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Article

Renormalizable and Unitary Model of Quantum Gravity †

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Abstract: We consider $R + R^2$ relativistic quantum gravity with the action where all possible terms quadratic in the curvature tensor are added to the Einstein-Hilbert term. This model was shown to be renormalizable in the work by K.S. Stelle. In this paper, we demonstrate that the $R + R^2$ model is also unitary contrary to the statements made in the literature, in particular in the work by Stelle. New expressions for the $R + R^2$ Lagrangian within dimensional regularization and the graviton propagator are derived. We demonstrate that the $R + R^2$ model is a good candidate for the fundamental quantum theory of gravity.

Keywords: modified theories of gravity; quantum gravity; renormalizability; unitarity

1. Introduction

Creation of the fundamental quantum theory of gravity is one of the most important tasks of modern theoretical physics.

Three of the four presently known fundamental interactions are perfectly described by Quantum Field Theory. Electromagnetic and weak nuclear interactions are unified within the Standard Model and strong nuclear interaction is currently described by Quantum Chromodynamics. But the fourth fundamental interaction—gravitation—is presently described only by classical General Relativity by Einstein and steadily escaped attempts at quantization.

It is well known that the problem arises because of the non-renormalizability [1] of General Relativity. In Reference [1] it was shown that General Relativity without matter fields is renormalizable at the one loop level but becomes unrenormalizable after inclusion of matter fields.

In 1977 K.S. Stelle showed [2] the renormalizability of the Lorentz invariant gravitational action, which includes, as well as the Einstein-Hilbert R-term, the R^2-terms with four derivatives of the metric. His proof used the specific covariant gauge where the structure of ultraviolet divergences is particularly simple. For more general gauges he made the assumption of the cohomological structure of divergences. Recently, this hypothesis was shown to be correct for the general class of background gauges [3]. Thus, we consider the renormalizability of gravity with four derivatives of the metric to be well established. We will call this $R + R^2$ model quadratic quantum gravity.

Stelle has also stated [2,4] that quantum gravity with four derivatives of the metrics is unphysical since it violates either unitarity or causality. Thus, according to him the model can serve only as an example of a renormalizable model or as an effective theory in some domain of energies. Since then, this model has been commonly considered in the literature to be unphysical.

In this article, we demonstrate that quadratic quantum gravity is, in fact, unitary. Hence, we state that $R+R^2$ gravity is a good candidate for the fundamental theory of quantum gravity.

We derive new expressions for the Lagrangian of quadratic quantum gravity and for the graviton propagator within dimensional regularization. These items were also discussed in our papers [5,6].

2. Results

Let us consider the Lorentz invariant $R+R^2$ action with all possible terms quadratic in the curvature tensor $R_{\mu\nu}$

$$S_{sym} = \int d^D x \mu^{-2\epsilon} \sqrt{-g} \left(-M_{Pl}^2 R + \alpha R_{\mu\nu} R^{\mu\nu} + \beta R^2 + \delta R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + M_{Pl}^2 \Lambda \right), \tag{1}$$

where the first R-term is the Einstein-Hilbert action. The last term, the Λ-term, can be omitted in the action since we will work within perturbation theory and it will not provide contributions in this case.

$M_{Pl}^2 = 1/(16\pi G)$ is the squared Planck mass, $R_{\mu\nu\rho\sigma}$, $R_{\mu\nu}$ and R are the Riemann tensor, the Ricci tensor and the Ricci scalar correspondingly. α, β and δ are dimensionless coupling constants, $D = 4-2\epsilon$ is the space time dimension. ϵ and μ are the parameters of dimensional regularization.

The Riemann tensor is

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}, \tag{2}$$

where Christoffell symbols are

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(\partial_\nu g_{\mu\beta} + \partial_\mu g_{\nu\beta} - \partial_\beta g_{\mu\nu} \right). \tag{3}$$

We would like to underline that the dimensional regularization developed in Reference [7–11] is presently the only practically available regularization of ultraviolet (and infrared) divergences which preserves gauge invariance of quantum gravity.

Usually the term with the coupling δ in the action (1) is missed in the literature [2,3,12]. This is due to the Gauss-Bonnet topological identity

$$\int d^4 x \sqrt{-g} \left(R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right) = 0, \tag{4}$$

which is valid for space-times topologically equivalent to flat space only in 4-dimensions but the dimension of the space within dimensional regularization is different from four. Thus, this extra term should be added to the action with an independent interaction constant to ensure renormalizability.

It may seem that the introduction of three new independent gravitational constants into the Lagrangian besides the Newton constant is too big a price for quantization but dimensional regularization unavoidably demands it.

We will use perturbation theory. Hence, we will consider the linearized theory around the flat space metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{5}$$

where the convention is used $\eta_{\mu\nu} = diag(+1,-1,-1,-1)$ in four dimensions. In dimensional regularization $\eta_{\mu\nu}\eta^{\mu\nu} = D$. It is understood that indexes are raised and lowered with the tensor $\eta_{\mu\nu}$.

Gauge transformations of the Lagrangian are generated by diffeomorphisms $x^\mu \to x^\mu + \zeta^\mu(x)$ and are

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \zeta_\nu + \partial_\nu \zeta_\mu + \left(h_{\lambda\mu}\partial_\nu + h_{\lambda\nu}\partial_\mu + (\partial_\lambda h_{\mu\nu}) \right) \zeta^\lambda, \tag{6}$$

where $\zeta_\mu(x)$ are arbitrary functions.

According to standard Faddeev-Popov quantization [13]—for the review see Reference [14]—one should add to the action the gauge fixing term which we choose in the form

$$S_{gf} = -\frac{1}{2\xi}\int d^D x F_\mu \partial_\nu \partial^\nu F^\mu, \tag{7}$$

where $F^\mu = \partial_\nu h^{\nu\mu}$, ξ is the gauge parameter. Of course, as usual, physical results do not depend on the specific choice of the gauge condition.

One also adds the ghost term

$$S_{ghost} = \int d^D x d^D y \overline{C}_\mu(x)\frac{\delta F^\mu(x)}{\delta\zeta_\nu(y)} C_\nu(y) = \tag{8}$$

$$\int d^D x \partial^\nu \overline{C^\mu}\left[\partial_\nu C_\mu + \partial_\mu C_\nu + h_{\lambda\mu}\partial_\nu C^\lambda + h_{\lambda\nu}\partial_\mu C^\lambda + (\partial_\lambda h_{\mu\nu})C^\lambda\right],$$

here $\overline{C}$ and C are ghost fields. As the result, one gets the generating functional for the Green functions of gravitons

$$Z(J) = N\int dh_{\mu\nu} dC_\lambda d\overline{C_\rho} \exp\left[i\left(S_{sym} + S_{gf} + S_{ghost} + d^D x \mu^{-2\epsilon} J_{\mu\nu} h^{\mu\nu}\right)\right], \tag{9}$$

where as usual N is the normalization factor and $J_{\mu\nu}$ is the source of gravitons.

We will work in perturbation theory, hence we make the shift of the fields

$$h_{\mu\nu} \to M_{Pl}\mu^{-\epsilon} h_{\mu\nu}. \tag{10}$$

Perturbative expansion goes in the inverse powers of the Plank mass M_{Pl} or in the Newton coupling constant $G \propto 1/M_{Pl}^2$.

We want to derive the graviton propagator. We make the Fourier transform to the momentum space and write the part of the Lagrangian which is quadratic in $h_{\mu\nu}$

$$\begin{aligned} Q_{\mu\nu\rho\sigma} = \frac{1}{4}\int d^D k\, h^{\mu\nu}(-k)\Big[&\left(k^2 + M_{Pl}^{-2}k^4(\alpha + 4\delta)\right)P^{(2)}_{\mu\nu\rho\sigma} \\ &+ k^2\left(-2 + 4M_{Pl}^{-2}k^2(\alpha + 3\beta + \delta)\right)P^{(0-s)}_{\mu\nu\rho\sigma} \\ &+ \frac{1}{\xi}M_{Pl}^{-2}k^4\left(P^{(1)}_{\mu\nu\rho\sigma} + 2P^{(0-w)}_{\mu\nu\rho\sigma}\right)\Big] h^{\rho\sigma}(k), \end{aligned} \tag{11}$$

here $P^{(i)}_{\mu\nu\rho\sigma}$ are the projectors to the spin-2, -1 and -0 components of the field h correspondingly:

$$P^{(2)}_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\Theta_{\mu\rho}\Theta_{\nu\sigma} + \Theta_{\mu\sigma}\Theta_{\nu\rho}\right) - \frac{1}{3}\Theta_{\mu\nu}\Theta_{\rho\sigma}, \tag{12}$$

$$P^{(1)}_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\Theta_{\mu\rho}\omega_{\nu\sigma} + \Theta_{\mu\sigma}\omega_{\nu\rho} + \Theta_{\nu\rho}\omega_{\mu\sigma} + \Theta_{\nu\sigma}\omega_{\mu\rho}\right), \tag{13}$$

$$P^{(0-s)}_{\mu\nu\rho\sigma} = \frac{1}{3}\Theta_{\mu\nu}\Theta_{\rho\sigma}, \tag{14}$$

$$P^{(0-w)}_{\mu\nu\rho\sigma} = \omega_{\mu\nu}\omega_{\rho\sigma}, \tag{15}$$

where $\Theta_{\mu\nu} = \eta_{\mu\nu} - k_\mu k_\nu / k^2$ and $\omega_{\mu\nu} = k_\mu k_\nu / k^2$ are the transverse and longitudinal projectors.

We would like to note that the expression (11) differs from the analogous one in Reference [12] where ϵ-dependent contributions are present. Our expression (11) does not contain ϵ-dependent terms.

To obtain the graviton propagator $D_{\mu\nu\rho\sigma}$ we should invert the matrix in the square brackets of (11):

$$[Q]_{\mu\nu\kappa\lambda} D^{\kappa\lambda\rho\sigma} = \frac{1}{2}(\delta^\rho_\mu \delta^\sigma_\nu + \delta^\sigma_\mu \delta^\rho_\nu). \tag{16}$$

Then the propagator has the form

$$D_{\mu\nu\rho\sigma} = \frac{1}{i(2\pi)^D} \left[\frac{4}{k^2} \left(\frac{1}{1 + M_{Pl}^{-2} k^2 (\alpha + 4\delta)} \right) P^{(2)}_{\mu\nu\rho\sigma} \right. \tag{17}$$

$$- \frac{2}{k^2} \left(\frac{1 + 2\epsilon \frac{1 - M_{Pl}^{-2} k^2 (\alpha + 4\beta)}{1 + M_{Pl}^{-2} k^2 (\alpha + 4\delta)}}{1 - \epsilon - M_{Pl}^{-2} k^2 \left((2\alpha + 6\beta + 2\delta) - \epsilon(\alpha + 4\beta) \right)} \right) P^{(0-s)}_{\mu\nu\rho\sigma}$$

$$\left. + 4\xi \frac{1}{M_{Pl}^{-2} k^4} \left(P^{(1)}_{\mu\nu\rho\sigma} + \frac{1}{2} P^{(0-w)}_{\mu\nu\rho\sigma} \right) \right].$$

Now we perform partial fractioning. The graviton propagator becomes

$$D_{\mu\nu\rho\sigma} = \frac{1}{i(2\pi)^D} \left[4 P^{(2)}_{\mu\nu\rho\sigma} \left(\frac{1}{k^2} - \frac{1}{k^2 - M_{Pl}^2 / (-\alpha - 4\delta)} \right) \right. \tag{18}$$

$$- 2 \frac{P^{(0-s)}_{\mu\nu\rho\sigma}}{1 - \epsilon} \left(1 + 2\epsilon \frac{1 - M_{Pl}^{-2} k^2 (\alpha + 4\beta)}{1 + M_{Pl}^{-2} k^2 (\alpha + 4\delta)} \right)$$

$$\left(\frac{1}{k^2} - \frac{1}{k^2 - M_{Pl}^2 (1 - \epsilon) / (2\alpha + 6\beta + 2\delta - \epsilon(\alpha + 4\beta))} \right)$$

$$\left. + \frac{4\xi}{M_{Pl}^{-2} k^4} \left(P^{(1)}_{\mu\nu\rho\sigma} + \frac{1}{2} P^{(0-w)}_{\mu\nu\rho\sigma} \right) \right].$$

We would like to underline that one of the poles in the term with $P^{(0-s)}_{\mu\nu\rho\sigma}$ depends on the parameter of dimensional regularization ϵ. Also residues of both poles in this term depend on the regularization parameter ϵ. Hence it should be clear that poles and residues of the tree level graviton propagator do not have physical meaning.

In 4-dimensional space one gets for the graviton propagator

$$D_{\mu\nu\rho\sigma} = \frac{4}{i(2\pi)^D} \left[\frac{P^{(2)}_{\mu\nu\rho\sigma} - \frac{1}{2} P^{(0-s)}_{\mu\nu\rho\sigma}}{k^2} - \frac{P^{(2)}_{\mu\nu\rho\sigma}}{k^2 - M_{Pl}^2 / (-\alpha - 4\delta)} \right. \tag{19}$$

$$\left. + \left(\frac{1}{2} \right) \frac{P^{(0-s)}_{\mu\nu\rho\sigma}}{k^2 - M_{Pl}^2 / (2\alpha + 6\beta + 2\delta)} + \frac{\xi}{M_{Pl}^{-2} k^4} \left(P^{(1)}_{\mu\nu\rho\sigma} + \frac{1}{2} P^{(0-w)}_{\mu\nu\rho\sigma} \right) \right],$$

Let us consider now classical four-derivative gravity. In this case, for a point particle with the energy-momentum tensor $T_{\mu\nu} = \delta^0_\mu \delta^0_\nu M \delta^3(x)$ the gravitational field is [4]

$$V(r) = \frac{M}{2\pi M_{Pl}^2}\left(-\frac{1}{4r} + \frac{e^{-m_2 r}}{3r} - \frac{e^{-m_0 r}}{12r}\right). \tag{20}$$

Here $m_2^2 = M_{Pl}^2/(-\alpha - 4\delta)$ and $m_0^2 = M_{Pl}^2/(2\alpha + 6\beta + 2\delta)$ are the squared masses of the massive spin-2 and spin-0 gravitons. The values of the coupling constants α, β and δ can be chosen to ensure the positivity of masses. In the works by Stelle [2,4] it was mentioned that the masses can be chosen to be large enough in order to have agreements with experiments.

Our propagator (19) reproduces this expression (20). It can be seen after the calculation of the tree level Feynman diagram describing an exchange of two point-like particles with a graviton.

The graviton propagator in Reference [2] contains some small technical errors. It dos not reproduce the expression (20). To see this, one can put all coupling constants, except the Newton coupling constant, equal to zero in the $R + R^2$ Lagrangian. Then the Lagrangian is reduced to the Lagrangian of General Relativity. Correspondingly, the graviton propagator should be reduced to the graviton propagator of General Relativity:

$$D_{\mu\nu\rho\sigma}(k) = \frac{1}{i(2\pi)^4}\frac{\frac{1}{2}\eta_{\mu\rho}\eta_{\nu\sigma} + \frac{1}{2}\eta_{\mu\sigma}\eta_{\nu\rho} - \frac{1}{2}\eta_{\mu\nu}\eta_{\rho\sigma} + terms \propto k}{k^2}, \tag{21}$$

where for simplicity one can take the gauge condition with $\xi = 0$.

Our propagator (19) reproduces the propagator (21) if one puts couplings α, β, δ equal to zero. The graviton propagator of Reference [2] in the corresponding limit gives, in the third term of the numerator of (21), the factor 1 instead of $1/2$.

The second term in the graviton propagator (19) has the non-standard minus sign. That is why it should be considered as the massive spin-2 ghost. To have renormalizability of the theory, one should shift all poles of propagators in Feynman integrals in the same way $k^2 \to k^2 + i0$. Thus, the ghost should be considered as the state with the negative metric [2]. That is why Stelle made the statement [2,4] about violation either unitarity or causality in the model with four derivatives of the metric.

However, this massive spin-2 ghost is unstable. It will unavoidably decay in two massless physical gravitons. The width of this decay is presumably small. But independently of the numerical value of the corresponding decay width, this spin-2 ghost particle does not appear as the asymptotic state of the S-matrix. Hence, only physical gravitons with the positive metric participate as external particles of S-matrix amplitudes. Thus, unitarity is preserved in the $R + R^2$ theory.

There is the statement that theories with ghosts are unstable, that is, they do not have a stable vacuum state [15]; for a brief review see Reference [16]. This statement is proved only for Quantum Mechanical systems. Quantum Field Theory is a quite different story and renomalizability plus unitarity should be enough to have a consistent theory.

It is necessary to mention that the S-matrix by construction satisfies the unitarity relation

$$S^+ S = 1 \tag{22}$$

in the theories with Hermitian Lagrangians, see Reference [17].

It can be seen that if one represents the S-matrix as the T-exponent within the operator formalism:

$$S = T\left(e^{i\int L(x)dx}\right). \tag{23}$$

Let us present the corresponding proof. One should introduce a function $g(x)$ with values in the interval $(0,1)$ which describes the intensity of gravitational interactions. If $g(x) = 0$ then interactions are switched off, if $g(x) = 1$ then interactions are switched on; if $0 < g(x) < 1$ then interactions are partly switched on. Substituting the real Lagrangian $L(x)$ by the product $L(x)g(x)$ one gets interactions switched on with intensity $g(x)$. Thus, the S-matrix becomes the functional

$$S(g) = T\left(exp\ \ i\int L(x)g(x)dx\right). \tag{24}$$

Then one should split the interaction region described by the function $g(x)$ with space-like surfaces $t = const$ into an infinitely large number of infinitely thin segments Δ_i. One gets

$$S(g) = T\left(exp\ \ i\int L(x)g(x)dx\right) = T\left(exp\ \ i\sum_j\int_{\Delta_j} L(x)g(x)dx\right) = \tag{25}$$
$$T\left(\prod_j exp\ \ i\int_{\Delta_j} L(x)g(x)dx\right).$$

$S(g)$ is defined as the limit

$$S(g) = \lim_{\Delta_j\to 0} T\left(\prod_j\left(1 + i\int_{\Delta_j} L(x)g(x)dx\right)\right). \tag{26}$$

The right hand side of (26) is a usual product taken in the chronological order of segments Δ_j. But for sufficiently small Δ_j each factor in this product is unitary up to small terms of higher orders. These higher orders can be neglected in the considered limit according to mathematics. That is why the whole product is unitary. Thus unitarity of $S(g)$ and hence of the matrix

$$S = \lim_{g(x)\to 1} S(g) \tag{27}$$

is proved.

Unitarity of the S-matrix in the presence of negative metric states was considered previously in References [18–22] (see also references therein). The question of causality was also studied there.

3. Discussions

We should note that the tree level graviton propagator (19) is essentially modified after the summation of one-loop corrections. As was mentioned, the second term of the expression (19) has the minus sign. Hence the one-loop correction (due to the Feynman diagram with the massless graviton in the loop) will move the pole of the spin-2 ghost from the real value $k^2 = M_{Pl}^2/(-\alpha - 4\delta)$ to the complex value $k^2 = M_{Pl}^2/(-\alpha - 4\delta) - i\Gamma$. Here, Γ is the width of the decay of the spin-2 ghost into the pare of physical gravitons. This complex pole will be located not on the physical but on the unphysical Riemann sheet. It is completely analogous to the well-known virtual level of the system of a neutron and a proton with opposite spins [23].

We would like to stress that we consider not pure R^2 theory but the theory where the R^2 terms are added to the Einstein-Hilbert Lagrangian. Gravitational constants α, β and δ for these terms can be chosen small enough in such a way that predictions of the considered theory on large scales will coincide practically with predictions of General Relativity. The R^2 terms are needed only to ensure renormalizability of the theory and this is achieved for any choice of the gravitational constants α, β and δ.

We have considered here only purely gravitational $R + R^2$ action. It was shown in Reference [2] that the inclusion of the matter fields in the theory is straightforward.

4. Conclusions

We have demonstrated unitarity of quadratic quantum gravity with the $R + R^2$ action. This model was previously proved to be renormalizable in the work by K.S. Stelle [2]. Hence, one can conclude that the considered model is a proper candidate for the fundamental quantum theory of gravitation.

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References

1. Hooft, G.; Veltman, M.J.G. One-loop divergencies in the theory of gravitation. *Ann. Inst. H. Poincare Phys. Theor.* **1974**, *A20*, 69.
2. Stelle, K.S. Renormalization of higher-derivative quantum gravity. *Phys. Rev. D* **1977**, *16*, 953. [CrossRef]
3. Barvinsky, A.O.; Blas, D.; Herrero-Valea, M.; Sibiryakov, S.M.; Steinwatchs, C.F. Renormalization of gauge theories in the background-field approach. *JHEP* **2018**, *2018*, 35.
4. Stelle, K.S. Classical gravity with higher derivatives. *Gen. Rel. Grav.* **1978**, *9*, 353. [CrossRef]
5. Larin, S.A. Higher-derivative relativistic quantum gravity. *Mod. Phys. Lett.* **2018**, *33*, 1850028.
6. Larin, S.A. Fourth-derivative relativistic quantum gravity. *EPJ Web Conf.* **2018**, *191*, 07002. [CrossRef]
7. Wilson, K.G.; Fisher, M.E. Critical exponents in 3.99 dimensions. *Phys. Rev. Lett.* **1972**, *28*, 240. [CrossRef]
8. Veltman, M. Regularization and renormalization of gauge fields. *Nucl. Phys. B* **1972**, *44*, 189–213.
9. Bollini, C.G.; Giambiagi, J.J. Lowest order "divergent" graphs in v-dimensional space. *Phys. Lett. B* **1972**, *40*, 566. [CrossRef]
10. Ashmore, J.F. A Method of Gauge Invariant Regularization. *Nuovo Cimento Lett.* **1972**, *4*, 289. [CrossRef]
11. Cicuta, G.M.; Montaldi, E. Analytic renormalization via continuous space dimension. *Nuovo Cimento Lett.* **1972**, *4*, 329. [CrossRef]
12. Salvio, A.; Strumia, A. Agravity. *JHEP* **2014**, *2014*, 80.
13. Faddeev, L.D.; Popov, V.N. Feynman diagrams for the Yang-Mills field. *Phys. Lett. B* **1967**, *25*, 30. [CrossRef]
14. Faddeev, L.D.; Slavnov, A.A. Gauge fields. Introduction to quantum theory. *Front. Phys.* **1990**, *83*, 1. [CrossRef]
15. Pais, A.; Uhlenbeck, G.E. On field theories with non-localized action. *Phys. Rev.* **1950**, *79*, 145. [CrossRef]
16. Smilga, A. Classical and quantum dynamics of higher-derivative systems. *Int. J. Mod. Phys.* **2017**, *A32*, 1730025. [CrossRef]
17. Bogolubov, N.N.; Shirkov, D.V. *Introduction to the Theory of Quantized Fields*; Nauka: Moscow, Russia, 1976.
18. Lee, T.D.; Wick, G.C. Negative metric and the unitarity of the S-matrix. *Nucl. Phys. B* **1969**, *9*, 209. [CrossRef]
19. Lee, T.D.; Wick, G.C. Unitarity in the Nθθ sector of soluble model with indefinite metric. *Nucl. Phys. B* **1969**, *10*, 1. [CrossRef]
20. Lee, T.D.; Wick, G.C. Finite Theory of Quantum Electrodynamics. *Phys. Rev. D* **1970**, *2*, 1033. [CrossRef]
21. Lee, T.D.; Wick, G.C. Questions of Lorentz Invariance in Field Theories With Indefinite Metric . *Phys. Rev.* **1971**, *D3*, 1046. [CrossRef]

22. Cutcosky, R.E.; Landshoff, P.V.; Olive, D.I.; Polkinghome, J.C. A non-analytic S-matrix. *Nucl. Phys. B* **1969**, *12*, 281–300.
23. Landau, L.D.; Lifshits, E.M. *Quantum Mechanics: Non-Relativistic Theory, Course of Theoretical Physics*; Pergamon Press: Oxford, UK, 1991; Volume 3.