

Article

Logic Gates Formed by Perturbations in an Asynchronous Game of Life

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Abstract: The game of life (GL), a type of two-dimensional cellular automaton, has been the subject of many studies because of its simple mechanism and complex behavior. In particular, the construction of logic circuits using the GL has helped to extend the concept of computation. Conventional logic circuits assume deterministic transitions due to the synchronicity of the classic GL. However, they are fragile to noise and cannot maintain the expected behavior in an environment with noise. In this study, a probabilistic logic gate model was constructed using perturbations in an asynchronous game of life (AGL). Since our asynchronous automaton had no heterogeneity in either the horizontal or vertical directions, it was symmetrical with respect to spatial structure. On the other hand, the construction of the logical gate was implemented to contain heterogeneity in the horizontal or vertical directions, which could allow an AND gate and an OR gate in a single system. It was based on the phase transition between connected and unconnected phases, which is newly discovered in this study. In the model, perturbations symmetrically entail operations successful and unsuccessful, and this symmetrical double action is given not to interfere with established operations but to make operations possible. Therefore, this model had a different meaning from logic gates that exclude perturbations or use them externally. The idea of this perturbation is analogous to the inherent noise that destroys and generates structures in biological swarms.



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Keywords: cellular automata; asynchronous updating; perturbations; logic gate; phase transition

1. Introduction

To date, various logic gates have been constructed for the GL. For example, there are logic gates that use collisions of gliders [1–4] or geographical constraints with fixed objects [5]. These logic gates are based on synchronous updating. In the classic GL, the state of the cells is updated using the states of neighbors at the same time step. Information is transmitted without delay or noise in such a synchronous GL, and its transitions are deterministic. Therefore, logic gates in a synchronous GL are fragile to perturbations and cannot maintain their behavior in environments with noise.

To address this issue, probabilistic logic gates are constructed in an AGL using perturbations. Information transmission, such as the interaction of molecules in the real world, is asynchronous, containing delay and noise [6]. The AGL is the application of asynchronicity to update the states of the cells. Specifically, the AGL is the normal GL with an asynchronous rate p_{async} , a probability that the states of the cells will not be updated. There are many studies about such asynchronous cellular automata [7–9].

Furthermore, in this study, perturbations were introduced into the GL system by stochastically reversing the states of cells. In cellular automaton research, perturbations are used to prove the robustness of the system, often by adding fluctuations to a steady state externally and observing their effects [10–13]. This evaluation assumes that perturbations are external to the structure of the system and that the perturbations and the structure are in opposition [14]. However, this assumption fails to capture the possibility that perturbations are intrinsic to the structure. This research aimed to perform operations with logic gates

based on a structure that does not exclude perturbations but rather is formed by internal perturbations. By making perturbations intrinsic to logic gate operations, it will be possible to approach robust computations performed in the real world with noise.

To determine the conditions under which logic gates are formed, a property of phase transitions was used. Several phase transitions with respect to asynchronous rates have been found so far [15]. One of them, which is related to this study, is a phase transition between the frozen phase and unfrozen phase. The frozen phase is the phase in which the GL system remains unchanged, with no cells in state 1 or only fixed objects or oscillators. In contrast, a phase in which the system continues to change after a certain time is called an unfrozen phase. As the asynchronous rate is increased, the GL system transitions from a frozen phase to an unfrozen phase near the critical point $p_{\text{async}} = 0.1269$ [16].

Based on the above, the difference between the phases shown in Figure 1 was the focus of this study. These patterns belong to the unfrozen phase and continue their transitions, showing similar patterns. The maze-like pattern (right column) is known as the labyrinth phase (LP) [13], and some phase transitions about the LP have been found so far [12,17]. However, the critical points of these phase transitions belong to the frozen phase, and the difference between the phases in the unfrozen phase has not been found. Therefore, to investigate the difference in these phases, a new definition is necessary.

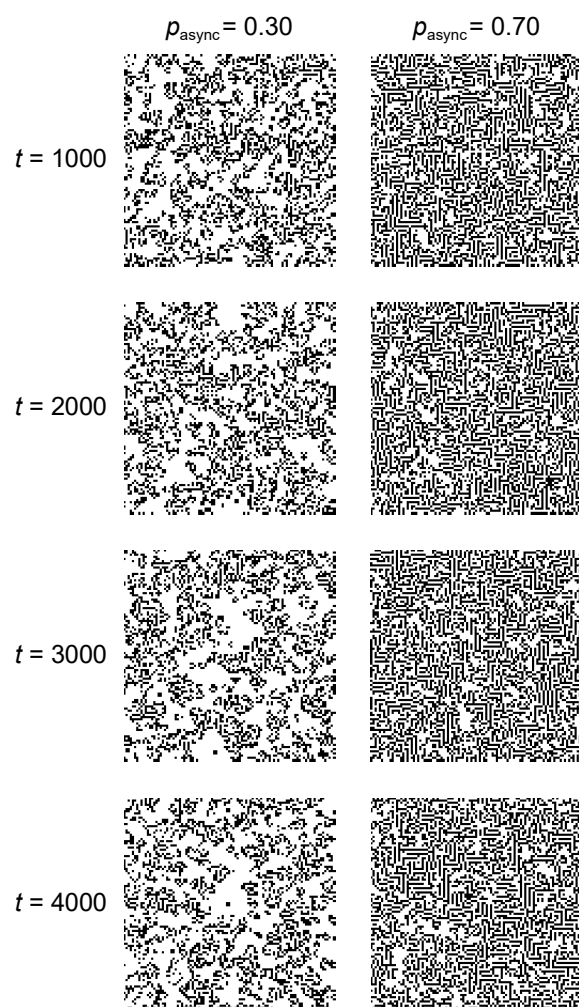


Figure 1. AGS systems. The systems show different patterns at $p_{\text{async}} = 0.30$ (left column) and $p_{\text{async}} = 0.70$ (right column). These systems continue the transition as in rows 1 to 4.

2. Materials and Methods

The maze-like pattern was defined as a “connected state” in this study. A connected state is determined by the following process:

1. A cell in state 1 at edge 1 of the system is set to x ;
2. If there is a cell in state 1 in the Moore neighborhood of x , then set it to x ;
3. Repeat process 2;
4. If x is at edge 2 of the system, then edge 1 and edge 2 are connected, and the system is determined to be in a connected state.

Figure 2 shows an example of a system in a connected state. The red cells are cells that have ever been set to x . This “path” runs from the upper edge to the lower edge of the system. This means that these edges are connected, and this system is in a connected state.

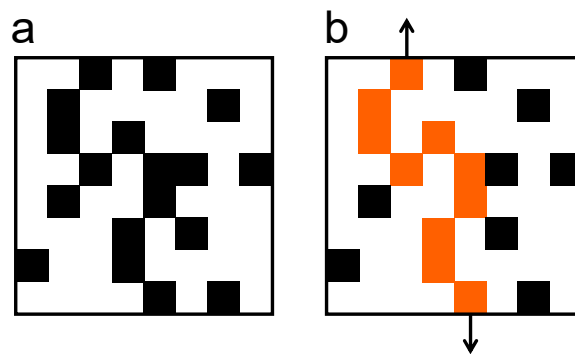


Figure 2. An example of a system in a connected state. (a) GL system. (b) This system is determined to be in a connected state because the “path” shown in red runs from one edge to another edge of the system.

In our experiments, perturbations were added by flipping the states of cells at the perturbation rate p_{noise} . The AGL, perturbation, and evaluation of connectivity were performed by the following program:

For each cell at site (i, j)

$$\text{temporary state}_{i,j}^{t+1} = \text{state}_{i,j}^t \quad \text{with probability } p_{\text{async}}$$

$$\text{temporary state}_{i,j}^{t+1} = f(\text{state}_{i,j}^t, \text{sum}_{i,j}^t) \quad \text{with probability } 1 - p_{\text{async}}$$

For each cell at site (i, j)

$$\begin{aligned} \text{If temporary state}_{i,j}^{t+1} = 0 \text{ then} \quad & \text{state}_{i,j}^{t+1} = 1 \text{ with probability } p_{\text{noise}} \\ & \text{state}_{i,j}^{t+1} = 0 \text{ with probability } 1 - p_{\text{noise}} \end{aligned}$$

$$\begin{aligned} \text{If temporary state}_{i,j}^{t+1} = 1 \text{ then} \quad & \text{state}_{i,j}^{t+1} = 0 \text{ with probability } p_{\text{noise}} \\ & \text{state}_{i,j}^{t+1} = 1 \text{ with probability } 1 - p_{\text{noise}} \end{aligned}$$

Evaluate whether the edges are connected

Repeat until T_{max}

where

$\text{state}_{i,j}^t$ is the current state : 0 or 1.

$\text{temporary state}_{i,j}^{t+1}$ is not the actual state of the cell. It is used to add perturbations and determine the next state.

$$\text{sum}_{i,j}^t = \sum_{m=-1, n=-1}^{m=+1, n=+1} \text{state}_{i+m, j+n}^t - \text{state}_{i,j}^t$$

$$\begin{array}{ll}
f(0, 0) = 0 & f(1, 0) = 0 \\
f(0, 1) = 0 & f(1, 1) = 0 \\
f(0, 2) = 0 & f(1, 2) = 1 \\
f(0, 3) = 1 & f(1, 3) = 1 \\
f(0, 4) = 0 & f(1, 4) = 0 \\
f(0, 5) = 0 & f(1, 5) = 0 \\
f(0, 6) = 0 & f(1, 6) = 0 \\
f(0, 7) = 0 & f(1, 7) = 0 \\
f(0, 8) = 0 & f(1, 8) = 0
\end{array}$$

Connectivity is evaluated by the following program:

For each cell at site (i_0, j_0) on edge 1,

i.e., $i_0 = 1, i_0 = N, j_0 = 1$, or $j_0 = N$,

1. Initialize the path list;
2. If state $_{i_0, j_0} = 1$, then $i_{\text{order}} = i_0, j_{\text{order}} = j_0$ add tuple (i_0, j_0) to the path list;
3. If state $_{i_{\text{order}}+m, j_{\text{order}}+n} = 1$ and $(i_{\text{order}} + m, j_{\text{order}} + n)$ is not included in the path list, then $i_{\text{order}} = i_{\text{order}} + m, j_{\text{order}} = j_{\text{order}} + n$ add tuple $(i_{\text{order}}, j_{\text{order}})$ to the path list repeat process 3 recursively;
4. If the cell at site $(i_{\text{order}}, j_{\text{order}})$ is on edge 2, i.e., $i_{\text{order}} = 1, i_{\text{order}} = N, j_{\text{order}} = 1$, or $j_{\text{order}} = N$, then edge 1 and edge 2 are connected

where

N is the system size

If $i = 1$, then the edge is on the left side

Or if $i = N$, then the edge is on the right side

If $j = 1$, then the edge is on the lower side

Or if $j = N$, then the edge is on the upper side

$$-1 \leq m \leq +1, -1 \leq n \leq +1$$

$$1 \leq i_{\text{order}} + m \leq N, 1 \leq j_{\text{order}} + n \leq N$$

3. Results

First, the existence of the phase transition of connectivity was examined. Figure 3 shows the percentage of trials in which the top and bottom edges and the left and right edges were simultaneously connected at least once by $T_{\text{max}} = 10,000$. No perturbations were added. The system was 100 by 100 cells and had periodic boundaries. One hundred trials were made at each asynchronous rate. There were two phases: a connected phase in which connections were likely to occur and an unconnected phase in which connections scarcely occur. As the asynchronous rate is increased, the GL system transitions from the connected phase to the unconnected phase near the critical point $p_{\text{async}} = 0.350$.

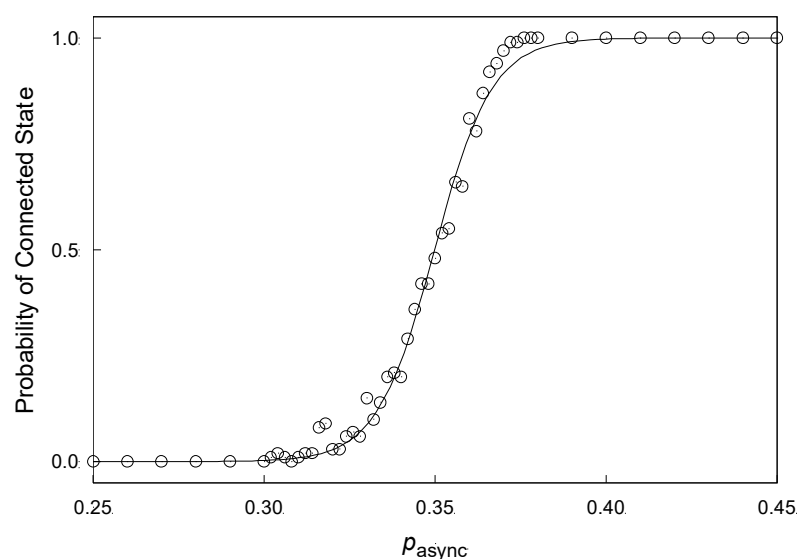


Figure 3. Phase transition between the connected phase and unconnected phase with respect to the asynchronous rate. The plots are data from experiments, and the curve is a sigmoid function fitted to the data.

Then, the effect of perturbations on the phase transition was examined. Figure 4 shows the connectivity at various perturbation rates. Five different perturbation rates were used. In each graph, there are phase transitions between the connected phase and unconnected phase, such as when there were no perturbations ($p_{\text{noise}} = 0.00\%$). The critical point decreased as the perturbation rate increased. This property can also be confirmed by the parameters of the fitting function shown in Table 1. Hence, the results for $p_{\text{noise}} = 0.00\%$ and $p_{\text{noise}} = 0.01\%$ were very different from those for $p_{\text{noise}} = 2.00\%$ at approximately $p_{\text{async}} = 0.350$. Considering this difference in the phases, a logic gate model was designed.

Table 1. Parameters of the fitting functions.

p_{noise} (%)	a	b
0.00	118.973	0.350
0.01	116.782	0.349
0.10	113.975	0.346
0.50	95.467	0.326
1.00	95.949	0.305
2.00	86.398	0.265

A fitting function is a sigmoid represented by $\frac{1}{1+\exp[-a(x-b)]}$.

In this study, the GL system was taken as an analogy with electric circuits; a connection of the edges of the system was read as an energization of the circuit. As mentioned above, simply put, if perturbations are added to the system, then connections occur, and if no perturbations are added, then no connections occur at certain values of the asynchronous rate. Then, a correspondence can be made with electric circuits that energize if there is a conductor and do not energize if there is no conductor. Since the connection is mapped to the energization, the presence/absence of perturbations can be mapped to the presence/absence of a conductor.

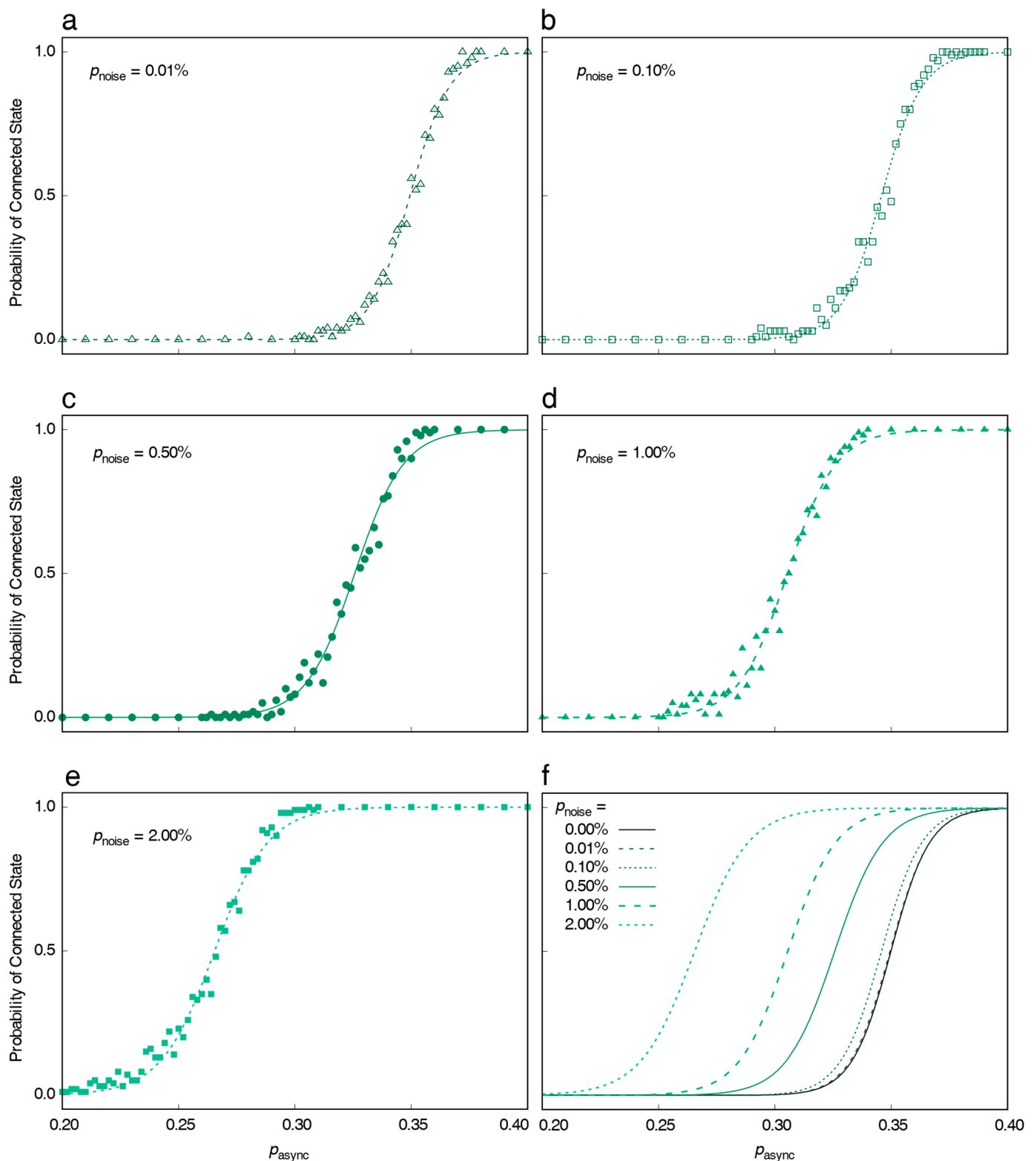


Figure 4. Phase transitions at various perturbation rates. (a–e) Plots of the experimental data and curves of the fitting functions. (f) Fitting functions for all perturbation rates.

Figure 5 represents our probabilistic logic gate model. If the input is 1, then perturbations are added to the input area at probability p_{noise} during a trial. If the input is 0, then no perturbations are added to the area. If connections occur during a trial, the output is 1. Otherwise, the output is 0. Note the arrangement of the input areas in the model.

When they are viewed horizontally, they are in parallel. If viewed vertically, they are in series. That is, when the connection between the left edge and right edge of the system is to be output, the system is a parallel circuit, as shown in Figure 6. When the connection between the upper edge and lower edge of the system is to be output, the system is a series circuit.

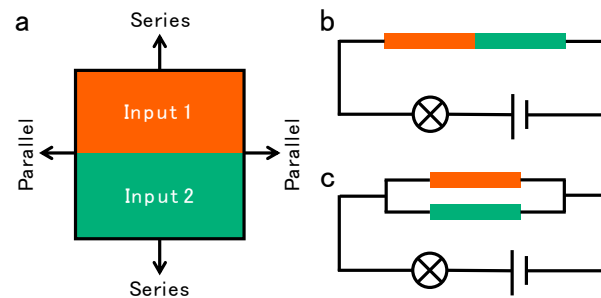


Figure 5. (a) Logic gate model. The arrangement of its input areas is the same as that of the conductors in (b) a series circuit when viewed vertically and that of (c) a parallel circuit when viewed horizontally.

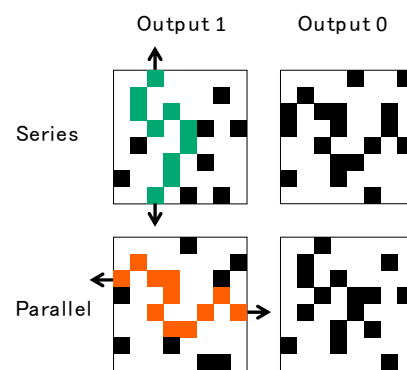


Figure 6. Examples of outputs in series and parallel circuits.

In electric circuits, if the inputs are to be the presence or absence of conductors and the output is to be energized or not, the series circuit becomes an AND gate and the parallel circuit becomes an OR gate. Therefore, experiments were performed to determine whether the logic gate model could be used to implement AND and OR gates.

Figure 7 shows the output of the model. Perturbations were added at $p_{\text{noise}} = 2.00\%$ if the input was 1. The output was set to 1 if it was connected at least once by $T_{\text{max}} = 5000$ and 0 otherwise. The system was 200 by 200 cells and had periodic boundaries. One hundred trials were made at each asynchronous rate. In both circuits, the outputs changed from 0 to 1 as the asynchronous rate increased from 0.30 to 0.40. In the middle region, the gap in the outputs between the two circuits became larger. In particular, at $p_{\text{async}} = 0.350$, fewer than 30% of the trials returned an output of 1 in the series circuit, whereas more than 70% of the trials returned an output of 1 in the parallel circuit. This is an appropriate region in which to establish both the AND gate and OR gate at the same asynchronous rate.

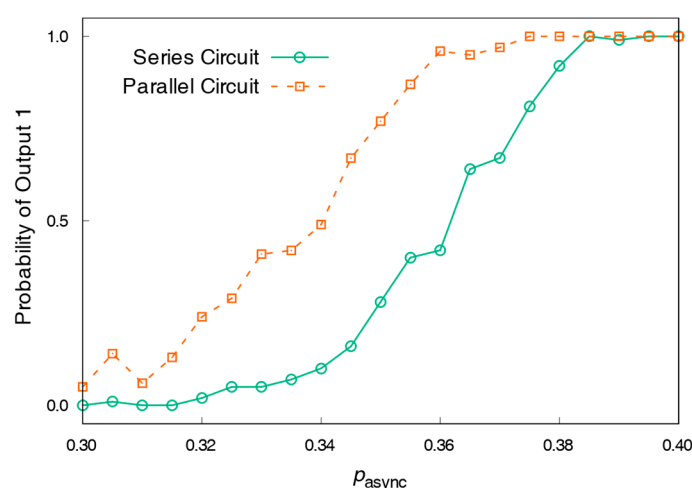


Figure 7. Outputs for the input (1, 0) in the series and parallel circuits of our model.

Figure 8 represents the outputs of two circuits for all input combinations at $p_{\text{async}} = 0.350$. The behavior of the probabilistic AND gate was obtained in a series circuit, while the behavior of the probabilistic OR gate was obtained in a parallel circuit.

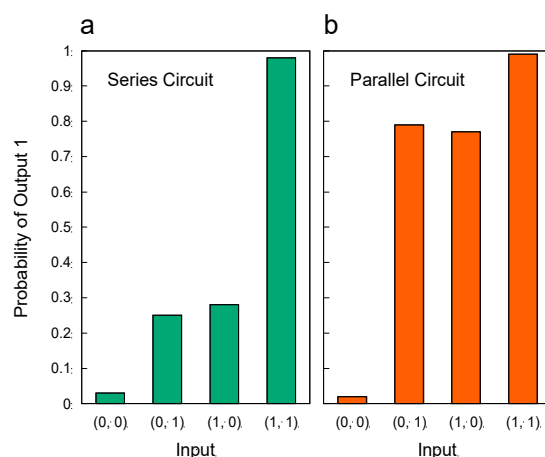


Figure 8. Outputs of the logic gate model at $p_{\text{async}} = 0.350$. (a) The series circuit behaved as a probabilistic AND gate, and (b) the parallel circuit behaved as a probabilistic OR gate.

4. Discussion

The aim of this study was to construct a robust logic gate using a GL. First the existence of a phase transition between connected and unconnected phases was discovered and the effect of perturbations was investigated. Then, the logic gate model was designed by using an electric circuit as an analogy. In our model, a probabilistic AND gate and probabilistic OR gate are implemented in one system around the critical point of the phase transition. The asymmetric behaviors in the vertical and horizontal directions are due to the arrangement of the input areas and the property of the phase transition. For applying this model to advanced calculations, there are two limitations. The first is the number of time steps and the computational cost required to return the output. In the model, the recursive function was called for 5000 or 10,000 time steps. Second, as the size of the system changes, the probability that the system becomes a connected state also changes; the edges are more difficult to be connected in a larger system. These constraints must be taken into account, especially when expanding the system.

In recent years, logic circuits based on two-dimensional materials have been studied in the field of nanotechnology [18]. The work of Liu et al. is similar to this work in that both AND and OR gates can be implemented with a single device, the behavior of the

device can be changed depending on the parameters, and the output is stable for specific perturbations. In comparison with Liu et al.'s work, the weak point of our model is that it does not have temporal properties. In other words, the perturbation rate p_{noise} and the rules of the GL do not change during a trial, making learning and memorization impossible. To address this issue, methods such as changing the local rule of cellular automata [19] and misrecognizing states [20] need to be considered.

On the other hand, what this research emphasizes is a method of using perturbations. In conventional logic gates in the GL, perturbations are considered to be external to the structure, and there are no perturbations in it. However, in the real world, are structure and perturbation in opposition? It is known that in groups of living creatures, even when there are no external enemies, individuals move within the group and continuously change their relative positions [21]. This perturbation is called “inherent noise”, and it has been suggested that inherent noise is an essential element for the formation of swarms [22]. The structure of the swarm does not oppose perturbations. Rather, it is destroyed and generated by perturbations and becomes dynamic. The structure of the connection in this study is also doubly affected by the perturbations of asynchronous updating and flipping states, which symmetrically entail the operation successful and unsuccessful. This symmetrical double constraint of perturbations is the reason our model works correctly. Our model is expected to be applied to explain and model the robust behavior of individuals in the swarm and fluctuating behavior when not in the swarm [23]. It can be considered a starting point for reconstructing the relationship between the two terms of structure and perturbation in the computation. The realization of inherent noise in cellular automata in a clearer form will be the subject of further study.

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