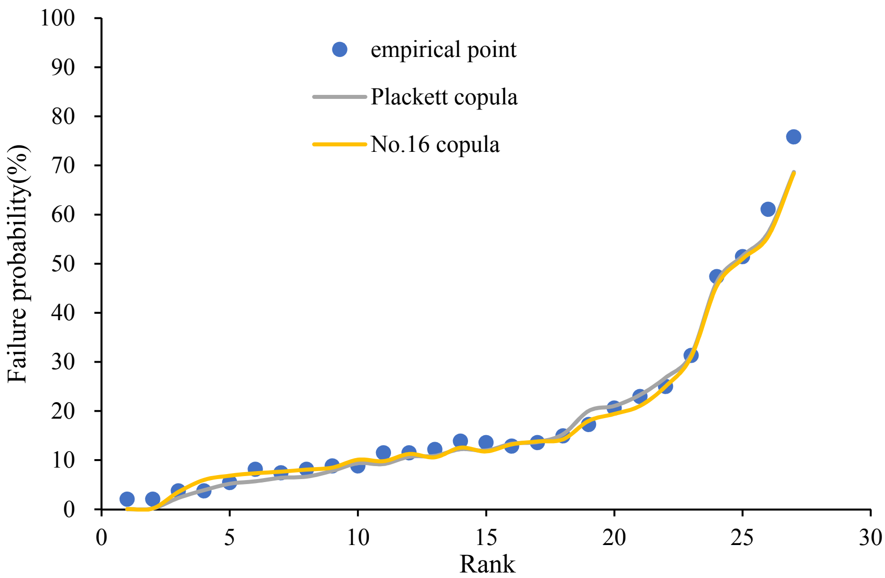


Figure 3. Fitting of the Plackett copula and the No.16 copula.

Table 3. The values of the parameters with corresponding AIC, BIC, and ε values for the chosen copulas.

Copula	θ	AIC	BIC	ε
Gaussian	−0.461	−1.34	1.25	262,504
Plackett	0.244	−4.17	−2.87	228,847
No.16	0.04	1.44	2.74	227,954
Frank	−2.974	−3.52	−2.22	228,574

4. Results and Discussion

In this section, the two preferred copulas, Plackett copula and No.16 copula, are used to evaluate the risk of the CPJM. The objective is to provide a more reasonable estimation of the probability of the CPJM failure. The two copulas chosen by the two methods are used for risk assessment in view of the physical condition of the irrigation water allocation strategy and reasonable engineering conditions. The water supply frequency of CWS and PWS is set to 50% (normal year), 75% (dry year), and 95% (extremely dry year); thus, nine combinations are proposed for the analysis of the joint probability, the conditional probability, and the corresponding return period of CPJM failure.

4.1. Plackett Copula and No.16 Copula for Evaluating the Probability of CPJM Failure

With the two copulas, the joint probability and corresponding return period estimated with Equations (10) and (12) with given frequencies of CWS and PWS are summarized in Tables 4 and 5.

Table 4. Joint probability of channel-pond joint water supply mode failure using the Plackett copula.

Channel Water Supply Frequency (%)	**Pond Water Supply Frequency (%) [Probability(%), Return Period(Year)]**		
	50	**75**	**95**
50	[16.54, 6]	[6.35, 16]	[1.03, 97]
75	[6.35, 16]	[2.39, 42]	[0.39, 256]
95	[1.03, 97]	[0.39, 256]	[0.07, 1429]

Table 5. Joint probability of channel-pond joint water supply mode failure with the No.16 copula.

Channel Water Supply Frequency (%)	**Pond Water Supply Frequency (%) [Probability(%), Return Period(Year)]**		
	50	**75**	**95**
50	[14.92, 7]	[4.33, 23]	[0.57, 175]
75	[4.33, 23]	[1.16, 86]	[0.16, 625]
95	[0.57, 175]	[0.16, 625]	[0.02, 5000]

Given the PWSF, the conditional probability and corresponding return period can be estimated with Equations (11) and (13), respectively. Assuming the water supply frequency of the PWS is 50% (normal year), 75% (dry year), and 95% (extremely dry year), the conditional probabilities and return period of CPJM failure are estimated in Tables 6 and 7.

Table 6. Conditional probability of channel-pond joint water supply mode failure using the Plackett copula.

Channel Water Supply Frequency (%)	**Pond Water Supply Frequency (%) [Probability(%), Return Period(Year)]**		
	50	**75**	**95**
50	[33.08, 3]	[25.39, 4]	[20.63, 5]
75	[12.69, 8]	[9.54, 10]	[7.87, 13]
95	[2.06, 49]	[1.57, 64]	[1.32, 76]

Table 7. Conditional probability of channel-pond joint water supply mode failure using the No.16 copula.

Channel Water Supply Frequency (%)	**Pond Water Supply Frequency (%) [Probability(%), Return Period(Year)]**		
	50	**75**	**95**
50	[29.84, 3]	[17.34, 6]	[11.42, 9]
75	[8.67, 11]	[4.64, 22]	[3.17, 32]
95	[1.14, 88]	[0.63, 159]	[0.45, 222]

Tables 4 and 5 show that the joint probability of failure is from 0.02%–16.54% as estimated by the two copulas, while the probability for the research region according to an existing study is 33.3% [72]. Therefore, taking PWS into consideration is necessary for objective failure analysis of the irrigation water allocation strategy and in water resource planning and management in irrigation districts. For example, given the dry year, as determined by rainfall frequency, the corresponding CWSF should be 97.31%, while the actual value is 89.29%, in reality, for the same year. Correspondingly, the CWS could be reduced by 301,937.2 m^3. This volume of saved water, which can be shifted to other fields, is equalto the benefit of agricultural water saving. In Tables 6 and 7, when the PWSF is 50%–95%, the conditional probability of CPJM failure ranges between 0.45%–33.08%, which is greater than the JP between 0.02%–16.54%. Therefore, with the same failure probability, the corresponding CWSF and PWSF are higher, and the CWS and PWS exhibit higher values for the conditional probability than for the joint probability. In fact, the predicted CWS corresponding to the CWSF calculated with Equation (11) is greater than the actual demand, which ensures irrigation water supply security to a certain extent.

The assurance rate of irrigation water in the irrigation district, where paddy rice is cultivated as a major crop, is between 75%–95%, which indicates that the designed return period ranges from four to 20 years. When the channel and pond water supply frequencies are between 75% and 95%, the return periods of the joint probability and the conditional probability of CPJM are 42–5000 years and 10–222 years, respectively. Therefore, the risk rarely happens in CPJM. The return period of irrigation system failure is greatly improved in the CPJM with an appropriate irrigation water allocation strategy. Thus, the CPJM helps to estimate the resistance of the actual irrigation water allocation strategy to drought damage compared with the single water supply model used in irrigation projects.

According to the information collected over 30 years, the annual average of CWS and PWS are 698,004 m^3 and 407,796 m^3, respectively. With a water supply frequency of 50%, the CWS and PWS are 634,206 m^3 and 403,700 m^3, respectively. Therefore, channels and ponds can generally supply water at a frequency of 50%, and the CPJM failure is at relatively low joint probabilities of 16.54% and 14.92%. Similarly, with the conditional probability analysis, the CPJM failure is 33.08% and 29.84% with a return period of three years when the CWSF is 50% under the condition that PWSF is 50%, which is consistent with the reality of the three-year drought return period in research region [72].

Based on the prediction of future hydrological conditions, the water supply capability of channels and ponds can be calculated and the risk of the CPJM is predictable. Optimal solutions can be obtained to fulfill different intentions, and the risks of CPJM can be analyzed at different frequencies of CWS based on the frequency of PWS.

4.2. Plackett Copula and No.16 Copula for Developing an Irrigation Water Allocation Strategy Based on Typical Hydrological Years

Based on the conditional probability analysis with the two copulas, four years, 10 years, and 20 years are set as the return period of CPJM failure based on the assurance rate of the irrigation water, and the frequencies of PWS are 17.86%, 67.86%, and 28.57%, which correspond to the typical years for the inconsistency between the rainfall frequency and the pond water supply frequency. Then, the calculation provides the frequencies of CWS shown in Tables 8 and 9 with the assistance of Equations (11) and (13).The values range from 2.46%–24.72%. In fact, the CWSF is actually 42.86%–89.29%, corresponding to the frequency of the typical years based on the frequency of statistical data. In this situation, the actual CWSF of all years exceeds the water demand when the PWSF is determined. Thus, the PWS plays an important role in guaranteeing irrigation.

Table 8. Channel water supply frequency under different typical years and corresponding pond water supply frequency of the conditional probability analysis with the Plackett copula.

Return Period (Years)	Channel Water Supply Frequency (%)		
	Typical Year (%)		
	50	75	95
4	21.88 *	13.14 **	20.01 ***
10	8.68 *	5.01 **	7.90 ***
20	4.33 *	2.46 **	3.93 ***

Note: *, ** and *** denote the pond water supply frequencies of 17.86%, 67.86% and 28.57%, respectively.

Table 9. Channel water supply frequency under different typical years and corresponding pond water supply frequency of the conditional probability analysis with No.16 copula.

Return Period (Years)	Channel Water Supply Frequency (%)		
	Typical Year (%)		
	50	75	95
4	24.72 *	15.88 **	22.81 ***
10	11.36 *	8.29 **	10.75 ***
20	6.67 *	5.34 **	6.43 ***

Note: *, ** and *** denote the pond water supply frequencies of 17.86%, 67.86% and 28.57%, respectively.

Meanwhile, a large amount of water could be saved by performing a full and reasonable estimation of the water supplied by ponds. For example, in the case of a typical 75% year with a PWSF of 67.86%, the corresponding water demand on the channel is 957,029 m^3, in contrast to the actual CWS of 1,132,046 m^3. Thus, the water supply can be reduced by 175,035 m^3. Water conservation is achieved, to some extent, in the CPJM. Meanwhile, if the frequency of the hydrological year can be predicted, the water management plan of the channel will be determined by applying the appropriate frequency to the PWSF.

In addition to a quantitative descriptive analysis of the CWS under different PWSF values and typical hydrological years using the conditional probability with the two copulas, it is also possible to design an irrigation water allocation strategy. For a certain failure probability of CPJM, the relation curve charts of CWSF and PWSF can be obtained, as shown in Figures 4 and 5. Each line in the figure represents an equal-potential line of CPJM failure. The CWSF can be obtained with estimation of PWSF and different predicted failure probability. The design chart shows that the failure probability increases with decreasing frequencies of CWS and PWS. This indicates that the probability that the CPWF or PWSF will exceed their respective values prescribed in the design as these prescribed values decrease. Under the chosen failure probabilities, the CWSF is no more than 60% under different PWSF conditions. Therefore, optimizing the use of PWS is important for decreasing CWS and allowing that water to be used by other industries. Clearly, ponds play an important role in the irrigation water allocation.

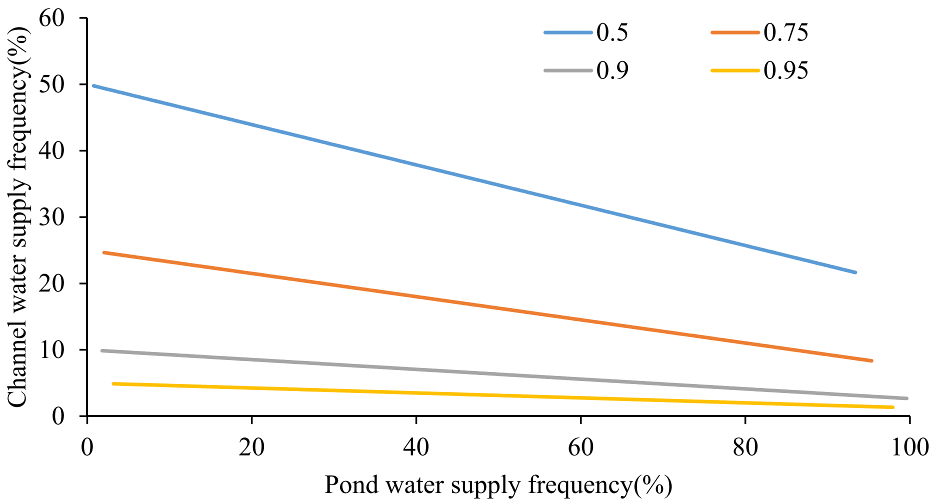


Figure 4. Channel water supply frequency with given pond water supply frequency under different mode failure probability based on the Plackett copula.

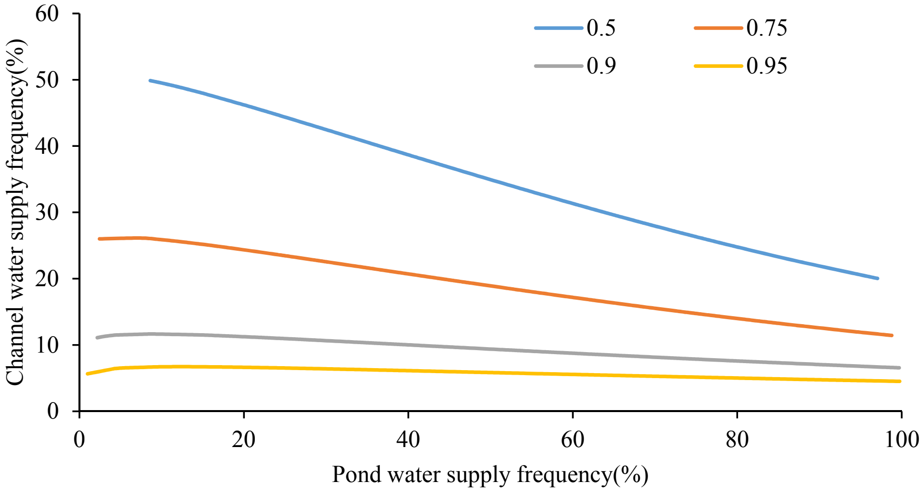


Figure 5. Channel water supply frequency with given pond water supply frequency under different mode failure probability based on the No.16 copula.

4.3. Discussion of the Two Copulas

The Plackett copula and No.16 copula can both be used to characterize the dependence structure between the CWS and PWS. They are consistent with the different standards for optimizing copulas. However, the results of the different copulas differ greatly based on data characteristics and estimation precision. Therefore, choosing suitable copulas for different research purposes is important in practice.

5. Conclusions

In this paper, a new irrigation water allocation mode, the channel-pond joint water supply mode, is proposed, and the Plackett copula and No.16 copula are used to evaluate the failure probability of the mode in the ZID. The following conclusions can be drawn from this study:

(1) Ignoring the action of ponds in the evaluation of irrigation water allocation strategy failure diverges from reality. The CPJM matches the real practice of water utilization and is more reasonable in terms of physical mechanisms, thereby avoiding the pointless enlargement of the engineering scale for the purpose of increasing irrigation reliability.

(2) The Placket copula and No.16 copula are optimized via the method of mathematical statistical analysis and the method to improve the estimation precision, respectively. Based on the two copulas, the joint probability, the conditional probability, and the corresponding return period are obtained. The results indicate that the risk is lower than that found by a previous study and that the failure probability analysis is more objective based on the actual water utilization behavior. Clearly, the CPJM is more realistic and practicable. The irrigation water allocation strategy based on CPJM can be determined for different hydrological years and predicted frequencies of CWS and PWS.

(3) The results of the chosen copulas with different methods may differ greatly. Therefore, it is necessary to choose suitable copulas for different research purposes in practice to avoid misestimation and unrealistic designs.

(4) We chose the CWS and the PWS as the two factors for analysis based on the computational complexity of the parameters with increasing variables. This method may have limitations when the irrigation system is influenced by more factors. Moreover, when the study region is very large and features a complex water balance mechanism, adequate data on the hydrology and dynamic features of the system are needed. For example, because ground water is also a significant portion of irrigation water, irrigation water management should incorporate both surface water and groundwater sectors. In such a situation, more complex models should be introduced. For example, since ground water is also a significant portion of irrigation water, it is desired to tackle the issue of irrigation water management from both surface and ground sectors. Under such a situation, more complex modes should be introduced.

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