Leakage Detection and Estimation Algorithm for Loss Reduction in Water Piping Networks

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Abstract: Water loss through leaking pipes constitutes a major challenge to the operational service of water utilities. In recent years, increasing concern about the financial loss and environmental pollution caused by leaking pipes has been driving the development of efficient algorithms for detecting leakage in water piping networks. Water distribution networks (WDNs) are disperse in nature with numerous number of nodes and branches. Consequently, identifying the segment(s) of the network and the exact leaking pipelines connected to this segment(s) where higher background leakage outflow occurs is a challenging task. Background leakage concerns the outflow from small cracks or deteriorated joints. In addition, because they are diffuse flow, they are not characterised by quick pressure drop and are not detectable by measuring instruments. Consequently, they go unreported for a long period of time posing a threat to water loss volume. Most of the existing research focuses on the detection and localisation of burst type leakages which are characterised by a sudden pressure drop. In this work, an algorithm for detecting and estimating background leakage in water distribution networks is presented. The algorithm integrates a leakage model into a classical WDN hydraulic model for solving the network leakage flows. The applicability of the developed algorithm is demonstrated on two different water networks. The results of the tested networks are discussed and the solutions obtained show the benefits of the proposed algorithm. A noteworthy evidence is that the algorithm permits the detection of critical segments or pipes of the network experiencing higher leakage outflow and indicates the probable pipes of the network where pressure control can be performed. However, the possible position of pressure control elements along such critical pipes will be addressed in future work.

Keywords: hydraulic model; pipe leakage; leakage detection; leakage estimation; water distribution systems; water loss

1. Introduction

Losses occur in nearly all water distribution networks, even though the amount of water loss in each distribution system varies between countries and also from a particular distribution system to the other. In South Africa, for instance, almost 37% of the total input volume is lost through leaking pipeline [1]. In some well-developed and monitored water distribution systems, about 7% of the total input volume into the network is lost through leaking pipes [2]. Elsewhere, in less monitored systems, more than 50% of the total input volume into the network is lost through leaking pipes [3,4]. Therefore, water loss through leaking pipes constitutes a major challenge to the operational services of water utilities and is recognised as a costly problem strongly linked with interrupted service, waste of energy and natural resources [5–7]. Furthermore the quality of drinking water is a major concern. Leaks can introduce infections into the water distribution system under low pressure conditions [8,9].
Furthermore, the financial cost associated with leaking pipes is on the high side [1] and cannot be overlooked. Certainly, in water piping networks, minimising the leakage reduces the energy wasted in water pumping while also increasing the revenue generated by the water utilities according to the relation [10]

\[ E = \sum_i \rho \times g \times Q_{i\text{-leak}} (H_i + \Delta H_i) \times T \]  

\[ \text{CO}_2 = E \times C_{\text{int}} \]  

where \( E \) is the energy consumption losses caused by water loss, \( Q_{i\text{-leak}} \) is the leakage outflow at location \( i \), \( H_i \) is the piezometric head at location \( i \), \( \Delta H_i \) is the head loss between the pumps and the water loss location, \( g \) is the acceleration due to gravity, \( \rho \) is the fluid density, \( T \) represents the duration of the water loss at location \( i \) and \( C_{\text{int}} \) denotes the intensity of carbon dioxide emission.

It is evidence from Equations (1) and (2) that reducing the leakage outflows will save more energy being injected into the network, which directly reduces the emission of carbon dioxide into the atmosphere.

In water distribution networks (WDNs), leakage through pipes have evolved into two major types, namely burst type and background type leakages. The former is characterised by quick pressure drop and can be easily detected by measuring instruments such as pressure sensors stationed at specific locations along the length of the pipe. Burst type leakage often surfaces on the ground and are usually reported by public or utility workers. Thus, the repair time is faster. Background leakages are not detectable by measuring instruments, does not surface on the ground and can go unreported for a very long period of time posing a major threat to water utilities. They are not characterised by a sudden pressure drop compared to pipe burst. In water distribution networks, background leakage is hidden and runs continuously along the length of the pipes and can only be controlled by reducing pressures at the pipe node. Although most leakage detection methodology can only detect the most probable leaks in the network. Detecting leaks beyond a certain level is typically uneconomical [11]. Background leakages increases with the pipe internal pressure, therefore, reducing the excessive pressure at strategic point(s) in the network is worthwhile in reducing water losses due to the leaking pipes. However, identifying the segments or nodes of the network where such pressure control is required is very essential. Most of the existing methodologies developed can mainly be used for detecting, and in some cases, localising burst type leakages using flow meters and pressure sensors stationed at specific locations on the pipe. Background type leakages are not detected by measuring instruments, thus, the techniques based on pressure sensor data are not effective in detecting background type leakages. This paper focuses on the development of an algorithm for the detection and estimation of background leakage outflows in WDNs. The proposed algorithm will help in estimating and reporting the pipes of the network experiencing higher background leakage outflows. These pipes are considered as critical pipes by the algorithm and recommends pressure control along the critical pipes. Selecting the pipes of the network where pressure control and monitoring is required is a benefit of the proposed algorithm. The rest of the paper is organised as follows. In Section 2, the background of the study as well as some past research efforts are briefly discussed. In Section 3, the proposed leakage detection algorithm and its formulations are discussed. The results of its application to two different water distribution networks are discussed in Section 4 while Section 5 presents the conclusion and future works.

2. Background and Related Works

There is a clear evidence that water loss through leaking pipes in water distribution systems is complex and has a significant impact on the water system. Therefore, understanding the hydraulic characteristics of leakage flow and its control is crucial to designing an effective leakage detection methodology. A typical leakage programme usually starts with water audit based on the available
flow measurements. To this objective, the network leakage is estimated based on either a 24 h zone
measurement or minimum night flow (MNF) analysis [9]. The former requires an isolated area
of the network supplied from one or two inflow points where the inflow into the area is measured and
monitored. The minimum night flow analysis involves flow measurements in the period of least
consumption typically between the hours of 02:00 h to 04:00 h [10]. During the MNF period,
the water demand is usually very low, the head losses in each pipe get reduced, the pressure head
and leakages reach their maximum values [10,12]. The water loss in the network is then obtained
by subtracting the measured legitimate night use from the minimum night flow measurements.
In a situation where the flow measurements in each pipe are not available, a hydraulic model with
the capability of estimating the network leakage outflows could prove invaluable and go a long way
in solving network leakage issues. Nevertheless, several definitions of leakage in WDNs exist. Initially,
leakage outflow is attributed to a flow through an orifice [13]. The orifice flow equation described
by Equation (3) is similar to the emitter features of the EPANET software widely used for WDNs
hydraulic simulation [14].

$$Q_{\text{leak}} = C_d h^n = C_d A (2gh)^n$$

where $Q_{\text{leak}}$ represents the leakage flow rate, $C_d$ is the leakage discharge coefficient, $h$ denotes
the pressure head, $A$, the area of the leak opening and $n$ is the pressure to leakage exponent usually 0.5.

An improved leakage equation is proposed by May [15] where the leakage discharge is expressed
in terms of the leakage opening area categorised as fixed area and variable area discharge (FAVAD).
The FAVAD equation can be described by the following equation

$$Q_{\text{leak}} = C_d A_{\text{leak}}^f (2gh)^n + C_d A_{\text{leak}}^v (2gh)^n$$

where $A_{\text{leak}}^f$ and $A_{\text{leak}}^v$ are the fixed and variable leak opening area. Equations (3) and (4) are widely
used in many research studies [16–24] to model and assesses network leakage outflows. However, the
research attempts in [16,25] have proved that the use of the orifice flow equation can lead to misleading
results based on some specific pipe conditions. For instance, Greyvenstein and van Zyl [16] reported
that the orifice flow equation can lead to erroneous results when the model pipe is made of flexible
material. Furthermore, when a negative pressure head occurs in the node of the network, a misleading
results can also be given by the orifice flow equation [25].

Most of these research works are dedicated to burst type leakage with a particular leak
opening area. For diffuse flow along the pipes (background leakage type) where the leak opening
is not visible and the area cannot be estimated, the previous mentioned leakage models are not sufficient
for estimating such type of leakage. While the methodology proposed by the authors in [26] may
be used, it requires the knowledge of the total leakage for parameter calibration. Also, most of the
methodologies developed in the past and recent years for leak detection and localisation can mainly
be used for burst type leakage detection. These research efforts delve into the use of flow meters
and pressure sensors stationed at specific locations on the pipe to detect leakages. For instance,
Aksela et al. [27] uses the knowledge of reported leak experience with the data collected from
flow meter readings to model and train the system. Farley et al. [28] presented a methodology
for the detection of pipe burst, achieved by identifying the optimal locations of pressure sensors.
Similarly, a sensor placement and leakage detection methodology to identify leakages in a WDS based
on the deviation of sensor pressure from an estimated pressure was presented by Perez et al. [29].
Some other research works employ the benefits of the artificial intelligence system for leakage detection
purposes [30–32]. Mounce et al. [30] proposed a leakage detection method based on an artificial neural
network to harmonise data obtained from different sensors to classify different types of leakage
in a WDS. The developed methodology is based on sensor time series data and thus requires a large
monitoring database. In many of these research works, the location of the pipe burst is identified
by using the arrival times and magnitudes of burst-induced transient waves at two or more points
where the pressure sensors were stationed. Background type leakages are not detected by measuring
instruments and go unreported for a long period of time, thus, the techniques based on pressure sensor data [28–31,33] are not effective in detecting background type leakages.

Background leakage is hidden and runs continuously along the length of the pipes in the network. It has been acknowledged that background leakage outflows posed the major threat to water utilities as they can neither be detected by measuring instruments nor surface on the ground for utility workers to notice. More recently, the battle of background leakage outflows has been a subject of discussion in the research community [34–42]. The current battle of background leakage assessment for water networks (BBLAWN) is an attempt to combine the methodologies for water distribution systems rehabilitation planning and sectorization [43–49] with pressure control management strategies [50–62] to achieve a reduced leakage ratio and lower the water distribution systems operational cost. Certainly, reducing the pipe pressure will significantly reduce the leakage outflows in the network. However, in the practical sense, it is not cost effective to reduce the entire network pressure. This is because some nodes of the network must have sufficient pressure to supplying and fully satisfying end users demands at the nodes. If the pressure head at a node is insufficient, a reduction in the water flowing from the tap is expected and, in the worst case, the discharge that can be drafted will be zero, regardless of the actual demand [25]. The proposed algorithm in this paper would help in specifying or determining which pipes of the network where such pressure control is necessary. Selecting the pipes of the network where pressure control and monitoring is required will be a benefit and will assist in supporting the BBLAWN for reduced leakage ratio in water network.

3. The Proposed Leakage Detection Algorithm

The process involved in the proposed leakage detection algorithm is briefly discussed in the pseudo code illustrated in Algorithm 1. The algorithm incorporates a leakage model into a classical water distribution network hydraulic simulation model to estimate the network flows, including leakage outflow at each node as well as at the pipe level. As shown in Algorithm 1, the process entails the hydraulic analysis of the water network and the leakage computation. The algorithm load and read the supplied water distribution network data and initialised. Afterwards, a hydraulic analysis is performed for such network based on the supplied data. The hydraulic analysis is achieved through modelling the water network topology and solving the resulting model using an iterative Newton-based methodology. During the hydraulic analysis of the network, the nodal leakage outflow is computed and the algorithm then checks if the estimated leakage outflow at the node is relatively low (or is less than a predefine tolerance), and if such is confirmed, it reports no leaking node as the flow rate in such node is less than tolerance or relatively low. Otherwise, it reports the leaking node number and search for all the pipes connected to this node. Thereafter, it computes the leakage flow in each pipe. Furthermore, it checks if the estimated leakage flow in each pipe is relatively high. Such pipe is tagged as critical pipes, the algorithm then recommends a pressure control along the critical pipes. In most cases, any pipe of the network with a relatively high background leakage flow above a predefine tolerance is tagged as a critical pipe where a pressure control or pressure adjustment is recommended.
Algorithm 1: Proposed leakage detection algorithm
1: Start |
2: Load network parameters
3: Read network parameters and initialise
4: for node $i = 1$ to $n_t$, ($n_t$: The number of nodes in the network)
5: for pipe $j = 1$ to $b$, ($b$: The number of pipes in the network)
6: Run hydraulic analysis and compute leakage vector $\vec{q}_{nleak}$
   if $\vec{q}_{nleak} < $ tolerance (or relatively low)
      Print “No leaking node”
   else
      i: Print “Leaking node ID”
      ii: Search for pipes connected to this node
      iii: Compute the pipe leakage vector $\vec{Q}_p$
      if $\vec{Q}_p < $ tolerance (or relatively low)
         Print “No leaking pipe”
      else
         Print “Leaking pipe ID”
         Tag leaking pipe as critical pipes and report critical pipe ID
         Display “Pressure control recommended along the critical pipe with ID…”
   end if
7: end for $j$
8: end for $i$
9: Stop |

3.1. WDN Topology and Model Formulation

A water distribution network (WDN) can be represented by a connected graph with a set of edges and a set of nodes. The former consist of pipes, pumps, and valves. The basic hydraulic equations describing the flow in a water distribution system are governed by two basic principles; namely the principle of mass continuity in the node and energy conservation around the hydraulic loop. For any water piping networks comprising of $b$ number of branches or pipes, $n$ number of junction nodes, $n_s$ number of source nodes or fixed-grade nodes (nodes with known pressure heads), and $n_l$ number of load nodes (nodes with unknown pressure heads), the total number of nodes in the network is $n = n_s + n_l$. The mass continuity equation can be written for each node, and the energy conservation equation can be written for any loop. Using graph theory, the continuity equation at any given node may be expressed as

$$q_i = \sum C_{ij} Q_j$$

where, $q_i \in \mathbb{R}^{n \times 1} = [q_1, q_2, \ldots, q_n]^T$ represents the column vector of nodal injection or demand and $Q_j \in \mathbb{R}^{b \times 1} = [Q_1, Q_2, \ldots, Q_b]^T$ is the column vector of pipe flows while $C_{ij}$ denotes the node-pipe connectivity matrix of dimension $(n \times b)$, whose elements are derived from

$$C_{ij} = \begin{cases} 
+1 & \text{if the flow in pipe } j \text{ enters node } i \\
-1 & \text{if the flow in pipe } j \text{ leaves node } i \\
0 & \text{if pipe } j \text{ is not incident to node } i 
\end{cases}$$
The node-pipe connectivity matrix $C$ may be decomposed into two sub-matrices as

$$C = \begin{bmatrix} C_s \\ C_l \end{bmatrix}$$  \hspace{1cm} (7)

where, $C_s$ denotes the source node-pipe connectivity matrix of dimension ($n_s \times b$) relating to the node with known pressure and $C_l$ is the load node-pipe connectivity matrix of dimension ($n_l \times b$) relating to the node with unknown pressure. If the pressure is given at the source nodes and demand (loads) are given at the load nodes, by decomposing Equation (5) and writing the flow at the load nodes only

$$C_l Q = -q$$  \hspace{1cm} (8)

Furthermore, the energy conservation concerns the pressure drop across the pipes. For a closed loop, the pressure drop across the pipes may be expressed as

$$D \Delta P = 0$$  \hspace{1cm} (9)

where $D \in \mathbb{R}^{m \times b}$ represents the loop-pipe incidence matrix and $m$ is the number of loops. The elements of matrix $D$ are derived from

$$D_{ij} = \begin{cases} +1 & \text{if pipe } j \text{ is in loop } i \text{ and is in the same direction} \\ -1 & \text{if pipe } j \text{ is in loop } i \text{ and is in the opposite direction} \\ 0 & \text{if pipe } j \text{ is not in loop } i \end{cases}$$  \hspace{1cm} (10)

In Equation (9), $\Delta P = [\Delta P_1, \ldots, \Delta P_b]^T$ represents the pressure drop vector across the pipes. The energy conservation may also be expressed as

$$\Delta P = \begin{bmatrix} C_s^T \\ C_l^T \end{bmatrix} \begin{bmatrix} P_s \\ P_l \end{bmatrix}$$  \hspace{1cm} (11)

where $P_s = [P_{s(1)}, \ldots, P_{s(n_s)}]^T$ denotes the column vector (dimension ($n_s \times 1$)) of the source pressure and $P_l = [P_{l(1)}, \ldots, P_{l(n_l)}]^T$ is the vector of the load pressure of dimension ($n_l \times 1$).

Another set of equations necessary for the solution of a piping network are the pipe-flow equations which relate the pressure drop across a given pipe to the flow in that pipe. Consider a network element shown in Figure 1, with two end nodes $i$ and $g$, the pressure drop due to the friction of the flow of water with the pipe wall is generally expressed as

$$\Delta P_j = P_i - P_g = k Q_j^a = k |Q_j|^{a-1}$$  \hspace{1cm} (12)

where $P_i$ and $P_g$ are the pressure at both ends of the pipe, $Q_j$ is the flow in the pipe $j$ and $k$ represents the pipe hydraulic resistance.

Substituting $\Delta P = k |Q|^{a-1}$ into Equation (11)

$$\text{diag}(k |Q|^{a-1}) Q - C_l^T P_l - C_s^T P_s = 0$$  \hspace{1cm} (13)
If we define a matrix $A$ as

$$A = diag(k|Q|^\alpha - 1)$$

Equation (13) may be written as

$$AQ - C_l^T P_l - C_s^T P_s = 0$$

(14)

Both Equations (8) and (14) are the steady state hydraulic model to be solved to estimate the pipe flow and the pressure at the load node, given the pressure at the source node and the demand at the load node. The system of equations described by Equation (15) is partly linear and partly non-linear.

$$\begin{align*}
AQ - C_l^T P_l - C_s^T P_s = 0 \\
C_l Q + q = 0
\end{align*}$$

(15)

Equation (15) can be solved by an iterative method. The matrix $A$ is a diagonal matrix of dimension ($b \times b$) whose elements are derived from the pressure drop relation as

$$A = \begin{bmatrix}
k_1|Q_1|^{\alpha - 1} & \cdots & \cdots & \cdots \\
\cdots & k_2|Q_2|^{\alpha - 1} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
\cdots & \cdots & \cdots & k_b|Q_b|^{\alpha - 1}
\end{bmatrix}$$

(16)

Both $\alpha$ and $k$ depend on the pressure drop or head loss model used [14,63]. $k$ is the vector of pipe hydraulic resistance, it depends on parameters as

$$k_j \in f(\varsigma, D, L)$$

(17)

where $\varsigma$ represents the pipe equivalent roughness coefficient, $D$ is the pipe diameter, $L$ is the pipe length and $\alpha$ is the the pressure exponent whose value depends on the pressure drop or head loss model used (1.85 for Hazen-William (HW) and 2 for both Darcy-Weisbach (DW) or Chezy-Manning (CM) head loss model) [14,63].

In any event, using the DW or the HW model, the hydraulic resistance for the $j^{th}$ pipe may be expressed as

$$k_j = \frac{8f_j L_j}{g\pi^2 D_j^5}$$

(18)

for DW model, and

$$k_j = L_j \left(\frac{3.59}{Chw_j}\right)^{1.852} \times \frac{1}{D_j^{4.87}}$$

(19)

for HW model. In Equation (18), $f_j$, and $g$ represent the frictional factor of the $j^{th}$ pipe and the acceleration due to gravity. In Equation (19), $Chw_j$ denotes the Hazen-William friction coefficient for the $j^{th}$ pipe. The variables $L_j$ and $D_j$ in both equations represent the length and diameter of the $j^{th}$ pipe.

The pipe frictional factor $f_j$ in Equation (18) depends on the Reynolds number ($Re$) as well as the equivalent roughness factor ($\varsigma$) and can be estimated using the Colebrook equation, the Jain’s formula or any other related expressions.
It is important to emphasize that in order to account for leakage flow in the model represented by Equation (15), the demand vector $q$ comprises of the normal demand and the nodal leakage flows. That is,

$$q = q_{\text{norm}} + q_{\text{leak}}$$

where $q_{\text{norm}}$ denotes the vector of the normal nodal demand and $q_{\text{leak}}$ is the vector of the nodal leakage flow.

### 3.2. WDN Hydraulic Model Solution

The classical pipe network analysis problems is to find a set of flow $Q$ and the pressure $P$ in a water distribution network with the input (nodal injection or demand) and the source pressure known. The system of equation in (15) may be solved applying Newton-Raphson iterative method. Define $f(x)$ as

$$f(x) = \begin{pmatrix} A & -C_l^T \\ C_l & 0 \end{pmatrix} \begin{pmatrix} Q \\ P_l \end{pmatrix} - \begin{pmatrix} C_l^T P_s \\ -q \end{pmatrix}$$

with $x = (Q, P_l)$, therefore,

$$f(Q, P_l) = \begin{pmatrix} f_1(Q, P_l) \\ f_2(Q, P_l) \end{pmatrix}$$

The system of non-linear equation of $f(Q, P_l) = 0$ may be solved by Newton-Raphson (NR) iterative method. Thus, at every iteration "r", the NR method is described as

$$x^{(r+1)} = x^{(r)} - J^{-1} f(x^{(r)})$$

where $J$ is the Jacobian matrix of the function $f(x)$. For the function above, the Jacobian matrix is given by

$$J = \begin{pmatrix} D & -C_l^T \\ C_l & 0 \end{pmatrix}$$

where $D \in \mathbb{R}^{b \times b}$ is a diagonal matrix whose elements are the partial derivatives of pressure drop component $A = \text{diag}(k|Q^{a-1})$ given as $\alpha A$. The elements of matrix $D$ for all the pipes in the network may be obtained as

$$D = \begin{bmatrix} ak_1|Q_1|^{a-1} & \cdots & \cdots & \cdots \\ \cdots & ak_2|Q_2|^{a-1} & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ \cdots & \cdots & \cdots & ak_b|Q_b|^{a-1} \end{bmatrix} = \alpha A$$

From Equation (23),

$$f(x^{(r+1)} - x^{(r)}) = -f(x^{(r)})$$

Substituting the value of $J$, $f(x^{(r)})$ and $x(Q, P_l)$ into Equation (26), one may write

$$\begin{pmatrix} DQ^{(r+1)} - DQ^{(r)} - C_l^T P_l^{(r+1)} + C_l^T P_l^{(r)} + AQ^{(r)} - C_l^T P_s^{(r)} \\ C_l Q^{(r+1)} - C_l Q^{(r)} + C_l Q^{(r)} \end{pmatrix} = \begin{pmatrix} C_l^T P_s \\ -q \end{pmatrix}$$

From Equation (27), one may rewrite

\[ DQ^{(r+1)} - C_t^T P_t^{(r+1)} = (D - A)Q^{(r)} + C_s^T P_s \]  

(28)

and,

\[ C_t Q^{(r+1)} = -q \]  

(29)

Multiplying both sides of Equation (28) by \( C_t D^{-1} \), one may write

\[ C_t Q^{(r+1)} - C_t^T D^{-1} C_t P_t^{(r+1)} = C_t D^{-1} [(D - A)Q^{(r)} + C_s^T P_s] \]  

(30)

Replacing \( C_t Q^{(r+1)} = -q \) in Equation (30), therefore

\[ -q - C_t^T D^{-1} C_t p_t^{(r+1)} = C_t D^{-1} [(D - A)Q^{(r)} + C_s^T P_s] \]  

(31)

Define matrix \( B \) (network admittance matrix) as

\[ B = C_t D^{-1} C_t^T \]  

(32)

Equation (31) may the be rewritten as

\[ p_t^{(r+1)} = B^{-1} [-q - C_t D^{-1} [(D - A)Q^{(r)} + C_s^T P_s]] \]  

Therefore, the estimate of the pressure at each iteration \( "r" \) is obtained as

\[ p_t^{(r+1)} = -B^{-1} [q + C_t D^{-1} [(D - A)Q^{(r)} + C_s^T P_s]] \]  

(33)

The admittance matrix \( B \) used is highly sparse, symmetric and must be handled using an efficient sparsity techniques. From Equation (28) as well, the estimate of the pipe flow at each iteration \( "r" \) may be expressed as

\[ Q^{(r+1)} = Q^{(r)} + D^{-1} [C_t^T P_t^{(r+1)} - AQ^{(r)} + C_s^T P_s] \]  

(34)

The Equations (33) and (34) give the iterative solution of the system of non-linear equation describe in Equation (15). The Newton-Raphson method is known to give a fast convergence provided a good initial solution is available [64]. The derived solution is closely similar to those obtained by [65–67]. The matrices \( C_t \) and \( C_s \) are derived from the topology of the water distribution networks.

3.3. Integrating a Leakage Model

In water distribution networks (WDNs), leakage occurs at the nodes as well as along the pipes. Previous research works have shown that leakage depends on the network pressure, therefore, the pressure-leakage relationship is defined in the vector \( \bar{q}_{\text{leak}} \). It should be noted that background leakage flow occurs continuously along the length of a pipe. If the leakage flow along a pipe \( j \) is denoted by \( \bar{Q}_j \), then \( \bar{Q}_j \) may be expressed as

\[ \bar{Q}_j = \begin{cases} \beta_j L_j (P_j)^\delta & \text{if } P_j > 0 \\ 0 & \text{if } P_j \leq 0 \end{cases} \]  

(35)

where, \( \beta_j \) is the background leakage discharge coefficient of the \( j^{th} \) pipe, \( L_j \) is the length of the \( j^{th} \) pipe, \( \delta \) is the leakage-pressure exponent reported to be equivalent to 1.18 for background leakage [68]. \( P_j \) is the pressure in pipe \( j \) computed as the mean of the pressure values at its end nodes.
In matrix form, defining a vector \( \bar{Q} = [\bar{Q}_1, \bar{Q}_2, \ldots, \bar{Q}_B]^T \) as the vector of the leakage flow along all the pipes, then

\[
\bar{Q} = \begin{cases} 
\beta L(\bar{P})^\delta & \text{if } \bar{P} > 0 \\
0 & \text{if } \bar{P} \leq 0
\end{cases}
\]  

(36)

where, \( \beta = [\beta_1, \ldots, \beta_B]^T \) is the vector of empirical constants relating to background leakage parameter, \( L = [L_1, \ldots, L_B]^T \) is the vector of the pipe length, and \( \bar{P} = [\bar{P}_1, \ldots, \bar{P}_B]^T \) is the vector of the average pressures along the pipes. The vector \( \bar{P} \) may be expressed using the topological incidence matrix as

\[
\bar{P} = \frac{1}{2} \psi^T \Psi
\]

(37)

where,

\[
\psi = |C|
\]

(38)

is the absolute of the network incidence matrix \( C \), which gives a matrix of ones. \( \Psi_l \) is the vector of the load node pressures.

If the vector of the nodal leakage is denoted by \( \bar{q}_{\text{leak}} \), then the elements of \( \bar{q}_{\text{leak}} \) may be computed from the topological incidence matrix as

\[
\bar{q}_{\text{leak}} = \frac{1}{2} \psi \bar{Q}
\]

(39)

4. Numerical Examples

The applicability of the developed leakage detection algorithm was demonstrated on two different water distribution networks derived from literature. For both networks, \( \beta = 2.0 \times 10^{-8} \) [34] was assumed for the pipes. All computations and the hydraulic analysis are done in MATLAB software environment.

4.1. Numerical Example 1

Figure 2 shows the schematic diagram of the case study water network used for the numerical example 1. The network consist of 1 supply node (tank node) and 8 demand or load nodes (non-tank nodes). The supply node (node 1) and the load nodes (which indexes from node 2 to node 9) are interconnected by a series of pipes with different length and diameter. The data for each pipe and node in this network is shown in Table 1 and Table 2.

Figure 2. The schematic diagram of the case study network 1 (Adapted from: [69]).
Table 1. Pipe data for the case study network 1 [69].

<table>
<thead>
<tr>
<th>Pipe ID</th>
<th>Start Node</th>
<th>End Node</th>
<th>Length (m)</th>
<th>Diameter (mm)</th>
<th>Chw</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>500</td>
<td>150</td>
<td>110</td>
</tr>
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Table 2. Node data for the case study network 1 [69].

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In Figure 3, the profile of the nodal leakage outflow for the case study network 1 is presented. From the figure, it may be seen that nodes 2, 3, 4 and 5 have the highest leakage outflow while the least leakage outflow occurs in nodes 7, 8 and 9. Therefore, nodes 2, 3, 4 and 5 may be initially considered as critical nodes of the network where pressure adjustment could be needed. The pipes connected to these nodes are pipes 1, 2, 3 for node 2, pipes 2, 5, 6 for node 3, pipes 3 and 4 for node 4 while pipes 4 and 5 are connected to node 5. If the leakage flow along any or all of these pipes is relatively high, therefore, a pressure control is required in such node of the network.

Figure 3. The profile of the nodal leakage outflow for the case study network 1.
Figure 4 shows the pattern of the discharge and the leakage flow rate in each pipe for the case study network 1. From the figure, it may be observed that pipe 1 has the highest flow rate. This is obvious as it is directly connected to the supply node (node 1) while the most downstream pipe (Pipe number 10) has the least flow rate. In terms of the leakage flow, pipes 8, 9 and 10 has the least leakage flow rates among other pipes in the network. The highest leakage flow rate is noticed in pipes 2 and 4. Also, the leakage flow in pipes 4 and 5 is higher than their corresponding actual discharges. Therefore, these pipes together with pipe 1 may be tagged as critical pipes for this network where pressure adjustment or pressure control is needed.

Figure 4. The pattern of the pipe discharge and leakage flow for the case study network 1.

In Figure 5, the profile of the estimated flow rate in each pipe before and after leakage outflow is presented. It is evidence that the flow rate in most of the pipes is reduced when leak occurs along pipes. In pipes 8, 9 and 10, their flow rate remain the same due to the fact that the leakage flow in those pipes is zero. In most cases, the presence of leak reduces the flows in each pipe.

Figure 5. The Pattern of the pipe flow rate before and after leak for the case study network 1.

To further establish which pipes of the network contribute mostly to water loss. The water loss volume through each pipe is computed for two minutes and the results obtained are illustrated in Figure 6. Minutes analysis was considered because the actual transients in water distribution systems tend to have high frequencies with time periodicities of seconds or minutes, by far smaller than the time increments of interest, generally hours [70]. Considering Figure 6, it may be observed that pipe 2
(connected between nodes 2 and 3) and pipe 4 (connected between nodes 4 and 5) are experiencing higher water loss volume. With evidence from Figure 3 that initially tagged nodes 2, 3, 4 and 5 as critical nodes with higher leakage outflow, thus, with the water loss volume result, one may safely conclude that pipes 2 and 4 experiencing the greatest loss volume and which are connected to these nodes can be tagged as critical pipes. Therefore, the pressure at their corresponding nodes may be adjusted or reduced to minimise the water loss rate.

Figure 6. Water loss volume in each pipe for the case study network 1.

4.2. Numerical Example 2

Figure 7 shows the schematic diagram of the case study water network used for the numerical example 2. As can be seen in Figure 7, the network consist of 1 supply node (tank node) and 45 demand or load nodes (non-tank nodes). The supply node (node 1) and the load nodes (which indexes from node 2 to node 46) are interconnected by a series of pipes with different length and diameter. The data for each pipe and node in this network is shown in Tables 3 and 4.

Figure 7. The schematic diagram of the case study network 2 (Adapted from: [71]).

Figure 8 shows the leakage profile for each node of the case study network 2, including the supply node. It is evidence that nodes 5, 6 and 41 of this network are experiencing the highest leakage outflow and may be considered as being the most critical nodes of the network. Thus, pressure control
might be needed to minimise the leakage outflow through these nodes as well as the overall leakage in the network. The pipes connected to these nodes include pipes (4, 5, 59, 60, 70, 71) for node 5, pipes (5, 6, 14, 46, 60, 61, 71) for node 6, and pipes (51, 52, 53) to node 41 respectively. Any pipe connected to these nodes with a relatively high leakage flow rate requires a pressure control at either one or both of its end nodes.

![Figure 8. The profile of the nodal leakage outflow for the case study network 2.](image)

**Table 3.** Pipe data for the case study network 2 [71].

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</tbody>
</table>

In Figure 9, the pattern of the nodal demand and the leakage level at the nodes is illustrated. The leakage level is compared to the actual demand at the nodes of the network. It is obvious that even though the background leakage outflow at each node is lower than the actual nodal demand, the leakage level can still pose a threat to the available flow at each consumer end. The flow delivered to the consumer end will be compromised due to the presence of the nodal leakage flow, though not significant.

Figure 9. Nodal demand vs the leakage outflow for each node for the case study network 2.

Figure 10 concerns the pattern of the pipe discharge and the leakage flow rate in each pipe. In a similar manner to the case study network 1, pipe 1 has the highest flow rate as it is directly connected to the supply node while pipe 38 has the least flow rate. Furthermore, it is evident that the leakage flow is relatively low compared to the actual flow in each pipe apart from pipes 24, 38, 41 and 89 having leakage flow higher than their actual discharge. Aside from searching for the pipes with the highest leakage flow, any pipe of the network having a leakage flow greater than its actual
discharge (such as pipes 24, 38, 41 and 89) of the case study network 2, may also be considered to be a critical pipe.

![Figure 10](image1.png)

**Figure 10.** The pattern of the pipe discharges and the leakage flow in each pipe for the case study network 2.

The result presented in Figure 11 shows that the leakage flow in each pipe of the case study network 2 is almost negligible when compared to the actual discharge in each pipe. Figure 11 illustrates the pipe discharge with and without the event of leak. The pattern of both flows (with and without leak) is almost the same because the leakage flow in each of the pipe is relatively low and almost negligible compared to the actual pipe discharge.

![Figure 11](image2.png)

**Figure 11.** The pattern of the pipe discharges before and after leakage flow for the case study network 2.

In Figure 12, the water loss volume through each pipe is illustrated. This figure shows which branches of the network contribute mostly to the water loss volume. Some pipes (branches) of the network are experiencing higher water loss when computed for two minutes of flow. Pipes (5, 13, 40, 51, 52, 53, 57, 58, 60 and 71) are experiencing higher water loss volume (greater than 0.04 m$^3$/s), and may be considered to be critical pipes. A noteworthy evidence from this figure is that even though the leakage flow rate in these pipes (the considered critical pipes) are relatively low compared to the actual discharge, those pipes still contribute largely to water loss volume in the network.
Therefore, pressure control in these pipes will minimise the leakage flow rate through these pipes, which adversely reduce the volume of water loss in the entire network.

![Figure 12. The volume of water loss through each pipe for the case study network 2.](image)

In Table 5, a comparison of the solution convergence of the hydraulic analysis stage of the algorithm for the case study networks is presented. An error tolerance of $10^{-5}$ [72] was used as the convergence criterion. It is observed that the hydraulic simulation method used has a faster convergence; 4 iterations for a network with 71 pipes.

| Table 5. Comparison of the solution/convergence history for the case study networks. |
|----------------------------------|------|------|
| Case Study | Network 1 | Network 2 |
| Number of pipes | 10 | 71 |
| Number of nodes | 9 | 46 |
| Number of iteration | 3 | 4 |

5. Conclusions

Water distribution networks (WDNs) are disperse in nature with numerous number of nodes and pipes. Consequently, identifying the segments of the network and the exact leaking pipelines connected to these segments where higher background leakage outflow occurs is a challenging task. In this work, an algorithm for detecting and estimating background leakage outflow in water distribution networks is developed and presented. The algorithm integrates a leakage model into a WDN hydraulic model for solving the network leakage flow. The applicability of the developed algorithm is demonstrated on two water supply networks. The results presented show that the developed algorithm permits the detection and estimation of critical segments or branches of the network experiencing higher background leakage outflow and a point of the network where pressure control may be performed. However, localising the exact point on the pipe where higher background leakage outflow occurs and the possible position of pressure control elements along such critical pipes will be addressed in future work. A noteworthy evidence is that, the algorithm may be used to assess the state of water networks.

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while Yskandar Hamam help with some improvement in the mathematical formulations. The manuscript was thoroughly reviewed by Yskandar Hamam while Adnan M. Abu-Mahfouz and Bolanle T. Abe also suggested some improvements in the paper.

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References


