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Cubic Intuitionistic q-Ideals of BCI-Algebras

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Abstract: In this paper, the notion of cubic intuitionistic q-ideals in BCI-algebras is introduced. A relationship between a cubic intuitionistic subalgebra, a cubic intuitionistic ideal, and a cubic intuitionistic q-ideal is discussed. Conditions for a cubic intuitionistic ideal to be a cubic intuitionistic q-ideal are provided. Characterizations of a cubic intuitionistic q-ideal are considered. The cubic intuitionistic extension property for a cubic intuitionistic q-ideal is established. Furthermore, the product of cubic intuitionistic subalgebras, ideals, and q-ideals are investigated.

Keywords: cubic set; cubic intuitionistic set; cubic intuitionistic subalgebra; cubic intuitionistic ideal; cubic intuitionistic q-ideal

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1. Introduction

In 2012, Jun et al. [1] presented cubic sets, and afterward, this idea was connected to a few algebraic structures ([2–18]). In 2017, extending the concept of a cubic set, Jun [19] established the idea of a cubic intuitionistic set. He introduced the notions of the (left, right) internal cubic intuitionistic set, the double left (right) internal cubic intuitionistic set, the cross left (right) internal cubic intuitionistic set, and the (cross) external cubic intuitionistic set and investigated related properties. He applied this theory to subalgebras and ideals in a BCK/BCI-algebra and obtained some useful results. He provided relations between a cubic ideal and a cubic intuitionistic subalgebra in a BCK/BCI-algebra. Senapati et al. [20] applied this theory to several ideals of BCI-algebras. Senapati et al. [21] also applied cubic intuitionistic theory to subalgebras and closed ideals of B-algebras.

The cubic sets consider only the membership intervals and do not weight the non-membership segment of the information entities. However, in reality, it is frequently difficult to express the estimation of the membership degree by an exact value in a fuzzy set. In such instances, it is probably simpler to depict vagueness and uncertainty in the real world using an interval value and an exact value, rather than unique interval/exact values. Therefore, the hybrid form of an interval value might be extremely valuable to depict the uncertainties because of one's reluctant judgment in a decision-making issue. For this reason, Jun presented the notion of the cubic intuitionistic set (CIS), which is described by two components simultaneously, where one represents the membership degrees by an interval-valued intuitionistic fuzzy set (IVIFS) and the other represents the membership degrees by the intuitionistic fuzzy set (IFS). Clearly, the benefit of the CIS is that it can generously contain more information to express the IVIFS and the IFS simultaneously. For example, assume a supervisor has to evaluate the work of his/her colleagues. The colleague furnishes him/her with his/her self-examined report saying that he/she had finished 10–20% and simultaneously has not accomplished 60–70%

of the work assigned to him/her. After studying his/her document from the supervisor, he/she gives his/her judgment under the IFS condition by saying that he/she disagrees with the completed work by 30% and agrees to the the incomplete work by 20%. Then, in that case, CIS is defined as $(\langle[0.10, 0.20], [0.60, 0.70]\rangle, \langle 0.30, 0.20\rangle)$. Consequently, this environment increases the level of precision by enhancing the scope of the membership interval by considering a fuzzy set membership value corresponding to it. As a result, it is a useful tool for dealing with the imprecise and ambiguous data during the decision-making process under an uncertain environment.

In this paper, we present the idea of cubic intuitionistic q-ideals in BCI-algebras. We examine the connection between a cubic intuitionistic subalgebra, a cubic intuitionistic ideal, and a cubic intuitionistic q-ideal and give conditions for a cubic intuitionistic ideal to be a cubic intuitionistic q-ideal. We set up characterizations of a cubic intuitionistic q-ideal and consider the cubic intuitionistic extension property for a cubic intuitionistic q-ideal. We introduce the product of cubic intuitionistic subalgebras, ideals, and q-ideals of BCI-algebras. Finally, we draw the conclusion and present some topics for future research.

2. Preliminaries

In this section, we express some essential ideas related to BCI-algebras and cubic intuitionistic sets over the universe of discourse T.

By a BCI-algebra, we denote an algebra T with a binary operation "$*$" and a constant zero fulfilling the accompanying axioms for all $l, j, k \in T$:

(i) $((l * j) * (l * k)) * (k * j) = 0$,
(ii) $(l * (l * j)) * j = 0$,
(iii) $l * l = 0$,
(iv) $l * j = 0$ and $j * l = 0$ imply $l = j$.

We can define a partial ordering "$\leq$" by $l \leq j$ if and only if $l * j = 0$.

If a BCI-algebra T fulfills $0 * l = 0$, for all $l \in T$, then we say that T is a BCK-algebra. Any BCK/BCI-algebra T fulfills the following axioms for all $l, j, k \in T$:

(a1) $(l * j) * k = (l * k) * j$,
(a2) $((l * k) * (j * k)) * (l * j) = 0$,
(a3) $l * 0 = l$
(a4) $l * j = 0 \Rightarrow (l * k) * (j * k) = 0, (k * j) * (k * l) = 0$.

Throughout this paper, T always means a BCK/BCI-algebra without any specification.
A non-empty subset S of T is known as a subalgebra of T if $l * j \in S$ for any $l, j \in S$. A nonempty subset I of T is known as an ideal of T if it fulfills:

(I_1) $0 \in I$ and
(I_2) $l * j \in I$ and $j \in I$ imply $l \in I$.

A non-empty subset I of T is said to be an q-ideal [22] of T if it fulfills (I_1) and:

(I_3) $l * (j * k) \in I$ and $j \in I$ imply $l * k \in I$, for all $l, j, k \in T$.

A BCI-algebra is said to be associative if $(l * j) * k = l * (j * k)$, for all $l, j, k \in T$.

A BCI-algebra is said to be quasi-associative if it fulfills the following inequality:

$$(l * j) * k \leq l * (j * k), \text{ for all } l, j, k \in T.$$

Given two closed subintervals $D_1 = [D_1^-, D_1^+]$ and $D_2 = [D_2^-, D_2^+]$ of $[0, 1]$, we characterize the order "$\ll$" and "$\gg$" as follows:

$$D_1 \ll D_2 \Leftrightarrow D_1^- \leq D_2^- \text{ and } D_1^+ \leq D_2^+$$
$$D_1 \gg D_2 \Leftrightarrow D_1^- \geq D_2^- \text{ and } D_1^+ \geq D_2^+.$$

We additionally characterize the refined maximum (rmax) and refined minimum (rmin) as:

$$\text{rmax}\{D_1, D_2\} = [\max\{D_1^-, D_2^-\}, \max\{D_1^+, D_2^+\}]$$
$$\text{rmin}\{D_1, D_2\} = [\min\{D_1^-, D_2^-\}, \min\{D_1^+, D_2^+\}].$$

Denote by $D[0,1]$ the set of all closed subintervals of $[0,1]$. In this paper, we use the interval-valued intuitionistic fuzzy set:

$$A = \{\langle l, M_A(l), N_A(l)\rangle : l \in T\}$$

in which $M_A(l)$ and $N_A(l)$ are closed subintervals of $[0,1]$ for all $l \in T$. Furthermore, we use the notations $M_A^-(l)$ and $M_A^+(l)$ to denote the left end point and the right end point of the interval $M_A(l)$, respectively, and thus, we get $M_A(l) = [M_A^-(l), M_A^+(l)]$. For effortlessness, we shall use the symbol $A(l) = \langle M_A(l), N_A(l)\rangle$ or $A = \langle M_A, N_A\rangle$ for the interval-valued intuitionistic fuzzy set $A = \{\langle l, M_A(l), N_A(l)\rangle : l \in T\}$.

Definition 1 ([19]). *Let T be a nonempty set. By a cubic intuitionistic set in T, we denote a structure $\widetilde{A} = \{\langle l, A(l), \lambda(l)\rangle : l \in T\}$ in which A is an interval-valued intuitionistic fuzzy set in T and λ is an intuitionistic fuzzy set in T.*

A cubic intuitionistic set $\widetilde{A} = \{\langle l, A(l), \lambda(l)\rangle : l \in T\}$ is basically shown by $\widetilde{A} = \langle A, \lambda\rangle$.

3. Cubic Intuitionistic q-Ideals

In what follows, we simply use T to mean a BCI-algebra, unless otherwise specified.

Definition 2 ([19]). *A cubic intuitionistic set $\widetilde{A} = \langle A, \lambda\rangle$ in T is known as a cubic intuitionistic subalgebra of T over the binary operator $*$ if it fulfills the following conditions:*

(a) $M_A(l * j) \gg \text{rmin}\{M_A(l), M_A(j)\}$,
(b) $N_A(l * j) \ll \text{rmax}\{N_A(l), N_A(j)\}$,
(c) $\mu_\lambda(l * j) \leq \max\{\mu_\lambda(l), \mu_\lambda(j)\}$,
(d) $\nu_\lambda(l * j) \geq \min\{\nu_\lambda(l), \nu_\lambda(j)\}$,

for all $l, j \in T$.

Definition 3 ([19]). *A cubic intuitionistic set $\widetilde{A} = \langle A, \lambda\rangle$ in T is known as a cubic intuitionistic ideal of T if it fulfills the following conditions for all $l, j \in T$:*

(a) $M_A(0) \gg M_A(l)$ *and* $N_A(0) \ll N_A(l)$,
(b) $\mu_\lambda(0) \leq \mu_\lambda(l)$ *and* $\nu_\lambda(0) \geq \nu_\lambda(l)$,
(c) $M_A(l) \gg \text{rmin}\{M_A(l * j), M_A(j)\}$,
(d) $N_A(l) \ll \text{rmax}\{N_A(l * j), N_A(j)\}$,
(e) $\mu_\lambda(l) \leq \max\{\mu_\lambda(l * j), \mu_\lambda(j)\}$,
(f) $\nu_\lambda(l) \geq \min\{\nu_\lambda(l * j), \nu_\lambda(j)\}$.

Definition 4. *A cubic intuitionistic set $\widetilde{A} = \langle A, \lambda\rangle$ in T is known as a cubic intuitionistic q-ideal of T if it fulfills Conditions (a) and (b) in Definition 3 and for all $l, j, k \in T$:*

(a) $M_A(l * k) \gg \text{rmin}\{M_A(l * (j * k)), M_A(j)\}$,
(b) $N_A(l * k) \ll \text{rmax}\{N_A(l * (j * k)), N_A(j)\}$,
(c) $\mu_\lambda(l * k) \leq \max\{\mu_\lambda(l * (j * k)), \mu_\lambda(j)\}$,
(d) $\nu_\lambda(l * k) \geq \min\{\nu_\lambda(l * (j * k)), \nu_\lambda(j)\}$.

We now outline the above definitions by utilizing the accompanying examples.

Example 1. *Let T=$\{0, a, b, c\}$ be a BCI-algebra with the following Cayley table:*

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Define a cubic intuitionistic set $\widetilde{A} = \langle A, \lambda\rangle$ in T as follows:

T	$A = \langle M_A, N_A\rangle$	$\lambda = (\mu_\lambda, \nu_\lambda)$
0	$\langle[0.5, 0.7], [0.1, 0.2]\rangle$	$(0.1, 0.8)$
a	$\langle[0.4, 0.6], [0.2, 0.3]\rangle$	$(0.2, 0.7)$
b	$\langle[0.2, 0.4], [0.3, 0.5]\rangle$	$(0.4, 0.5)$
c	$\langle[0.2, 0.4], [0.3, 0.5]\rangle$	$(0.4, 0.5)$

All the conditions of Definition 4 have been satisfied by the set A with the above values. A few results are calculated below.
$M_A(a * b) = M_A(c) = [0.2, 0.4] = \text{rmin}\{[0.5, 0.7], [0.2, 0.4]\} = \text{rmin}\{M_A(a * (c * b)), M_A(c)\}$,
$M_A(a * 0) = [0.4, 0.6] \gg [0.2, 0.4] = \text{rmin}\{[0.2, 0.4], [0.2, 0.4]\} = \text{rmin}\{M_A(a * (c * 0)), M_A(c)\}$,
$N_A(a * b) = N_A(c) = [0.3, 0.5] = \text{rmax}\{[0.1, 0.2], [0.3, 0.5]\} = \text{rmax}\{N_A(a * (c * b)), N_A(c)\}$,
$N_A(a * 0) = [0.2, 0.3] \ll [0.3, 0.5] = \text{rmax}\{[0.3, 0.5], [0.3, 0.5]\} = \text{rmax}\{N_A(a * (c * 0)), N_A(c)\}$,
$\mu_\lambda(a * b) = \mu_\lambda(c) = 0.4 = \max\{0.1, 0.4\} = \max\{\mu_\lambda(a * (c * b)), \mu_\lambda(c)\}$,
$\mu_\lambda(a * 0) = \mu_\lambda(a) = 0.2 \le 0.4 = \max\{0.4, 0.4\} = \max\{\mu_\lambda(a * (c * 0)), \mu_\lambda(0)\}$,
$\nu_\lambda(a * b) = \nu_\lambda(c) = 0.5 = \min\{0.5, 0.8\} = \min\{\nu_\lambda(a * (c * b)), \nu_\lambda(c)\}$,
$\nu_\lambda(a * 0) = \nu_\lambda(a) = 0.7 \ge 0.5 = \min\{0.5, 0.5\} = \min\{\nu_\lambda(a * (c * 0)), \nu_\lambda(c)\}$.
Thus, $\widetilde{A} = \langle A, \lambda\rangle$ is a cubic intuitionistic q-ideal of T.

Proposition 1. *If $\widetilde{A} = \langle A, \lambda\rangle$ is a cubic intuitionistic q-ideal of T, then it is order reversing.*

Proof. Let $l, j \in T$ be such that $l \le j$. Then, $l * j = 0$. This implies $M_A(l) = M_A(l * 0) \gg \text{rmin}\{M_A(l * (j * 0)), M_A(j)\} = \text{rmin}\{M_A(l * j), M_A(j)\} = \text{rmin}\{M_A(0), M_A(j)\} \gg M_A(j)$, $N_A(l) = N_A(l * 0) \ll \text{rmax}\{N_A(l * (j * 0)), N_A(j)\} = \text{rmax}\{N_A(l * j), N_A(j)\} = \text{rmax}\{N_A(0), N_A(j)\} \ll N_A(j)$, $\mu_\lambda(l) = \mu_\lambda(l * 0) \le \max\{\mu_\lambda(l * (j * 0)), \mu_\lambda(j)\} = \max\{\mu_\lambda(l * j), \mu_\lambda(j)\} = \max\{\mu_\lambda(0), \mu_\lambda(j)\} \le \mu_\lambda(j)$ and $\nu_\lambda(l) = \nu_\lambda(l * 0) \ge \min\{\nu_\lambda(l * (j * 0)), \nu_\lambda(j)\} = \min\{\nu_\lambda(l * j), \nu_\lambda(j)\} = \min\{\nu_\lambda(0), \nu_\lambda(j)\} \ge \nu_\lambda(j)$. This completes the proof. □

Proposition 2. *Every cubic intuitionistic q-ideal $\widetilde{A} = \langle A, \lambda\rangle$ of a BCI-algebra T fulfills the following inequalities for all $l, j \in T$:*

1. $M_A(l * j) \gg M_A(l * (0 * j))$, $N_A(l * j) \ll N_A(l * (0 * j))$, $\mu_\lambda(l * j) \le \mu_\lambda(l * (0 * j))$, *and* $\nu_\lambda(l * j) \ge \nu_\lambda(l * (0 * j))$,
2. $M_A(0 * l) \gg M_A(0 * (0 * l))$, $N_A(0 * l) \ll N_A(0 * (0 * l))$, $\mu_\lambda(0 * l) \le \mu_\lambda(0 * (0 * l))$, *and* $\nu_\lambda(0 * l) \ge \nu_\lambda(0 * (0 * l))$.

Proof. The proof is straightforward. □

We now examine the relationship between a cubic intuitionistic q-ideal and a cubic intuitionistic subalgebra.

Theorem 1. *If $\widetilde{A} = \langle A, \lambda\rangle$ is a cubic intuitionistic q-ideal of T, then it is a cubic intuitionistic subalgebra.*

Proof. Let $\widetilde{A} = \langle A, \lambda \rangle$ be a cubic intuitionistic q-ideal of T and $l, j, k \in T$. Then, substituting j for k in Definition 4, we get $M_A(l * j) \gg \mathrm{rmin}\{M_A(l * (j * j)), M_A(j)\} = \mathrm{rmin}\{M_A(l * 0), M_A(j)\} = \mathrm{rmin}\{M_A(l), M_A(j)\}$, $N_A(l * j) \ll \mathrm{rmax}\{N_A(l * (j * j)), N_A(j)\} = \mathrm{rmax}\{N_A(l * 0), N_A(j)\} = \mathrm{rmax}\{N_A(l), N_A(j)\}$, $\mu_\lambda(l * j) \leq \max\{\mu_\lambda(l * (j * j)), \mu_\lambda(j)\} = \max\{\mu_\lambda(l * 0), \mu_\lambda(j)\} = \max\{\mu_\lambda(l), \mu_\lambda(j)\}$ and $\nu_\lambda(l * j) \geq \min\{\nu_\lambda(l * (j * j)), \nu_\lambda(j)\} = \min\{\nu_\lambda(l * 0), \nu_\lambda(j)\} = \min\{\nu_\lambda(l), \nu_\lambda(j)\}$. Therefore, $\widetilde{A}$ is a cubic intuitionistic subalgebra of T. $\square$

Note that every cubic intuitionistic q-ideal of a BCI-algebra T is a cubic intuitionistic ideal of T by putting $k = 0$ in Definition 4 and using (a3). However, the converse is not true, as seen in the following example.

Example 2. *Let T=$\{0, a, b, c\}$ be a BCI-algebra with the following Cayley table:*

$*$	0	a	b	c
0	0	c	b	a
a	a	0	c	b
b	b	a	0	c
c	c	b	a	0

Define a cubic intuitionistic set $\widetilde{A} = \langle A, \lambda \rangle$ in T as follows:

T	$A = \langle M_A, N_A \rangle$	$\lambda = (\mu_\lambda, \nu_\lambda)$
0	$\langle [0.6, 0.8], [0.1, 0.2] \rangle$	$(0.1, 0.9)$
a	$\langle [0.4, 0.6], [0.2, 0.3] \rangle$	$(0.3, 0.6)$
b	$\langle [0.3, 0.4], [0.4, 0.5] \rangle$	$(0.5, 0.4)$
c	$\langle [0.2, 0.4], [0.4, 0.5] \rangle$	$(0.5, 0.4)$

Then, $\widetilde{A} = \langle A, \lambda \rangle$ is a cubic intuitionistic ideal of T, but not a cubic intuitionistic q-ideal of T since $M_A(c * a) = M_A(b) = [0.4, 0.6] \ll [0.6, 0.8] = M_A(0) = \mathrm{rmin}\{M_A(c * (0 * a)), M_A(0)\}$, $N_A(c * a) = [0.2, 0.3] \gg [0.1, 0.2] = \mathrm{rmax}\{N_A(c * (0 * a)), N_A(0)\}$, $\mu_\lambda(c * a) = 0.3 \nleq 0.1 = \max\{\mu_\lambda(c * (0 * a)), \mu_\lambda(0)\}$, and $\nu_\lambda(c * a) = 0.6 \ngeq 0.9 = \min\{\mu_\lambda(c * (0 * a)), \mu_\lambda(0)\}$.

We give a condition for a cubic intuitionistic ideal to be a cubic intuitionistic q-ideal.

Theorem 2. *Every cubic intuitionistic ideal of an associative BCI-algebra T is a cubic intuitionistic q-ideal of T.*

Proof. Assume that $\widetilde{A}$ is a cubic intuitionistic ideal of an associative BCI-algebra T. For any $l, j, k \in T$, we have $M_A(l * k) \gg \mathrm{rmin}\{M_A((l * k) * j), M_A(j)\} = \mathrm{rmin}\{M_A((l * j) * k), M_A(j)\} = \mathrm{rmin}\{M_A(l * (j * k)), M_A(j)\}$, $N_A(l * k) \ll \mathrm{rmax}\{N_A((l * k) * j), N_A(j)\} = \mathrm{rmax}\{N_A((l * j) * k), N_A(j)\} = \mathrm{rmax}\{N_A(l * (j * k)), N_A(j)\}$, $\mu_\lambda(l * k) \leq \max\{\mu_\lambda((l * k) * j), \mu_\lambda(j)\} = \max\{\mu_\lambda((l * j) * k), \mu_\lambda(j)\} = \max\{\mu_\lambda(l * (j * k)), \mu_\lambda(j)\}$, and $\nu_\lambda(l * k) \geq \min\{\nu_\lambda((l * k) * j), \nu_\lambda(j)\} = \min\{\nu_\lambda((l * j) * k), \nu_\lambda(j)\} = \min\{\nu_\lambda(l * (j * k)), \nu_\lambda(j)\}$. Hence, $\widetilde{A} = \langle A, \lambda \rangle$ is a cubic intuitionistic q-ideal of T. $\square$

Corollary 1. *Let T be a BCI-algebra, which fulfills any one of the subsequent assertions:*

(a) $0 * l = l$, *for all* $l \in T$
(b) $l * j = j * l$, *for all* $l, j \in T$.

Then, every cubic intuitionistic ideal is a cubic intuitionistic q-ideal.

Corollary 2. *Let T be a quasi-associative BCI-algebra, which fulfills any one of the following conditions, for all $l, j, k, p \in T$,*

(a) $0 * (0 * l) = l$,
(b) $0 * (j * l) = l * j$,
(c) $l * j = 0 \Rightarrow l = j$,
(d) $k * l = k * j \Rightarrow l = j$,
(e) $l * k = j * k \Rightarrow l = j$,
(f) $(l * j) * (l * k) = k * j$,
(g) $(j * l) * (k * l) = j * k$,
(l) $(l * j) * (l * k) = 0 * (j * k)$,
(i) $(l * j) * (k * p) = (l * k) * (j * p)$,

then each cubic intuitionistic ideal is a cubic intuitionistic q-ideal.

Theorem 3. *Let $\widetilde{A} = \langle A, \lambda \rangle$ be a cubic intuitionistic ideal of T that fulfills: $M_A(l * j) \gg M_A(l)$, $N_A(l * j) \ll N_A(l)$, $\mu_\lambda(l * j) \leq \mu_\lambda(l)$, and $\nu_\lambda(l * j) \geq \nu_\lambda(l)$, for all $l, j \in T$. Then $\widetilde{A}$ is a cubic intuitionistic q-ideal of T.*

Proof. Assume that $\widetilde{A}$ is a cubic intuitionistic ideal of T and $l, j, k \in T$. Then, by using (a1) and from Definition 3, we get:

$$\begin{aligned} M_A(l*k) &\gg \text{rmin}\{M_A((l*k)*(j*k)), M_A(j*k)\} \\ &= \text{rmin}\{M_A((l*(j*k))*k), M_A(j*k)\} \\ &\gg \text{rmin}\{M_A(l*(j*k)), M_A(j)\}, \\ \mu_\lambda(l*k) &\leq \max\{\mu_\lambda((l*k)*(j*k)), \mu_\lambda(j*k)\} \\ &= \max\{\mu_\lambda((l*(j*k))*k), \mu_\lambda(j*k)\} \\ &\leq \max\{\mu_\lambda(l*(j*k)), \mu_\lambda(j)\}. \end{aligned}$$

Similarly, we get $N_A(l * k) \ll \text{rmax}\{N_A(l * (j * k)), N_A(j)\}$, and $\nu_\lambda(l * k) \geq \min\{\nu_\lambda(l * (j * k)), \nu_\lambda(j)\}$. Hence, $\widetilde{A} = \langle A, \lambda \rangle$ is a cubic intuitionistic q-ideal of T. □

The sets $\{l \in T : M_A(l) = M_A(0)\}$, $\{l \in T : N_A(l) = N_A(0)\}$, $\{l \in T : \mu_\lambda(l) = \mu_\lambda(0)\}$, and $\{l \in T : \nu_\lambda(l) = \nu_\lambda(0)\}$ are indicated by T_{M_A}, T_{N_A}, T_{μ_λ}, and T_{ν_λ}, respectively. These four sets are also q-ideals of T.

Theorem 4. *Let $\widetilde{A} = \langle A, \lambda \rangle$ be a cubic intuitionistic q-ideal of T. Then, the sets T_{M_A}, T_{N_A}, T_{μ_λ}, and T_{ν_λ} are q-ideals of T.*

Proof. Assume that $\widetilde{A}$ is a cubic intuitionistic q-ideal of T. Then, it is obvious that $0 \in T_{M_A} \cap T_{N_A} \cap T_{\mu_\lambda} \cap T_{\nu_\lambda}$. Let $l, j, k \in T$ be such that $l * (j * k) \in T_{M_A} \cap T_{N_A} \cap T_{\mu_\lambda} \cap T_{\nu_\lambda}$ and $j \in T_{M_A} \cap T_{N_A} \cap T_{\mu_\lambda} \cap T_{\nu_\lambda}$. Then, $M_A(l * (j * k)) = M_A(0) = M_A(j)$, $N_A(l * (j * k)) = N_A(0) = N_A(j)$, $\mu_\lambda(l * (j * k)) = \mu_\lambda(0) = \mu_\lambda(j)$, and $\nu_\lambda(l * (j * k)) = \nu_\lambda(0) = \nu_\lambda(j)$. Thus:

$$\begin{aligned} M_A(l*k) &\gg \text{rmin}\{M_A(l*(j*k)), M_A(j)\} = \text{rmin}\{M_A(0), M_A(0)\} = M_A(0), \\ N_A(l*j) &\ll \text{rmax}\{N_A(l*(j*k)), N_A(j)\} = \text{rmax}\{N_A(0), N_A(0)\} = N_A(0), \\ \mu_\lambda(l*k) &\leq \max\{\mu_\lambda(l*(j*k)), \mu_\lambda(j)\} = \max\{\mu_\lambda(0), \mu_\lambda(0)\} = \mu_\lambda(0), \\ \nu_\lambda(l*k) &\geq \min\{\nu_\lambda(l*(j*k)), \nu_\lambda(j)\} = \min\{\nu_\lambda(0), \nu_\lambda(0)\} = \nu_\lambda(0). \end{aligned}$$

Since $\widetilde{A} = \langle A, \lambda \rangle$ is a cubic intuitionistic q-ideal of T, we have $M_A(l * k) = M_A(0)$, $N_A(l * k) = N_A(0)$, $\mu_\lambda(l * k) = \mu_\lambda(0)$, and $\nu_\lambda(l * k) = \nu_\lambda(0)$, i.e., $l * k \in T_{M_A} \cap T_{N_A} \cap T_{\mu_\lambda} \cap T_{\nu_\lambda}$. Hence, the sets T_{M_A}, T_{N_A}, T_{μ_λ}, and T_{ν_λ} are q-ideals of T. □

Let $\widetilde{A} = \langle A, \lambda \rangle$ be a cubic intuitionistic set in a nonempty set T. Given $([s_1, t_1], [s_2, t_2]) \in D[0,1] \times D[0,1]$ and $[\theta_1, \theta_2] \in [0,1] \times [0,1]$, we consider the sets:

$$\begin{aligned} M_A[s_1, t_1] &= \{l \in T | M_A(l) \gg [s_1, t_1]\}, \\ N_A[s_2, t_2] &= \{l \in T | N_A(l) \ll [s_2, t_2]\}, \\ \mu_\lambda(\theta_1) &= \{l \in T | \mu_\lambda(l) \leq (\theta_1)\}, \\ \mu_\lambda(\theta_2) &= \{l \in T | \mu_\lambda(l) \geq (\theta_2)\}. \end{aligned}$$

Theorem 5. *Let $\widetilde{A} = \langle A, \lambda \rangle$ be a cubic intuitionistic q-ideal of T, then the sets $M_A[s,t]$, $N_A[s,t]$, $\mu_\lambda(\theta)$, and $\nu_\lambda(\theta)$ are q-ideals of T for all $[s,t] \in D[0,1]$ and $\theta \in [0,1]$.*

Proof. Assume that $\widetilde{A}$ is a cubic intuitionistic q-ideal of T. For any $[s,t] \in D[0,1]$ and $\theta \in [0,1]$, let $l \in T$ be such that $l \in M_A[s,t] \cap N_A[s,t] \cap \mu_\lambda(\theta) \cap \nu_\lambda(\theta)$. Then, $M_A(l) \gg [s,t]$, $N_A(l) \ll [s,t]$, $\mu_\lambda(l) \leq \theta$, and $\nu_\lambda(l) \geq \theta$. Now, $M_A(0) = M_A(j*0) \gg \mathrm{rmin}\{M_A(j*(l*0)), M_A(l)\} = \mathrm{rmin}\{M_A(j*0), M_A(l)\} = \mathrm{rmin}\{M_A(0), M_A(l)\} \gg [s,t]$, $N_A(0) = N_A(j*0) \ll \mathrm{rmax}\{N_A(j*(l*0)), N_A(l)\} = \mathrm{rmax}\{N_A(j*0), N_A(l)\} = \mathrm{rmax}\{N_A\ (0), N_A(l)\} \ll [s,t]$, $\mu_\lambda(0) = \mu_\lambda(j*0) \leq \max\{\mu_\lambda(j*(l*0)), \mu_\lambda(l)\} = \max\{\mu_\lambda(j*0), \mu_\lambda(l)\} = \max\{\mu_\lambda(0), \mu_\lambda(l)\} \leq \theta$ and $\nu_\lambda(0) = \nu_\lambda(j*0) \geq \min\{\nu_\lambda(j*(l*0)), \nu_\lambda(l)\} = \min\{\nu_\lambda(j*0), \nu_\lambda(l)\} = \min\{\nu_\lambda(0), \nu_\lambda(l)\} \geq \theta$. Thus, $0 \in M_A[s,t] \cap N_A[s,t] \cap \mu_\lambda(\theta) \cap \nu_\lambda(\theta)$. Now, letting $l*(j*k), j \in M_A[s,t] \cap N_A[s,t] \cap \mu_\lambda(\theta) \cap \nu_\lambda(\theta)$. This implies that:

$$\begin{aligned} M_A(l*k) &\gg \mathrm{rmin}\{M_A(l*(j*k)), M_A(j)\} \gg [s,t] \\ N_A(l*j) &\ll \mathrm{rmax}\{N_A(l*(j*k)), N_A(j)\} \ll [s,t] \\ \mu_\lambda(l*k) &\leq \max\{\mu_\lambda(l*(j*k)), \mu_\lambda(j)\} \leq \theta \\ \nu_\lambda(l*k) &\geq \min\{\nu_\lambda(l*(j*k)), \nu_\lambda(j)\} \geq \theta. \end{aligned}$$

Therefore, $l*k \in M_A[s,t] \cap N_A[s,t] \cap \mu_\lambda(\theta) \cap \nu_\lambda(\theta)$. Hence, $M_A[s,t]$, $N_A[s,t]$, $\mu_\lambda(\theta)$, and $\nu_\lambda(\theta)$ are q-ideals of T. $\square$

Theorem 6. *Let $\widetilde{A} = \langle A, \lambda \rangle$ be a cubic intuitionistic set in T such that the non-empty sets $M_A[s_1, t_1]$, $N_A[s_2, t_2]$, $\mu_\lambda(\theta_1)$, and $\nu_\lambda(\theta_2)$ are q-ideals of T for all $([s_1, t_1], [s_2, t_2]) \in D[0,1] \times D[0,1]$, and $(\theta_1, \theta_2) \in [0,1] \times [0,1]$. Then, $\widetilde{A}$ is a cubic intuitionistic q-ideal of T.*

Proof. Suppose that for every $([s_1, t_1], [s_2, t_2]) \in D[0,1] \times D[0,1]$ and $(\theta_1, \theta_2) \in [0,1] \times [0,1]$, $M_A[s_1, t_1]$, $N_A[s_2, t_2]$, $\mu_\lambda(\theta_1)$, and $\nu_\lambda(\theta_2)$ are non-empty q-ideals of T. Assume that $M_A(0) \ll M_A(d)$, that is $[M_A^-(0), M_A^+(0)] \ll [M_A^-(d), M_A^+(d)]$ for some $d \in T$. If we take $\tilde{s}_d = \frac{1}{2}[M_A^-(0) + M_A^-(d)]$, $\tilde{t}_d = \frac{1}{2}[M_A^+(0) + M_A^+(d)]$, then $M_A(0) = [M_A^-(0), M_A^+(0)] \ll [\tilde{s}_d, \tilde{t}_d] \ll [M_A^-(d), M_A^+(d)] = M_A(d)$. Hence, $0 \notin M_A[\tilde{s}_d, \tilde{t}_d]$. This is a contradiction, and so, $M_A(0) \gg M_A(l)$ for all $l \in T$. Similarly, $N_A(0) \ll N_A(l)$, $\mu_\lambda(0) \leq \mu_\lambda(l)$, and $\nu_\lambda(0) \geq \nu_\lambda(l)$ for all $l \in T$.

Now, let $d, f, g \in T$ be such that $M_A(d*g) \ll \mathrm{rmin}\{M_A(d*(f*g)), M_A(f)\}$. Suppose that $M_A(d*g) = [(d*g)^-, (d*g)^+]$, $M_A(f) = [f^-, f^+]$, and $M_A(d*(f*g)) = [(d*(f*g))^-, (d*(f*g))^+]$. Assume that $\tilde{s}_0 = \frac{1}{2}((d*g)^- + \min\{(d*(f*g))^-, f^-\})$, $\tilde{t}_0 = \frac{1}{2}((d*g)^+ + \min\{(d*(f*g))^+, f^+\})$. Then, $(d*g)^- \ll \tilde{s}_0 \ll \min\{(d*(f*g))^-, f^-\}$ and $(d*g)^+ \ll \tilde{t}_0 \ll \min\{(d*(f*g))^+, f^+\}$, which implies that:

$$\begin{aligned} M_A(d*g) &= [(d*g)^-, (d*g)^+] \ll [\tilde{s}_0, \tilde{t}_0] \\ &\ll [\min\{(d*(f*g))^-, f^-\}, \min\{(d*(f*g))^+, f^+\}] \\ &= \mathrm{rmin}\{M_A(d*(f*g)), M_A(f)\}. \end{aligned}$$

Thus, $d*g \notin M_A[\tilde{s}_0, \tilde{t}_0]$, but $d*(f*g), f \in M_A[\tilde{s}_0, \tilde{t}_0]$. This is a contradiction, and hence, M_A fulfills $M_A(l*k) \gg \mathrm{rmin}\{M_A(l*(j*k)), M_A(j)\}$, for all $l, j, k \in T$. Similarly, we can prove that

$N_A(l*j) \ll \text{rmax}\{N_A(l*(j*k)), N_A(j)\}$, $\mu_\lambda(l*k) \leq \max\{\mu_\lambda(l*(j*k)), \mu_\lambda(j)\}$, and $\nu_\lambda(l*k) \geq \min\{\nu_\lambda(l*(j*k)), \nu_\lambda(j)\}$, for all $l, j, k \in$. Therefore, $\widetilde{A} = \langle A, \lambda \rangle$ forms a cubic intuitionistic q-ideal of T. □

The characterizations of cubic intuitionistic q-ideal are given by the following theorem.

Theorem 7. *If $\widetilde{A} = \langle A, \lambda \rangle$ is a cubic intuitionistic ideal of T, then the accompanying assertions are identical:*

(i) *$\widetilde{A}$ is a cubic intuitionistic q-ideal of T,*
(ii) *$M_A(l*j) \gg M_A(l*(0*j))$, $N_A(l*j) \ll N_A(l*(0*j))$, $\mu_\lambda(l*j) \leq \mu_\lambda(l*(0*j))$ and $\nu_\lambda(l*j) \geq \nu_\lambda(l*(0*j))$, for all $l, j \in T$,*
(iii) *$M_A((l*j)*k) \gg M_A(l*(j*k))$, $N_A((l*j)*k) \ll N_A(l*(j*k))$, $\mu_\lambda((l*j)*k) \leq \mu_\lambda(l*(j*k))$ and $\nu_\lambda((l*j)*k) \geq \nu_\lambda(l*(j*k))$, for all $l, j, k \in T$.*

Proof. $(i) \Rightarrow (ii)$ Let $\widetilde{A} = \langle A, \lambda \rangle$ be a cubic intuitionistic q-ideal of T. Then, for all $l, j \in T$, we have:

$$\begin{aligned}
M_A(l*j) &\gg \text{rmin}\{M_A(l*(0*j)), M_A(0)\} = M_A(l*(0*j)), \\
N_A(l*j) &\ll \text{rmax}\{N_A(l*(0*j)), N_A(0)\} = N_A(l*(0*j)), \\
\mu_\lambda(l*j) &\leq \max\{\mu_\lambda(l*(0*j)), \mu_\lambda(0)\} = \mu_\lambda(l*(0*j)), \\
\nu_\lambda(l*j) &\geq \min\{\nu_\lambda(l*(0*j)), \nu_\lambda(0)\} = \mu_\lambda(l*(0*j)).
\end{aligned}$$

Therefore, $(ii)$is fulfilled.
$(ii) \Rightarrow (iii)$ Let (ii) be fulfilled. For all $l, j, k \in T$, we have $((l*j)*(0*k))*(l*(j*k)) = ((l*j)*(l*(j*k)))*(0*k) \leq ((j*k)*j)*(0*k) = ((j*j)*k)*(0*k) = (0*k)*(0*k) = 0$. It follows from Proposition 1 that:

$$\begin{aligned}
M_A((l*j)*(0*k))*(l*(j*k)) &\gg M_A(0), \\
N_A((l*j)*(0*k))*(l*(j*k)) &\ll N_A(0), \\
\mu_\lambda((l*j)*(0*k))*(l*(j*k)) &\leq \mu_\lambda(0), \\
\nu_\lambda((l*j)*(0*k))*(l*(j*k)) &\geq \nu_\lambda(0).
\end{aligned}$$

Since $\widetilde{A} = \langle A, \lambda \rangle$ is a cubic intuitionistic ideal of T, we have:

$$\begin{aligned}
M_A((l*j)*(0*k))*(l*(j*k)) &= M_A(0), \\
N_A((l*j)*(0*k))*(l*(j*k)) &= N_A(0), \\
\mu_\lambda((l*j)*(0*k))*(l*(j*k)) &= \mu_\lambda(0) \\
\nu_\lambda((l*j)*(0*k))*(l*(j*k)) &= \nu_\lambda(0).
\end{aligned}$$

Using (ii), we get:

$$\begin{aligned}
M_A((l*j)*k) &\gg M_A((l*j)*(0*k)) \\
&= \text{rmin}\{M_A(((l*j)*(0*k))*(l*(j*k))), M_A(l*(j*k))\} \\
&= \text{rmin}\{M_A(0), M_A(l*(j*k))\} = M_A(l*(j*k)), \\
N_A((l*j)*k) &\ll N_A((l*j)*(0*k)) \\
&= \text{rmax}\{N_A(((l*j)*(0*k))*(l*(j*k))), N_A(l*(j*k))\} \\
&= \text{rmax}\{N_A(0), N_A(l*(j*k))\} = N_A(l*(j*k)), \\
\mu_\lambda((l*j)*k) &\leq \mu_\lambda((l*j)*(0*k)) \\
&= \max\{\mu_\lambda(((l*j)*(0*k))*(l*(j*k))), \mu_\lambda(l*(j*k))\} \\
&= \max\{\mu_\lambda(0), \mu_\lambda(l*(j*k))\} = \mu_\lambda(l*(j*k)),
\end{aligned}$$

$$\begin{aligned}\nu_\lambda((l*j)*k) &\geq \nu_\lambda((l*j)*(0*k)),\\ &= \min\{\nu_\lambda(((l*j)*(0*k))*(l*(j*k))), \nu_\lambda(l*(j*k))\}\\ &= \min\{\nu_\lambda(0), \nu_\lambda(l*(j*k))\} = \nu_\lambda(l*(j*k)).\end{aligned}$$

Hence, the inequality (iii) is also fulfilled.
$(iii) \Rightarrow (i)$ Let (iii) be valid. For all $l, j, k \in$, we have:

$$\begin{aligned}M_A(l*k) &\gg \text{rmin}\{M_A((l*k)*j), M_A(j)\}\\ &= \text{rmin}\{M_A((l*j)*k), M_A(j)\}\\ &\gg \text{rmin}\{M_A(l*(j*k)), M_A(j)\},\\ N_A(l*k) &\ll \text{rmax}\{N_A((l*k)*j), N_A(j)\}\\ &= \text{rmax}\{N_A((l*j)*k), N_A(j)\}\\ &\ll \text{rmax}\{N_A(l*(j*k)), N_A(j)\},\\ \mu_\lambda(l*k) &\leq \max\{\mu_\lambda((l*k)*j), \mu_\lambda(j)\}\\ &= \max\{\mu_\lambda((l*j)*k), \mu_\lambda(j)\}\\ &\leq \max\{\mu_\lambda(l*(j*k)), \mu_\lambda(j)\},\\ \nu_\lambda(l*k) &\geq \min\{\nu_\lambda((l*k)*j), \nu_\lambda(j)\}\\ &= \min\{\nu_\lambda((l*j)*k), \nu_\lambda(j)\}\\ &\geq \min\{\nu_\lambda(l*(j*k)), \nu_\lambda(j)\}.\end{aligned}$$

Therefore, $\widetilde{A} = \langle A, \lambda\rangle$ is a cubic intuitionistic q-ideal of T. Therefore, the assertion (i) holds. The proof is complete. □

Next, we provide some other characterizations of the cubic intuitionistic q-ideal in the following theorem.

Theorem 8. *If $\widetilde{A} = \langle A, \lambda\rangle$ is a cubic intuitionistic ideal of T, then the accompanying assertions are identical:*

(i) *$\widetilde{A}$ is a cubic intuitionistic q-ideal of T,*
(ii) $M_A((l*k)*j) \gg M_A((l*k)*(0*j))$, $N_A((l*k)*j) \ll N_A((l*k)*(0*j))$, $\mu_\lambda((l*k)*j) \leq \mu_\lambda((l*k)*(0*j))$, *and* $\nu_\lambda((l*k)*j) \geq \nu_\lambda((l*k)*(0*j))$, *for all* $l, j, k \in T$,
(iii) $M_A(l*j) \gg \text{rmin}\{M_A((l*k)*(0*j)), M_A(k)\}$, $N_A(l*j) \ll \text{rmax}\{N_A((l*k)*(0*j)), N_A(k)\}$, $\mu_\lambda(l*j) \leq \max\{\mu_\lambda((l*k)*(0*j)), \mu_\lambda(k)\}$, *and* $\nu_\lambda(l*j) \geq \min\{\nu_\lambda((l*k)*(0*j)), \nu_\lambda(k)\}$, *for all* $l, j, k \in T$.

Proof. $(i) \Rightarrow (ii)$ This is the same as the above theorem.
$(ii) \Rightarrow (iii)$ Assume that (ii) is valid. For all $l, j, k \in$, we have $M_A(l*j) \gg \text{rmin}\{M_A((l*j)*k), M_A(k)\} = \text{rmin}\{M_A((l*k)*j), M_A(k)\} \gg \text{rmin}\{M_A((l*k)*(0*j)), M_A(k)\}$, $N_A(l*j) \ll \text{rmax}\{N_A((l*j)*k), N_A(k)\} = \text{rmax}\{N_A((l*k)*j), N_A(k)\} \ll \text{rmax}\{N_A((l*k)*(0*j)), N_A(k)\}$, $\mu_\lambda(l*j) \leq \max\{\mu_\lambda((l*j)*k), \mu_\lambda(k)\} = \max\{\mu_\lambda((l*k)*j), \mu_\lambda(k)\} \leq \max\{\mu_\lambda((l*k)*(0*j)), \mu_\lambda(k)\}$ and $\nu_\lambda(l*j) \geq \min\{\nu_\lambda((l*j)*k), \nu_\lambda(k)\} = \min\{\nu_\lambda((l*k)*j), \nu_\lambda(k)\} \geq \min\{\nu_\lambda((l*k)*(0*j)), \nu_\lambda(k)\}$. Therefore, (iii) is fulfilled.
$(iii) \Rightarrow (i)$ Assume that (iii) is valid. If we take $k = 0$ in (iii), then:

$$\begin{aligned}&M_A(l*j) \gg \text{rmin}\{M_A((l*0)*(0*j)), M_A(0)\} = \text{rmin}\{M_A(l*(0*j)), M_A(0)\} = M_A(l*(0*j)),\\ &N_A(l*j) \ll \text{rmax}\{N_A((l*0)*(0*j)), N_A(0)\} = \text{rmax}\{N_A(l*(0*j)), N_A(0)\} = N_A(l*(0*j)),\\ &\mu_\lambda(l*j) \leq \max\{\mu_\lambda((l*0)*(0*j)), \mu_\lambda(0)\} = \max\{\mu_\lambda(l*(0*j)), \mu_\lambda(0)\} = \mu_\lambda(l*(0*j)),\\ &\nu_\lambda(l*j) \geq \min\{\nu_\lambda((l*0)*(0*j)), \nu_\lambda(0)\} = \min\{\nu_\lambda(l*(0*j)), \nu_\lambda(0)\} = \nu_\lambda(l*(0*j)).\end{aligned}$$

It follows from Theorem 7 that $\widetilde{A} = \langle A, \lambda \rangle$ is a cubic intuitionistic q-ideal of T. The proof is complete. □

Theorem 9. *(Cubic intuitionistic extension property for a cubic intuitionistic q-ideal) Let $\widetilde{A} = \langle A, \lambda \rangle$ and $\widetilde{B} = \langle B, \vartheta \rangle$ be cubic intuitionistic ideals of T such that $\widetilde{A} \lesssim \widetilde{B}$, and $M_A(0) = M_B(0)$, $N_A(0) = N_B(0)$, $\mu_\lambda(0) = \mu_\vartheta(0)$, and $\nu_\lambda(0) = \nu_\vartheta(0)$. If $\widetilde{A}$ is a cubic intuitionistic q-ideal of T, then so is $\widetilde{B} = \langle B, \vartheta \rangle$.*

Proof. Let $l, j \in T$. If we take $r = l * (0 * j)$, then $(l * a) * (0 * j) = (l * (0 * j)) * r = 0$. Using Theorem 7, we have:

$$\begin{aligned}
M_A((l * r) * j) &\gg M_A((l * r) * (0 * j)) = M_A(0) = M_B(0)\\
N_A((l * r) * j) &\ll N_A((l * r) * (0 * j)) = N_A(0) = N_B(0)\\
\mu_\lambda((l * r) * j) &\leq \mu_\lambda((l * r) * (0 * j)) = \mu_\lambda(0) = \mu_\vartheta(0)\\
\nu_\lambda((l * r) * j) &\geq \nu_\lambda((l * r) * (0 * j)) = \nu_\lambda(0) = \nu_\vartheta(0).
\end{aligned}$$

Thus, $M_B((l * r) * j) \gg M_A((l * r) * j) \gg M_B(0) \gg M_B(r)$, $N_B((l * r) * j) \ll N_A((l * r) * j) \ll N_B(0) \ll N_B(r)$, $\mu_\vartheta((l * r) * j) \leq \mu_\lambda((l * r) * j) \leq \mu_\vartheta(0) \leq \mu_\vartheta(r)$, and $\nu_\vartheta((l * r) * j) \geq \nu_\lambda((l * r) * j) \geq \nu_\vartheta(0) \geq \nu_\vartheta(r)$. Since $\widetilde{B} = \langle B, \vartheta \rangle$ is a cubic intuitionistic ideal, it follows that:

$$\begin{aligned}
M_B(l * j) &\gg \mathrm{rmin}\{M_B((l * j) * r), M_B(r)\} = M_B(r) = M_B(l * (0 * j))\\
N_B(l * j) &\ll \mathrm{rmax}\{N_B((l * j) * r), N_B(r)\} = N_B(r) = N_B(l * (0 * j))\\
\mu_\vartheta(l * j) &\leq \max\{\mu_\vartheta((l * j) * r), \mu_\vartheta(r)\} = \mu_\vartheta(r) = \mu_\vartheta(l * (0 * j))\\
\nu_\vartheta(l * j) &\geq \min\{\nu_\vartheta((l * j) * r), \nu_\vartheta(r)\} = \nu_\vartheta(r) = \nu_\vartheta(l * (0 * j)).
\end{aligned}$$

Using Theorem 7, we conclude that $\widetilde{B} = \langle B, \vartheta \rangle$ is a cubic intuitionistic q-ideal of T. □

4. Product of Cubic Intuitionistic Subalgebras, Ideals, and q-Ideals

In this section, we will provide some new definitions on the Cartesian product of cubic intuitionistic subalgebras, ideals, and q-ideals in BCI-algebras.

Definition 5. *Let $\widetilde{A} = \langle A, \lambda \rangle$ and $\widetilde{B} = \langle B, \vartheta \rangle$ be two cubic intuitionistic sets of T and S, respectively. Then, the Cartesian product $\widetilde{A} \times \widetilde{B} = ([M_A \times M_B, N_A \times N_B], [\mu_\lambda \times \mu_\vartheta, \nu_\lambda \times \nu_\vartheta])$ of $T \times S$ is defined by $(M_A \times M_B)(l, j) = \mathrm{rmin}\{M_A(l), M_B(j)\}$, $(N_A \times N_B)(l, j) = \mathrm{rmax}\{N_A(l), N_B(j)\}$, $(\mu_\lambda \times \mu_\vartheta)(l, j) = \max\{\mu_\lambda(l), \mu_\vartheta(j)\}$, and $(\nu_\lambda \times \nu_\vartheta)(l, j) = \min\{\nu_\lambda(l), \nu_\vartheta(j)\}$, where $M_A \times M_B \mid T \times S \to D[0, 1]$, $N_A \times N_B \mid T \times S \to D[0, 1]$, $\mu_\lambda \times \mu_\vartheta \mid T \times S \to [0, 1]$, and $\nu_\lambda \times \nu_\vartheta \mid T \times S \to [0, 1]$ for all $(l, j) \in T \times S$.*

Remark 1. *Let T and S be BCI-algebras. We define $*$ on $T \times S$ by $(l, j) * (u, v) = (l * u, j * v)$ for every (l, j), (u, v) belonging to $T \times S$, then it is clear that $(T \times S, *, (0, 0))$ is a BCI-algebra.*

Definition 6. *A cubic intuitionistic set $\widetilde{A} \times \widetilde{B} = ([M_A \times M_B, N_A \times N_B], [\mu_\lambda \times \mu_\vartheta, \nu_\lambda \times \nu_\vartheta])$ of $T \times S$ is called a cubic intuitionistic subalgebra if it satisfies the following conditions, for all (l_1, j_1) and $(l_2, j_2) \in T \times S$,*

(i) $(M_A \times M_B)((l_1, j_1) * (l_2, j_2)) \gg \mathrm{rmin}\{(M_A \times M_B)(l_1, j_1), (M_A \times M_B)(l_2, j_2)\}$,
(ii) $(N_A \times N_B)((l_1, j_1) * (l_2, j_2)) \ll \mathrm{rmax}\{(N_A \times N_B)(l_1, j_1), (N_A \times N_B)(l_2, j_2)\}$,
(iii) $(\mu_\lambda \times \mu_\vartheta)((l_1, j_1) * (l_2, j_2)) \leq \max\{(\mu_\lambda \times \mu_\vartheta)(l_1, j_1), (\mu_\lambda \times \mu_\vartheta)(l_2, j_2)\}$,
(iv) $(\nu_\lambda \times \nu_\vartheta)((l_1, j_1) * (l_2, j_2)) \geq \min\{(\nu_\lambda \times \nu_\vartheta)(l_1, j_1), (\nu_\lambda \times \nu_\vartheta)(l_2, j_2)\}$.

Definition 7. *A cubic intuitionistic set $\widetilde{A} \times \widetilde{B} = ([M_A \times M_B, N_A \times N_B], [\mu_\lambda \times \mu_\vartheta, \nu_\lambda \times \nu_\vartheta])$ of $T \times S$ is called a cubic intuitionistic q-ideal if it satisfies the following conditions: for all (l_1, j_1), (l_2, j_2), and $(l_3, j_3) \in T \times S$,*

(i) $(M_A \times M_B)(0, 0) \gg (M_A \times M_B)(l, j)$ *and* $(N_A \times N_B)(0, 0) \ll (N_A \times N_B)(l, j)$,

(ii) $(\mu_\lambda \times \mu_\vartheta)(0,0) \leq (\mu_\lambda \times \mu_\vartheta)(l,j)$ *and* $(\nu_\lambda \times \nu_\vartheta)(0,0) \geq (\nu_\lambda \times \nu_\vartheta)(l,j)$,
(iii) $(M_A \times M_B)((l_1,j_1) * (l_3,j_3)) \gg \text{rmin}\{(M_A \times M_B)((l_1,j_1) * ((l_2,j_2) * (l_3,j_3))), (M_A \times M_B)(l_2,j_2)\}$,
(iv) $(N_A \times N_B)((l_1,j_1) * (l_3,j_3)) \ll \text{rmax}\{(N_A \times N_B)((l_1,j_1) * ((l_2,j_2) * (l_3,j_3))), (N_A \times N_B)(l_2,j_2)\}$,
(v) $(\mu_\lambda \times \mu_\vartheta)((l_1,j_1) * (l_3,j_3)) \leq \max\{(\mu_\lambda \times \mu_\vartheta)((l_1,j_1) * ((l_2,j_2) * (l_3,j_3))), (\mu_\lambda \times \mu_\vartheta)(l_2,j_2)\}$,
(vi) $(\nu_\lambda \times \nu_\vartheta)((l_1,j_1) * (l_3,j_3)) \geq \min\{(\nu_\lambda \times \nu_\vartheta)((l_1,j_1) * ((l_2,j_2) * (l_3,j_3))), (\nu_\lambda \times \nu_\vartheta)(l_2,j_2)\}$.

For the Cartesian product of two subalgebras, we have the following theorem.

Theorem 10. *Let $\widetilde{A} = \langle A, \lambda \rangle$ and $\widetilde{B} = \langle B, \vartheta \rangle$ be cubic intuitionistic subalgebras of T and S, respectively, then $\widetilde{A} \times \widetilde{B}$ is a cubic intuitionistic subalgebra of $T \times S$.*

Proof. For any (l_1,j_1) and $(l_2,j_2) \in T \times S$, we have:

$$\begin{aligned}
(M_A \times M_B)((l_1,j_1) * (l_2,j_2)) &= (M_A \times M_B)(l_1 * l_2, j_1 * j_2) \\
&= \text{rmin}\{M_A(l_1 * l_2), M_B(j_1 * j_2)\} \\
&\gg \text{rmin}\{\text{rmin}\{M_A(l_1), M_A(l_2)\}, \text{rmin}\{M_B(j_1), M_B(j_2)\}\} \\
&= \text{rmin}\{\text{rmin}\{M_A(l_1), M_B(j_1)\}, \text{rmin}\{M_A(l_2), M_B(j_2)\}\} \\
&= \text{rmin}\{(M_A \times M_B)(l_1,j_1), (M_A \times M_B)(l_2,j_2)\}, \\
(N_A \times N_B)((l_1,j_1) * (l_2,j_2)) &= (N_A \times N_B)(l_1 * l_2, j_1 * j_2) \\
&= \text{rmax}\{N_A(l_1 * l_2), N_B(j_1 * j_2)\} \\
&\ll \text{rmax}\{\text{rmax}\{N_A(l_1), N_A(l_2)\}, \text{rmax}\{N_B(j_1), N_B(j_2)\}\} \\
&= \text{rmax}\{\text{rmax}\{N_A(l_1), N_B(j_1)\}, \text{rmax}\{N_A(l_2), N_B(j_2)\}\} \\
&= \text{rmax}\{(N_A \times N_B)(l_1,j_1), (N_A \times N_B)(l_2,j_2)\},
\end{aligned}$$

$$\begin{aligned}
(\mu_\lambda \times \mu_\vartheta)((l_1,j_1) * (l_2,j_2)) &= (\mu_\lambda \times \mu_\vartheta)(l_1 * l_2, j_1 * j_2) \\
&= \max\{\mu_\lambda(l_1 * l_2), \mu_\vartheta(j_1 * j_2)\} \\
&\leq \max\{\max\{\mu_\lambda(l_1), \mu_\lambda(l_2)\}, \max\{\mu_\vartheta(j_1), \mu_\vartheta(j_2)\}\} \\
&= \max\{\max\{\mu_\lambda(l_1), \mu_\vartheta(j_1)\}, \max\{\mu_\lambda(l_2), \mu_\vartheta(j_2)\}\} \\
&= \max\{(\mu_\lambda \times \mu_\vartheta)(l_1,j_1), (\mu_\lambda \times \mu_\vartheta)(l_2,j_2)\}, \\
\text{and} \quad (\nu_\lambda \times \nu_\vartheta)((l_1,j_1) * (l_2,j_2)) &= (\nu_\lambda \times \nu_\vartheta)(l_1 * l_2, j_1 * j_2) \\
&= \min\{\nu_\lambda(l_1 * l_2), \nu_\vartheta(j_1 * j_2)\} \\
&\geq \min\{\min\{\nu_\lambda(l_1), \nu_\lambda(l_2)\}, \min\{\nu_\vartheta(j_1), \nu_\vartheta(j_2)\}\} \\
&= \min\{\min\{\nu_\lambda(l_1), \nu_\vartheta(j_1)\}, \min\{\nu_\lambda(l_2), \nu_\vartheta(j_2)\}\} \\
&= \min\{(\nu_\lambda \times \nu_\vartheta)(l_1,j_1), (\nu_\lambda \times \nu_\vartheta)(l_2,j_2)\}.
\end{aligned}$$

Hence, $\widetilde{A} \times \widetilde{B}$ is a cubic intuitionistic subalgebra of $T \times S$. □

For the Cartesian product of two ideals of a BCI-algebra, we have the following theorem.

Theorem 11. *Let $\widetilde{A}$ and $\widetilde{B}$ be cubic intuitionistic ideals of T, then $\widetilde{A} \times \widetilde{B}$ is a cubic intuitionistic ideal of $T \times T$.*

Proof. For any $(x,y) \in T \times T$, we have $(M_A \times M_B)(0,0) = \text{rmin}\{M_A(0), M_B(0)\} \gg \text{rmin}\{M_A(l), M_B(j)\} = (M_A \times M_B)(l,j)$, $(N_A \times N_B)(0,0) = \text{rmax}\{N_A(0), N_B(0)\} \ll \text{rmax}\{N_A(l), N_B(j)\} = (N_A \times N_B)(l,j)$, $(\mu_\lambda \times \mu_\vartheta)(0,0) = \max\{\mu_\lambda(0), \mu_\vartheta(0)\} \leq \max\{\mu_\lambda(l), \mu_\vartheta(j)\} = (\mu_\lambda \times \mu_\vartheta)(l,j)$, and $(\nu_\lambda \times \nu_\vartheta)(0,0) = \min\{\nu_\lambda(0), \nu_\vartheta(0)\} \geq \min\{\nu_\lambda(l), \nu_\vartheta(j)\} = (\nu_\lambda \times \nu_\vartheta)(l,j)$.

Let (l_1, j_1) and $(l_2, j_2) \in T \times T$. Then:

$$\begin{aligned}
(M_A \times M_B)(l_1, j_2) &= \operatorname{rmin}\{M_A(l_1), M_B(j_1)\} \\
&\gg \operatorname{rmin}\{\operatorname{rmin}\{M_A(l_1 * l_2), M_A(l_2)\}, \operatorname{rmin}\{M_B(j_1 * j_2), M_B(j_2)\}\} \\
&= \operatorname{rmin}\{\operatorname{rmin}\{M_A(l_1 * l_2), M_B(j_1 * j_2)\}, \operatorname{rmin}\{M_A(l_2), M_B(j_2)\}\} \\
&= \operatorname{rmin}\{(M_A \times M_B)(l_1 * l_2, j_1 * j_2), (M_A \times M_B)(l_2, j_2)\} \\
&= \operatorname{rmin}\{(M_A \times M_B)((l_1, j_1) * (l_2, j_2)), (M_A \times M_B)(l_2, j_2)\}, \\
(N_A \times N_B)(l_1, j_2) &= \operatorname{rmax}\{N_A(l_1), N_B(j_1)\} \\
&\ll \operatorname{rmax}\{\operatorname{rmax}\{N_A(l_1 * l_2), N_A(l_2)\}, \operatorname{rmax}\{N_B(j_1 * j_2), N_B(j_2)\}\} \\
&= \operatorname{rmax}\{\operatorname{rmax}\{N_A(l_1 * l_2), N_B(j_1 * j_2)\}, \operatorname{rmax}\{N_A(l_2), N_B(j_2)\}\} \\
&= \operatorname{rmax}\{(N_A \times N_B)(l_1 * l_2, j_1 * j_2), (N_A \times N_B)(l_2, j_2)\} \\
&= \operatorname{rmax}\{(N_A \times N_B)((l_1, j_1) * (l_2, j_2)), (N_A \times N_B)(l_2, j_2)\}, \\
(\mu_\lambda \times \mu_\vartheta)(l_1, j_2) &= \max\{\mu_\lambda(l_1), \mu_\vartheta(j_1)\} \\
&\leq \max\{\max\{\mu_\lambda(l_1 * l_2), \mu_\lambda(l_2)\}, \max\{\mu_\vartheta(j_1 * j_2), \mu_\vartheta(j_2)\}\} \\
&= \max\{\max\{\mu_\lambda(l_1 * l_2), \mu_\vartheta(j_1 * j_2)\}, \max\{\mu_\lambda(l_2), \mu_\vartheta(j_2)\}\} \\
&= \max\{(\mu_\lambda \times \mu_\vartheta)(l_1 * l_2, j_1 * j_2), (\mu_\lambda \times \mu_\vartheta)(l_2, j_2)\} \\
&= \max\{(\mu_\lambda \times \mu_\vartheta)((l_1, j_1) * (l_2, j_2)), (\mu_\lambda \times \mu_\vartheta)(l_2, j_2)\}, \\
\text{and}(\nu_\lambda \times \nu_\vartheta)(l_1, j_2) &= \min\{\nu_\lambda(l_1), \nu_\vartheta(j_1)\} \\
&\geq \min\{\min\{\nu_\lambda(l_1 * l_2), \nu_\lambda(l_2)\}, \min\{\nu_\vartheta(j_1 * j_2), \nu_\vartheta(j_2)\}\} \\
&= \min\{\min\{\nu_\lambda(l_1 * l_2), \nu_\vartheta(j_1 * j_2)\}, \min\{\nu_\lambda(l_2), \nu_\vartheta(j_2)\}\} \\
&= \min\{(\nu_\lambda \times \nu_\vartheta)(l_1 * l_2, j_1 * j_2), (\nu_\lambda \times \nu_\vartheta)(l_2, j_2)\} \\
&= \min\{(\nu_\lambda \times \nu_\vartheta)((l_1, j_1) * (l_2, j_2)), (\nu_\lambda \times \nu_\vartheta)(l_2, j_2)\}.
\end{aligned}$$

Hence, $\widetilde{A} \times \widetilde{B}$ is a cubic intuitionistic ideal of $T \times T$. □

For the Cartesian product of two q-ideals of a BCI-algebra, we have the following theorem.

Theorem 12. *Let $\widetilde{A}$ and $\widetilde{B}$ be cubic intuitionistic q-ideals of T, then $\widetilde{A} \times \widetilde{B}$ is a cubic intuitionistic q-ideal of $T \times T$.*

Proof. Let (l_1, j_1), (l_2, j_2), $(l_3, j_3) \in T \times T$. Then:

$$\begin{aligned}
& (M_A \times M_B)((l_1, j_1) * (l_3, j_3)) \\
= \;& (M_A \times M_B)(l_1 * l_3, j_1 * j_3) = \operatorname{rmin}\{M_A(l_1 * l_3), M_B(j_1 * j_3)\} \\
\gg \;& \operatorname{rmin}\{\operatorname{rmin}\{M_A(l_1 * (l_2 * l_3)), M_A(l_2)\}, \operatorname{rmin}\{M_B(j_1 * (j_2 * j_3)), M_B(j_2)\}\} \\
= \;& \operatorname{rmin}\{\operatorname{rmin}\{M_A(l_1 * (l_2 * l_3)), M_B(j_1 * (j_2 * j_3))\}, \operatorname{rmin}\{M_A(l_2), M_B(j_2)\}\} \\
= \;& \operatorname{rmin}\{(M_A \times M_B)(l_1 * (l_2 * l_3), j_1 * (j_2 * j_3)), (M_A \times M_B)(l_2, j_2)\} \\
= \;& \operatorname{rmin}\{(M_A \times M_B)((l_1, j_1) * ((l_2, j_2) * (l_3, j_3))), (M_A \times M_B)(l_2, j_2)\}, \\
& (N_A \times N_B)((l_1, j_1) * (l_3, j_3)) \\
= \;& (N_A \times N_B)(l_1 * l_3, j_1 * j_3) = \operatorname{rmax}\{N_A(l_1 * l_3), N_B(j_1 * j_3)\} \\
\ll \;& \operatorname{rmax}\{\operatorname{rmax}\{N_A(l_1 * (l_2 * l_3)), N_A(l_2)\}, \operatorname{rmax}\{N_B(j_1 * (j_2 * j_3)), N_B(j_2)\}\} \\
= \;& \operatorname{rmax}\{\operatorname{rmax}\{N_A(l_1 * (l_2 * l_3)), N_B(j_1 * (j_2 * j_3))\}, \operatorname{rmax}\{N_A(l_2), N_B(j_2)\}\} \\
= \;& \operatorname{rmax}\{(N_A \times N_B)(l_1 * (l_2 * l_3), j_1 * (j_2 * j_3)), (N_A \times N_B)(l_2, j_2)\} \\
= \;& \operatorname{rmax}\{(N_A \times N_B)((l_1, j_1) * ((l_2, j_2) * (l_3, j_3))), (N_A \times N_B)(l_2, j_2)\},
\end{aligned}$$

$$
\begin{aligned}
& (\mu_\lambda \times \mu_\vartheta)((l_1, j_1) * (l_3, j_3)) \\
= \; & (\mu_\lambda \times \mu_\vartheta)(l_1 * l_3, j_1 * j_3) = \max\{\mu_\lambda(l_1 * l_3), \mu_\vartheta(j_1 * j_3)\} \\
\leq \; & \max\{\max\{\mu_\lambda(l_1 * (l_2 * l_3)), \mu_\lambda(l_2)\}, \max\{\mu_\vartheta(j_1 * (j_2 * j_3)), \mu_\vartheta(j_2)\}\} \\
= \; & \max\{\max\{\mu_\lambda(l_1 * (l_2 * l_3)), \mu_\vartheta(j_1 * (j_2 * j_3))\}, \max\{\mu_\lambda(l_2), \mu_\vartheta(j_2)\}\} \\
= \; & \max\{(\mu_\lambda \times \mu_\vartheta)(l_1 * (l_2 * l_3), j_1 * (j_2 * j_3)), (\mu_\lambda \times \mu_\vartheta)(l_2, j_2)\} \\
= \; & \max\{(\mu_\lambda \times \mu_\vartheta)((l_1, j_1) * ((l_2, j_2) * (l_3, j_3))), (\mu_\lambda \times \mu_\vartheta)(l_2, j_2)\},
\end{aligned}
$$

and

$$
\begin{aligned}
& (\nu_\lambda \times \nu_\vartheta)((l_1, j_1) * (l_3, j_3)) \\
= \; & (\nu_\lambda \times \nu_\vartheta)(l_1 * l_3, j_1 * j_3) = \min\{\nu_\lambda(l_1 * l_3), \nu_\vartheta(j_1 * j_3)\} \\
\geq \; & \min\{\min\{\nu_\lambda(l_1 * (l_2 * l_3)), \nu_\lambda(l_2)\}, \min\{\nu_\vartheta(j_1 * (j_2 * j_3)), \nu_\vartheta(j_2)\}\} \\
= \; & \min\{\min\{\nu_\lambda(l_1 * (l_2 * l_3)), \nu_\vartheta(j_1 * (j_2 * j_3))\}, \min\{\nu_\lambda(l_2), \nu_\vartheta(j_2)\}\} \\
= \; & \min\{(\nu_\lambda \times \nu_\vartheta)(l_1 * (l_2 * l_3), j_1 * (j_2 * j_3)), (\nu_\lambda \times \nu_\vartheta)(l_2, j_2)\} \\
= \; & \min\{(\nu_\lambda \times \nu_\vartheta)((l_1, j_1) * ((l_2, j_2) * (l_3, j_3))), (\nu_\lambda \times \nu_\vartheta)(l_2, j_2)\}.
\end{aligned}
$$

Hence, $\widetilde{A} \times \widetilde{B}$ is a cubic intuitionistic q-ideal of $T \times T$. □

5. Conclusions

Recently, Jun [19] studied a unique extension of cubic sets and its applications in BCK/BCI-algebras. In this paper, we have applied this new notion of the cubic intuitionistic set to q-ideals of a BCI-algebra and studied a few of their related characteristics in detail. In the future, these definitions and fundamental results can be applied to some different algebraic structures, for example Lie algebras and lattices. There are more topics today that could take advantage of this cubic intuitionistic set theory. Like for example in DNA identification [23], in the genetic algorithm in order to formalize the procedures [24], in genomics [25], in fuzzy logic-based networks, and in particular, in complex networks [26].

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References

1. Jun, Y.B.; Kim, C.S.; Yang, K.O. Cubic sets. *Ann. Fuzzy Math. Inform.* **2012**, *4*, 83–98.
2. Jana, C.; Senapati, T. Cubic G-subalgebras of G-algebras. *Ann. Pure Appl. Math.* **2015**, *10*, 105–115.
3. Jun, Y.B.; Jung, S.T.; Kim, M.S. Cubic subgroups. *Ann. Fuzzy Math. Inform.* **2011**, *2*, 9–15.
4. Jun, Y.B.; Kim, C.S.; Kang, M.S. Cubic subalgebras and ideals of BCK/BCI-algebras. *Far East J. Math. Sci.* **2010**, *44*, 239–250.
5. Jun, Y.B.; Kim, C.S.; Kang, J.G. Cubic q-ideals of BCI-algebras. *Ann. Fuzzy Math. Inform.* **2011**, *1*, 25–31.
6. Jun, Y.B.; Lee, K.J.; Kang, M.S. Cubic structures applied to ideals of BCI-algebras. *Comput. Math. Appl.* **2011**, *62*, 3334–3342. [CrossRef]
7. Jun, Y.B.; Khan, A. Cubic ideals in semigroups. *Honam Math. J.* **2013**, *35*, 607–623. [CrossRef]
8. Jun, Y.B.; Muhiuddin, G.; Ozturk, M.A.; Roh, E.H. Cubic soft ideals in BCK/BCI-algebras. *J. Comput. Anal. Appl.* **2017**, *22*, 929–940.
9. Muhiuddin, G.; Al-roqi, A.M. Cubic soft sets with applications in BCK/BCI-algebras. *Ann. Fuzzy Math. Inform.* **2014**, *8*, 291–304.
10. Muhiuddin, G.; Feng, F.; Jun, Y.B. Subalgebras of BCK/BCI-algebras based on cubic soft sets. *Sci. World J.* **2014**, *2014*, 458638. [CrossRef]
11. Senapati, T.; Kim, C.S.; Bhowmik, M.; Pal, M. Cubic subalgebras and cubic closed ideals of B-algebras. *Fuzzy Inf. Eng.* **2015**, *7*, 129–149. [CrossRef]

12. Senapati, T. Cubic BF-subalgebras of BF-algebras. *Anal. Univ. Oradea Fasc. Math.* **2016**, *23*, 97–105.
13. Senapati, T. Cubic structure of BG-subalgebras of BG-algebras. *J. Fuzzy Math.* **2016**, *24*, 151–162.
14. Senapati, T.; Shum, K.P. Cubic implicative ideals of BCK-algebras. *Mo. J. Math. Sci.* **2017**, *29*, 125–138.
15. Senapati, T.; Shum, K.P. Cubic commutative ideals of BCK-algebras. *Mo. J. Math. Sci.* **2018**, *30*, 5–19.
16. Amat, S.; Busquier, S.; Legaz, M.J.; Ruiz, J. Unifying the classical approach with new technologies: An innovative proposal for teaching mathematics in engineering. *Int. J. Interact. Multimed. Artif. Intell.* **2015**, *3*, 110–132. [CrossRef]
17. Sarria, I.; Gonzalez, R.; Magrenan, A.A.; Narvaez, S.P.; Orcos, L. Games Math. Adaptive Video Game to Evaluate Basic Mathematic Concepts. In *Learning Technology for Education Challenges*; LTEC 2017, Communications in Computer and Information Science; Uden, L., Liberona, D., Liu, Y., Eds.; Springer: Cham, Switzerland, 2017; Volume 734, pp. 27–35.
18. Behl, R.; Sarria, I.; Crespo, R.G.; Magrenan, A.A. Highly efficient family of iterative methods for solving nonlinear models. *J. Comput. Appl. Math.* **2019**, *346*, 110–132. [CrossRef]
19. Jun, Y.B. A novel extension of cubic sets and its applications in BCK/BCI-algebras. *Ann. Fuzzy Math. Inform.* in press.
20. Senapati, T.; Jun, Y.B.; Shum, K.P. Cubic intuitionistic subalgebras and closed cubic intuitionistic ideals of B-algebras. *J. Intell. Fuzzy Syst.* in press.
21. Senapati, T.; Jun, Y.B.; Muhiuddin, G.; Shum, K.P. Cubic intuitionistic structures applied to ideals of BCI-algebras. *Analele Stiintifice ale Universitatii Ovidius Constanta* in press.
22. Liu, Y.L.; Meng, J.; Zhang, X.H.; Yue, Z.C. q-ideals and a-ideals in BCI-algebras. *Southeast Asian Bull. Math.* **2000**, *24*, 243–253. [CrossRef]
23. Santos-Sierra, A.D.; Sanchez-Avila, C.; Jara-Vera, V. A fuzzy DNA based algorithm for identification and authentication in an iris detection system. In Proceedings of the IEEE 42nd International Carnahan Conference on Security Technology (ICCST'08), Prague, Czech Republic, 13–16 October 2008; pp. 226–232.
24. Cheng, C.B.; Cheng, C.J.; Lee, E.S. Neuro-fuzzy and genetic algorithm in multiple response optimization. *Comput. Math. Appl.* **2002**, *44*, 1503–1514. [CrossRef]
25. Ressom, H.; Natarajan, P.; Varghese, R.S.; Musavi, M.T. Applications of fuzzy logic in genomics. *Fuzzy Sets Syst.* **2005**, *152*, 125–138. [CrossRef]
26. Bergner, A.; Frasca, M.; Sciuto, G.; Buscarino, A.; Ngamga, E.J.; Fortuna, L. Kurths, J. Remote synchronization in star networks. *Phys. Rev. E* **2012**, *85*, 026208. [CrossRef] [PubMed]