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Generalized Fuzzy Linguistic Bicubic B-Spline Surface Model for Uncertain Fuzzy Linguistic Data

Mohd Syafiq Bidin [1], Abd. Fatah Wahab [1,*], Mohammad Izat Emir Zulkifly [2,*] and Rozaimi Zakaria [3]

1 Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu, Kuala Terengganu 21300, Malaysia
2 Faculty of Science, Department of Mathematical Sciences, Universiti Teknologi Malaysia, Johor Bahru 81310, Malaysia
3 Faculty of Science and Natural Resources, Universiti Malaysia Sabah, Kota Kinabalu 88450, Malaysia
* Correspondence: fatah@umt.edu.my (A.F.W.); izatemir@utm.my (M.I.E.Z.)

Abstract: A fuzzy linguistic data set that is uncertain is difficult to analyze and describe in the form of a smooth and continuous generic figure. Therefore, the study aims to develop a new model of a B-spline surface using a different approach of a crisp and fuzzy linguistic point relation with three types of linguistic function: low L, medium M_i and high H. These linguistic functions are defined first to introduce the fuzzy linguistic point relation. Then, a new algorithm of the fuzzy linguistic bicubic B-spline surface model is presented to convert fuzzy linguistic data into fuzzy linguistic control points. In addition, a numerical example of fuzzy linguistic data is considered at the end of this study to visualize the suggested model. Thus, the relation between the fuzzy linguistic data points can be analyzed to present another area of knowledge in which symmetry phenomena occur. The symmetry here plays an important role in solving the uncertain fuzzy linguistic data problem by using the suggested model.

Keywords: B-spline surface; fuzzy linguistic point relation; fuzzy linguistic data; fuzzy linguistic control point; fuzzy linguistic B-spline

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1. Introduction

Real data from real phenomena and scenarios are difficult to describe and analyze using existing methods. There are even more problems when they involve linguistic terms that are uncertain by nature where they carry their own meaning depending on the individual's perception [1]. The use of linguistic terms in daily activities has various interpretations and assumptions. For example, the intonation of the use of sentences or words in conversation sometimes has a different meaning. Therefore, the relationship between linguistics and the uncertainty inherent in the concept of fuzzy sets is inseparable [2]. To deal with uncertain information, Zadeh [3] proposed the concept of the fuzzy set, which has been widely used in different fields. Then, it was followed by the concept of linguistic variables and its application, also proposed by Zadeh [4–6].

Language is one of the primary expressions of human intelligence. What makes language so difficult for artificial intelligence (AI) is its ambiguity, which refers to the possibility of interpreting linguistic units in different ways, and ubiquitous quality in natural language [7]. In the twenty-first century, the activities of human life have been revolutionized by the widespread application of the internet, and the capabilities of computing systems and intelligence that are driving a new era of AI. A variety of intelligent human behaviors such as memory, emotion, perception, judgment, reasoning, recognition, proof, communication, understanding, design, thinking, learning, creating and others, can be realized artificially by using machines or building systems and networks. However, these are built on certainty or precision and are very limited by their formal axiom systems, which cannot simulate the uncertainty of the human thought processes due to their precision [8].

The problems of linguistic terms can be translated into the form of linguistic data sets and need to be solved using appropriate models to produce clear visuals in the form of curves and surfaces. This method requires the acquisition of real data from the real world to be modeled with geometric modeling. Then, the data can be simulated and analyzed for the research conducted. However, the developed model will not lead to any change if the problem of data sets that are fuzzy and have uncertainty is not resolved first. Existing geometric functions in the field of geometric modeling cannot model a data set into curves and surfaces when there is an ambiguous property of the data set obtained. Geometric modeling can only be performed when the data set has full membership, while data sets that do not have full membership are ignored [9].

To solve this problem, a new model needs to be proposed using a fuzzy linguistic approach and combined with the spline function in geometric modeling. This research paper will define a fuzzy linguistic point relation where two sets of fuzzy linguistic data are connected to obtain the exact membership value, which can then form a clear and smooth surface. From the result of this surface, the process of evaluation and analysis can be carried out more easily and smoothly. Research in this field is still new, where the study of curves has only been conducted by Hussain et al. [1,2] and Wahab and Hussain [10]. Therefore, this paper presents another area of knowledge in which symmetry plays an important role in solving the problem of fuzzy linguistic data that have an uncertain nature. The process of developing a new model of the fuzzy linguistic bicubic B-spline surface will be discussed in the following sections.

2. Introduction to B-Spline Function

In 1946, Schoenberg [11] introduced the B-spline or basis spline for the uniform knot cases. Then, Boor [12] began using the B-spline function as a tool to present geometry curves and introduced a recursive assessment of the B-spline known as De Boor's algorithm in 1960. He became one of the most influential proponents of the B-spline in approximation theory. Dempski [13] and Farin [14] discussed that through the representations of curves and surfaces, a set of data can be modeled with B-spline functions. A curve approximates a set of control points without necessarily passing through any control points when polynomials are fitted to the path. Another method for geometric modeling is interpolation where a smooth curve is constructed through each data point, according to Salomon [15]. A B-spline surface can be obtained by taking a bidirectional net of control points, two knot vectors and the product of the univariate functions as follows (Piegl and Tiller [16])

$$Bs(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} P_{i,j} N_{i,p}(u) N_{j,q}(v) \tag{1}$$

where $N_{i,p}(u)$ and $N_{j,q}(v)$ are the basic function of crisp B-splines with degree p and q in parameters u and v, respectively.

The B-spline surface is a tensor product basis function $N_{i,p}(u)N_{j,q}(v)$. The set of control points is often referred to as the control network, and the value of u and v is between 0 and 1. Both u and v are vectors that determine the intersection direction of the B-spline curve for the formation of the B-spline surface in the form of rows, i and columns, j. Hence, the B-spline surface maps a square unit to a surface patch, as shown in Figure 1.

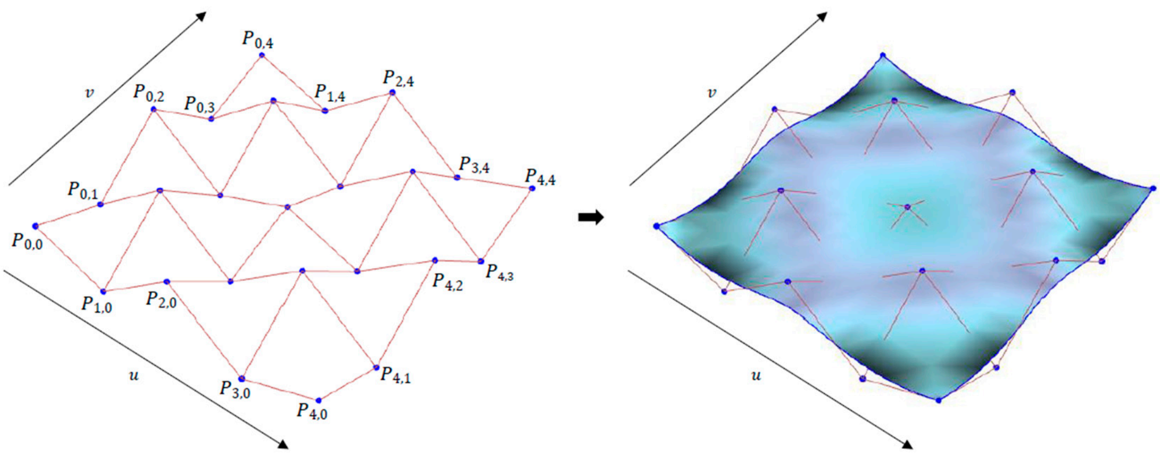


Figure 1. Formation of control network from crisp control points to B-spline surface.

3. Fuzzy Set Approach to B-Spline

Using fuzzy set theory (Zadeh, [3]), Wahab et al. [17,18] introduced a new approach of the fuzzy B-spline surface model using fuzzy control points, defined as

$$\widetilde{Bs}(s,t) = \sum_{i=0}^{n}\sum_{j=0}^{m} \widetilde{P}_{i,j} N_{i,p}(s) N_{j,q}(t) \tag{2}$$

where $N_{i,p}(s)$ and $N_{j,p}(t)$ are the B-spline basic functions with degree p and q, respectively, in parameters s and t, of which the value is between 0 and 1. Each knot vectors specified must qualify conditions $s = n + p + 1$ and $t = m + q + 1$. Therefore, the B-spline surface is a surface in the form of a tensor product. $\widetilde{P}_{i,j} = \left\langle \widetilde{P}_{i,j}^{\leftarrow}, P_{i,j}, \widetilde{P}_{i,j}^{\rightarrow} \right\rangle$ is the (i,j)th fuzzy control point in row i and column j that will form the fuzzy control network for surface patch formation.

Figure 2a shows an example of bicubic shape of the fuzzy B-spline surface model with fuzzy control points while without fuzzy control points in Figure 2b. The model is formed by 16 fuzzy control points that are also known as fuzzy data points. These fuzzy control points form the fuzzy control network that illustrate this surface model.

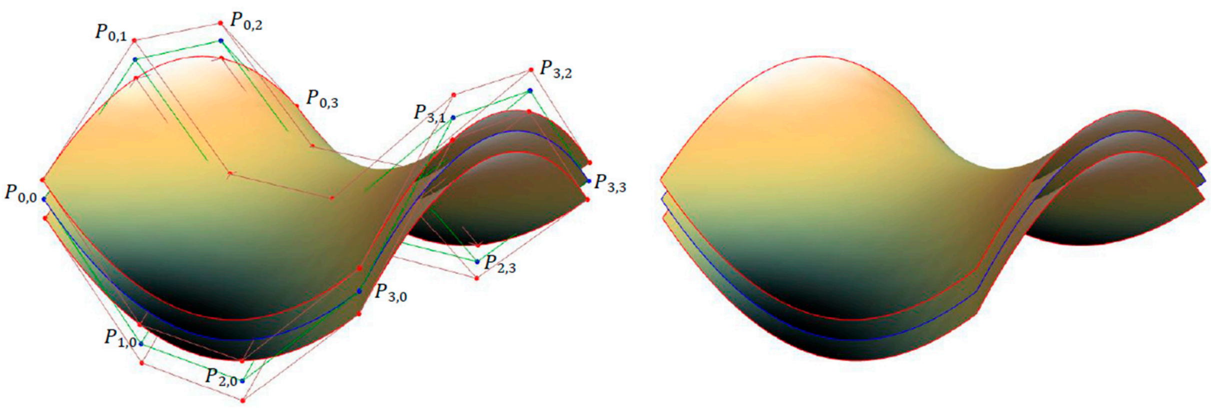


Figure 2. Example of bicubic fuzzy B-spline surface models (**a**) with fuzzy control points and (**b**) without fuzzy control points.

The objective of this study is to introduce a new model of the fuzzy linguistic bicubic B-spline surface based on fuzzy set theory and the B-spline function in geometric modeling.

The use of fuzzy set theory can solve problems in modeling uncertainty data, while the model was developed geometrically through the production of curves and surfaces using the B-spline function corresponding to the study conducted to obtain clear and smooth visualization results. Thus, a new model was developed with a combination of all these concepts, which, subsequently, can be applied to linguistic data from the real world for the purpose of forecasting, data modeling and others.

This study is further structured as follows. Section 4 reviews the existing studies on fuzzy sets and linguistics. Then, Section 5 presents the introduction of fuzzy linguistic data processing and the definition of fuzzy set theory. Section 6 defines linguistic variables with an example of Body Mass Index (BMI) while Section 7 presents new definitions for fuzzy linguistic point relations. The fuzzy linguistic model with its algorithm is proposed in Section 8. Section 9 discusses a numerical example and visualization of the model. Lastly, conclusions and recommendations for further research are depicted in Section 10.

4. Literature Review

Linguistics is the scientific analysis of language, as opposed to the comprehensible but impressionistic analysis of language. Semantic studies examine how a meaning is managed in its journey from human thought to a word or sentence [19]. Kracht [20] said that language is a set of signs defined as a quadruple $(\pi, \mu, \lambda, \sigma)$, where π is its exponent (or phonological structure), μ its morphological structure, λ its syntactic structure and σ its meaning (or semantic structure). A text is certainly more than a sequence of sentences, and the study of discourse is indeed a very important one. Furthermore, sentences are so complicated that we need a long time to study them. In addition to that, Aikhenvald [21] discussed the properties of evidence systems that are related to discourse, and also the details of cross-linguistic study that will provide valuable insights into the nature of human cognition. Maynard [22] studied Japanese primarily from the perspective of discourse and conversation analysis by consistently analyzing real-life Japanese, part of contemporary Japanese culture, in a constant dynamic flow of being produced, consumed and interpreted. In particular, several studies have also been conducted to investigate concepts roughly corresponding to anger in languages such as Kovecses [23,24], Soriano [25], King [26] and Matsuki [27]. In normal situations, uncertain data are omitted from the data set regardless of their effect on any outcome. Therefore, the evaluation and analysis process of the visualized data will be incomplete. The data set should be filtered if there is an element of uncertainty so that it can be used to generate a model of a curve or surface to be studied.

To overcome this problem, linguistic fuzzy sets are used, which are a generalization of fuzzy set theory from Zadeh [3] and further extended in [4–6]. The concept of linguistic variables has been widely used and has become the main reference of researchers. The study discussed about linguistic variables characterized by a quintuple $(\mathcal{X}, T(\mathcal{X}), U, G, M)$, where $\mathcal{X}$ is the name of the variable, $T(\mathcal{X})$ is the term set of $\mathcal{X}$ (the collection of its linguistic values), U is a universe of discourse, G is a syntactic rule that generates the term in $T(\mathcal{X})$ and M is a semantic rule that associates with each linguistic value X and its meaning $M(X)$ where M denotes a fuzzy subset of U. Bonissone [28] proposed a solution for the problem of how to associate labels with unlabeled fuzzy sets based on semantic similarity (linguistic approximation) and also how to perform arithmetic operations with fuzzy numbers. In addition to that, Delgado et al. [29] defined an aggregation operation between linguistic labels based on their meaning that can be performed without any reference to this semantic representation. Therefore, it is very useful from a computational point of view because it can be implemented as a simple table or procedure. Bordogna and Pasi [30] introduced a solution to the problem of numerical query weights by defining an existing weighted Boolean retrieval model and a linguistic extension formalized in fuzzy set theory, where the numerical query weights are replaced by linguistic descriptors that determine the importance level of the terms, while Khoury et al. [31] proposed the use of a hybrid statistical fuzzy methodology to represent subject–verb–object triplets that calculates the membership of subject–verb pairs and verb–object pairs in various domains.

To deal with an unbalanced set of linguistic terms, Herrera et al. [32] presented a fuzzy linguistic methodology by developing a representation model for unbalanced linguistic information that uses the concept of linguistic hierarchy as the basis of representation. This methodology is built on the concept of linguistic hierarchy and on a 2-tuple fuzzy linguistic representation model that also consists of representation algorithms and computational approaches for unbalanced linguistic information. The linguistic application of fuzzy set theory to natural language analysis formulated a working definition of fuzziness that can be used in future research on fuzzy linguistics clarified by Ma [33]. Ramos-Soto and Pereira-Fariña [34] proposed a new understanding of interpretability in the context of the linguistic description of fuzzy data by approaching this concept from a natural language generation (NLG) perspective as opposed to focusing on the classical notions of interpretability for fuzzy systems and linguistic summaries or descriptions on data (LDD). In particular, many researchers have also used the fuzzy linguistic approach in multi-criteria decision making such as Tong and Bonissone [35], Herrera and Martinez [36], Rodriguez et al. [37,38], Wang et al. [39] and Nguyen et al. [40]. However, in order to evaluate and analyze a model, linguistic fuzzy data sets need to be visualized first as a curve or surface model.

The modeling of fuzzy linguistic data sets can be demonstrated through the construction of curves and surfaces using existing functions in computer-aided geometric design (CAGD). Geometric modeling is a process when a set of data is translated into a real 2-dimensional or 3-dimensional form to produce a smooth and clear visualization. Among the studies that have been conducted is that by Jaccas et al. [41,42] who discussed a fuzzy logic approach to curve and surface design in the context of CAGD. Gallo and Spagnuolo [43] and Gallo et al. [44] presented a new approach for modeling spatial data, which takes into account the uncertainty that may be associated with the original data set. The resulting model has explicitly embedded uncertainty properties that allow users to visualize and reason about the reconstructed model at different levels, as well as use different model layers according to the accuracy required by the specific analysis. In addition to that, Anile et al. [45] introduced an innovative approach to model fuzzy and sparse data by generalizing B-spline to fuzzy B-spline where its power depends on the possibility of being used as an approximation function for both fuzzy and crisp data. In addition, Wahab et al. [17,18] and Wahab and Ali [46] used fuzzy set theory and its properties through the concept of fuzzy numbers to introduce the fuzzy control point for fuzzy curve and fuzzy surface models, which can then solve the uncertainty problem in CAGD. This study was later continued by Zakaria et al. [47], Zakaria and Wahab [48,49], Karim et al. [50] and Bakar et al. [51].

Further studies in the field of fuzzy set theory and CAGD have been conducted, specifically by Zulkifly and Wahab [52,53] and Zulkifly et al. [54,55], by introducing a new concept of intuitionistic fuzzy sets with geometric modeling. Shah and Wahab [56] and Shah et al. [57] introduced fuzzy topological digital space concepts with their properties. Zakaria et al. [58,59], Wahab et al. [60] and Adesah et al. [61] extended the study of geometric modeling with fuzzy set theory to a type-2 fuzzy set. In the field of fuzzy linguistics, Hussain et al. [1,2] introduced fuzzy linguistic control points (FLCP), which can be used to generate spline models through linguistic terms. The FLCP theorem has been redefined using modifiers or linguistic hedges to solve the uncertainty problem of linguistic data in geometric modeling. The FLCP was blended with spline basis functions to produce several spline models characterized by fuzzy linguistics. Then, Wahab and Hussain [10] introduced new fuzzy spline models such as fuzzy linguistic Bézier and fuzzy linguistic B-spline by using the definition of fuzzy linguistic control points. The control points are redefined through the fuzzy linguistic approach to produce a new set of control points with linguistic terms. Bidin et al. [62] introduced a new approach for the fuzzy linguistic point relation that can generate a new model called as fuzzy linguistic cubic B-spline curve model. The presented method shows that the model can be generated using directive linguistic terms and functions.

However, previous studies on fuzzy linguistics blended with spline functions only involve a curve form, while a surface form has not yet been extensively studied. Therefore, this paper discusses and introduces a new model of fuzzy linguistic data visualized with spline functions in geometric modeling. There are three fuzzy linguistic variables, which are low L, medium M_i and high H, that are discussed in the section to define the fuzzy linguistic point relation. Then, a new algorithm is introduced to build a new model called the fuzzy linguistic bicubic B-spline surface model. Lastly, this new model is visualized using numerical examples of linguistic data sets with randomly selected membership values. From the results of visualization, the evaluation and analysis process is expected to be easier to perform and provide great benefits in various fields, especially the problem of ambiguity in the expression of human intelligence.

5. Preliminaries

Using a fuzzy linguistic set [63], we introduce the fuzzy linguistic control points using the fuzzy linguistic point relation. From any real phenomena or scenarios, we will obtain real data, but due to many factors, the data collected come with uncertainty. Therefore, a linguistic function is used to measure the real data into a linguistic data set. Then, the linguistic data set will visualize various curve (2-D) and surface (3-D) images as in the process described in Figure 3.

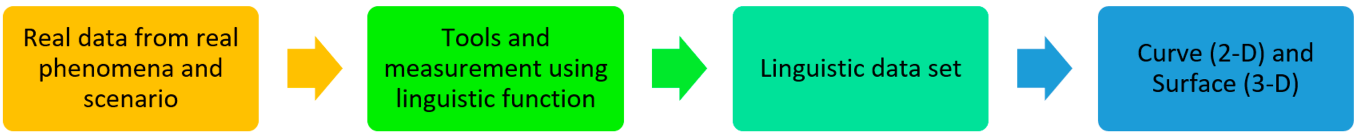


Figure 3. Fuzzy linguistic data process.

Several definitions related to the concepts of fuzzy sets, fuzzy numbers and fuzzy control points are briefly discussed as follows (Wahab and Husain [10]):

Definition 1. *Let X be a non-empty set. A fuzzy set A in X is characterized by a membership function $\mu_A : X \to [0,1]$ and $\mu_A(X)$ is called as the degree of membership of element x in A for $x \in X$ can be defined as*

$$\mu_A(X) = \begin{cases} 1 & if \;\; x \in A \\ c & if \;\; x \widetilde{\in} A \\ 0 & if \;\; x \notin A \end{cases} \tag{3}$$

where $\mu_A(x) = 1$ is a full membership function, $\mu_A(x) = c$ is a non-full membership function, $\mu_A(x) = 0$ is a non-membership function and c is the membership grade value between 0 and 1.

A fuzzy set can also be described as a set of ordered pairs and can be written as $\widetilde{A} = \{x, \mu_A(x)\}$. If $X = \{x_1, x_2, \ldots, x_n\}$ is a finite set and $\widetilde{A} \subseteq X$, then the fuzzy set A can be written as

$$\widetilde{A} = \{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), \ldots, (x_n, \mu_A(x_n))\} \subseteq X \tag{4}$$

Definition 2. *Let A be a fuzzy set in the universe of discourse X and μ_A be the membership of A. Then, the α-cut A_α of A in X is defined as*

$$A_\alpha = \{x_i \subseteq X | \mu_A(x_i) \geq \alpha\} \tag{5}$$

where $\alpha \subseteq (0,1)$.

Definition 3. *Let a fuzzy set A on the real line be the set of all normal, convex, continuous data, and the membership function in the closed interval and the support $(A) = \{x|\mu_A(x)\rangle 0\}$ is limited.*

A generalized fuzzy number $\widetilde{A}$ with the membership function $\mu_{\widetilde{A}}(x),\ x \in R$ can be defined as

$$\mu_{\widetilde{A}}(x) = \begin{cases} l_{\widetilde{A}}(x), & a \le x \le b \\ 1, & b \le x \le c \\ r_{\widetilde{A}}(x), & c \le x \le d \\ 0, & otherwise \end{cases} \tag{6}$$

where $l_{\widetilde{A}}(x)$ is the left membership function of the increasing function $[a,b]$ and $r_{\widetilde{A}}(x)$ is the right membership function of the decreasing function $[c,d]$ such that $l_{\widetilde{A}}(a) = r_{\widetilde{A}}(d) = 0$ and $l_{\widetilde{A}}(b) = r_{\widetilde{A}}(c) = 1$.

If $l_{\widetilde{A}}(x)$ and $r_{\widetilde{A}}(x)$ are linear, then $\widetilde{A} = (a,b,c,d)$ is a trapezoidal fuzzy number. If $b = c$, then the trapezoidal fuzzy number is defined as triangular fuzzy number written as $\widetilde{A} = (a,b,d)$ or $\widetilde{A} = (a,c,d)$.

Definition 4. *Fuzzy set $\widetilde{P}$ in space of S is said to be a set of fuzzy control points if, and only if, for every α-level set chosen, there exist pointed, which is $P = \left\langle \widetilde{P}_i^{\leftarrow}, P_i, \widetilde{P}_i^{\rightarrow} \right\rangle$, in S with every P_i is a crisp point and membership function $\mu_P : S \to (0,1]$, which is defined as $\mu_P(P_i) = 1$.*

$$\mu_P(P_i^{\leftarrow}) = \begin{cases} 0 & if\ P_i^{\leftarrow} \notin S \\ c \in (0,1) & if\ P_i^{\leftarrow} \widetilde{\in} S \\ 1 & if\ P_i^{\leftarrow} \in S \end{cases} \text{ and } \mu_P(P_i^{\rightarrow}) = \begin{cases} 0 & if\ P_i^{\rightarrow} \notin S \\ c \in (0,1) & if\ P_i^{\rightarrow} \widetilde{\in} S \\ 1 & if\ P_i^{\rightarrow} \in S \end{cases} \tag{7}$$

where $\mu_P\left(P_i^{\leftarrow}\right)$ is the left membership grade value, $\mu_P\left(P_i^{\rightarrow}\right)$ is the right membership grade value and c is the membership grade value between 0 and 1.

Fuzzy set $\widetilde{P}$ can generally be written as

$$\widetilde{P} = \left\{ \widetilde{P}_i : i = 0,1,2,\ldots,n \right\} \tag{8}$$

where with $P = \left\langle \widetilde{P}_i^{\leftarrow}, P_i, \widetilde{P}_i^{\rightarrow} \right\rangle$ and $\widetilde{P}_i^{\rightarrow}$ being the left fuzzy control point, crisp control point and right fuzzy control point, respectively.

6. Linguistic Variables

A linguistic variable is defined as a variable whose values are words or sentences in a natural or artificial language. According to Zadeh [4], a linguistic variable is characterized by a quintuple $(\mathcal{X}, T(\mathcal{X}), U, G, M)$, where $\mathcal{X}$ is the name of the variable, $T(\mathcal{X})$ is the term set of $\mathcal{X}$ (the collection of its linguistic values), U is a universe of discourse, G is a syntactic rule that generates the term in $T(\mathcal{X})$ and M is a semantic rule that associates each linguistic value X with its meaning $M(X)$ where M denotes a fuzzy subset of U.

As an example from weight categories based on body mass index (BMI), let $X = \{x_i | x_i \text{ BMI}\} \ \exists\ A_i \subset X$ subject to $A_i = \{A_1, A_2, A_3, A_4\}$, where A_1, A_2, A_3 and A_4 are underweight, normal, overweight, obesity and severe obesity, respectively, namely as a linguistic model of BMI shown in Figure 4.

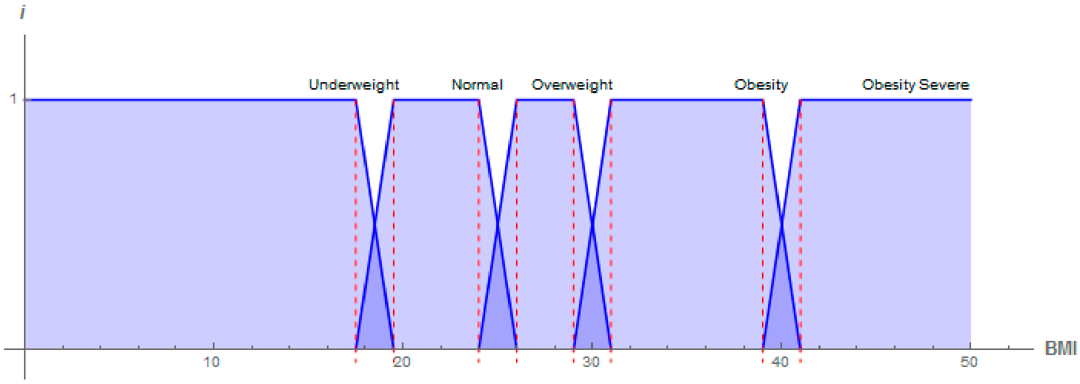


Figure 4. Linguistic model of BMI.

7. Fuzzy Linguistic Point Relation

There are three types of fuzzy linguistic functions: low L_i, medium M_i and high H_i, as shown in Figure 5.

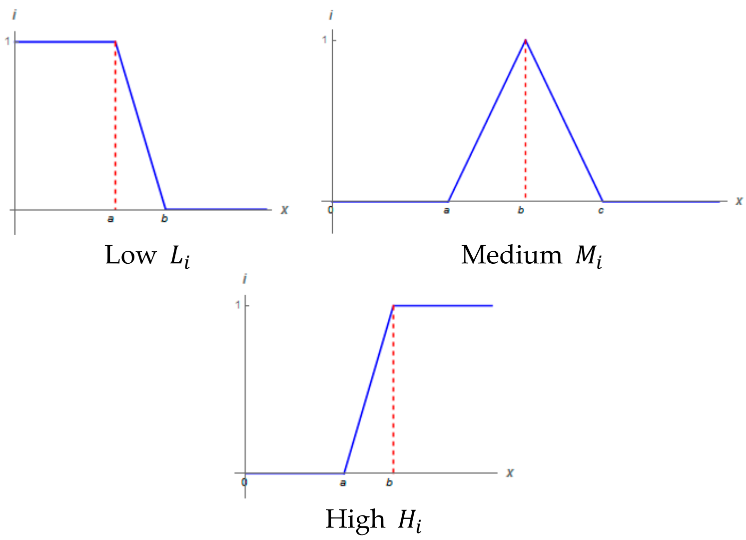


Figure 5. Three types of fuzzy linguistic function.

These fuzzy linguistic functions are also called linguistic behaviours where a, b and c are the elements of X that display the increase and decrease points for each function. Low L_i and high H_i have only one linguistic set, but there are probably more than one linguistic set for medium M_i, where linguistic set $LS = \{L, M_1, M_2, \ldots, M_n, H\}$ is shown in Figure 6. L is the one and only low linguistic set, M_1 is the first medium linguistic set, M_2 is the second medium linguistic set, M_n is the subsequent medium linguistic set and H is the one and only high linguistic set.

Referring to Figures 5 and 6, we can briefly state the definition for each linguistic function for low L_i, medium M_i and high H_i as follows (Bidin et al. [62]):

Definition 5. *Let low linguistic function L_i in domain $L \to [0,1]$ be defined by a linguistic set, then*

$$L_i(x) = \begin{cases} 1 & , 0 \le x \le a \\ \frac{b-x}{b-a} & , a \le x \le b \\ 0 & , x \ge b \end{cases} \tag{9}$$

where $L_i(x)$ is called the degree of membership of $x \in X$.

Definition 6. *Let medium linguistic function M_i in domain $M \to [0,1]$ be defined by a linguistic set, then*

$$M_i(x) = \begin{cases} 0 & , x \le a \\ \frac{x-a}{b-a} & , a \le x \le b \\ \frac{c-x}{c-b} & , b \le x \le c \\ 0 & , x \ge c \end{cases} \tag{10}$$

where $M_i(x)$ is called the degree of membership of $x \in X$.

Definition 7. *Let high linguistic function H_i in domain $H \to [0,1]$ be defined by a linguistic set, then*

$$H_i(x) = \begin{cases} 0 & , x \le a \\ \frac{x-a}{b-a} & , a \le x \le b \\ 1 & , x \ge b \end{cases} \tag{11}$$

where $H_i(x)$ is called the degree of membership of $x \in X$.

Definition 8. *Let X be a universal set of linguistics where $\forall x \in X$ and there exist linguistic functions $[L_i, M_i, H_i]$ such that there a set of ordered pairs $(x, L_i(x))$ or $(x, M_i(x))$ or $(x, H_i(x))$ when $L_i(x),\ M_i(x),\ H_i(x) \in [0,1]$*

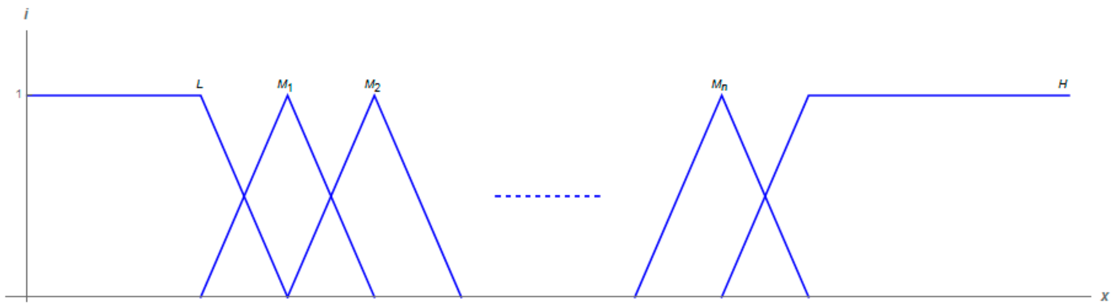


Figure 6. Fuzzy linguistic functions in the BMI linguistic model.

From Definition 5, the low linguistic function L_i in Figure 7 in accordance to (9) can be defined as $x_i \in X$ there $L_i(x_i) \in [0,1]$ exists such that $(x_i, L_i(x_i))$ where x_i is an element in X of the low linguistic function L_i.

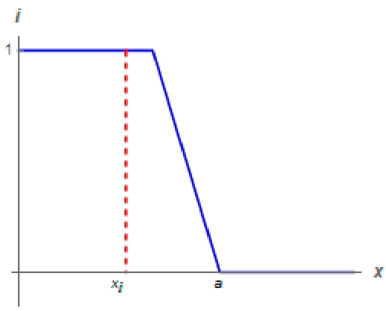


Figure 7. $x_i \in X$ of low linguistic function L_i.

Let X and Y be any collection of universal space, then $\widetilde{L_X} \times \widetilde{L_Y}$ is a fuzzy linguistic relation. Suppose that L_X and L_Y in Figure 8 are two low linguistic functions on $X = \{\ x_i\ : i = 0, 1, 2, \ldots, n\}$ and $Y = \{\ y_i\ : i = 0, 1, 2, \ldots, n\}$, respectively. Then

$$\widetilde{L_X} \times \widetilde{L_Y} = \left\{ ((x, y), \mu_{\widetilde{L_X} \times \widetilde{L_Y}}(x, y) \middle| \mu_{\widetilde{L_X} \times \widetilde{L_Y}}(x, y) \le 1) \right. \tag{12}$$

where $\mu_{\widetilde{L_X} \times \widetilde{L_Y}}(x, y)$ is the degree of membership of element $(x, y) \in X \times Y$.

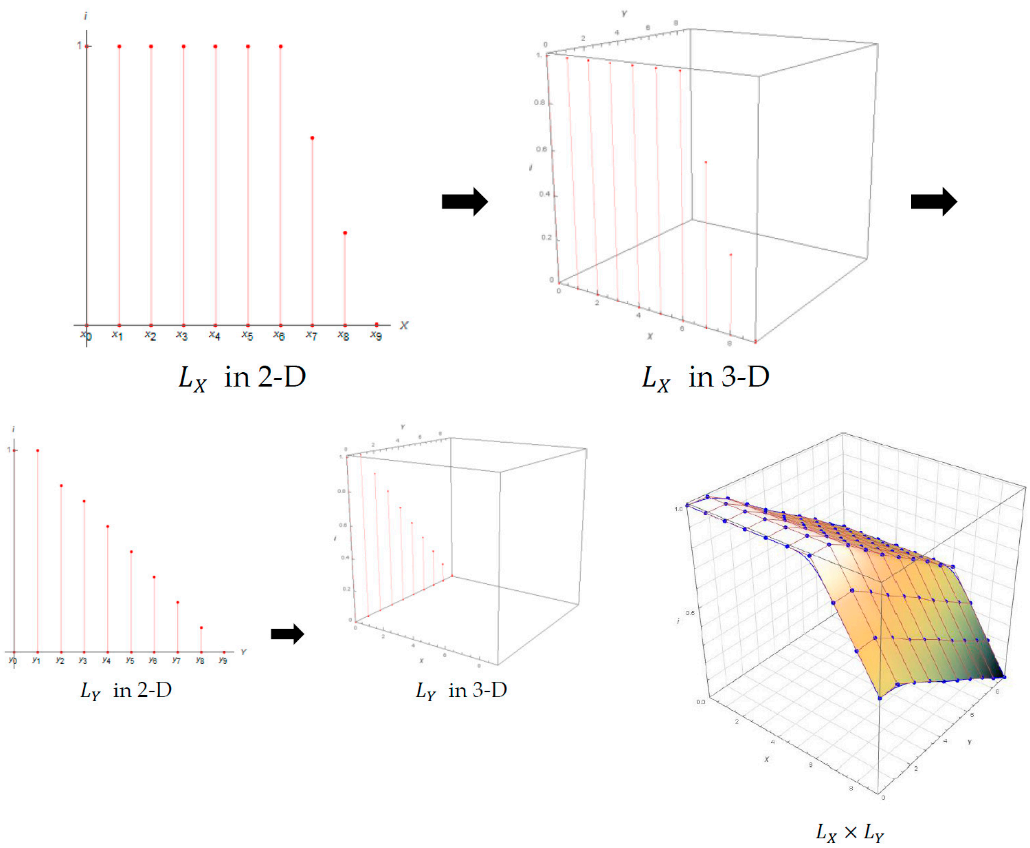


Figure 8. Fuzzy linguistic point relation of low linguistic functions L_X and L_Y.

From definition 6, the medium linguistic function M_i in Figure 9 in accordance to (10) can be defined as $x_i \in X$ and $M_i(x_i) \in [0,1]$ exists such that $(x_i, M_i(x_i))$ where x_i is an element in X of the medium linguistic function M_i.

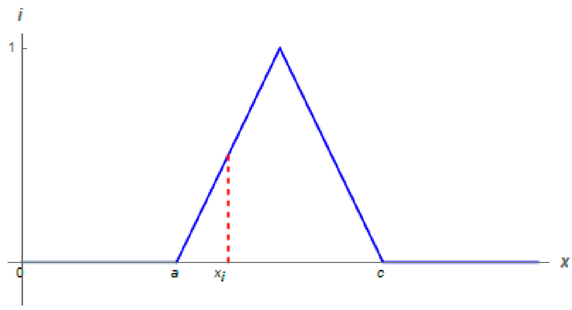


Figure 9. $x_i \in X$ of medium linguistic function M_i.

Let X and Y be any collection of universal spaces, then $\widetilde{M_X} \times \widetilde{M_Y}$ is a fuzzy linguistic relation. Suppose that M_X and M_Y in Figure 10 are two medium linguistic functions on $X = \{\, x_i \ : i = 0, 1, 2, \ldots, n\}$ and $Y = \{\, y_i \ : i = 0, 1, 2, \ldots, n\}$, respectively. Then

$$\begin{aligned} \widetilde{M_X} \times \widetilde{M_Y} &= \{((x,y), \mu_{X\times Y}(x,y) | \mu_{X\times Y}(x,y) \le 1\} \\ \widetilde{M_X} \times \widetilde{M_Y} &= \left\{((x,y), \mu_{\widetilde{M_X}\times\widetilde{M_Y}}(x,y) \middle| \mu_{\widetilde{M_X}\times\widetilde{M_Y}}(x,y) \le 1\right\} \end{aligned} \tag{13}$$

where $\mu_{\widetilde{M_X}\times\widetilde{M_Y}}(x,y)$ is the degree of membership of element $(x,y) \in X \times Y$.

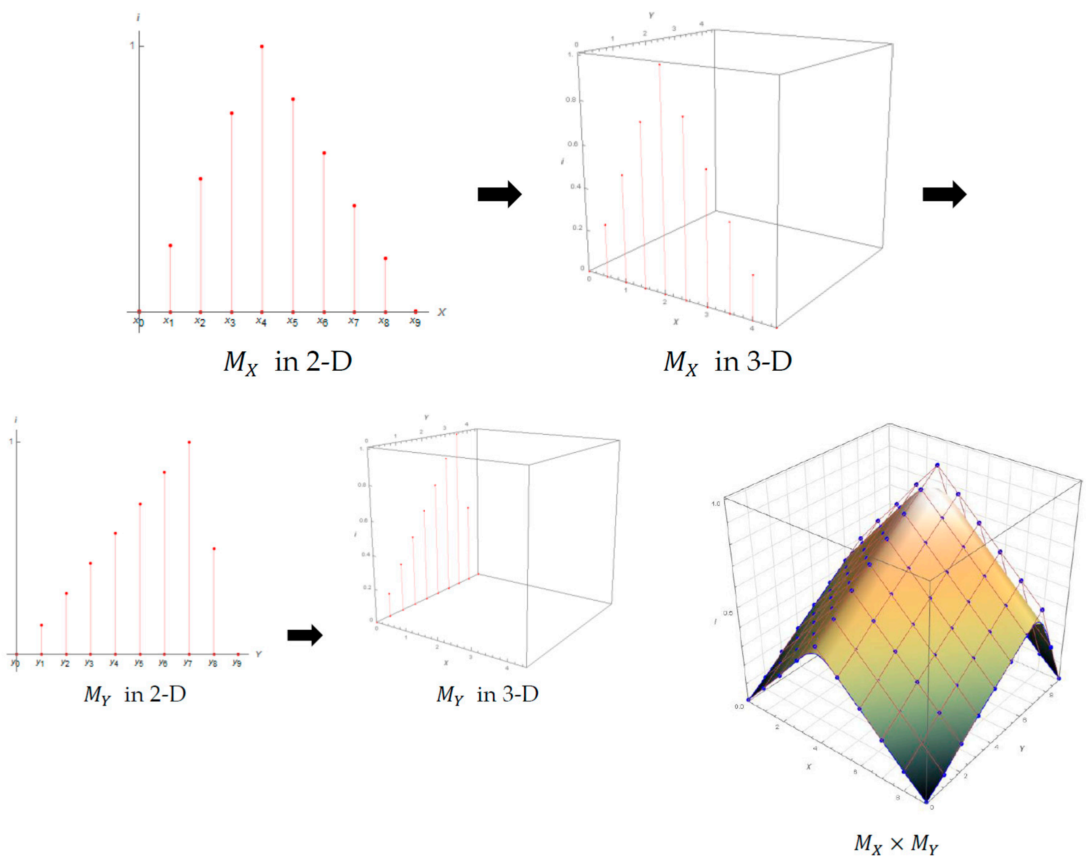


Figure 10. Fuzzy linguistic point relation of medium linguistic functions M_X and M_Y.

From Definition 7, the high linguistic function H_i in Figure 11 in accordance to (11) can be defined as $x_i \in X$ and $H_i(x_i) \in [0,1]$ exists such that $(x_i, H_i(x_i))$ where x_i is an element in X of the high linguistic function H_i.

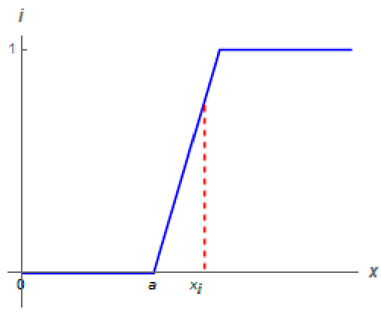


Figure 11. $x_i \in X$ of high linguistic function H_i.

Let X and Y be any collection of universal space, then $\widetilde{H_X} \times \widetilde{H_Y}$ is a fuzzy linguistic relation. Suppose that H_X and H_Y in Figure 12 are two high linguistic functions on $X = \{\, x_i \;: i = 0,1,2,\ldots,n\}$ and $Y = \{\, y_i \;: i = 0,1,2,\ldots,n\}$, respectively. Then

$$\widetilde{H_X} \times \widetilde{H_Y} = \left\{((x,y), \mu_{\widetilde{H_X} \times \widetilde{H_Y}}(x,y) \middle| \mu_{\widetilde{H_X} \times \widetilde{H_Y}}(x,y) \le 1\right\} \tag{14}$$

where $\mu_{\widetilde{H_X} \times \widetilde{H_Y}}(x,y)$ is the degree of membership of element $(x,y) \in X \times Y$.

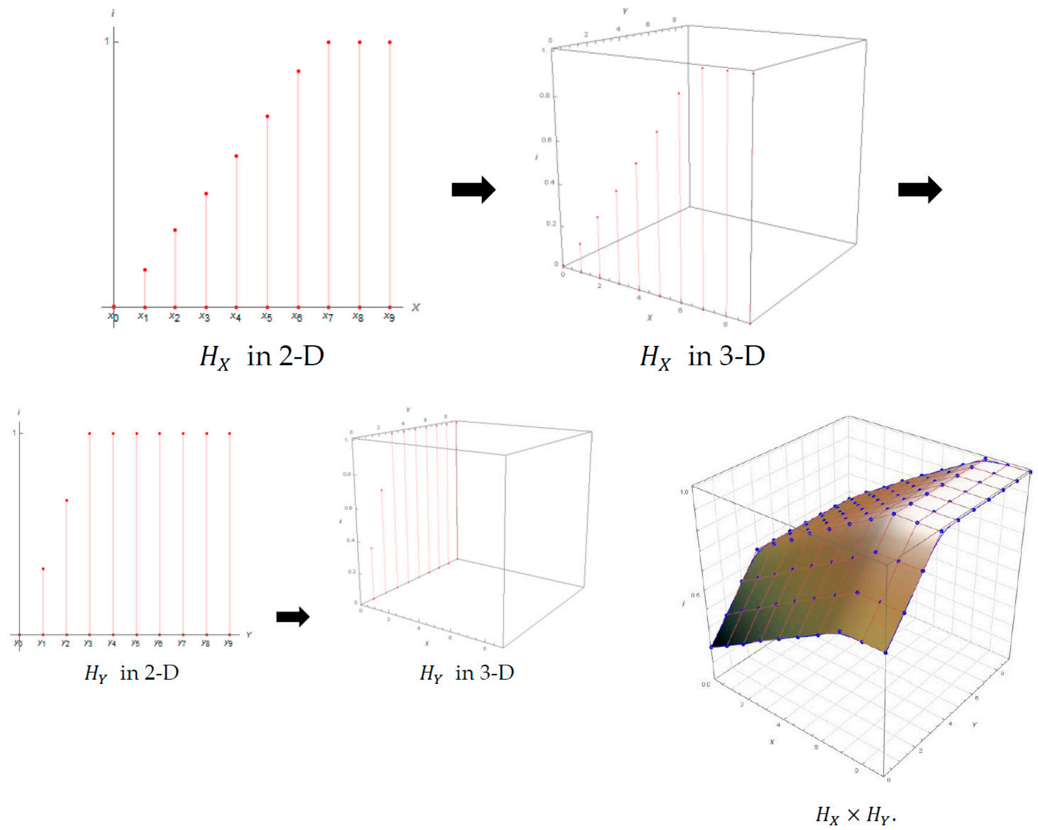


Figure 12. Fuzzy linguistic point relation of high linguistic functions H_X and H_Y.

Then, from Equations (12)–(14), the fuzzy linguistic point relations denoted as $L_i(x, y)$, $M_i(x, y)$ and $H_i(x, y)$, respectively, generate a fuzzy linguistic surface where

$$\forall(x, y) \in L_i \text{ or } (x, y) \in M_i \text{ or } (x, y) \in H_i \text{ and } L_i(x, y) \text{ or } M_i(x, y) \text{ or } H_i(x, y) \text{ exist in } [0, 1]. \tag{15}$$

$\widetilde{L_X} \times \widetilde{L_Y}$, $\widetilde{M_X} \times \widetilde{M_Y}$ and $\widetilde{H_X} \times \widetilde{H_Y}$ are a fuzzy linguistic set in $X \times Y$ that can be defined as

$$\begin{aligned} \mu_{\widetilde{L_X} \times \widetilde{L_Y}}(x, y) &= \left\{ \min \mu_{\widetilde{L_X} \times \widetilde{L_Y}}(x, y), max \mu_{\widetilde{L_X} \times \widetilde{L_Y}}(x, y) \right\} \\ \mu_{\widetilde{L_X} \times \widetilde{L_Y}}(x, y) &= \left\{ \min \mu_{\widetilde{L_X} \times \widetilde{L_Y}}(x, y), max \mu_{\widetilde{L_X} \times \widetilde{L_Y}}(x, y) \right\} \\ \mu_{\widetilde{L_X} \times \widetilde{L_Y}}(x, y) &= \left\{ \min \mu_{\widetilde{L_X} \times \widetilde{L_Y}}(x, y), max \mu_{\widetilde{L_X} \times \widetilde{L_Y}}(x, y) \right\} \end{aligned} \tag{16}$$

In the next section, a new model of the fuzzy linguistic bicubic B-spline surface is developed to generate a visualization through the fuzzy linguistic point relation and B-spline function.

8. Fuzzy Linguistic Bicubic B-Spline Surface Model

Based on the fuzzy linguistic point relation implemented with the B-spline basis function, a model of the fuzzy linguistic bicubic B-spline surface can be represented as $\widetilde{LBs}(s, t)$ and generated as follows

$$\widetilde{LBs}(s, t) = \sum_{i=0}^{n} \sum_{j=0}^{m} \widetilde{P}_{i,j} N_{i,p}(s) N_{j,q}(t) \tag{17}$$

where $\widetilde{P}_{i,j}$ is a fuzzy linguistic control point, $N_{i,p}(s)$ and $N_{j,p}(t)$ are the B-spline basic functions with degree p and q, respectively, in parameters $s, t \in [0, 1]$, $s = n + p + 1$ as well as $t = m + q + 1$ and these are knot values for each vector, $i = 0, 1, 2, \ldots, n$ and $j = 0, 1, 2, \ldots, m$.

After all the definition processes in the previous section, the next processes represented by Algorithm 1 below are applied to obtain the result of the fuzzy linguistic bicubic B-spline surface model.

Algorithm 1 Fuzzy linguistic bicubic B-spline surface modeling

Step 1: Define all fuzzy data points. Let $\widetilde{P}_{i,j} = \left\{ \widetilde{P}_{(0,0)}, \widetilde{P}_{(1,0)}, \widetilde{P}_{(2,0)}, \ldots, \widetilde{P}_{n,m} \right\} \in X \times Y$ where $\widetilde{P}_0 = \{x_0, y_0, \widetilde{z}_0\}, \widetilde{P}_1 = \{x_1, y_1, \widetilde{z}_1\}, \ \widetilde{P}_2 = \{x_2, y_2, \widetilde{z}_2\}, \ \ldots, \ \widetilde{P}_n = \{x_n, y_n, \widetilde{z}_n\}$.

Step 2: Define all fuzzy point relations where $\mu_L : L \to I = [0,1]$, $\mu_M : M \to I = [0,1]$ and $\mu_H : H \to I = [0,1]$.

Step 3: Define all fuzzy control points where

$$\widetilde{P}_{i,j} = \begin{bmatrix} (x_0, y_0), \mu_{\widetilde{P}_{(0,0)}}(x_0, y_0) & (x_0, y_1), \mu_{\widetilde{P}_{(1,0)}}(x_0, y_1) & \cdots & (x_0, y_m), \mu_{\widetilde{P}_{(0,m)}}(x_0, y_m) \\ (x_1, y_0), \mu_{\widetilde{P}_{(1,0)}}(x_1, y_0) & (x_1, y_1), \mu_{\widetilde{P}_{(1,1)}}(x_1, y_1) & \cdots & (x_1, y_m), \mu_{\widetilde{P}_{(1,m)}}(x_1, y_m) \\ \vdots & \vdots & \ddots & \vdots \\ (x_n, y_0), \mu_{\widetilde{P}_{(n.0)}}(x_n, y_0) & (x_n, y_1), \mu_{\widetilde{P}_{(n.1)}}(x_n, y_1) & \cdots & (x_n, y_m), \mu_{\widetilde{P}_{(n.m)}}(x_n, y_m) \end{bmatrix}$$

Step 4: Define all linguistic functions $\left[L_{i,}, M_i, H_i\right]$ to form fuzzy linguistic control points.

Step 5: Develop fuzzy linguistic bicubic B-spline surface model using fuzzy linguistic control points blended with B-spline basis function.

From the algorithm, the fuzzy linguistic bicubic B-spline surface model can be illustrated through each step. The visualization of the model can be seen in the next section.

9. Numerical Example and Visualization

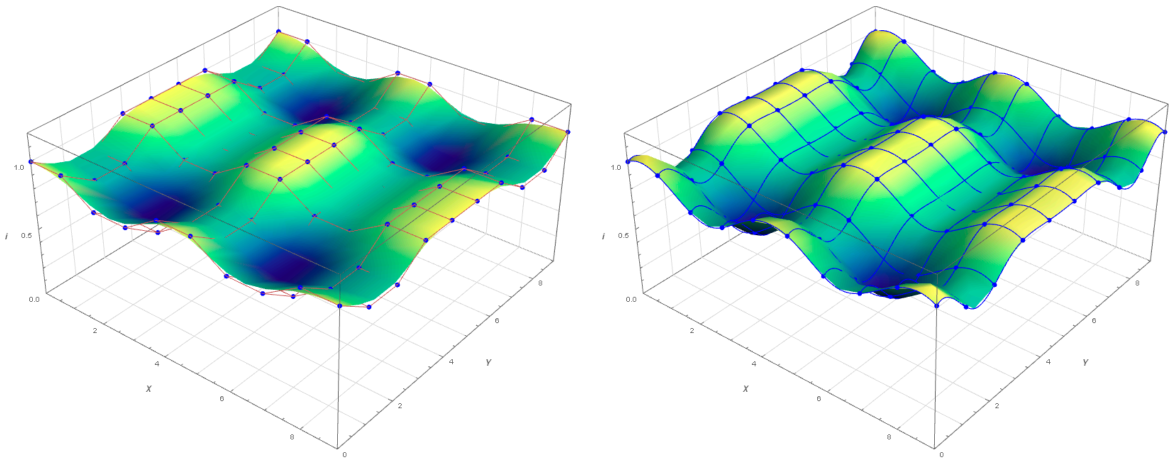
In this section, we discuss a numerical example of the fuzzy linguistic bicubic B-spline surface model. B-spline surfaces are generated through linguistic commands appearing on the fuzzy linguistic point relation. Based on the algorithm from the previous section, let linguistic data $LD = \widetilde{P}_{i,j}$, then

$$LD = \begin{bmatrix} (x_0, y_0), \mu_{\widetilde{P}_{(0,0)}}(x_0, y_0) & (x_0, y_1), \mu_{\widetilde{P}_{(1,0)}}(x_0, y_1) & \cdots & (x_0, y_9), \mu_{\widetilde{P}_{(0,9)}}(x_0, y_9) \\ (x_1, y_0), \mu_{\widetilde{P}_{(1,0)}}(x_1, y_0) & (x_1, y_1), \mu_{\widetilde{P}_{(1,1)}}(x_1, y_1) & \cdots & (x_1, y_9), \mu_{\widetilde{P}_{(1,9)}}(x_1, y_9) \\ \vdots & \vdots & \ddots & \vdots \\ (x_n, y_0), \mu_{\widetilde{P}_{(9,0)}}(x_9, y_0) & (x_9, y_1), \mu_{\widetilde{P}_{(9,1)}}(x_9, y_1) & \cdots & (x_9, y_9), \mu_{\widetilde{P}_{(9,9)}}(x_9, y_9) \end{bmatrix} = \widetilde{P}_{i,j}$$

Table 1 is a numerical example of linguistic data with randomly selected membership values that generate the final result of the fuzzy linguistic bicubic B-spline surface model in Figure 13 through linguistic commands appearing on the fuzzy linguistic point relation introduced in this paper.

Table 1. Numerical example of linguistic data.

(x_n, y_n)	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
x_0	$(0,0), 1,00$	$(0,1), 0.84$	$(0,2), 0.84$	$(0,3), 1.00$	$(0,4), 1.00$	$(0,5), 1.00$	$(0,6), 1.00$	$(0,7), 0.84$	$(0,8), 0.84$	$(0,9), 1.00$
x_1	$(1,0), 1.00$	$(1,1), 0.84$	$(1,2), 0.84$	$(1,3), 1.00$	$(1,4), 1.00$	$(1,5), 1.00$	$(1,6), 1.00$	$(1,7), 0.84$	$(1,8), 0.84$	$(1,9), 1.00$
x_2	$(2,0), 0.84$	$(2,1), 0.67$	$(2,2), 0.67$	$(2,3), 0.84$	$(2,4), 0.84$	$(2,5), 0.84$	$(2,6), 0.84$	$(2,7), 0.67$	$(2,8), 0.67$	$(2,9), 0.84$
x_3	$(3,0), 0.84$	$(3,1), 0.67$	$(3,2), 0.67$	$(3,3), 0.84$	$(3,4), 0.84$	$(3,5), 0.84$	$(3,6), 0.84$	$(3,7), 0.67$	$(3,8), 0.67$	$(3,9), 0.84$
x_4	$(4,0), 1.00$	$(4,1), 0.84$	$(4,2), 0.84$	$(4,3), 1.00$	$(4,4), 1.00$	$(4,5), 1.00$	$(4,6), 1.00$	$(4,7), 0.84$	$(4,8), 0.84$	$(4,9), 1.00$
x_5	$(5,0), 1.00$	$(5,1), 0.84$	$(5,2), 0.84$	$(5,3), 1.00$	$(5,4), 1.00$	$(5,5), 1.00$	$(5,6), 1.00$	$(5,7), 0.84$	$(5,8), 0.84$	$(5,9), 1.00$
x_6	$(6,0), 0.84$	$(6,1), 0.67$	$(6,2), 0.67$	$(6,3), 0.84$	$(6,4), 0.84$	$(6,5), 0.84$	$(6,6), 0.84$	$(6,7), 0.67$	$(6,8), 0.67$	$(6,9), 0.84$
x_7	$(7,0), 0.84$	$(7,1), 0.67$	$(7,2), 0.67$	$(7,3), 0.84$	$(7,4), 0.84$	$(7,5), 0.84$	$(7,6), 0.84$	$(7,7), 0.67$	$(7,8), 0.67$	$(7,9), 0.84$
x_8	$(8,0), 1.00$	$(8,1), 0.84$	$(8,2), 0.84$	$(8,3), 1.00$	$(8,4), 1.00$	$(8,5), 1.00$	$(8,6), 1.00$	$(8,7), 0.84$	$(8,8), 0.84$	$(8,9), 1.00$
x_9	$(9,0), 1.00$	$(9,1), 0.84$	$(9,2), 0.84$	$(9,3), 1.00$	$(9,4), 1.00$	$(9,5), 1.00$	$(9,6), 1.00$	$(9,7), 0.84$	$(9,8), 0.84$	$(9,9), 1.00$



(a) (b)

Figure 13. Example of (**a**) fuzzy linguistic bicubic B-spline surface model and (**b**) fuzzy linguistic interpolation bicubic B-spline surface model.

Figure 13 show the final results of the fuzzy linguistic bicubic B-spline surface model from the numerical example of linguistic data that are generated through a linguistic command appearing on the fuzzy linguistic point relation introduced in this paper.

10. Conclusions

The main purpose of this study is to introduce a new model, namely, the fuzzy linguistic bicubic B-spline surface to model uncertain fuzzy linguistic data. This model is based on the fuzzy set theory approach and the B-spline in geometric modeling. Firstly, this study discusses a new approach to fuzzy linguistics by defining three types of fuzzy linguistic points: low L, medium M_i and high H. Then, based on these definitions, a new algorithm is developed. Finally, by using a random numerical example of linguistic data, a surface model of the B-spline is generated and can be used for analytical purposes for further studies.

This study only discusses in generalized form and shows the final results for the proposed model. Thus, more in-depth studies are needed with new definitions and theories to display the processes in more detail. Real data are required to apply this generalized model in order to produce the expected visualization and analyzation. The presented method shows that the model can be generated by directive linguistic terms and functions.

This model has limitations if the linguistic data used are fuzzy in nature, which leads to complexity. Therefore, advanced methods for the fuzzy set theory approach should be used to obtain more accurate results. The specific meaning of ‘complex’ in complex uncertain data is the uncertainty stack of two collected data point arguments. It is impossible to accurately model complex uncertain data using a suitable curve or surface such as a B-spline unless a new definition is formulated for a B-spline function with a complex uncertain meaning [59].

This study could also be extended to other spline models such as Bézier and NURBS (Non-Uniform Rational B-Spline), and also in future works, especially in the development of artificial intelligence (AI) technology for greater beneficial impact. The basic problem of computational linguistics is the modeling of basic linguistic processes such as language comprehension, production and learning. It includes the main problems of AI systems such as perception, communication, knowledge, planning, reasoning and learning. Each of these items has a linguistic function as discussed in this study. If this problem can be applied using the model introduced, it should be able to improve the weaknesses of the existing AI technology.

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