symmetry

MDPI

Article

Super-Weak Force and Neutrino Masses

Zoltán Trócsányi [1,2]

[1] Institute for Theoretical Physics, ELTE Eötvös Loránd University, Pázmány Péter Sétány 1/A, H-1117 Budapest, Hungary; Zoltan.Trocsanyi@cern.ch
[2] MTA-DE Particle Physics Research Group, P.O. Box 105, H-4010 Debrecen, Hungary

Received: 31 October 2019; Accepted: 2 January 2020; Published: 6 January 2020

Abstract: We consider an anomaly free extension of the standard model gauge group $G_{\rm SM}$ by an abelian group to $G_{\rm SM} \otimes U(1)_Z$. The condition of anomaly cancellation is known to fix the Z-charges of the particles, but two. We fix one remaining charge by allowing for all possible Yukawa interactions of the known left-handed neutrinos and new right-handed ones that obtain their masses through interaction with a new scalar field with spontaneously broken vacuum. We discuss some of the possible consequences of the model.

Keywords: gauge symmetry; extension of the standard model of particle interactions; neutrino masses

1. Introduction

The remarkable experimental success of the standard model of elementary particle interactions [1] leaves very little room for the explanation of the observed deviations from it. This success story has culminated in the discovery of the Higgs particle [2,3], which could not have happened without the immense theoretical input to the design of the accelerator and the experiments. With this discovery, a new era of particle physics has also arrived as there is no established model that can guide us to new discoveries. Therefore, theories that might incorporate the existing deviations from the standard model are desirable.

The most outstanding experimental observations that cannot be explained by the standard model are the (i) abundance of dark matter in the universe; (ii) non-vanishing neutrino masses; (iii) leptogenesis (Baryogenesis can be explained in the standard model provided leptogenesis occurs, which is called lepto-baryogenesis); (iv) accelerating expansion of the universe, signaling the existence of dark energy [4] (There are numerous other deviations of experimental results from precision predictions, but to date none has reached the significance of discovery). In addition to (i)–(iv), (v) inflation in the early universe is also considered a fairly established fact, although there is no direct proof for it. All these facts have to be explained by such an extension of the standard model that respects (a) the high precision confirmation of the standard model at collider experiments (b) and the lack of finding new particles beyond the Higgs boson by the LHC experiments [5,6]. There is one more feature of the standard model, the metastability of a vacuum [7,8] that does not necessarily require new physics, but, if new physics exist, it should not worsen the stability, but possibly push the vacuum to the stability region.

In addition to the experimental success of the standard model, it is also highly efficient being based on the concepts of local gauge invariance and spontaneous symmetry breaking [9,10]. The only exception of economical description is the relatively large number of Yukawa couplings of the fermions needed to explain their masses. The generation of the fermion masses, however, is also highly efficient in the sense that it uses the same spontaneous symmetry breaking of the scalar field to which all other particles owe their masses. In this spirit, it is reasonable to expect that the non-vanishing masses of the neutrinos should be explained by Yukawa couplings too. In addition, the choice of the

gauge groups and number of family replications look arbitrary and presently these are determined by phenomenology only.

Clearly, the neutrino masses must play a fundamental role in the possible extensions of the standard model. As the gauge and mass eigenstates of the neutrinos differ, they must feel a second force to the gauge interaction. The second force can be a Yukawa coupling to a scalar. Such explanation of neutrino masses in general requires the assumption of the existence of right-handed neutrinos and perhaps a new scalar field.

In the spirit of economy and level of arbitrariness explained above, in this article, we propose an extension of the zoo of particles in the standard model with three right-handed neutrinos and the gauge symmetry of the standard model Lagrangian $G_{\rm SM} = SU(3)_{\rm c} \otimes SU(2)_{\rm L} \otimes U(1)_Y$ to $G_{\rm SM} \otimes U(1)_Z$. Such extensions have already been considered in the literature extensively (for an incomplete set of popular examples and their studies, see [11–13]). In particular, it was shown that the charge assignment of the matter fields is constrained by the requirement of anomaly cancellations up to two free charges [14]. To define the model completely, one has to take a specific choice for these remaining free charges. In this article, we propose that the mechanism for the generation of neutrino masses fixes the values of the $U(1)_Z$ charges up to an overall scale that can be embedded in the $U(1)_Z$ coupling.

The difference between our proposal and existing studies is two-fold. The model proposed here introduces a new force along the same principles as the known forces are included in the standard model: all renormalizable terms that are allowed by the underlying gauge symmetry are present, but no other symmetry than the extra $U(1)_Z$ is assumed. Our primary goal is not the prediction of new observable phenomena at collider experiments, but first focus only on the unexplained phenomena (i–iv), with respecting the observations (a) and (b). As the deviations from the standard model are related to the intensity and cosmic frontiers of particle physics, we assume that the new $U(1)_Z$ interaction is secluded from the standard model by a small coupling. Thus, we propose the model in a region of the parameter space that has received little attention before.

2. Definition of the Model

2.1. Fermion Sector

We consider the usual three fermion families of the standard model extended with one right-handed Dirac neutrino in each family (We find it natural to assume one extra neutrino in each family although known observations do not exclude other possibilities). We introduce the notation

$$\psi^f_{q,1} = \begin{pmatrix} U^f \\ D^f \end{pmatrix}_{\rm L} \qquad \psi^f_{q,2} = U^f_{\rm R} \qquad \psi^f_{q,3} = D^f_{\rm R}\,; \qquad \psi^f_{l,1} = \begin{pmatrix} \nu^f \\ \ell^f \end{pmatrix}_{\rm L} \qquad \psi^f_{l,2} = \nu^f_{\rm R} \qquad \psi^f_{l,3} = \ell^f_{\rm R} \quad (1)$$

for the chiral quark fields ψ_q and chiral lepton fields ψ_l. In Equation (1), L and R denote the left and right-handed projections of the same field (The Weyl spinors of $\nu_{\rm L}$ and $\nu_{\rm R}$ can be embedded into different Dirac spinors, leading to Majorana neutrinos, without essential changes in the model. However, the negative results of the experiments searching for neutrinoless double β-decay make the Majorana nature of neutrinos increasingly unlikely),

$$\psi_{\rm L/R} \equiv \psi_{\mp} = \frac{1}{2}\left(1 \mp \gamma_5\right)\psi \equiv P_{\rm L/R}\psi\,. \quad (2)$$

Then, the field content in family f ($f = 1, 2$ or 3) consists of two quarks, U_f, D_f, a neutrino ν_f and a charged lepton ℓ_f, where U_f is the generic notation for the u-type quarks $U_1 = {\rm u}$, $U_2 = {\rm c}$, $U_3 = {\rm t}$, while D_f is that for d-type quarks, $D_1 = {\rm d}$, $D_2 = {\rm s}$, and $D_3 = {\rm b}$. The charged leptons ℓ_f can be $\ell_1 = e$, $\ell_2 = \mu$ or $\ell_3 = \tau$ and ν_f are the corresponding neutrinos, $\nu_1 = \nu_e$, $\nu_2 = \nu_\mu$, $\nu_3 = \nu_\tau$.

For a matrix $U \in G_{\rm SM} \otimes U(1)_Z$, the three generic fields in Equation (1) transform as

$$\begin{aligned} U\psi_1(x) &= \mathrm{e}^{\mathrm{i}\boldsymbol{T}\cdot\boldsymbol{\alpha}(x)}\,\mathrm{e}^{\mathrm{i}\,y_1\beta(x)}\,\mathrm{e}^{\mathrm{i}\,z_1\zeta(x)}\,\psi_1(x), \quad \text{where} \quad \boldsymbol{T} = \frac{1}{2}(\tau_1,\tau_2,\tau_3), \\ U\psi_j(x) &= \qquad\qquad \mathrm{e}^{\mathrm{i}\,y_j\beta(x)}\mathrm{e}^{\mathrm{i}\,z_j\zeta(x)}\psi_j(x), \quad \text{where} \quad j = 2,3, \end{aligned} \tag{3}$$

and $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$, with α_i, β, $\zeta \in \mathbb{R}$. The matrices τ_i are the Pauli matrices, y_j is the hypercharge, while z_j denotes the Z-charge of the field ψ_j. There is a lot of freedom how to choose the Z-charges. In this article, we make two assumptions that fix these completely. The first is that the charges do not depend on the families, which is also the case in the standard model (Several recent observations hint at violation of lepton flavor universality, which may be taken into account in our model by choosing family dependent Z-charges. However, those results are controversial at present, so we neglect them). With this assumption, the assignment for the Z-charges of the fermions can be expressed using two free numbers Z_1 and Z_2 of the U quark fields if we want a model free of gauge and gravity anomalies. The rest of the charges must take values as given in Table 1 [14].

Table 1. Assignments for the representations (for $SU(N)$) and charges (for $U(1)$) of fermion and scalar fields of the complete model. The charges y_j denote the eigenvalue of $Y/2$, with Y being the hypercharge operator and z_j denote the supercharges of the fields ψ_j of Equation (1) ($j = 1, 2, 3$). The right-handed Dirac neutrinos $\nu_{\rm R}$ are sterile under the $G_{\rm SM}$ group. The sixth column gives a particular realization of the $U(1)_Z$ charges, motivated below, and the last one is added for later convenience.

field	$SU(3)_{\rm c}$	$SU(2)_{\rm L}$	y_j	z_j	z_j	$r_j = z_j/z_\phi - y_j$
$U_{\rm L}$, $D_{\rm L}$	3	2	$\frac{1}{6}$	Z_1	$\frac{1}{6}$	0
$U_{\rm R}$	3	1	$\frac{2}{3}$	Z_2	$\frac{7}{6}$	$\frac{1}{2}$
$D_{\rm R}$	3	1	$-\frac{1}{3}$	$2Z_1 - Z_2$	$-\frac{5}{6}$	$-\frac{1}{2}$
$\nu_{\rm L}$, $\ell_{\rm L}$	1	2	$-\frac{1}{2}$	$-3Z_1$	$-\frac{1}{2}$	0
$\nu_{\rm R}$	1	1	0	$Z_2 - 4Z_1$	$\frac{1}{2}$	$\frac{1}{2}$
$\ell_{\rm R}$	1	1	-1	$-2Z_1 - Z_2$	$-\frac{3}{2}$	$-\frac{1}{2}$
ϕ	1	2	$\frac{1}{2}$	z_ϕ	1	$\frac{1}{2}$
χ	1	1	0	z_χ	-1	-1

The Dirac Lagrangian summed over the family replications,

$$\begin{aligned} \mathcal{L}_{\rm D} &= \mathrm{i}\sum_{f=1}^{3}\sum_{j=1}^{3}\left(\overline{\psi}^f_{q,j}(x)\not{D}_j\psi^f_{q,j}(x) + \overline{\psi}^f_{l,j}(x)\not{D}_j\psi^f_{l,j}(x)\right), \\ D^\mu_j &= \partial^\mu + \mathrm{i}g_{\rm L}\,\boldsymbol{T}\cdot\boldsymbol{W}^\mu + \mathrm{i}g_Y\,y_j B^\mu + \mathrm{i}g_Z\,z_j Z^\mu \end{aligned} \tag{4}$$

is invariant under local $G = G_{\rm SM} \otimes U(1)_Z$ gauge transformations, provided the five gauge fields introduced in the covariant derivative transform as

$$\begin{aligned} \boldsymbol{T}\cdot\boldsymbol{W}^\mu(x) &\xrightarrow{G} \boldsymbol{T}\cdot\boldsymbol{W}'^\mu(x) = U(x)\,\boldsymbol{T}\cdot\boldsymbol{W}^\mu(x)\,U^\dagger(x) + \frac{\mathrm{i}}{g_{\rm L}}\left[\partial^\mu U(x)\right]U^\dagger(x) \\ B^\mu &\xrightarrow{G} B'^\mu(x) = B^\mu(x) - \frac{1}{g_Y}\,\partial^\mu\beta(x) \qquad Z^\mu \xrightarrow{G} Z'^\mu(x) = Z^\mu(x) - \frac{1}{g_Z}\,\partial^\mu\zeta(x), \end{aligned} \tag{5}$$

where $U(x) = \exp\left[\mathrm{i}\boldsymbol{T}\cdot\boldsymbol{\alpha}\,(x)\right]$. The gauge invariant kinetic term for these vector fields is

$$\mathcal{L}_{B,Z,W} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}\boldsymbol{W}_{\mu\nu}\cdot\boldsymbol{W}^{\mu\nu}, \tag{6}$$

with $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \equiv \partial_{[\mu} B_{\nu]}$, $Z_{\mu\nu} = \partial_{[\mu} Z_{\nu]}$ and $\boldsymbol{W}_{\mu\nu} = \partial_{[\mu} \boldsymbol{W}_{\nu]} - g\, \boldsymbol{W}_\mu \times \boldsymbol{W}_\nu$. The field strength $\boldsymbol{T} \cdot \boldsymbol{W}_{\mu\nu}$ transforms covariantly under G transformations, $\boldsymbol{T} \cdot \boldsymbol{W}_{\mu\nu} \xrightarrow{G} U(x)\, \boldsymbol{T} \cdot \boldsymbol{W}_{\mu\nu}\, U^\dagger(x)$, but $B_{\mu\nu}$ and $Z_{\mu\nu}$ are invariant, hence a kinetic mixing term of the $U(1)$ fields is also allowed by gauge invariance:

$$-\frac{\epsilon}{2} B_{\mu\nu} Z^{\mu\nu}\,. \tag{7}$$

We can get rid of this mixing term by redefining the $U(1)$ fields using the transformation

$$\begin{pmatrix} B'_\mu \\ Z'_\mu \end{pmatrix} = \begin{pmatrix} 1 & \sin\theta_Z \\ 0 & \cos\theta_Z \end{pmatrix} \begin{pmatrix} B_\mu \\ Z_\mu \end{pmatrix} \qquad \sin\theta_Z = \epsilon\,. \tag{8}$$

In terms of the redefined fields, the covariant derivative becomes

$$D_j^\mu = \partial^\mu + \mathrm{i} g_\mathrm{L}\, \boldsymbol{T} \cdot \boldsymbol{W}^\mu + \mathrm{i} g_Y\, y_j B'^\mu + \mathrm{i}(g'_Z\, z_j - g'_Y\, y_j) Z'^\mu, \tag{9}$$

where $g'_Y = g_Y \tan\theta_Z = \epsilon g_Y + \mathrm{O}(\epsilon^3)$ and $g'_Z = g_Z / \cos\theta_Z = g_Z + \mathrm{O}(\epsilon^2)$. Thus, the effect of the kinetic mixing is to change the couplings of the matter fields to the vector field Z^μ. Note that we cannot immediately combine the coupling factor $(g'_Z\, z_j - g'_Y\, y_j)$ into a single product of a coupling and a charge. We shall discuss this issue further below.

Gauge symmetry forbids mass terms for gauge bosons. Fermion masses must also be absent because

$$m\, \bar\psi\psi = m\, \bar\psi_\mathrm{L} \psi_\mathrm{R} + m\, \bar\psi_\mathrm{R} \psi_\mathrm{L},$$

but the ψ_L, ψ_R fields transform differently under G. Thus, the G-invariant Lagrangian describes massless fields in contradiction to observation.

2.2. Scalar Sector

To solve the puzzle of missing masses, we proceed similarly as in the standard model, but, in addition to the usual Brout–Englert–Higgs (BEH) field ϕ, which is an $SU(2)_\mathrm{L}$-doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + \mathrm{i}\phi_2 \\ \phi_3 + \mathrm{i}\phi_4 \end{pmatrix}. \tag{10}$$

We also introduce another complex scalar χ that transforms as a singlet under G_SM transformations. The gauge invariant Lagrangian of the scalar fields is

$$\mathcal{L}_{\phi,\chi} = [D_{\phi\,\mu}\phi]^* D_\phi^\mu \phi + [D_{\chi\,\mu}\chi]^* D_\chi^\mu \chi - V(\phi,\chi), \tag{11}$$

where the covariant derivative for the scalar s ($s = \phi, \chi$) is

$$D_s^\mu = \partial^\mu + \mathrm{i} g_\mathrm{L}\, \boldsymbol{T} \cdot \boldsymbol{W}^\mu + \mathrm{i} g_Y\, y_s B'^\mu + \mathrm{i}(g'_Z\, z_s - g'_Y\, y_s) Z'^\mu \tag{12}$$

and the potential energy

$$V(\phi,\chi) = V_0 - \mu_\phi^2 |\phi|^2 - \mu_\chi^2 |\chi|^2 + \left(|\phi|^2, |\chi|^2\right) \begin{pmatrix} \lambda_\phi & \frac{\lambda}{2} \\ \frac{\lambda}{2} & \lambda_\chi \end{pmatrix} \begin{pmatrix} |\phi|^2 \\ |\chi|^2 \end{pmatrix}, \tag{13}$$

in addition to the usual quartic terms, introduces a coupling term $-\lambda|\phi|^2|\chi|^2$ of the scalar fields in the Lagrangian. For the doublet, $|\phi|$ denotes the length $\sqrt{|\phi^+|^2 + |\phi^0|^2}$. The value of the additive constant V_0 is irrelevant for particle dynamics but may be relevant for inflationary scenarios, hence we allow

for its non-vanishing value. In order for this potential energy to be bounded from below, we have to require the positivity of the self-couplings, $\lambda_\phi, \lambda_\chi > 0$. The eigenvalues of the coupling matrix are

$$\lambda_\pm = \frac{1}{2}\left(\lambda_\phi + \lambda_\chi \pm \sqrt{(\lambda_\phi - \lambda_\chi)^2 + \lambda^2}\right), \tag{14}$$

while the corresponding un-normalized eigenvectors are

$$u^{(+)} = \begin{pmatrix} \frac{2}{\lambda}(\lambda_+ - \lambda_\chi) \\ 1 \end{pmatrix} \quad \text{and} \quad u^{(-)} = \begin{pmatrix} \frac{2}{\lambda}(\lambda_- - \lambda_\chi) \\ 1 \end{pmatrix}. \tag{15}$$

As $\lambda_+ > 0$ and $\lambda_- < \lambda_+$, in the physical region, the potential can be unbounded from below only if $\lambda_- < 0$ and $u^{(-)}$ points into the first quadrant, which may occur only when $\lambda < 0$. In this case, to ensure that the potential is bounded from below, one also has to require that the coupling matrix be positive definite, which translates into the condition

$$4\lambda_\phi\lambda_\chi - \lambda^2 > 0\,. \tag{16}$$

With these conditions satisfied, we can find the minimum of the potential energy at field values $\phi = v/\sqrt{2}$ and $\chi = w/\sqrt{2}$ where the vacuum expectation values (VEVs) are

$$v = \sqrt{2}\sqrt{\frac{2\lambda_\chi\mu_\phi^2 - \lambda\mu_\chi^2}{4\lambda_\phi\lambda_\chi - \lambda^2}} \qquad w = \sqrt{2}\sqrt{\frac{2\lambda_\phi\mu_\chi^2 - \lambda\mu_\phi^2}{4\lambda_\phi\lambda_\chi - \lambda^2}}\,. \tag{17}$$

Using the VEVs, we can express the quadratic couplings as

$$\mu_\phi^2 = \lambda_\phi v^2 + \frac{\lambda}{2}w^2 \qquad \mu_\chi^2 = \lambda_\chi w^2 + \frac{\lambda}{2}v^2 \tag{18}$$

so those are both positive if $\lambda > 0$. If $\lambda < 0$, the constraint (16) ensures that the denominators of the VEVs in Equation (17) are positive, so the VEVs have non-vanishing real values only if

$$2\lambda_\chi\mu_\phi^2 - \lambda\mu_\chi^2 > 0 \quad \text{and} \quad 2\lambda_\phi\mu_\chi^2 - \lambda\mu_\phi^2 > 0 \tag{19}$$

simultaneously, which can be satisfied if at most one of the quadratic couplings is smaller than zero. We summarize the possible cases for the signs of the couplings in Table 2.

Table 2. Possible signs of the couplings in the scalar potential $V(\phi,\chi)$ in order to have two non-vanishing real VEVs. Θ is the step function, $\Theta(x) = 1$ if $x > 0$ and 0 if $x < 0$.

$\Theta(\lambda)$	$\Theta(\lambda_\phi)$	$\Theta(\lambda_\chi)$	$\Theta(4\lambda_\phi\lambda_\chi - \lambda^2)$	$\Theta(\mu_\phi^2)\,\Theta(\mu_\chi^2)$	$\Theta(2\lambda_\chi\mu_\phi^2 - \lambda\mu_\chi^2)\Theta(2\lambda_\phi\mu_\chi^2 - \lambda\mu_\phi^2)$
1	1	1	unconstrained	1	unconstrained
0	1	1	1	1	unconstrained
0	1	1	1	0	1

After spontaneous symmetry breaking of $G \to SU(3)_c \otimes U(1)_Q$, we use the following convenient parametrization for the scalar fields:

$$\phi = \frac{1}{\sqrt{2}}\,e^{i\boldsymbol{T}\cdot\boldsymbol{\xi}(x)/v}\begin{pmatrix} 0 \\ v + h'(x) \end{pmatrix} \quad \text{and} \quad \chi(x) = \frac{1}{\sqrt{2}}\,e^{i\eta(x)/w}\big(w + s'(x)\big)\,. \tag{20}$$

We can use the gauge invariance of the model to choose the unitary gauge when

$$\phi'(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h'(x) \end{pmatrix} \quad \text{and} \quad \chi'(x) = \frac{1}{\sqrt{2}}(w + s'(x)), \tag{21}$$

and the vector fields are transformed according to Equation (5). With this gauge choice, the scalar kinetic term contains quadratic terms of the gauge fields from which one can identify mass parameters of the massive standard model gauge bosons proportional to the vacuum expectation value v of the BEH field and also that of a massive vector boson Z'^{μ} proportional to w. We can diagonalize the mass matrix (quadratic terms) of the two real scalars (h' and s') by the rotation

$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta_S & -\sin\theta_S \\ \sin\theta_S & \cos\theta_S \end{pmatrix} \begin{pmatrix} h' \\ s' \end{pmatrix}, \tag{22}$$

where, for the scalar mixing angle $\theta_S \in (-\frac{\pi}{4}, \frac{\pi}{4})$, we find

$$\sin(2\theta_S) = -\frac{\lambda v w}{\sqrt{(\lambda_\phi v^2 - \lambda_\chi w^2)^2 + (\lambda v w)^2}}. \tag{23}$$

The masses of the mass eigenstates h and s are

$$M_{h/H} = \left(\lambda_\phi v^2 + \lambda_\chi w^2 \mp \sqrt{(\lambda_\phi v^2 - \lambda_\chi w^2)^2 + (\lambda v w)^2}\right)^{1/2}, \tag{24}$$

where $M_h \leq M_H$ by convention. At this point, either h or H can be the standard model Higgs boson. A more detailed analysis of this scalar sector but within a different $U(1)_Z$ model can be found in Ref. [15] and for the present model in Ref. [16].

2.3. Fermion Masses

We already discussed that explicit mass terms of fermions would break $SU(2)_{\mathrm{L}} \otimes U(1)_Y$ invariance. However, we can introduce gauge-invariant fermion-scalar Yukawa interactions (We distinguish the hypercharge Y from the index referring to Yukawa terms using different type of letters)

$$\mathcal{L}_{\mathrm{Y}} = -\left[c_D \bar{Q}_{\mathrm{L}} \cdot \phi \, D_{\mathrm{R}} + c_U \bar{Q}_{\mathrm{L}} \cdot \tilde{\phi} \, U_{\mathrm{R}} + c_\ell \bar{L}_{\mathrm{L}} \cdot \phi \, \ell_{\mathrm{R}}\right] + \text{h.c.}, \tag{25}$$

where h.c. means Hermitian conjugate terms and the parameters c_D, c_U, c_ℓ are called Yukawa couplings that are matrices in family indices and summation over the families is understood implicitly. The dot product abbreviates scalar products of $SU(2)$ doublets:

$$\bar{Q}_{\mathrm{L}} \cdot \phi \equiv (\bar{U}, \bar{D})_{\mathrm{L}} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} \quad \bar{Q}_{\mathrm{L}} \cdot \tilde{\phi} \equiv (\bar{U}, \bar{D})_{\mathrm{L}} \begin{pmatrix} \phi^{(0)\,*} \\ -\phi^{(+)\,*} \end{pmatrix} \tag{26}$$

and $\bar{L} \equiv (\bar{\nu}_\ell, \bar{\ell})$. The Z-charge of the BEH field is constrained by $U(1)_Z$ invariance of the Yukawa terms to $z_\phi = Z_2 - Z_1$, which works simultaneously for all three terms.

After spontaneous symmetry breaking and fixing the unitary gauge, this Yukawa Lagrangian becomes

$$\mathcal{L}_{\mathrm{Y}} = -\frac{1}{\sqrt{2}}(v + h(x))\left[c_D \, \bar{D}_{\mathrm{L}} D_{\mathrm{R}} + c_U \, \bar{U}_{\mathrm{L}} U_{\mathrm{R}} + c_\ell \, \bar{\ell}_{\mathrm{L}} \ell_{\mathrm{R}}\right] + \text{h.c.} \tag{27}$$

We see that there are mass terms with mass matrices $M_i = \frac{c_i v}{\sqrt{2}}$, where $i = D, U, \ell$:

$$\mathcal{L}_{\rm Y} = -\left(1 + \frac{h(x)}{v}\right)\left[\bar{D}_{\rm L}\, M_D\, D_{\rm R} + \bar{U}_{\rm L}\, M_U\, U_{\rm R} + \bar{\ell}_{\rm L}\, M_\ell\, \ell_{\rm R}\right] + \text{h.c.} \tag{28}$$

The general complex matrices M_i can be diagonalized employing bi-unitary transformations. The diagonal elements on the basis of mass eigenstates provide the mass parameters of the fermions. Due to the bi-unitary transformation, the left- and right-handed components of the fermion field are different linear combinations of the mass eigenstates.

The neutrino oscillation experiments suggest non-vanishing neutrino masses and the weak and mass eigenstates of the left-handed neutrinos do not coincide. In principle, the charge assignment of our model allows for the following gauge invariant Yukawa terms of dimension four operators for the neutrinos

$$\mathcal{L}^{\nu}_{\rm Y} = -\sum_{i,j}\left((c_\nu)_{ij}\bar{L}_{i,\rm L}\cdot\tilde{\phi}\,\nu_{j,\rm R} + \frac{1}{2}(c_{\rm R})_{ij}\,\overline{\nu^c_{i,\rm R}}\nu_{j,\rm R}\,\chi\right) + \text{h.c.} \tag{29}$$

for arbitrary values of Z_1 and Z_2 if the superscript c denotes the charge conjugate of the field, $\nu^c = -\mathrm{i}\gamma_2\nu^*$, and the Z-charge of the right-handed neutrinos and the new scalar satisfy the relation $z_\chi = -2z_{\nu_{\rm R}}$. There are two natural choices to fix the Z-charges: (i) the left- and right-handed neutrinos have the same charge; or (ii) those have opposite charges (We explain in Section 2.5 the reason for considering this choice being natural). In the first case, we have

$$Z_2 - 4Z_1 = -3Z_1, \tag{30}$$

which is solved by $Z_1 = Z_2$, and it leads to the charge assignment of the $U(1)_{B-L}$ extension of the standard model, studied in detail (see for instance, [17] and references therein). In the second case,

$$Z_2 - 4Z_1 = 3Z_1, \tag{31}$$

which is solved by $Z_1 = Z_2/7$. As the overall scale of the Z-charges depends only on the value of the gauge coupling g'_Z, we set Z_2 freely. For instance, choosing $Z_2 = 7/6$ implies $Z_1 = 1/6$ and the Z-charge of the BEH scalar is

$$z_\phi = 1, \tag{32}$$

while that of the new scalar is

$$z_\chi = -1 = -z_\phi\,. \tag{33}$$

While we cannot exclude the infinitely many cases when the magnitudes of Z-charges of the left- and right-handed neutrinos differ, we find it natural to assume that Equation (31) is valid. The corresponding Z-charges are given explicitly in the sixth column of Table 1.

After the spontaneous symmetry breaking of the vacuum of the scalar fields, Equation (29) leads to the following mass terms for the neutrinos:

$$\mathcal{L}^{\nu}_{\rm Y} = -\frac{1}{2}\sum_{i,j}\left[\left(\overline{\nu_{\rm L}},\,\overline{\nu^c_{\rm R}}\right)_i M(h,s)_{ij}\begin{pmatrix}\nu^c_{\rm L}\\ \nu_{\rm R}\end{pmatrix}_j + \text{h.c.}\right], \tag{34}$$

where

$$M(h,s)_{ij} = \begin{pmatrix} 0 & m_{\rm D}\left(1+\frac{h}{v}\right)\\ m_{\rm D}\left(1+\frac{h}{v}\right) & M_{\rm M}\left(1+\frac{s}{w}\right)\end{pmatrix}_{ij}, \tag{35}$$

with complex $m_{\rm D}$ and real $M_{\rm M}$ being symmetric 3×3 matrices, so $M(0,0)$ is a complex symmetric 6×6 matrix. The diagonal elements of the mass matrix $M(0,0)$ provide Majorana mass terms for the

left-handed and right-handed neutrinos. Thus, we conclude that the model predicts *vanishing masses of the left-handed neutrinos* at the fundamental level.

The off-diagonal elements represent interaction terms that look formally like Dirac mass terms, $-\sum_{i,j}\overline{\nu_{i,\mathrm{L}}}(m_\mathrm{D})_{ij}\nu_{j,\mathrm{R}}+$ h.c. After spontaneous symmetry breaking the quantum numbers of the particles $\nu^c_{i,\mathrm{L}}$ and $\nu_{i,\mathrm{R}}$ being identical, they can mix. Thus, the propagating states will be a mixture of the left- and right-handed neutrinos, providing effective masses for the left-handed ones. Those states can be obtained by the diagonalization of the full matrix $M(0,0)$, for which a possible parametrization is given for instance in Ref. [18].

In order to understand the structure of the matrix $M(0,0)$ better, we first diagonalize the matrices m_D and M_M separately by a unitary transformation and an orthogonal one. Defining

$$\nu'_{\mathrm{L},i}=\sum_j(U_\mathrm{L})_{ij}\nu_{\mathrm{L},j}\quad\text{and}\quad\nu'_{\mathrm{R},i}=\sum_j(O_\mathrm{R})_{ij}\nu_{\mathrm{R},j}, \tag{36}$$

we can rewrite the neutrino Yukawa Lagrangian as

$$\mathcal{L}^\nu_\mathrm{Y}=-\frac{1}{2}\sum_{i,j}\left[\left(\overline{\nu'_\mathrm{L}},\overline{\nu'^c_\mathrm{R}}\right)_i M'(h,s)_{ij}\begin{pmatrix}\nu'^c_\mathrm{L}\\ \nu'_\mathrm{R}\end{pmatrix}_j+\text{h.c.}\right], \tag{37}$$

where

$$M'(h,s)=\begin{pmatrix}0 & mV\left(1+\frac{h}{v}\right)\\ V^\dagger m\left(1+\frac{h}{v}\right) & M\left(1+\frac{s}{w}\right)\end{pmatrix}. \tag{38}$$

In Equation (38), m and M are real diagonal matrices, while $V=U_\mathrm{L}^TO_\mathrm{R}$ is a unitary matrix, $VV^\dagger=1$, so $M'(0,0)$ is Hermitian with real eigenvalues that are the masses of the mass eigenstates of neutrinos. In general, $M'(0,0)$ may have 15 independent parameters: m_i and M_i ($i=1,2,3$), while there are three Euler angles and six phases V. Three phases can be absorbed into the definition of ν'_L.

Assuming the hierarchy $m_i\ll M_j$, we can integrate out the right-handed (heavy) neutrinos and obtain an effective higher dimensional operator with Majorana mass terms for the left-handed neutrinos

$$\mathcal{L}^\nu_{\mathrm{dim}-5}=-\frac{1}{2}\sum_i m_{\mathrm{M},i}\left(1+\frac{h}{v}\right)^2\left(\overline{\nu'^c_{i,\mathrm{L}}}\nu'_{i,\mathrm{L}}+\text{h.c.}\right). \tag{39}$$

The Majorana masses $m_{\mathrm{M},i}$, i.e., eigenstates of the matrix $m_\mathrm{D}^\dagger M_\mathrm{M}^{-1}m_\mathrm{D}$, are suppressed by the ratios m_i/M_i as compared to m_i. The latter has a similar role in the Lagrangian as the mass parameters of the charged leptons, so one may assume $m_i\sim\mathrm{O}(100\,\mathrm{keV})$, while the masses of the right-handed neutrinos can be naturally around $\mathrm{O}(100\,\mathrm{GeV})$, so that $m_i/M_i\sim\mathrm{O}(10^{-6\pm1})$ and $m_{\mathrm{M},i}\lesssim 0.1\,\mathrm{eV}$. Thus, if $m_i\ll M_i$, then the mixing between the light and heavy neutrinos will be very small, the $\nu'_{i,\mathrm{L}}$ can be considered as the mass eigenstates that are mixtures of the left-handed weak eigenstates, and whose masses can be small naturally as suggested by phenomenological observations.

As we can only observe neutrinos together with their flavors through their charged current interactions, it is more natural to use the flavor eigenstates than the mass eigenstates. In the flavor basis, the couplings of the leptons to the W boson are diagonal:

$$\mathcal{L}^{(\ell)}_\mathrm{CC}=-\frac{g_\mathrm{L}}{\sqrt{2}}\sum_f\overline{\nu_\mathrm{L}}^f W^\dagger\ell^f_\mathrm{L}+\text{h.c.} \tag{40}$$

with summation over the three lepton flavors $f = e$, μ and τ. The same charged current interactions in mass basis $\nu_{\mathrm{L},i} = (U_{\mathrm{PMNS}})_{if}\nu_{\mathrm{L}}^{f}$, contain the Pontecorvo–Maki–Nakagawa–Sakata matrix U_{PMNS},

$$\mathcal{L}_{\mathrm{CC}}^{(\ell)} = -\frac{g_{\mathrm{L}}}{\sqrt{2}}\sum_{i,f=1}^{3}\overline{\nu_{\mathrm{L},i}}\,(U_{\mathrm{PMNS}})_{if}\,W^{+}\,\ell_{\mathrm{L}}^{f} + \mathrm{h.c.} \tag{41}$$

just like the charged current quark interactions contain the Cabibbo–Kobayashi–Maskawa matrix. If the heavy neutrinos are integrated out, then the matrix U_{L} coincides with the PMNS matrix. For propagating degrees of freedom, such as in the case of traveling neutrinos over macroscopic distances, one should use mass eigenstates $\nu_{\mathrm{L},i}$ and the PMNS matrix becomes the source of neutrino oscillations in flavor space. However, in the case of elementary particle scattering processes involving the left-handed neutrinos, one can work using the flavor basis, i.e., with Equation (40) because the effect of their masses can be neglected.

2.4. Re-Parametrization into Right-Handed and Mixed Couplings

Having set the Z-charges of the matter fields, we can re-parametrize the couplings to Z' using the new coupling

$$g'_{ZY} = g'_{Z} - g'_{Y} = \frac{g_Z - g_Y\sin\theta_Z}{\cos\theta_Z} = g_Z - \epsilon g_Y + \mathrm{O}(\epsilon^2), \tag{42}$$

with ϵ being the strength of kinetic mixing. Then, the covariant derivative in Equation (9) becomes

$$D_j^{\mu} = \partial^{\mu} + \mathrm{i}g_{\mathrm{L}}\,\boldsymbol{T}\cdot\boldsymbol{W}^{\mu} + \mathrm{i}\,y_j g_Y B'^{\mu} + \mathrm{i}\left(r_j g'_Z + y_j g'_{ZY}\right)Z'^{\mu}, \tag{43}$$

where $r_j = z_j - y_j$ and its values are given explicitly in the last column of Table 1. Thus, if a $U(1)_Z$ extension of G_{SM} is free of gauge and gravity anomalies and the Z-charges of left and right-handed fields are the opposite, then it is equivalent to a $U(1)_{\mathrm{R}}$ extension with tree-level mixed coupling g'_{ZY} [19], related to the kinetic mixing parameter ϵ by Equation (42).

Particle phenomenology of the standard model suggests that the interaction of the fermions through the Z' vector boson must be suppressed significantly. The origin of such a suppression can be either a small coupling to Z' or the large mass of Z'. Usual studies in the literature focus on the latter case. Here, we suggest to focus on the former possibility.

The complete Lagrangian is the sum of the pieces given in Equations (4), (6), (11), (25) and (29),

$$\mathcal{L} = \mathcal{L}_{\mathrm{D}} + \mathcal{L}_{B,Z,W} + \mathcal{L}_{\phi,\chi} + \mathcal{L}_{\mathrm{Y}} + \mathcal{L}_{\mathrm{Y}}^{\nu}, \tag{44}$$

with covariant derivative given in Equation (43), i.e., the kinetic mixing of Equation (7) is also taken into account.

2.5. Mixing in the Neutral Gauge Sector

The neutral gauge fields of the standard model and the Z' mix, which leads to mass eigenstates A_μ, Z_μ and T_μ (not to be confused with the isospin components T_i, $i = 1, 2, 3$). The mixing is described by a 3×3 mixing matrix as

$$\begin{pmatrix} W_\mu^3 \\ B'_\mu \\ Z'_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{\mathrm{W}}\cos\theta_T & -\cos\theta_{\mathrm{W}}\sin\theta_T & \sin\theta_{\mathrm{W}} \\ -\sin\theta_{\mathrm{W}}\cos\theta_T & \sin\theta_{\mathrm{W}}\sin\theta_T & \cos\theta_{\mathrm{W}} \\ \sin\theta_T & \cos\theta_T & 0 \end{pmatrix}\begin{pmatrix} Z_\mu \\ T_\mu \\ A_\mu \end{pmatrix}. \tag{45}$$

For the Weinberg mixing angle θ_W, we have the usual value $\sin\theta_W = g_Y/\sqrt{g_L^2+g_Y^2}$. We introduce the notion of reduced coupling defined by $\gamma_i = g_i/g_L$, i.e., $\gamma_L = 1$. Then, we have

$$\sin\theta_W = \frac{\gamma_Y}{\sqrt{1+\gamma_Y^2}}, \qquad \cos\theta_W = \frac{1}{\sqrt{1+\gamma_Y^2}} \tag{46}$$

and, for the mixing angle θ_T of the Z' boson, we find

$$\begin{aligned} \sin\theta_T &= \left[\frac{1}{2}\left(1 - \frac{1-\kappa^2-\tau^2}{\sqrt{(1+\kappa^2+\tau^2)^2-4\tau^2}}\right)\right]^{1/2}, \\ \cos\theta_T &= \left[\frac{1}{2}\left(1 + \frac{1-\kappa^2-\tau^2}{\sqrt{(1+\kappa^2+\tau^2)^2-4\tau^2}}\right)\right]^{1/2}, \end{aligned} \tag{47}$$

so $\tan(2\theta_T) = 2\kappa/(1-\kappa^2-\tau^2)$, with

$$\kappa = \frac{\gamma'_Y - 2\gamma'_Z}{\sqrt{1+\gamma_Y^2}} \quad \tau = 2\frac{\gamma'_Z\tan\beta}{\sqrt{1+\gamma_Y^2}} \tag{48}$$

and

$$\tan\beta = \frac{w}{v} \tag{49}$$

is the ratio of the scalar vacuum expectation values (not a scalar mixing angle). For small values of the new couplings γ'_{ZY} and γ'_Z, implying small κ, we have

$$\theta_T = \kappa + O(\tau^2, \kappa^3)\,. \tag{50}$$

The charged current interactions remain the same as in the standard model. The neutral current Lagrangian can be written in the form

$$\mathcal{L}_{NC} = \mathcal{L}_{QED} + \mathcal{L}_Z + \mathcal{L}_T, \tag{51}$$

where the first term is the usual Lagrangian of QED,

$$\mathcal{L}_{QED} = -eA_\mu J^\mu_{em} \quad J^\mu_{em} = \sum_{f=1}^{3}\sum_{j=1}^{3} e_j\left(\overline{\psi}^f_{q,j}(x)\gamma^\mu\psi^f_{q,j}(x) + \overline{\psi}^f_{l,j}(x)\gamma^\mu\psi^f_{l,j}(x)\right). \tag{52}$$

The second one is a neutral current coupled to the Z^0 boson,

$$\mathcal{L}_Z = -eZ_\mu\left(\cos\theta_T J^\mu_Z + \sin\theta_T J^\mu_T\right) = -eZ_\mu J^\mu_Z + O(\theta_T), \tag{53}$$

and the third one is the neutral current coupled to the T^0 boson,

$$\mathcal{L}_T = -eT_\mu\left(-\sin\theta_T J^\mu_Z + \cos\theta_T J^\mu_T\right). \tag{54}$$

In Equation (52), e is the electric charge unit and e_j is the electric charge of field ψ_j in units of e. In Equations (53) and (54), J^μ_Z is the usual neutral current,

$$J^\mu_Z = \sum_{f=1}^{3}\sum_{j=1}^{3}\frac{T_3 - \sin^2\theta_W\, e_j}{\sin\theta_W\cos\theta_W}\left(\overline{\psi}^f_{q,j}(x)\gamma^\mu\psi^f_{q,j}(x) + \overline{\psi}^f_{l,j}(x)\gamma^\mu\psi^f_{l,j}(x)\right), \tag{55}$$

while the new neutral current has the same dependence on fermion dynamics with different coupling strength:

$$J_T^\mu = \sum_{f=1}^{3}\sum_{j=1}^{3} \frac{\gamma'_Z r_j + \gamma'_{ZY} y_j}{\sin\theta_W} \left(\overline{\psi}^f_{q,j}(x)\gamma^\mu \psi^f_{q,j}(x) + \overline{\psi}^f_{l,j}(x)\gamma^\mu \psi^f_{l,j}(x)\right). \tag{56}$$

We can rewrite these currents as vector–axialvector currents using the non-chiral fields ψ_f

$$J_X^\mu = \sum_f \overline{\psi}_f(x)\gamma^\mu \left(v_f^{(X)} - a_f^{(X)}\gamma_5\right)\psi_f(x) \quad X = Z \text{ or } T \tag{57}$$

with vector couplings $v_f^{(X)}$ and axialvector couplings $a_f^{(X)}$ given in Appendix A and the summation runs over all quark and lepton flavors. Clearly, the QED current J^μ_{em} can also be written using non-chiral fields in the form of Equation (57) with $v_f^{(\text{em})} = e_f$ and $a_f^{(\text{em})} = 0$.

As the dependence on the couplings and charges of the neutral currents in Equations (55) and (56) are very different for different fermion fields, the only way that the standard model phenomenology is not violated by the extended model is if θ_T is small, which supports the expansion used in Equation (53). The choice for the Z-charges made in Equation (31) leads to the current J_T^μ being chiral, which we find natural as it mixes with the other chiral current J_Z^μ according to Equations (53) and (54).

To define the perturbation theory of this model explicitly, we present the Feynman rules in Appendix A.

2.6. Masses of the Gauge Bosons

The photon is massless, while the masses of the massive neutral bosons are

$$M_Z = M_W \frac{\cos\theta_T}{\cos\theta_W}\left[(1-\kappa\tan\theta_T)^2 + (\tau\tan\theta_T)^2\right]^{1/2} \tag{58}$$

and

$$M_T = M_W \frac{\sin\theta_T}{\cos\theta_W}\left[(1+\kappa\cot\theta_T)^2 + (\tau\cot\theta_T)^2\right]^{1/2}, \tag{59}$$

where $M_W = \frac{1}{2}v g_L$, and we assumed $M_T < M_Z$. Indeed, in order to have M_Z within the experimental uncertainty of the known measured value, we need $\theta_T \simeq 0$, which justifies the expansions at $\kappa = 0$,

$$M_Z = \frac{M_W}{\cos\theta_W}\left(1 + \mathrm{O}(\kappa^2)\right) \simeq \frac{M_W}{\cos\theta_W} \tag{60}$$

and

$$M_T = \frac{M_W}{\cos\theta_W}\tau\left(1 + \mathrm{O}(\kappa^2)\right) \simeq M_{Z'}, \tag{61}$$

where we used Equation (50) and $M_{Z'} = w g'_Z$. Thus, τ can also be written as the ratio of the masses of the two massive neutral gauge bosons,

$$\tau = \frac{M_{Z'}}{M_W}\cos\theta_W \simeq \frac{M_T}{M_Z}, \tag{62}$$

justifying our assumption on the hierarchy of masses. In fact, unless $w \gg v$, we find $M_T \ll M_Z$.

2.7. Free Parameters

There are five parameters in the scalar sector, λ_ϕ, λ_χ, λ, v and w that has to be determined experimentally, while the values of μ_ϕ and μ_χ (at tree level) are given in Equation (18). However, it is more convenient to use parameters that can be measured more directly, for instance,

$$M_h \;\; M_H \;\; \sin\theta_S \;\; v = (\sqrt{2}G_\mathrm{F})^{-1/2} \text{ and } \tan\beta, \tag{63}$$

of which we know two from measurements: one of the scalar masses and Fermi's constant.

In addition to the neutrino Yukawa couplings (or neutrino masses and PMNS mixing parameters), there are five free parameters in the model that we choose as the mass of the new scalar particle M_h or M_H (the other being fixed by the mass of the Higgs boson), the scalar and vector mixing angles $\sin\theta_S$ and $\sin\theta_T$, the ratio of the vacuum expectation values $\tan\beta$ and τ that is essentially the new gauge coupling. It can be shown [16] that, requiring stable vacuum up to the Planck scale, the Higgs particle coincides with the scalar h and according to a one-loop analysis of the running scalar couplings M_H falls into the range [144,558] GeV.

The other parameters can be expressed in terms of the free ones as follows: $w = v\tan\beta$,

$$\begin{aligned}
\lambda_\phi &= \frac{1}{2v^2}\left(M_{h/H}^2\cos^2\theta_S + M_{H/h}^2\sin^2\theta_S\right),\\
\lambda_\chi &= \frac{1}{2w^2}\left(M_{H/h}^2\cos^2\theta_S + M_{h/H}^2\sin^2\theta_S\right),\\
\lambda &= \sin(2\theta_S)\,\frac{M_H^2 - M_h^2}{2vw}
\end{aligned} \tag{64}$$

(first indices are to be used if $\lambda_\phi v^2 < \lambda_\chi w^2$, the second ones otherwise). The new parameters in the gauge sector can be expressed as

$$\begin{gathered}
\tan\theta_Z = \frac{\tau - \kappa\tan\beta}{\tan\beta\sin\theta_\mathrm{W}} \quad \gamma'_Z = \frac{\tau}{2\tan\beta\cos\theta_\mathrm{W}} \quad \gamma'_Y = \frac{\tau - \kappa\tan\beta}{\tan\beta\cos\theta_\mathrm{W}} \quad \gamma'_{ZY} = \frac{2\kappa\tan\beta - \tau}{2\tan\beta\cos\theta_\mathrm{W}},\\
\kappa = \cot(2\theta_T)\left(\sqrt{1 + (1-\tau^2)\tan^2(2\theta_T)} - 1\right) = (1-\tau^2)\sin\theta_T + \mathrm{O}(\theta_T^3)\,.
\end{gathered} \tag{65}$$

3. Discussion

Our hope in devising this model is to explain the established experimental observations listed in the introduction. We envisage the following scenario:

- The lightest new particle is a natural candidate for WIMP dark matter if it is sufficiently stable.
- Majorana neutrino mass terms for the right-handed neutrinos and Yukawa interactions between the left- and right-handed neutrinos and the BEH vacuum are generated by the spontaneous symmetry breaking of the scalar fields as outlined in Section 2.3. This scenario provides a possible origin of neutrino oscillations and effective Majorana mass terms for the left-handed neutrinos.
- The neutrino Yukawa terms provide a source for the PMNS matrix as shown in Section 2.3, which can have a CP-violating phase yielding stronger CP violation in the lepton sector than there is in the quark sector.
- The vacuum of the χ scalar has a charge $z_j = -1$ (or $r_j = -1$) that may be a source of the current accelerated expansion of the universe.
- The second scalar together with the established BEH field can cause hybrid inflation.

At present, we consider these possible consequences of the model that need further studies to find out if they fulfill. Before exploring that the model makes these explanations credible, we have to find answer to the following question: *Is there any region of the parameter space of the model that is not*

excluded by experimental results, both established in standard model phenomenology and elsewhere? Of course, answering this question requires studies well beyond the scope of a single article.

4. Conclusions

In this paper, we collected the well established experimental observations that cannot be explained by the standard model of particle interactions. We have then proposed an anomaly free extension by a $U(1)_Z$ gauge group, which is the simplest possible model. We also assumed the existence of a new complex scalar field with Z-charge only (i.e., neutral with respect to the standard model interactions) and three right-handed neutrinos. In order to fix the Z-charges of the particle spectrum, we assumed that the left- and right-handed neutrinos have opposite Z-charges. Thus, such a model predicts the existence of (i) a massive neutral vector boson; (ii) a massive scalar particle and (iii) three massive right-handed neutrinos. The left-handed neutrinos remain massless as in the standard model, but their Yukawa interactions with the BEH field and the right-handed neutrinos provide a field theoretical basis for explaining neutrino oscillations and predict effective Majorana masses for the propagating mass eigenstates.

We have discussed how the new neutral gauge field Z^μ mixes with those of the standard model (B^μ and W_3^μ) and argued that the mixing results in a new vector boson T^0 of a small mass related to the small new gauge coupling and small mixing with the standard model vector fields. We also presented the Feynman rules of the model in unitary gauge and collected the new free parameters.

In order for the predictions of the model be credible, we have to answer whether there is any region of the parameter space that is not excluded by experimental results established in standard model phenomenology or elsewhere. To answer such a question with satisfaction, studies well beyond the scope of a single article are needed, which forecasts an exciting research project.

Funding: This work was supported by grant K 125105 of the National Research, Development and Innovation Fund in Hungary.

Acknowledgments: I am grateful to G. Cynolter, D. Horváth, S. Iwamoto, A. Kardos and S. Katz for their constructive criticism on the manuscript.

Conflicts of Interest: The author declares no conflict of interest.

Abbreviations

BEH	Brout–Englert–Higgs
PMNS	Pontecorvo–Maki–Nakagawa–Sakata
QCD	quantum chromodynamics
QED	quantum electrodynamics
SM	standard model
SSB	spontaneous symmetry breaking
UV	ultra-violet
VEV	vacuum expectation value

Appendix A. Feynman Rules

The Feynman rules of the model are obtained from the complete Lagrangian in Equation (44). For studying the UV behaviour of the model, it is convenient to use the Feynman rules before SSB, while for low energy phenomenology the rules after SSB are needed. In this paper, we present only the latter in a unitary gauge. The propagators of the new fields are related trivially to those of the standard fields. Thus, we present only the vertices, neglecting the rules related to QCD, which are unchanged.

- Gauge field–fermion interactions $V_\alpha \bar{f}_i f_j$: $-ie\gamma_\alpha(C^- P_- + C^+ P_+)$, where $C^\pm$ depends on the type of the gauge boson participating in the interaction, the flavor f of fermions and family number i and j as follows:

$V\bar{f}_i f_j$	C^+	C^-
$\gamma\bar{f}_i f_j$	$e_f\delta_{ij}$	$e_f\delta_{ij}$
$Z\bar{f}_i f_j$	$(g_f^+\cos\theta_T + h_f^+\sin\theta_T)\delta_{ij}$	$(g_f^-\cos\theta_T + h_f^-\sin\theta_T)\delta_{ij}$
$T\bar{f}_i f_j$	$(-g_f^+\sin\theta_T + h_f^+\cos\theta_T)\delta_{ij}$	$(-g_f^-\sin\theta_T + h_f^-\cos\theta_T)\delta_{ij}$
$W^+\bar{u}_i d_j$	0	$\frac{1}{\sqrt{2}\sin\theta_W}V_{ij}$
$W^-\bar{d}_j u_i$	0	$\frac{1}{\sqrt{2}\sin\theta_W}V_{ij}^\dagger$
$W^+\bar{\nu}_i \ell_j$	0	$\frac{1}{\sqrt{2}\sin\theta_W}\delta_{ij}$
$W^-\bar{\ell}_j \nu_i$	0	$\frac{1}{\sqrt{2}\sin\theta_W}\delta_{ij}$,

where

$$g_f^+ = -\frac{\sin\theta_W}{\cos\theta_W}e_f \quad g_f^- = \frac{T_f^3 - \sin\theta_W^2 e_f}{\sin\theta_W\cos\theta_W} \quad h_f^\pm = \frac{\gamma'_Z R_f^\pm + \gamma'_{ZY}(e_f - R_f^\mp)}{\sin\theta_W}, \tag{A1}$$

where $R_f^+ = 1/2$ for U_f or ν_f, $R_f^+ = -1/2$ for D_f or ℓ_f and $R_f^- = 0$. The vector and axial vector couplings of the Z^0 boson read as

$$\begin{aligned} v_f^{(Z)} &= \frac{1}{2}\left(g_f^- + g_f^+\right)\cos\theta_T + \frac{1}{2}\left(h_f^- + h_f^+\right)\sin\theta_T \\ &= \frac{\left(T_f^3 - 2(\sin\theta_W)^2 e_f\right)\cos\theta_T + \left(\kappa e_f + \gamma'_Y(R_f^+ - e_f)\cos\theta_W\right)\sin\theta_T}{2\sin\theta_W\cos\theta_W} \\ &= \frac{T_f^3 - 2(\sin\theta_W)^2 e_f}{2\sin\theta_W\cos\theta_W} + \mathrm{O}(\theta_T), \\ a_f^{(Z)} &= \frac{1}{2}\left(g_f^- - g_f^+\right)\cos\theta_T + \frac{1}{2}\left(h_f^- - h_f^+\right)\sin\theta_T = \frac{T_f^3\cos\theta_T - \kappa R_f^+\sin\theta_T}{2\sin\theta_W\cos\theta_W} \\ &= \frac{T_f^3}{2\sin\theta_W\cos\theta_W} + \mathrm{O}(\theta_T), \end{aligned}$$

while those of the T^0 boson are

$$\begin{aligned} v_f^{(T)} &= \frac{\left(\kappa e_f + \gamma'_Y(R_f^+ - e_f)\cos\theta_W\right)\cos\theta_T - \left(T_f^3 - 2(\sin\theta_W)^2 e_f\right)\sin\theta_T}{2\sin\theta_W\cos\theta_W}, \\ a_f^{(T)} &= -\frac{\kappa R_f^+\cos\theta_T + T_f^3\sin\theta_T}{2\sin\theta_W\cos\theta_W}. \end{aligned} \tag{A2}$$

- $H\bar{f}_i f_j$ vertex: ieC, where

$$C = -\delta_{ij}\frac{1}{2\sin\theta_W}\frac{m_{f,i}}{M_W}.$$

- $S\overline{\nu^c}_{\mathrm{R},i}\nu_{\mathrm{R},j}$ vertex: ieC, where
$$C = -\delta_{ij}\frac{1}{2\sin\theta_{\mathrm{W}}\tan\beta}\frac{m_{\nu_{\mathrm{R}},i}}{M_W}\,.$$
- Gauge field interactions:
  - The cubic gauge field interactions of fields $V_{1,\alpha}$, $V_{2,\beta}$ and $V_{3,\gamma}$ with all-incoming kinematics, $p^\mu + q^\mu + r^\mu = 0$ are $\Gamma_{\alpha,\beta,\gamma}(p,q,r) = \mathrm{i}eCV_{\alpha,\beta,\gamma}(p,q,r)$, where
$$V_{\alpha,\beta,\gamma}(p,q,r) = (p-q)_\gamma g_{\alpha\beta} + (q-r)_\alpha g_{\beta\gamma} + (r-p)_\beta g_{\alpha\gamma},$$
  while C depends on the type of the gauge bosons participating in the interaction as follows:

$V_1V_2V_3$	C
γW^+W^-	1
ZW^+W^-	$\frac{\cos\theta_{\mathrm{W}}}{\sin\theta_{\mathrm{W}}}\cos\theta_T$
TW^+W^-	$-\frac{\cos\theta_{\mathrm{W}}}{\sin\theta_{\mathrm{W}}}\sin\theta_T$

  - The quartic gauge field interactions of fields $V_{1,\alpha}$, $V_{2,\beta}$, $V_{3,\gamma}$ and $V_{4,\delta}$ are $\Gamma_{\alpha,\beta,\gamma,\delta} = \mathrm{i}e^2C\left[2g_{\alpha\beta}g_{\gamma\delta} - g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}\right]$, where C again depends on the type of the gauge bosons participating in the interaction as follows:

$V_1V_2V_3V_4$	C
$W^+W^-\gamma\gamma$	-1
$W^+W^-\gamma Z$	$-\frac{\cos\theta_{\mathrm{W}}}{\sin\theta_{\mathrm{W}}}\cos\theta_T$
$W^+W^-\gamma T$	$\frac{\cos\theta_{\mathrm{W}}}{\sin\theta_{\mathrm{W}}}\sin\theta_T$
W^+W^-ZZ	$-\left(\frac{\cos\theta_{\mathrm{W}}}{\sin\theta_{\mathrm{W}}}\cos\theta_T\right)^2$
W^+W^-TZ	$\left(\frac{\cos\theta_{\mathrm{W}}}{\sin\theta_{\mathrm{W}}}\right)^2\cos\theta_T\sin\theta_T$
W^+W^-TT	$-\left(\frac{\cos\theta_{\mathrm{W}}}{\sin\theta_{\mathrm{W}}}\sin\theta_T\right)^2$
$W^+W^+W^-W^-$	$\frac{1}{(\sin\theta_{\mathrm{W}})^2}$

- Scalar interactions: We denote the standard model Higgs boson by $\mathcal{H}$, while the new one by $\mathcal{S}$.
  - Cubic scalar interactions can be either of the form i$e\frac{C}{3!}S^3$ where C depends on the type of the scalar boson participating in the interaction:

SSS	C
$\mathcal{HHH}$	$-\frac{3}{2}\frac{M_h^2\cos^2\theta_S + M_H^2\sin^2\theta_S}{\sin\theta_{\mathrm{W}}M_W}$
$\mathcal{SSS}$	$-\frac{3}{2}\frac{M_h^2\sin^2\theta_S + M_H^2\cos^2\theta_S}{\sin\theta_{\mathrm{W}}M_W\tan\beta}$

or of the form $ie\frac{C}{2!}SSS'$, where C depends on the type of the S boson participating in the interaction:

SSS'	C
$\mathcal{HHS}$	$-\sin\theta_S\cos\theta_S\dfrac{M_H^2-M_h^2}{2\sin\theta_W M_W}$
$\mathcal{SSH}$	$-\sin\theta_S\cos\theta_S\dfrac{M_H^2-M_h^2}{2\sin\theta_W M_W\tan\beta}$.

Recall that $M_{H/h}$ is the mass of the heavier/lighter scalar.

– The quartic scalar interactions are either of the form $ie^2\frac{C}{4!}S^4$, where C depends on the type of the scalar bosons participating in the interaction as follows:

$SSSS$	C
$\mathcal{HHHH}$	$-\dfrac{3}{4}\dfrac{M_h^2\cos^2\theta_S+M_H^2\sin^2\theta_S}{(\sin\theta_W M_W)^2}$
$\mathcal{SSSS}$	$-\dfrac{3}{4}\dfrac{M_h^2\sin^2\theta_S+M_h^2\cos^2\theta_S}{(\sin\theta_W M_W\tan\beta)^2}$

or of the form $ie^2\frac{C}{2!2!}\mathcal{H}^2\mathcal{S}^2$, where

$$C=-\frac{3}{4}\frac{M_h^2-M_h^2}{(\sin\theta_W M_W)^2\tan\beta}\,.$$

- Mixed gauge field-scalar interactions:

  – The cubic gauge field-scalar interactions of fields $V_{1,\alpha}$, $V_{2,\beta}$ and S are $ig_{\alpha\beta}C$, where C depends on the types of the fields participating in the interaction as follows:

V_1V_2S	C
$W^+W^-\mathcal{H}$	$\dfrac{M_W}{\sin\theta_W}$
$ZZ\mathcal{H}$	$\dfrac{M_W}{\sin\theta_W}\dfrac{(\cos\theta_T-\kappa\sin\theta_T)^2}{(\cos\theta_W)^2}$
$TT\mathcal{H}$	$\dfrac{M_W}{\sin\theta_W}\dfrac{(\sin\theta_T+\kappa\cos\theta_T)^2}{(\cos\theta_W)^2}$
$TZ\mathcal{H}$	$\dfrac{M_W}{\sin\theta_W}\dfrac{(\sin\theta_T+\kappa\cos\theta_T)(\kappa\sin\theta_T-\cos\theta_T)}{(\cos\theta_W)^2}$
$ZZ\mathcal{S}$	$\dfrac{M_W}{\sin\theta_W\tan\beta}\dfrac{(\tau\sin\theta_T)^2}{(\cos\theta_W)^2}$
$TT\mathcal{S}$	$\dfrac{M_W}{\sin\theta_W\tan\beta}\dfrac{(\tau\cos\theta_T)^2}{(\cos\theta_W)^2}$
$TZ\mathcal{S}$	$\dfrac{M_W}{\sin\theta_W}\dfrac{\tau^2\sin\theta_T\cos\theta_T}{(\cos\theta_W)^2}$.

– Quartic gauge field-scalar interactions $V_\alpha V_\beta SS$: $ie^2 g_{\alpha\beta} C$, where C depends on the type of the gauge boson participating in the interaction as follows:

V_1V_2SS	C
$W^+W^-\mathcal{H}\mathcal{H}$	$\frac{1}{2(\sin\theta_W)^2}$
$ZZ\mathcal{H}\mathcal{H}$	$\frac{(\cos\theta_T - \kappa\sin\theta_T)^2}{2(\cos\theta_W\sin\theta_W)^2}$
$TT\mathcal{H}\mathcal{H}$	$\frac{(\sin\theta_T + \kappa\cos\theta_T)^2}{2(\cos\theta_W\sin\theta_W)^2}$
$TZ\mathcal{H}\mathcal{H}$	$\frac{(\sin\theta_T + \kappa\cos\theta_T)(\kappa\sin\theta_T - \cos\theta_T)}{2(\cos\theta_W\sin\theta_W)^2}$
$ZZ\mathcal{S}\mathcal{S}$	$\frac{(\tau\sin\theta_T)^2}{2(\cos\theta_W\sin\theta_W\tan\beta)^2}$
$TT\mathcal{S}\mathcal{S}$	$\frac{(\tau\cos\theta_T)^2}{2(\cos\theta_W\sin\theta_W\tan\beta)^2}$
$TZ\mathcal{S}\mathcal{S}$	$\frac{\tau^2\sin\theta_T\cos\theta_T}{2(\cos\theta_W\sin\theta_W\tan\beta)^2}$

References

1. Weinberg, S. A Model of Leptons. *Phys. Rev. Lett.* **1967**, *19*, 1264–1266, doi:10.1103/PhysRevLett.19.1264. [CrossRef]
2. Aad, G.; Abajyan, T.; Abbott, B.; Abdallah, J.; Khalek, S.A.; Abdelalim, A.A.; Abdinov, O.; Aben, R.; Abi, B.; Abolins, M.; et al. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. *Phys. Lett.* **2012**, *B716*, 1–29, doi:10.1016/j.physletb.2012.08.020. [CrossRef]
3. Chatrchyan, S.; Khachatryan, V.; Sirunyan, A.M.; Tumasyan, A.; Adam, W.; Aguilo, E.; Bergauer, T.; Dragicevic, M.; Erö, J.; Fabjan, C.; et al. Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC. *Phys. Lett.* **2012**, *B716*, 30–61, doi:10.1016/j.physletb.2012.08.021. [CrossRef]
4. Tanabashi, M.; Hagiwara, K.; Hikasa, K.; Nakamura, K.; Sumino, Y.; Takahashi, F.; Tanaka, J.; Agashe, K.; Aielli, G.; Amsler, C.; et al. Review of Particle Physics. *Phys. Rev.* **2018**, *D98*, 030001, doi:10.1103/PhysRevD.98.030001. [CrossRef]
5. Standard Model Physics. 2019. Available online: https://twiki.cern.ch/twiki/bin/view/AtlasPublic/StandardModelPublicResults (accessed on 5 January 2020).
6. Summaries of CMS Cross Section Measurements. 2019. Available online: https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsCombined (accessed on 5 January 2020).
7. Bezrukov, F.; Shaposhnikov, M. Standard Model Higgs boson mass from inflation: Two loop analysis. *J. High Energy Phys.* **2009**, *7*, 89, doi:10.1088/1126-6708/2009/07/089. [CrossRef]
8. Degrassi, G.; Di Vita, S.; Elias-Miro, J.; Espinosa, J.R.; Giudice, G.F.; Isidori, G.; Strumia, A. Higgs mass and vacuum stability in the Standard Model at NNLO. *J. High Energy Phys.* **2012**, *8*, 98, doi:10.1007/JHEP08(2012)098. [CrossRef]
9. Englert, F.; Brout, R. Broken Symmetry and the Mass of Gauge Vector Mesons. *Phys. Rev. Lett.* **1964**, *13*, 321–323, doi:10.1103/PhysRevLett.13.321. [CrossRef]
10. Higgs, P.W. Broken Symmetries and the Masses of Gauge Bosons. *Phys. Rev. Lett.* **1964**, *13*, 508–509, doi:10.1103/PhysRevLett.13.508. [CrossRef]
11. Schabinger, R.M.; Wells, J.D. A Minimal spontaneously broken hidden sector and its impact on Higgs boson physics at the large hadron collider. *Phys. Rev.* **2005**, *D72*, 093007, doi:10.1103/PhysRevD.72.093007. [CrossRef]
12. Pospelov, M.; Ritz, A.; Voloshin, M.B. Secluded WIMP Dark Matter. *Phys. Lett.* **2008**, *B662*, 53–61, doi:10.1016/j.physletb.2008.02.052. [CrossRef]

13. Basso, L.; Belyaev, A.; Moretti, S.; Shepherd-Themistocleous, C.H. Phenomenology of the minimal B-L extension of the Standard model: Z′ and neutrinos. *Phys. Rev.* **2009**, *D80*, 055030, doi:10.1103/PhysRevD.80.055030. [CrossRef]
14. Appelquist, T.; Dobrescu, B.A.; Hopper, A.R. Nonexotic neutral gauge bosons. *Phys. Rev.* **2003**, *D68*, 035012, doi:10.1103/PhysRevD.68.035012. [CrossRef]
15. Duch, M.; Grzadkowski, B.; McGarrie, M. A stable Higgs portal with vector dark matter. *J. High Energy Phys.* **2015**, *9*, 162, doi:10.1007/JHEP09(2015)162. [CrossRef]
16. Péli, Z.; Trócsányi, Z. Stability of the vacuum as constraint on $U(1)$ extensions of the standard model. *arXiv* **2019**, arXiv:1902.02791.
17. Basso, L. Phenomenology of the Minimal B-L Extension of the Standard Model at the LHC. Ph.D. Thesis, University of Southampton, Southampton, UK, 2011.
18. Blennow, M.; Fernandez-Martinez, E. Parametrization of Seesaw Models and Light Sterile Neutrinos. *Phys. Lett.* **2011**, *B704*, 223–229, doi:10.1016/j.physletb.2011.09.028. [CrossRef]
19. Del Aguila, F.; Masip, M.; Perez-Victoria, M. Physical parameters and renormalization of U(1)-a x U(1)-b models. *Nucl. Phys.* **1995**, *B456*, 531–549, doi:10.1016/0550-3213(95)00511-6. [CrossRef]