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A New Method to Support Decision-Making in an Uncertain Environment Based on Normalized Interval-Valued Triangular Fuzzy Numbers and COMET Technique

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Abstract: Multi-criteria decision-making (MCDM) plays a vibrant role in decision-making, and the characteristic object method (COMET) acts as a powerful tool for decision-making of complex problems. COMET technique allows using both symmetrical and asymmetrical triangular fuzzy numbers. The COMET technique is immune to the pivotal challenge of rank reversal paradox and is proficient at handling vagueness and hesitancy. Classical COMET is not designed for handling uncertainty data when the expert has a problem with the identification of the membership function. In this paper, symmetrical and asymmetrical normalized interval-valued triangular fuzzy numbers (NIVTFNs) are used for decision-making as the solution of the identified challenge. A new MCDM method based on the COMET method is developed by using the concept of NIVTFNs. A simple problem of MCDM in the form of an illustrative example is given to demonstrate the calculation procedure and accuracy of the proposed approach. Furthermore, we compare the solution of the proposed method, as interval preference, with the results obtained in the Technique for Order of Preference by Similarity to Ideal solution (TOPSIS) method (a certain preference number).

Keywords: Multi-criteria decision-making; the COMET method; triangular fuzzy number

1. Introduction

Decision-making is the most critical and fundamental tool in which decision-makers use to compare and rank different objects and alternatives based on a few particular criteria to make the best possible decision. Our daily life is full of different experiences and exposures, which lead us to numerous problems and situations where we need to follow the basics principles of operational research. It is a discipline that deals with the applications of advanced analytical methods of decision-making, which help make better decisions than any other technique. The fuzzy set theory [1] is the important field of mathematics, which provides a platform for multi-criteria decision-making (MCDM) to make decisions of such problems of daily life in complex situations. This theory was introduced by Zadeh [1] in 1965, which opened new corridors for decision-making. Bellman and Zadeh [2] used fuzzy logic for the decision-making process for the first time, and then it became one of the most vital fields for decision-making. With time, fuzzy sets theory gone through many developments and come across several applications of MCDM like control [3], effectiveness and user experience in online advertising [4], intelligent systems [5], assessment of web components [6], satellite image analysis [7], evaluation of death possibilities in patients with acute coronary syndrome [8], carbon dioxide geological storage [9] and many more. For decision-making, different techniques are used, and a general assessment of alternatives is preferred for the MCDM problems.

MCDM helps to make best possible decision by following different approaches in fuzzy environments such as triangular fuzzy numbers (TFNs) [10–13], hesitant fuzzy numbers [14–17], trapezoidal fuzzy numbers [18], generalized fuzzy numbers (GFNs) [19,20], interval-valued triangular fuzzy numbers (IVTFNs) [12,21], intuitionistic fuzzy numbers [22,23] and linguistic fuzzy sets [24–26]. With the inception of the new techniques in MCDM to achieve optimal solution, many methods like TOPSIS (The Technique for Order of Preference by Similarity to Ideal solution) [27–29], AHP (Analytic Hierarchy Process) [30–33], ANP (Analytic Network Process) [34–36], COMET (Characteristics Object Method) [33,37–41] etc. were developed and modified under different fuzzy environments such as TFNs, GFNs, trapezoidal fuzzy numbers, hesitant fuzzy numbers, linguistic fuzzy sets etc. for decision-making. Different approaches of MCDM follow the preference aggregation process and generally prefer relationship of outer ranking [12,42]. These approaches include the family of ELECTRE [43,44], PROMETHEE [45–47], NEAT F-PROMETHEE [48], REGIME [8,47], ARGUS [47], NAIADE [49], ORESTE [50], TACTIC [47,51], MELCHIOR, PAMSSEM [47,51] etc. It is important to mention here that numerous methods of MCDM ignore few important factors like ambiguity, fuzziness and vagueness of data [29,52]. However, the utmost explanation to such problems is the usage of fuzzy set theory which provides suitable solution for the problems of MCDM in the uncertain environments.

The COMET [37–39] was designed to handling uncertainty and vagueness of data in the MCDM problems. COMET is a distance-based method, where the final assessment is obtained as a combination of distances from a decision variant to nearest characteristic objects and their preference values [53,54]. Symmetric and asymmetric triangular fuzzy numbers are involved while solving the problems of MCDM. It is worth noticing that symmetric numbers are used when we are using equal division of the domain due to an increased lack of information. In the operational interpretation, COMET is different from TOPSIS [28] because COMET exploits the reference values of nearest characteristic objects and not just two as in TOPSIS (positive and negative ideal solution; PIS and NIS). The COMET method uses the representative values for each alternative and is very operative in the modeling of nonlinearity [5,14].

The COMET technique is helpful for a decision-maker to better comparisons, analysis, and decision-making processes, especially while dealing with complex problems with many alternatives [2] as it is entirely independent of the number of decisional variants [33]. Such techniques prefer the interpretation for the survival of an association between specific components of the MCDM problem. It is also worth emphasizing here that COMET makes assessments between the characteristic objects (COs), which are more suitable, and more comfortable, than the direct evaluations between alternatives. This comparison is made due to Weber–Fechner law [55], which is more helpful to control the difference between two decisional variants if it is too small where it would be not very easy to distinguish between those specific alternatives. The final decision-making [14,56] is acquired based on activated COs and preference values of respective COs. This property ensures that this method is free of rank reversal paradox [33]. Since the inception of the fuzzy set theory, many research accomplishments have been made to enhance different methods of decision-making; however, COMET is considered to be one of the prominent efforts for MCDM.

Interval-valued fuzzy set (IVFS) is distinguished as one of the significant generalization of the fuzzy set, which has proved the utility widely applied in many fields of daily life with practical implications [56–58]. In 2002, Yao and Lin [59] defined interval-valued triangular fuzzy numbers (IVTFNs), which are a useful extension of interval-valued fuzzy numbers (IVFNs) [57]. IVTFNs help handle the vagueness and uncertainty in various decision problems [11,18,19], which provide another handling form to make the optimal decision of MCDM problems. Gitinavard et al. [56] used this extension of the fuzzy set to industrial decisional problems in soft computing based on the multi-criteria group assessment method. Lee et al. [58] used IVFNs for supplier selection, which represents the application of IVFNs with a different range of variety. The theory of normalized interval-valued triangular fuzzy numbers (NIVTFNs) is critical in dealing with the environment in which DMs feel hesitation in providing their assessments in a discrete structure.

In this paper, we propose a new approach that combines the advantages of NIVFNs and the COMET method. Previously, obtained COMET extensions were provided to solve decisional problems under uncertainty using hesitant fuzzy sets (HFS), where the source of uncertainty was that expert known a set of possible values of the membership for one element. The main contribution of this work is dealing with another source of uncertainty. The main difficulty of establishing the membership function is because the data from an expert can have a margin of error, or the chosen shape of the function is not entirely adequate. In the COMET method, we should identify the membership function the best as we can. Therefore, by using NIVFNs, the expert can provide more safety guarantee that NIVFNs will cover the right membership function than by simple TFN. It is easy to prove because we can simplify and say that a NIVFN is a TFN with an added error margin. It is worth noticing that this connection eliminates dangerous paradoxes in decision-making areas and a new source of uncertainty.

The rest of this paper is organized as follows. Some crucial definitions and basic concepts related to TFNs, IVTFNs, NIVTFNs with some basic operations are discussed in Section 2. In Section 3, the COMET methodology in the context of NIVTFNs is developed to deal with vague and uncertain environments in the MCDM problems. A simple example is given in Section 4 to demonstrate the practical feasibility study of the proposed approach. The paper is ended by Section 5 with some conclusions related to research.

2. Preliminaries

In this section, we will focus on some important concepts which can play pivotal role in understanding the proposed study.

Definition 1. *Basic operations on two intervals [8,60].*

For any two intervals $A = [a_1, a_2]$, $B = [b_1, b_2]$ and $\lambda \in R$, the following basic operations on A and B can be defined as

$$
\begin{aligned}
&A \oplus B = [a_1 + b_1, a_2 + b_2] \\
&A \ominus B = [a_1 - b_2, a_2 - b_1] \\
&A \otimes B = [\min(a_1b_1, a_1b_2, a_2b_1, a_2b_2), \max(a_1b_1, a_1b_2, a_2b_1, a_2b_2)] \\
&\lambda A = [\lambda a_1, \lambda a_2] \\
&A^\lambda = [a_1^\lambda, a_2^\lambda]
\end{aligned}
$$

Definition 2. *Triangular Fuzzy Number [8,10,11,59].*

A fuzzy number $\tilde{A}(a, m, b)$ over the set of real numbers R is called a TFN if its membership function is represented by

$$
\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{m-a} & a \le x < m \\ 1 & x = m \\ \frac{b-x}{b-m} & m < x \le b \\ 0 & x > b \end{cases}
$$

The membership function $\mu_{\tilde{A}}(x)$ satisfies the following characteristics:

$$
\begin{aligned}
x_2 &> x_1 \Rightarrow \mu_{\tilde{A}}(x_2) > \mu_{\tilde{A}}(x_1) \; \forall \, x_1, x_2 \in [a, m] \\
x_2 &> x_1 \Rightarrow \mu_{\tilde{A}}(x_2) < \mu_{\tilde{A}}(x_1) \; \forall \, x_1, x_2 \in [m, b]
\end{aligned}
$$

If $m - a = m - b$ then it is a symmetrical TFN otherwise we call it asymmetrical TFN.

Definition 3. *Interval-valued fuzzy number [11,57,61].*

An interval-valued fuzzy number (IVFN) can be expressed as in the following form $\tilde{A} = \{x, [\mu_{\tilde{A}}{}^{L}(x), \mu_{\tilde{A}}{}^{U}(x)]\}, x \in R,\ \mu_{\tilde{A}}{}^{L},\ \mu_{\tilde{A}}{}^{U} : R \to [0,1]$ and $\mu_{\tilde{A}}{}^{L} \leq \mu_{\tilde{A}}{}^{U}$ where $\mu_{\tilde{A}}{}^{L}(x)$ and $\mu_{\tilde{A}}{}^{U}(x)$ are known as the lower and upper degrees of membership function $\mu_{\tilde{A}}(x)$ where $\mu_{\tilde{A}}(x) = [\mu_{\tilde{A}}{}^{L}(x), \mu_{\tilde{A}}{}^{U}(x)], x \in R$.

Definition 4. *Interval-valued triangular fuzzy number [12,21].*

The IVTFN can be defined as $\tilde{A} = [\tilde{A}_x^L, \tilde{A}_x^U]$, where $\tilde{A}_x^L = (a_1^L, b_1^L, c_1^L; w_{\tilde{A}}^L)$ and $\tilde{A}_x^U = (a_1^U, b_1^U, c_1^U; w_{\tilde{A}}^U)$ are two fuzzy numbers satisfying $a_1^U \leq a_1^L$, $c_1^L \leq c_1^U$ and $w_{\tilde{A}}^L \leq w_{\tilde{A}}^U$. The numbers $w_{\tilde{A}}^L$ and $w_{\tilde{A}}^U$ are called the heights of $\tilde{A}_x^L$ and $\tilde{A}_x^U$ respectively.

Definition 5. *Normalized interval-valued triangular fuzzy number [10,30].*

A NIVTFN number is an IVTFN with the following two characteristics:

$$b_1^L = b_1^U \text{ and } w_{\tilde{A}}^L = w_{\tilde{A}}^U = 1.$$

A NIVTFN is represented by $A = [A_x^L,\ A_x^U]$ and is expressed as $A = (a_1^U, a_1^L, b_1, c_1^L, c_1^U)$. The core of the NIVTFN $A = (a_1^U, a_1^L, b_1, c_1^L, c_1^U)$ is defined as the set of all points x in R such that $\mu_{\tilde{A}}{}^{L}(x) = \mu_{\tilde{A}}{}^{U}(x) = 1$. Since $\mu_{\tilde{A}}{}^{L}(b_1) = \mu_{\tilde{A}}{}^{U}(b_1) = 1$, therefore, b_1 is called the core of the NIVTFN $A = (a_1^U, a_1^L, b_1, c_1^L, c_1^U)$. The graphs of the IVTFN $\tilde{A}$ and NIVTFN A can be seen in Figures 1 and 2 respectively.

Definition 6. *Geometric mean*

Let $I_1 = (a_1^U, a_1^L, b_1, c_1^L, c_1^U), I_2 = (a_2^U, a_2^L, b_2, c_2^L, c_2^U), ..., I_n = (a_n^U, a_n^L, b_n, c_n^L, c_n^U)$ be n NIVTFNs. Then, the geometric mean of $I_1, I_2, ..., I_n$ can be defined as

$$G(I_1, I_2, ..., I_n) = \left(\left(\prod_{i=1}^{n} a_i^U\right)^{1/n}, \left(\prod_{i=1}^{n} a_i^L\right)^{1/n}, \left(\prod_{i=1}^{n} b_i\right)^{1/n}, \left(\prod_{i=1}^{n} c_i^L\right)^{1/n}, \left(\prod_{i=1}^{n} c_i^U\right)^{1/n} \right). \quad (1)$$

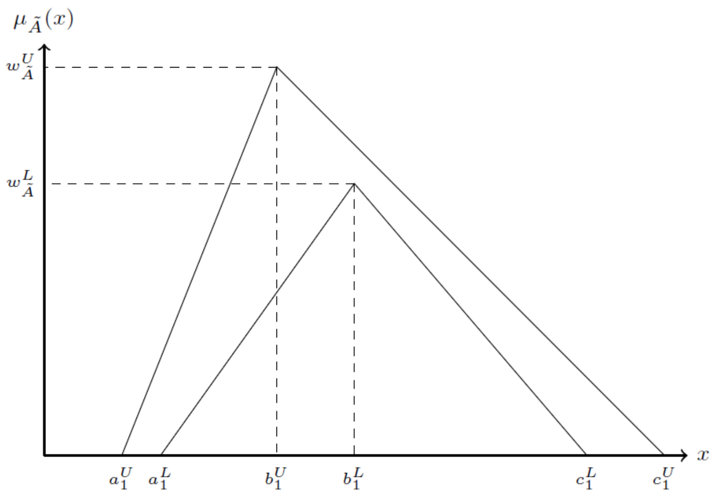

Figure 1. Interval-valued triangular fuzzy numbers example (symmetrical and asymmetrical case).

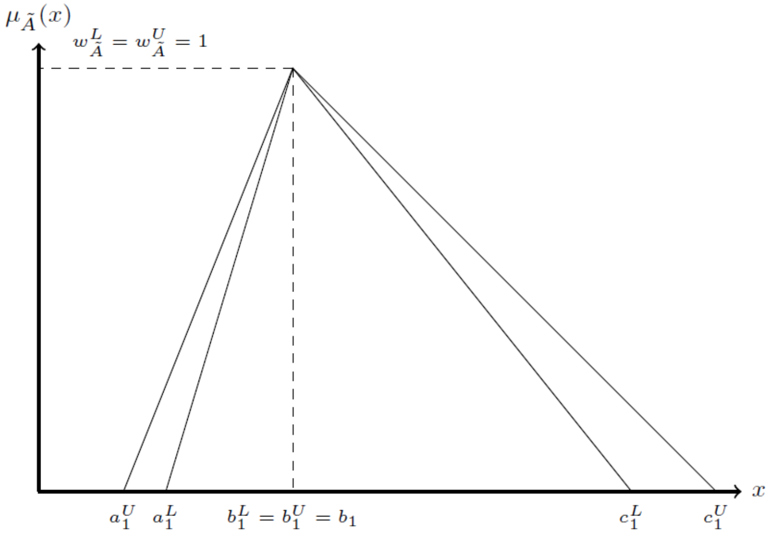

Figure 2. Normalized interval-valued triangular fuzzy numbers example (asymmetrical case).

3. COMET Method with NIVTFNs

This section is devoted to a theoretical description of the proposed approach to solving MCDM problems with the use of NIVTFNs and COMET. The whole procedure has been divided into five colliding steps described below. Let $A_j (j = 1, 2, \ldots .m)$ be a set of alternatives and $C_i (i = 1, 2, 3 \ldots n)$ be the set of criteria. The whole decision-making process by using the COMET method and NIVTFNs is presented in Figure 3.

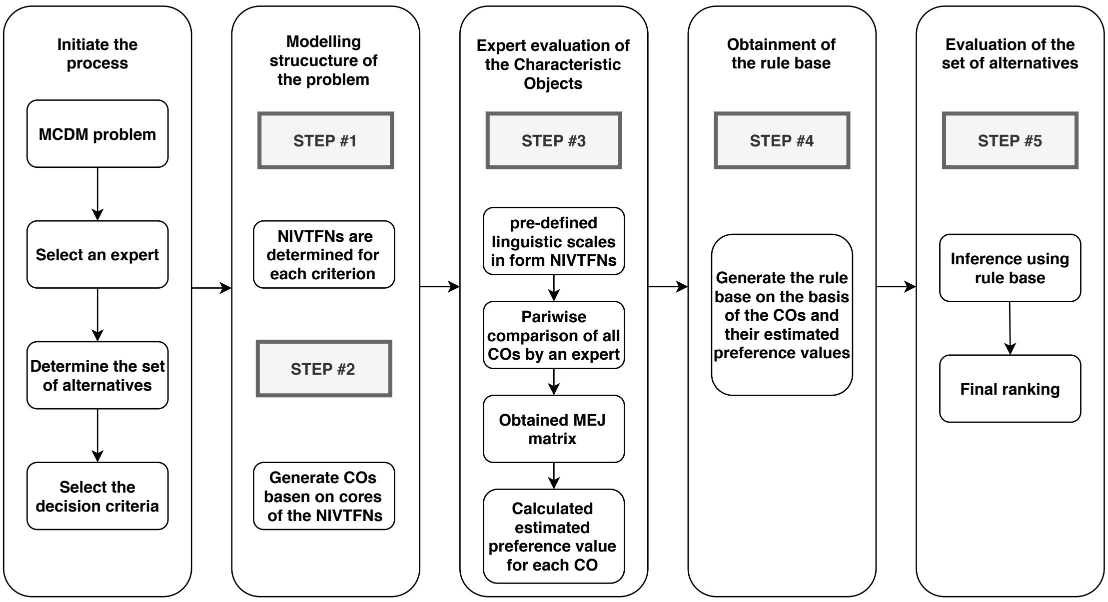

Figure 3. The flowchart with proposed approach by using NIVFNs and the COMET method.

Step 1: Define the space of the problem

Let $\mathcal{C}$ be the family of all NIVTFNs and $N_i = \{N_{i1}, N_{i2}, ..., N_{ic_i}\}$ be a collection of some NIVTFNs which are selected for each criterion C_i $(i = 1, 2, ..., n)$. As a result, the following families of NIVTFNs can be obtained for each criterion as follows:

$$\begin{gathered} C_1 = \{N_{11}, N_{12}, ..., N_{1c_1}\} \\ C_2 = \{N_{21}, N_{22}, ..., N_{2c_2}\} \\ \vdots \\ C_n = \{N_{n1}, N_{n2}, ..., N_{nc_n}\} \end{gathered}$$

Now, we need to find the core of each NIVTFNs selected for each criterion C_i $(i = 1, 2, ..., n)$. Afterwards, the core of each criterion is obtained which can be described as the core of each NIVTFN involved in the families as mentioned above, i.e.

$$\begin{gathered} C(C_1) = \{C(N_{11}), C(N_{12}), ..., C(N_{1c_1})\}\,; \\ C(C_2) = \{C(N_{21}), C(N_{22}), ..., C(N_{2c_2})\}\,; \\ \vdots \\ C(C_n) = \{C(N_{n1}), C(N_{n2}), ..., C(N_{nc_n})\}\,. \end{gathered}$$

Step 2: Generate the COs

The all possible COs can be obtained by taking the Cartesian product of all $C(C_i)(i = 1, 2, ..., n)$ as follow:

$$CO = C(C_1) \times C(C_2) \times ... \times C(C_n)$$

As the result of this, the following ordered sets are obtained containing all the cores of respective NIVTFNs as:

$$\begin{gathered} CO_1 = \{C(N_{11}), C(N_{21}), ..., C(N_{n1})\} \\ CO_2 = \{C(N_{11}), C(N_{21}), ..., C(N_{n2})\} \\ \vdots \\ CO_s = \{C(N_{1c_1}), C(N_{2c_2}), ..., C(N_{nc_n})\} \end{gathered}$$

where $s = \prod_{i=1}^{n} c_i$ is the count of all the COs.

Step 3: Rank and evaluate the COs

Collect the opinion of expert on the importance of all the COs via pairwise comparisons as represented by square matrix called the matrix of expert judgment (MEJ). The experts are requested to provide their assessments about $CO_l (1 \leq l \leq s)$ by using the pre-defined linguistic scales in the form of NIVTFNs which can express the relative importance of one CO over another. The $MEJ = [I_{ij}]_{s \times s}$ can expressed as

$$MEJ = \begin{bmatrix} I_{11} & I_{12} & \cdots & I_{1s} \\ I_{21} & I_{22} & \cdots & I_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ I_{s1} & I_{s2} & \cdots & I_{ss} \end{bmatrix}.$$

Each I_{ij} is NIVTFN which denotes the degree to which CO_i is preferred to CO_j.

Step 4: Preference values of COs

In this step, we will find two vectors known as SJ and P. The vector SJ called the vector of summed judgments is found by calculating the geometric mean of the corresponding elements in the form of NIVTFNs from the MEJ. This is represented by $SJ = [v_1, v_2, ...\ v_s]$, where each $v_l = (a_l, b_l, c_l, d_l, e_l)$ is NIVTFN and is obtained by taking the geometric mean $G(I_{l1}, I_{l2}, \cdots, I_{ls})$ of

$I_{l1}, I_{l2}, \cdots, I_{ls}$ $(1 \leq l \leq s)$ as discussed in Equation (1). The next vector $P = [P_1, P_2, ..., P_s]$ which actually contains the preference values of all the COs can be computed by the following formula

$$P_l = \frac{w_l}{\sum\limits_{l=1}^{s} w_l}, \; 1 \leq l \leq s, \tag{2}$$

$$\text{where } w_l = \frac{1}{5}\left(\frac{a_l}{\sum\limits_{l=1}^{s} a_l} + \frac{b_l}{\sum\limits_{l=1}^{s} b_l} + \frac{c_l}{\sum\limits_{l=1}^{s} c_l} + \frac{d_l}{\sum\limits_{l=1}^{s} d_l} + \frac{e_l}{\sum\limits_{l=1}^{s} e_l}\right). \tag{3}$$

Step 5: Inference in a fuzzy model and final ranking

As every alternative can be represented with a set of crisp numbers such as

$$A_j = \left\{a_{1j}, a_{2j}, ..., a_{nj}\right\}, \; j = 1, 2, \ldots .m \tag{4}$$

where the following conditions must be satisfied for each element of $A_j(j = 1, 2, ..., m)$.

$$\begin{array}{c} a_{1j} \in [C(N_{11}), C(N_{1c_1})] \\ a_{2j} \in [C(N_{21}), C(N_{2c_2})] \\ \vdots \\ a_{nj} \in [C(N_{n1}), C(N_{nc_n})] \end{array} \tag{5}$$

To get the final ranking of the alternatives corresponding to each criterion for each $j = 1, 2, ..., m$, we proceed as follows:

$$\begin{array}{l} a_{1j} \in [C(N_{1k_1}), C(N_{1(k_1+1)})] \\ a_{2j} \in [C(N_{2k_2}), C(N_{2(k_2+1)})] \\ \qquad\vdots \\ a_{nj} \in [C(N_{nk_n}), C(N_{n(k_n+1)})] \\ k_i = 1, 2 ... (c_i - 1) \end{array}$$

The activated rules (COs) i.e., the group of those COs where the membership function of each alternative A_j $(1 \leq j \leq m)$ is non-zero are

$$\begin{array}{c} \left(C(N_{1k_1}), C(N_{2k_2}), ..., C(N_{nk_n})\right); \\ \Big(C(N_{1k_1}), C(N_{2k_2}), ..., C(N_{n(k_n+1)})\Big); \\ \vdots \\ \Big(C(N_{1(k_1+1)}), C(N_{2(k_2+1)}), ..., C(N_{n(k_n+1)})\Big). \end{array}$$

The number of COs are obviously 2^n where $1 \leq 2^n \leq s$. Let $p_1, p_2, ..., p_{2^n}$ be the approximate values of preference of the activated rules (COs) which were already calculated in Step 4. We denote $N_i(a_{ij}) = \{N_{ik_i}(a_{ij}) \mid a_{ij} \in A_j, \; k_i = 1, 2 ... (c_i - 1)\}$ the value of each family of NIVTFNs at $a_{ij} \in A_j$ where $i = 1, 2, \ldots .n$ and $j = 1, 2, \ldots .m$. It should be noted that each member of this family is an interval of the form $[N'_{ik_i}(a_{ij}), N''_{ik_i}(a_{ij})]$ where $N'_{ik_i}(a_{ij}) \leq N''_{ik_i}(a_{ij})$ for each $i = 1, 2, \ldots .n$ and $j = 1, 2, \ldots .m$.

By using Definition 1, the preference value of each alternative $A_j (j = 1, 2, ..., m)$ in the form of interval can be computed as sum of the product of the preference values of all the COs and the fulfillment degrees of corresponding elements of A_j, i.e.

$$\begin{aligned}
&A_j = p_1 \left(N_{1k_1}(a_{1j}) \otimes N_{2k_2}\ (a_{2j}) \otimes \ldots \otimes N_{nk_n}(a_{nj}) \right) \oplus \\
&p_2 \left(N_{1k_1}(a_{1j}) \otimes N_{2k_2}(a_{2j}) \otimes \ldots \otimes N_{n(k_n+1)}(a_{nj})\ \right) \oplus \ldots \oplus \\
&p_{2^n} \left(N_{1(k_1+1)}(a_{1j})\ \otimes N_{2(k_2+1)}(a_{2j}) \otimes \ldots \otimes N_{n(k_n+1)}(a_{nj}) \right) = [I_j, I'_j]
\end{aligned} \tag{6}$$

The final preference value $Pr(A_j)(j = 1, 2, ..., m)$ of each alternative A_j $(1 \le j \le m)$ can be found by calculating the mean value of the corresponding preference interval $[I_j, I'_j]$, i.e.

$$Pr(A_j) = \frac{I_j + I'_j}{2},\ j = 1, 2, ..., m.$$

Finally, the final ranking of alternatives is obtained by sorting the final preference values of alternatives. The greater the preference value, the better the alternative A_j $(1 \le j \le m)$.

4. An Illustrative Example

In this section, we solve an illustrative example by using proposed approach. This example and presented calculations are intended to help the reader to understand the presented method. It will allow using the given technique to various types of problems by readers with a lower level of expertise in fuzzy sets and their extensions.

Let us consider the problem of selecting the new tank to buy by the government for the army. A tank is used as a primary armored fighting vehicle for front-line combat. The basic parameters providing good combat value and maneuverability are firepower, strong armor, good quality tracks, and a powerful engine. Let us say that we should analyze ten offers (alternatives). Each one was assessed separately in the three criteria, according to Firepower (FP), Battlefield Maneuverability (BM), and Engine Power (EP). The offers performance is presented in the form of a decision matrix with established three criteria and reference ranking by using expert knowledge, which can be seen in Table 1. The detailed calculation procedure will be presented in the next 5 steps.

Table 1. The performance matrix and reference ranking of the alternatives.

Alternatives	C_1 FP	C_2 BM	C_3 EP	Reference Ranking
A_1	84	8	3	2
A_2	65	7	3.5	8
A_3	73	6	3.7	7
A_4	76	8	4.2	6
A_5	80	7	3.5	4
A_6	61	6	3.8	9
A_7	80	6.5	3.7	5
A_8	85	8	4.5	1
A_9	59	6	3.5	10
A_{10}	79	8	4	3

Step 1: Suppose that N_1, N_2, and N_3 represent the three families of subsets of $\mathcal{C}$ selected for the criteria C_1, C_2 and C_3 respectively, where

$$\begin{aligned}
&N_1 = \{N_{11}, N_{12}, N_{13}\} = \{(30, 40, 40, 58, 65), (40, 42, 70, 90, 100), (70, 80, 100, 100, 100)\}, \\
&N_2 = \{N_{21}, N_{22}\} = \{(0, 0, 0, 4, 5), (3, 4, 9.5, 9.5)\}, \\
&N_3 = \{N_{31}, N_{32}, N_{33}\} = \{(0, 0, 0, 1.8, 2.5), (0, 0.5, 3, 4.8, 5.5), (3, 3.5, 5.5, 5.5, 5.5)\}.
\end{aligned}$$

The graphical representations of the families N_1, N_2, and N_3 for each criterion C_1, C_2 and C_3 can be seen in Figures 4–6 respectively. The cores for each family with respect to each criterion is determined as $C(N_1) = \{40, 70, 100\}$, $C(N_2) = \{0, 9.5\}$ and $C(N_3) = \{0, 3, 5.5\}$.

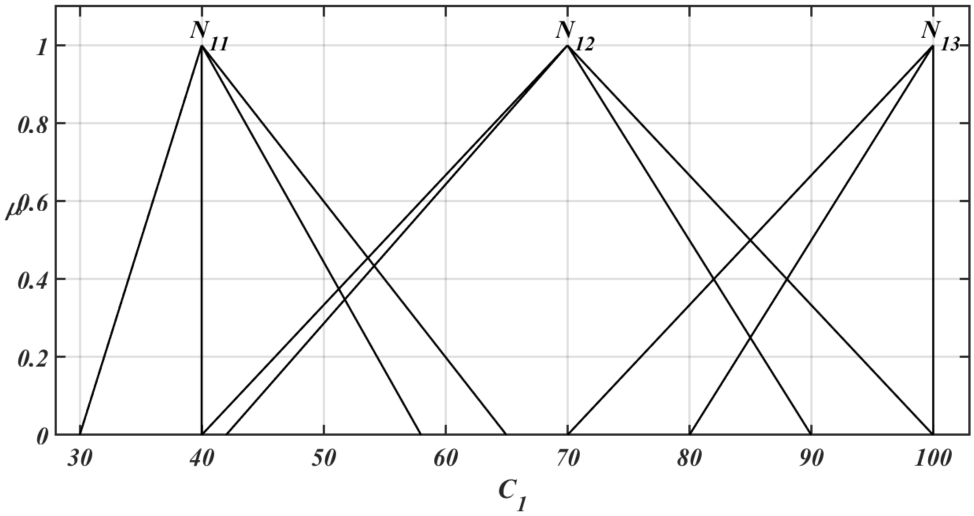

Figure 4. Graphical representation of NIVTFNs for C_1.

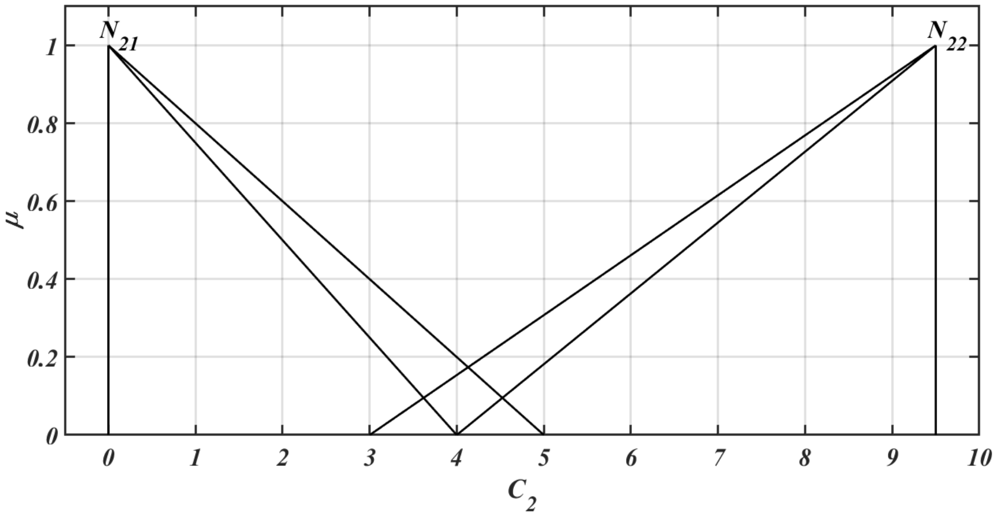

Figure 5. Graphical representation of NIVTFNs for C_2.

Step 2: The solution of COMET is obtained for different numbers of COs which can be obtained by taking the Cartesian product of the sets $C(N_1)$, $C(N_2)$ and $C(N_3)$. The list of all the COs with their set values are given as under:

$$\begin{array}{lll} CO_1 = \{40, 0, 0\} & CO_2 = \{40, 0, 3\} & CO_3 = \{40, 0, 5.5\} \\ CO_4 = \{40, 9.5, 0\} & CO_5 = \{40, 9.5, 3\}, & CO_6 = \{40, 9.5, 5.5\} \\ CO_7 = \{70, 0, 0\} & CO_8 = \{70, 0, 3\} & CO_9 = \{70, 0, 5.5\} \\ CO_{10} = \{70, 9.5, 0\} & CO_{11} = \{70, 9.5, 3\} & CO_{12} = \{70, 9.5, 5.5\} \\ CO_{13} = \{100, 0, 0\} & CO_{14} = \{100, 0, 3\} & CO_{15} = \{100, 0, 5.5\} \\ CO_{16} = \{100, 9.5, 0\} & CO_{17} = \{100, 9.5, 3\} & CO_{18} = \{100, 9.5, 5.5\} \end{array}$$

Step 3: For the comparison of COs, suppose that the expert provides his/her pairwise judgments in the form of pre-defined linguistic scales in the form of NIVTFNs as expressed in Table 2. The most preferred CO will get linguistic variable "absolutely important", the largest weaker CO will get linguistic variable "weakly important" and the COs with same comparison will get the linguistic variable "equally important".

Table 2. Pre-defined linguistic scales in the form of NIVTFNs.

Sr. No	Variable	Value
1	Weekly Important (WI)	$(0, 0, 0, 0.1, 0.2)$
2	Equally Important (EI)	$(0.2, 0.2, 0.2, 0.3, 0.4)$
3	Fairly Important (FI)	$(0.3, 0.3, 0.3, 0.4, 0.5)$
4	Strongly Important (SI)	$(0.7, 0.7, 0.7, 0.8, 0.9)$
5	Absolutely Important (AI)	$(0.8, 0.8, 0.8, 0.9, 1)$

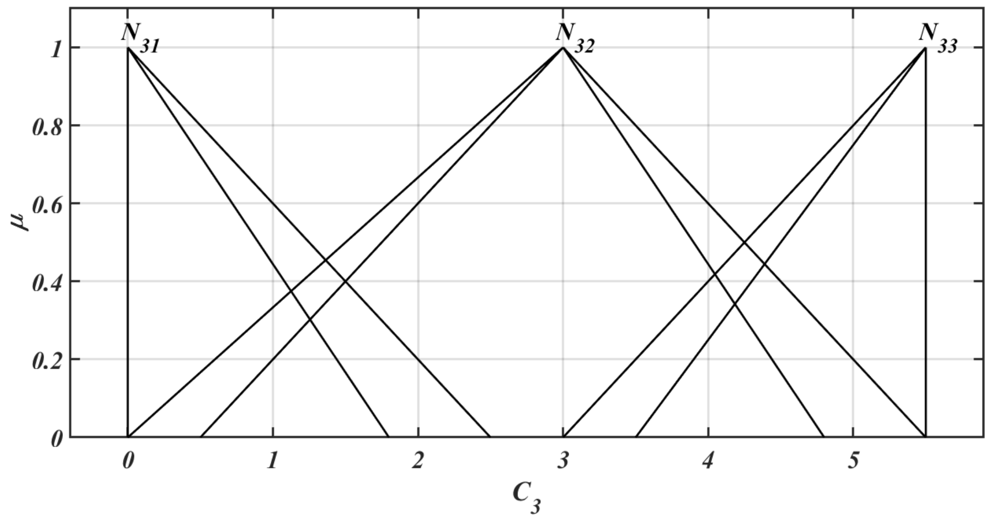

Figure 6. Graphical representation of NIVTFNs for C_3.

As a result, the matrix $MEJ = [I_{ij}]_{18\times 18}$ is obtained which can be seen in Tables 3 and 4.

Table 3. Matrix of Expert Judgments part (1/2).

	CO_1	CO_2	CO_3	CO_4	CO_5	CO_6	CO_7	CO_8	CO_9
CO_1	EI	WI	WI	WI	WI	WI	WI	WI	WI
CO_2	FI	EI	WI	WI	WI	WI	FI	WI	WI
CO_3	FI	FI	EI	FI	WI	WI	FI	FI	WI
CO_4	FI	FI	WI	EI	WI	WI	FI	FI	WI
CO_5	SI	FI	FI	FI	FI	WI	FI	FI	FI
CO_6	SI	FI	FI	FI	FI	EI	SI	FI	FI
CO_7	FI	WI	WI	WI	WI	WI	EI	WI	WI
CO_8	FI	FI	WI	WI	WI	WI	FI	EI	WI
CO_9	FI	FI	FI	FI	WI	WI	FI	FI	EI
CO_{10}	FI	FI	FI	FI	WI	WI	FI	FI	WI
CO_{11}	SI	FI	FI	FI	FI	WI	SI	FI	FI
CO_{12}	AI	SI	FI	FI	FI	FI	SI	FI	FI
CO_{13}	FI	EI	WI	WI	WI	WI	FI	WI	WI
CO_{14}	FI	FI	FI	FI	WI	WI	FI	FI	WI
CO_{15}	SI	FI	FI	FI	FI	WI	FI	FI	FI
CO_{16}	SI	FI	FI	FI	EI	WI	FI	FI	FI
CO_{17}	AI	SI	FI	FI	FI	FI	AI	FI	FI
CO_{18}	AI	AI	AI	SI	FI	FI	AI	SI	FI

Table 4. Matrix of Expert Judgments part (2/2).

	CO_{10}	CO_{11}	CO_{12}	CO_{13}	CO_{14}	CO_{15}	CO_{16}	CO_{17}	CO_{18}
CO_1	WI	WI	WI	WI	WI	WI	WI	WI	WI
CO_2	WI	WI	WI	EI	WI	WI	WI	WI	WI
CO_3	WI	WI	WI	FI	WI	WI	WI	WI	WI
CO_4	WI	WI	WI	FI	WI	WI	WI	WI	WI
CO_5	FI	WI	WI	FI	WI	WI	EI	WI	WI
CO_6	FI	FI	WI	FI	FI	FI	FI	WI	WI
CO_7	WI	WI	WI	WI	WI	WI	WI	WI	WI
CO_8	WI	WI	WI	FI	WI	WI	WI	WI	WI
CO_9	FI	WI	WI	FI	FI	WI	WI	WI	WI
CO_{10}	EI	WI	WI	FI	FI	WI	WI	WI	WI
CO_{11}	FI	EI	WI	FI	FI	FI	FI	WI	WI
CO_{12}	FI	FI	EI	SI	FI	FI	FI	FI	WI
CO_{13}	WI	WI	WI	EI	WI	WI	WI	WI	WI
CO_{14}	WI	WI	WI	FI	EI	WI	WI	WI	WI
CO_{15}	FI	WI	WI	FI	FI	EI	FI	WI	WI
CO_{16}	FI	WI	WI	FI	FI	WI	EI	WI	WI
CO_{17}	FI	FI	WI	SI	FI	FI	FI	EI	WI
CO_{18}	FI	FI	FI	FI	SI	FI	FI	FI	EI

Step 4: Now, we calculate the vector $SJ = [v_1, v_2, ..., v_{18}]$ based on MEJ as mentioned in Step 4. The first component v_1 can be computed by using Equation (1) as follows:

$v_1 = G(I_{11}, I_{12}, ..., I_{118}) = (0, 0, 0, 0.1063, 0.2079)$

On similar lines, the remaining components of the vector SJ are obtained as follows:

$$
\begin{array}{ll}
v_2 = (0,0,0,0.1318,0.2392) & v_3 = (0,0,0,0.1687,0.2821) \\
v_4 = (0,0,0,0.1562,0.2681) & v_5 = (0,0,0,0.2331,0.3506) \\
v_6 = (0,0,0,0.3325,0.4466) & v_7 = (0,0,0,0.1148,0.2187) \\
v_8 = (0,0,0,0.1446,0.2548) & v_9 = (0,0,0,0.2126,0.3286) \\
v_{10} = (0,0,0,0.1968,0.3123) & v_{11} = (0,0,0,0.3030,0.4192) \\
v_{12} = (0,0,0,0.4127,0.5210) & v_{13} = (0,0,0,0.1318,0.2392) \\
v_{14} = (0,0,0,0.1822,0.2968) & v_{15} = (0,0,0,0.2763,0.3930) \\
v_{16} = (0,0,0,0.2518,0.3689) & v_{17} = (0,0,0,0.3867,0.5013) \\
v_{18} = (0.4094,0.4094,0.4094,0.5175,0.6230) &
\end{array}
$$

By using Equation (2), the preference values of all the COs are then computed as

$P = [0.0114, 0.0136, 0.0166, 0.0182, 0.0217, 0.0293, 0.0121, 0.0262, 0.0201, 0.0189, 0.0271, 0.0353, 0.0136, 0.0177, 0.0250, 0.0232, 0.0373, 0.6326]^T$

where $w_l (1 \leq l \leq s)$ are obtained in the following by using Equation (3).

$$
\begin{array}{lll}
w_1 = 0.0116 & w_2 = 0.0138 & w_3 = 0.0169 \\
w_4 = 0.0185 & w_5 = 0.0221, & w_6 = 0.0299 \\
w_7 = 0.0124 & w_8 = 0.0267 & w_9 = 0.0205 \\
w_{10} = 0.0192 & w_{11} = 0.0276 & w_{12} = 0.0360 \\
w_{13} = 0.0138 & w_{14} = 0.0180 & w_{15} = 0.0255 \\
w_{16} = 0.0236 & w_{17} = 0.0380, & w_{18} = 0.6442
\end{array}
$$

Step 5: The preference interval indicating the approximate preference value of the first alternative $A_1 = \{84, 4, 3\}$ is computed by using Formula (6), which is given as follows:

$$\begin{aligned} A_1 = \; & [(N_{11}(84) \otimes N_{21}(4) \otimes N_{31}(3))P_1 \oplus (N_{11}(84) \otimes N_{21}(4) \otimes N_{32}(3))P_2 \oplus \\ & (N_{11}(84) \otimes N_{21}(4) \otimes N_{33}(3))P_3 \oplus (N_{11}(84) \otimes N_{22}(4) \otimes N_{31}(3))P_4 \oplus \\ & (N_{11}(84) \otimes N_{22}(4) \otimes N_{32}(3))P_5 \oplus (N_{11}(84) \otimes N_{22}(4) \otimes N_{33}(3))P_6 \oplus \\ & (N_{12}(84) \otimes N_{21}(4) \otimes N_{31}(3))P_7 \oplus (N_{12}(84) \otimes N_{21}(4) \otimes N_{32}(3))P_8 \oplus \\ & (N_{12}(84) \otimes N_{21}(4) \otimes N_{33}(3))P_9 \oplus (N_{12}(84) \otimes N_{22}(4) \otimes N_{31}(3))P_{10} \oplus \\ & (N_{12}(84) \otimes N_{22}(4) \otimes N_{32}(3))P_{11} \oplus (N_{12}(84) \otimes N_{22}(4) \otimes N_{33}(3))P_{12} \oplus \\ & (N_{13}(84) \otimes N_{21}(4) \otimes N_{31}(3))P_{13} \oplus (N_{13}(84) \otimes N_{21}(4) \otimes N_{32}(3))P_{14} \oplus \\ & (N_{13}(84) \otimes N_{21}(4) \otimes N_{33}(3))P_{15} \oplus (N_{13}(84) \otimes N_{22}(4) \otimes N_{31}(3))P_{16} \oplus \\ & (N_{13}(84) \otimes N_{22}(4) \otimes N_{32}(3))P_{17} \oplus (N_{13}(84) \otimes N_{22}(4) \otimes N_{33}(3))P_{18}] \\ & = [0.0193 \; 0.0906] \end{aligned}$$

The final preference value of A_1 can be found by calculating the mean score value as

$$Pr(A_1) = \frac{0.0193 + 0.0906}{2} = 0.05495$$

Similarly, the final preference values of all the remaining alternatives are obtained which are presented in Table 5.

Table 5. Ranking of Alternatives with COMET using NIVTFNs (Rank), TOPSIS ($TOPSIS_R$), and reference ranking ($Reference_R$), where RC_i relative closeness.

Alternatives	Preference Intervals	$P_r(A_i)$	Rank	RC_i	$TOPSIS_R$	$Reference_R$
A_1	$[0.0193, 0.0906]$	0.05495	2	0.8469	2	2
A_2	$[0.0126, 0.0242]$	0.01840	8	0.6521	8	8
A_3	$[0.0120, 0.0274]$	0.01970	7	0.7321	7	7
A_4	$[0.0144, 0.0720]$	0.04320	4	0.7655	6	6
A_5	$[0.0084, 0.0741]$	0.04125	5	0.8052	3	4
A_6	$[0.0065, 0.0189]$	0.01270	9	0.6106	9	9
A_7	$[0.0071, 0.0725]$	0.03980	6	0.8043	4	5
A_8	$[0.0809, 0.1894]$	0.13515	1	0.8584	1	1
A_9	$[0.0054, 0.0186]$	0.01200	10	0.5904	10	10
A_{10}	$[0.0114, 0.0982]$	0.05480	3	0.7963	5	3

However, to compare result of our proposed method, TOPSIS method is applied to solve the same problem. The positive ideal solution $d_i^+(i = 1, ..., 10)$ and negative ideal solution $d_i^-(i = 1, ..., 10)$ are determined, and the relative closeness coefficients $RC_i(i = 1, ..., 10)$ and final ranking are presented in Table 5.

The final ranking obtained by TOPSIS method is $A_8 \succ A_1 \succ A_5 \succ A_7 \succ A_{10} \succ A_4 \succ A_3 \succ A_2 \succ A_6 \succ A_9$ and the most desirable alternative is A_8. However, by using the COMET method with NIVTFNs, the ranking of the alternatives is $A_8 \succ A_1 \succ A_{10} \succ A_4 \succ A_5 \succ A_7 \succ A_3 \succ A_2 \succ A_6 \succ A_9$ which adequately match as those with the ranking obtained in the TOPSIS method as well as the reference ranking $A_8 \succ A_1 \succ A_{10} \succ A_5 \succ A_7 \succ A_4 \succ A_3 \succ A_2 \succ A_6 \succ A_9$.

In the proposed approach, the ranking has been determined as an average of the received intervals. The best and the worst alternatives have the same place in the ranking, and correlation is very high ($\rho = 0.9636$). However, some important differences are also observed in the ranking order, i.e., the alternatives A_4, A_5, A_7 and A_{10}. The proposed approach takes into account the new type of uncertainty compared to the previous extension of COMET. The result obtained here is the interval number, the so-called preference interval. Based on uncertain data, it is not possible to obtain a certain solution. Let us look at the assessment of alternatives A_4 and A_5 (Figure 7). The new quality of our solution is justified and possible discrepancies. The interval measure for A_4 is greater, but only in 74.07% of the cases for random values from these orders, we will obtain such a dependence. The solutions obtained by TOPSIS are the certain numbers and the any difference in the ranking has not explanation. The example above shows how a complete decision-making process can be carried out in order to make the result more knowledgeable about alternatives than other methods. This approach ensures that the phenomenon of rank reversal is avoided.

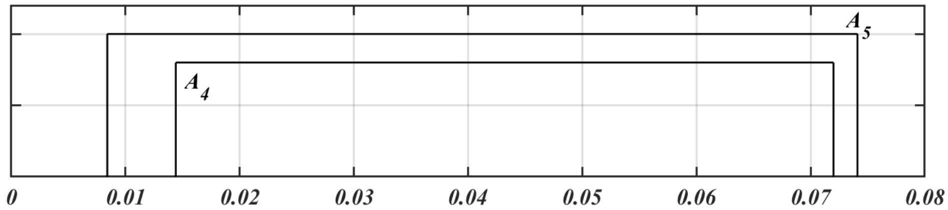

Figure 7. Graphical representation of the assessment of alternatives A_4 and A_5.

5. Conclusions

The uncertainty and diversity of assessment information provided by the DMs can be well reflected and modeled using NIVTFNs. The NIVTFNs are very useful to express vagueness and uncertainty more accurately as compared to fuzzy sets. Therefore, we synthesis a new method based on the COMET method and NIVTFNs. In that way, we obtained a helpful technique for solving MCDM problems under uncertain environment. In this study, we observed the difference between the proposed approach and TOPSIS methods for decision-making under uncertainty. We show that using preference intervals is more accurate and can justify the difference between rankings in an uncertain environment. Results of this approach are still free of rank reversal paradox due to the application of COMET, and it also permits decision-making under uncertain environment, especially where imprecise pieces of evidence and information are main hurdles for the decision-maker.

The presented approach is also following actual research trends in terms of methodological backgrounds. This paper provides theoretical manipulations of the extended approach of MCDM. It establishes a degree of membership in the form of interval instead of a crisp number, which is more suitable to tackle uncertainty during decision-making processes. To illustrate the whole calculation procedure of the COMET method using NIVTFNs, we studied a simple example. Future work will be geared towards the formulation of a comprehensive COMET-based system to support decision-making with an awareness of practical relevance and utility. Moreover, further research direction on the use of this approach and how to compare different rankings in the uncertain environment. As the next future works, we research the COMET method and:

- interval-valued intuitionistic fuzzy sets,
- hesitant fuzzy linguistic term sets,
- hesitant intuitionistic fuzzy linguistic term sets,
- etc.

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