

# Fuzzy Analytical Hierarchy Process Method

A descriptive adopted Fuzzy Analytical Hierarchy Process approach steps are written in the following headings. Authors adopted these steps to perform prioritization for the various models that were selected as the alternatives.

**Triangular Fuzzy Numbers:** Many lingual data were transformed into perceptible data in their respective Triangular Fuzzy Numbers (TFN). Triangular fuzzy numbers were used to restrict the vagueness/ambiguity of the parameter. M on F is fuzzy numbers and they are called TFN, membership equation of M and F is given in equations (1-2):

$$\mu_a(x) = F \rightarrow [0,1] \quad (1)$$

$$\mu_a(x) = \begin{cases} \frac{x}{mi-lo} - \frac{b}{mi-lo} & x \in [lo, mi] \\ \frac{x}{mi-up} - \frac{u}{mi-up} & x \in [mi, up] \\ 0 & \text{Otherwise} \end{cases} \quad (2)$$

Here, *lo*, *mi*, and *up* represent the *minor*, *average*, and *highest limit*, respectively, in the triangular membership function. Equations (3-5) help to aggregate TFN values.

Let us assume, M1 and M2 are two TFN numbers, M1= (lo1, mi1, up1) and M2= (lo2, mi2, up2).

The entire equation can be depicted as:

$$(lo_1, mi_1, up_1) + (lo_2, mi_2, up_2) = (lo_1 + lo_2, mi_1 + mi_2, up_1 + up_2) \quad (3)$$

$$(lo_1, mi_1, up_1) \times (lo_2, mi_2, up_2) = (lo_1 \times lo_2, mi_1 \times mi_2, up_1 \times up_2) \quad (4)$$

$$(lo_1, mi_1, up_1)^{-1} = (\frac{1}{up_1}, \frac{1}{mi_1}, \frac{1}{lo_1}) \quad (5)$$

Alpha cut method was adopted for defuzzification of fuzzified weights in this work. Function of alpha cut method has been shown in following mathematical statement (6-8).

$$\gamma\alpha, \beta(\eta_{ij}) = [\beta. \eta\alpha(lo_{ij}) + (1 - \beta). \eta\alpha(up_{ij})] \quad (6)$$

where  $0 \leq \alpha \leq 1$  and  $0 \leq \beta \leq 1$   
such that,

$$\eta\alpha(lo_{ij}) = (m_{ij} - lo_{ij}).\alpha + lo_{ij} \quad (7)$$

$$\eta\alpha(up_{ij}) = up_{ij} - (up_{ij} - m_{ij}).\alpha \quad (8)$$

For abolishing the uncertainty and ambiguities that arise due to the differing opinions of the practitioners and academicians, authors have used a hybrid version of AHP (also known as fuzzy AHP) which includes the fuzzy set theory with AHP method in this paper to evaluate the malware analysis approach.

**Analytic Hierarchy Process:** It is one of the most important techniques for assessment with multi criteria having multiple levels. It also helps in making decisions in case of multiple conflicting criteria [7]. In daily life, the multiple criteria problems can be solved by using AHP such as a selection of one criterion from different criteria.

For assessment, firstly, methods or attributes of the concerned problem should be identified. Secondly, a hierarchy of these methods should be created. The next step is to prepare a pair-wise comparison matrix that helps a person in taking decisions easily. The input came up with pair-wise comparisons to produce the decision matrix. Number *a<sub>ij</sub>* demonstrates the corresponding significance of standard *i* (*A<sub>i</sub>*) in comparison with standard *j* (*A<sub>j</sub>*). Saaty proposed a decision matrix that is used in AHP technique through pair-wise comparisons [8], as shown in equation (9).

$$A = [a_{ij}] = \begin{matrix} & \begin{matrix} L_1 & L_2 & \dots & L_n \end{matrix} \\ \begin{matrix} L_1 \\ L_2 \\ \vdots \\ L_n \end{matrix} & \begin{bmatrix} 1 & a_{11} & \dots & a_{1n} \\ 1/a_{21} & 1 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/a_{n1} & 1/a_{n2} & \dots & 1 \end{bmatrix} \end{matrix} \quad (9)$$

Here,  $i = 1, 2, 3, \dots, n$  and  $j = 1, 2, 3, \dots, n$  and  $a_{ij} = 1$  when  $i=j$ ; and  $a_{ij}=1/a_{ji}$  when  $i \neq j$

Where  $[a_{ij}]$  denotes the corresponding significance of two criteria,  $L_i$  and  $L_j$ . The linguistic scale for membership functions (1 to 9) is given in the table s1.

Table s1: Linguistic Variable and Corresponding Numeric Value

S.No.	Linguistic Values	Numeric Values	Fuzzified Numbers (TFNs)	1/
1	Equal Important (Eq)	1	(1,1,1)	(1,1,1)
2	Intermediate Value between Equal and Weakly (E & W)	2	(1,2,3)	(1/3,1/2,1)
3	Weakly Important (WI)	3	(2,3,4)	(1/4,1/3,1/2)
4	Intermediate Value between Weakly and Essential (W & E)	4	(3,4,5)	(1/5,1/4,1/3)
5	Essential Important (EI)	5	(4,5,6)	(1/6,1/5,1/4)
6	Intermediate Value between Essential and Very Strongly (E & VS)	6	(5,6,7)	(1/7,1/6,1/5)
7	Very Strongly Important (VS)	7	(6,7,8)	(1/8,1/7,1/6)
8	Intermediate Value between Very Strongly and Extremely (VS & ES)	8	(7,8,9)	(1/9,1/8,1/7)
9	Extremely Important (ES)	9	(7,9,9)	(1/9,1/9,1/7)

Table s1 describes the linguistic rate and defines the corresponding numeric values. After constructing the pair-wise matrix of the expert input, Consistency Ratio (CR) is calculated to curb the outcome of the AHP technique. CR is calculated with the help of equations(10-11).

$$CR = \frac{CI}{RI} \quad (10)$$

Where, Consistency Index (CI) is calculated from the equation (11).

$$CI = \frac{\lambda}{(n-1)} \quad (11)$$

Here, table s2 shows the amount of total responses (n) and Random Index (RI).

Table s2: Random Index

N	1	2	3	4	5	6	7	8	9
Random Index (RI)	0.00	0.00	0.58	0.90	1.12	1.24	1.35	1.41	1.49

If CR is less than 0.1, then the weight of each input is calculated. The refined pair-wise matrices are arranged and, if CR is greater than or equal to 0, the process is repeated. Aggregating the pair-wise comparison matrix is done from equation (12); thereafter, the CR value is verified.

$$m_{ij} = (B_{ij1} \cdot B_{ij2} \dots \dots B_{ijk})1/k \quad (12)$$

Here,  $B_{ijk}$  shows the decision of expert's  $k$  for the significance of two criteria, i.e.,  $L_i$  and  $L_j$ . The aggregated pair-wise comparison matrix is shown in equation 13.

$$\rho_{\alpha,\beta}(\tilde{A}) = \rho_{\alpha,\beta}[\tilde{a}_{ij}] = \begin{matrix} & L1 & L2 & \dots\dots\dots L_n \\ \begin{matrix} L1 \\ L2 \\ \vdots \\ L_n \end{matrix} & \begin{bmatrix} 1 & \rho_{\alpha,\beta}(\tilde{a}_{11}) \dots\dots & \rho_{\alpha,\beta}(\tilde{a}_{1i}) \\ 1/\rho_{\alpha,\beta}(\tilde{a}_{21}) & 1 \dots\dots & \rho_{\alpha,\beta}(\tilde{a}_{2i}) \\ \vdots & \vdots & \vdots \\ 1/\rho_{\alpha,\beta}(\tilde{a}_{j1}) & 1/\rho_{\alpha,\beta}(\tilde{a}_{j2}) \dots\dots & 1 \end{bmatrix} \end{matrix} \quad (13)$$

After aggregating the pair-wise comparison matrix, CR is calculated and verified, again. Thereafter, the next step is to determine the eigenvalue and eigenvector of the pair-wise comparison matrix. Estimation of the eigenvector is significant for determining the aggregated weightage of specific norm. Let us assume that  $W$  represents the *eigenvector*,  $I$  represents *unitary matrix*, and  $\lambda$  represents the *eigenvalue* of pair-wise comparison matrix  $\tilde{A}$  or  $[\tilde{a}_{ij}]$ , then the equation is.

$$[(\rho_{\alpha,\beta} \times \tilde{A}) - \lambda \times I] \cdot W = 0 \quad (14)$$

Where  $\tilde{A}$  is a matrix containing numeric value of the  $(\tilde{A})$ . Equation (14) is positioned on the linear metamorphosis of vectors. The eigenvectors is calculated as in equation (15).

$$[(\rho_{\alpha,\beta} \times \tilde{A}) - \lambda \times I] \cdot W = \begin{bmatrix} 1 & \rho_{\alpha,\beta}(\tilde{a}_{11}) \dots\dots & \rho_{\alpha,\beta}(\tilde{a}_{1i}) \\ 1/\rho_{\alpha,\beta}(\tilde{a}_{21}) & 1 \dots\dots & \rho_{\alpha,\beta}(\tilde{a}_{2i}) \\ \vdots & \vdots & \vdots \\ 1/\rho_{\alpha,\beta}(\tilde{a}_{j1}) & 1/\rho_{\alpha,\beta}(\tilde{a}_{j2}) \dots\dots & 1 \end{bmatrix} \quad (15)$$

For producing an identity matrix that cancels out each other, the practitioners need to multiply the eigenvalue  $\lambda$  with unitary matrix  $I$ . For that reason, the notation  $\lambda I$  is neglected in this case. Employing equations (14-15) results are shown in equation (16).

$$\begin{bmatrix} 1 & \rho_{\alpha,\beta}(\tilde{a}_{11}) \dots\dots & \rho_{\alpha,\beta}(\tilde{a}_{1i}) \\ 1/\rho_{\alpha,\beta}(\tilde{a}_{21}) & 1 \dots\dots & \rho_{\alpha,\beta}(\tilde{a}_{2i}) \\ \vdots & \vdots & \vdots \\ 1/\rho_{\alpha,\beta}(\tilde{a}_{j1}) & 1/\rho_{\alpha,\beta}(\tilde{a}_{j2}) \dots\dots & 1 \end{bmatrix} \times \begin{bmatrix} W1 \\ W2 \\ \vdots \\ Wn \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (16)$$

For bringing the weightage of specific norm with respect to all other possible norms is produced by employing equations (1-16). The work assesses the dependent weights and priorities (ranks) through the hierarchy described in figure 2, after calculating the stand alone weights.