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A New Solution to the Strong CP Problem

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Abstract: We suggest a new solution to the strong CP problem. The solution is based on the proper use of the boundary conditions for the QCD-generating functional integral. We expand the perturbative boundary conditions to both perturbative and nonperturbative fields integrated into the QCD-generating functional integral. It allows us to nullify the CP odd term in the QCD Lagrangian and, thus, to solve the strong CP problem. The presently popular solution to the strong CP problem of introducing axions violates the principle of renormalizability of the Quantum Field Theory, which is very successful phenomenologically. Our solution obeys the principle of renormalizability of the Quantum Field Theory and does not involve new exotic particles like axions.

Keywords: quantum chromodynamics; renormalizability; axions; CP violation; charge conjugation C; space reflection P

1. Introduction

The strong CP problem for a long time is considered, in fact, an unsolved, or at least not completely satisfactorily solved, outstanding problem of the Quantum Field Theory and Elementary Particle Physics. For an excellent review of the subject, see [1], where one can find various aspects of this problem. The most popular present solution [2,3] to the strong CP problem introduces new particles—axions [4,5]. Axions became so popular that they are even considered real candidates for the dark matter of the universe. But, presently, only restrictions on their possible properties are established, in spite of the numerous experimental efforts to discover such exotic particles; see, e.g., [6–8]. In addition, the axion solution of the strong CP problem violates the fundamental principle of renormalizability of the Quantum Field Theory. This basic principle is, presently, one of the most phenomenologically successful principles of Elementary Particles Theory. For example, it ensured, in Quantum Electrodynamics, the agreement between the theory and the experiment for the anomalous magnetic moment of the electron within ten decimal points. This impressive agreement convinces us that renormalizable Quantum Field Theory is a correct physical theory. Therefore, in our opinion, it seems to be interesting to find a solution to the strong CP problem that also obeys the principle of renormalizability of the Quantum Field Theory. In addition, it is desirable to find a solution that does not introduce new exotic particles like axions. This is the goal of this present paper.

To find such a solution, we will use, in a proper way (a proper way in our opinion), the boundary conditions in the generating functional of Green functions of Quantum Chromodynamics. It is well-established what kind of boundary conditions are imposed on the fields of the theory in the functional integral within the perturbative approach. These are known boundary conditions that produce the correct form of the perturbative propagators of the fields of the Lagrangian in the considered theory. The derivation of the perturbative propagators using the boundary conditions of the generating functional of Green functions of Quantum Chromodynamics can be found in [9]. We will assume that the same boundary conditions are valid for all fields of the theory that are integrated into the functional integral. We will suppose that perturbation theory calibrates the whole nonperturbative functional integral.

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Our solution will obey the principle of renormalizability of the Quantum Field Theory and will not involve new exotic particles like axions.

The CP problem is the question of why the strong interaction does not violate the charge–parity (CP) symmetry, which is the combination of charge conjugation (C) and parity (P) symmetries. The CP symmetry states that the laws of physics should be the same if a particle is replaced by its antiparticle and its spatial coordinates are inverted (P). However, the weak interaction is known to violate the CP symmetry, and there is no fundamental reason why the strong interaction should not perform in the same way.

The CP problem is also related to the origin of the matter–antimatter asymmetry in the universe, which is another unsolved mystery in physics. The CP violation involves scalar fields that couple to the quarks and induce a complex phase in the quark mass matrix. This phase could affect the properties of the neutron stars and black holes in X-ray binaries, such as their mass, radius, magnetic field, and spin; see, e.g., [10]. Another possible connection is that some models of CP violation involve new particles that have spin-1/2 and interact with the standard model particles via a new force [11]. These particles could affect the X-ray spectrum or the gravitational waves emitted by the system.

The CP violation in the early universe could have generated primordial magnetic fields that were amplified by the collapse of stars into neutron stars. These fields could then explain the existence of magnetars powered by extremely strong magnetic fields; see, e.g., [12]. However, this scenario is highly speculative and requires more theoretical and observational support.

2. Materials and Methods

In this present work, we will deal with the Quantum Chromodynamics (QCD) generating functional of Green functions, which will be the basic object of our considerations:

$$Z(J) = \int d\Phi \;\; exp\left(i \int d^4x \big(L_{QCD} + J_k \cdot \Phi_k\big)\right), \tag{1}$$

where $d\Phi$ denotes the integration measure of the functional integral $Z(J)$ over all fields Φ_k of the theory, gluons, and quarks. J_k are the sources of the fields. The symbol J in $Z(J)$ denotes the full set of sources J_k of the fields.

Within perturbation theory, the QCD Lagrangian L_{QCD} is invariant, particularly under the combined symmetry transformations CP, where C is the charge conjugation operator and P is the space reflection. More precisely, the QCD Lagrangian within perturbation theory is invariant under both the charge conjugation C and the space reflection C.

The essence of the CP problem is that in full nonperturbative QCD, one can add to the QCD Lagrangian the CP odd gauge invariant term, which seems to be not forbidden from the first principles:

$$\Delta L_\theta = \frac{\theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}. \tag{2}$$

It is invariant under the charge conjugation C and is not invariant under the space reflection P; hence, it is also noninvariant under the combined CP transformation. But, this term is forbidden by experiments with a rather high precision, as we will see below. The dual field strength tensor $\tilde{G}^a_{\mu\nu}$ in (2) is defined in the standard way:

$$\tilde{G}^a_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^a_{\rho\sigma}. \tag{3}$$

The θ-term in (2) is purely nonperturbative since it is invisible in perturbation theory because it can be rewritten as a total derivative:

$$\Delta L_\theta = \theta \partial_\mu K_\mu. \tag{4}$$

Here, K_μ is the known Chern–Simons current:

$$K_\mu = \frac{1}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} \left(A^a_\nu \partial_\rho A^a_\sigma + \frac{1}{3} f^{abc} A^a_\nu A^b_\rho A^c_\sigma \right). \tag{5}$$

The θ-term can be discarded within perturbation theory. It can be easily seen in the Euclidean space since the fields of the theory decrease in the Euclidean space at the time infinities and the total derivative (4) does not contribute to the QCD action. But, with the discovery of instantons [13], it was realized that the field configurations with the instanton boundary conditions give nonzero nonperturbative contributions to the action. In particular, the one instanton contribution looks like

$$\Delta S_\theta = \int d^4x \Delta L_\theta = \theta. \tag{6}$$

The key notion, here, is the famous topological charge that has the following form:

$$\mathcal{V} = \int d^4x \partial_\mu K_\mu = \int d^3x K_0(\vec{x}, t)|_{t=-\infty}^{t=+\infty} = \mathcal{K}(t \to +\infty) - \mathcal{K}(t \to -\infty), \tag{7}$$

where $\mathcal{K}$ is the Pontryagin number. The topological charge is zero for perturbative fields, i.e., in perturbation theory. But, instanton fields, for example in the $A_0 = 0$ gauge, interpolate between the zero gluon fields $A_i(\vec{x}, t = -\infty) = 0,\ \ i = 1, 2, 3$ and the nonzero gluon fields $A_i(\vec{x}, t = +\infty) = U^+ \partial_i U,\ \ i = 1, 2, 3$. Here, the matrix U is the Polyakov hedgehog:

$$U(\vec{x}) = exp\left(-\frac{i\pi\vec{x}\cdot\vec{\sigma}}{\sqrt{\vec{x}^2 + \rho^2}}\right). \tag{8}$$

For this instanton configuration, the Pontryagin number and, correspondingly, the topological charge are equal to unity:

$$\mathcal{V} = \mathcal{K}(t = +\infty) = 1. \tag{9}$$

Thus, the θ-term gives the nonzero nonperturbative contribution to the QCD action.

In the full QCD, with quarks, there are also contributions to the CP odd part of the QCD Lagrangian from the imaginary phases of the quark mass matrix. The phases can be rotated away by the chiral transformations of the quark fields. But, there is the famous axial anomaly [14,15]. It generates noninvariance of the measure of the Feynman functional integral under chiral transformations [16]. Therefore, the phases of the quark mass matrix arise before the $G\tilde{G}$ term in the Lagrangian. Hence, the parameter that determines the value of the CP violation is in fact

$$\theta + arg(det\mathcal{M}), \tag{10}$$

where $\mathcal{M}$ is the quark mass matrix.

Below, we shall use the same symbol θ for this parameter to simplify the notations, assuming that it already includes the effects of the quark mass matrix.

Probably the most essential phenomenological effect of the θ-term is a nonzero electric dipole moment of the neutron d_n. The electric dipole moment is given by the effective interaction Lagrangian

$$L_{nEDM} = \frac{d_n}{2}\bar{n}i\gamma_5\sigma_{\mu\nu}nF^{\mu\nu}, \tag{11}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the photon field strength tensor, n stands for the neutron field, and $\sigma_{\mu\nu} = \frac{1}{21}[\gamma_{mu}\gamma_{nu}]$ is the standard antisymmetric product of the Dirac gamma matrices.

The θ-term generates the following electric dipole moment of the neutron d_n:

$$< n(p_f)\gamma(k)|eJ_\mu^{em}A^\mu i\int d^4x\Delta L_\theta|n(p_i) >= d_n\bar{n}(p_f)\gamma_5\sigma_{\mu\nu}n(p_i)k^\mu\epsilon^\mu(k). \tag{12}$$

Here, J_μ is the quark electromagnetic current. $k^\mu = p_f^\mu - p_i^\mu$, where p_i^μ is the incoming momentum of the neutron and p_f^μ is the outgoing momentum of the neutron. $\epsilon^\mu(k)$ is the photon polarization.

The matrix element on the left-hand side of Equation (12) is zero in perturbative theory and is calculated within purely nonperturbative QCD. There are several nonperturbative methods by which the electric dipole moment of the neutron d_n was estimated; for a detailed overview, see [1] and the references therein. Here, we give a short summary of the results of the corresponding nonperturbative approaches. The performed bag model calculations produced the following result: $d_n \approx \theta 2.7 \cdot 10^{-16}$ e · cm. Shortly after this result, the chiral logarithms approach was used to obtain the following estimate: $d_n \approx \theta 5.2 \cdot 10^{-16}$ e · cm. The approach of chiral perturbation theory was further developed to produce the slightly lower result $d_n \approx \theta 3.3 \cdot 10^{-16}$ e · cm. Last but not least, the calculations based on the QCD sum rules method gave the following, again slightly lower, estimate: $d_n \approx \theta 1.2 \cdot 10^{-16}$ e · cm. All these results have considerable uncertainties of the order of 50 percent because of the essential difficulties of nonperturbative QCD calculations.

But anyway, the average theoretical value for d_n can be confidently estimated within the same order of 50 percent uncertainty as

$d_{n,theor} \approx \theta \cdot 10^{-16}$ e · cm.

This should be compared with the most recent experimental value [17] for the electric dipole moment of the neutron d_n, which is

$d_n = (0.0 \pm 1.1) \times 10^{-26}$ e · cm.

Thus, one obtains an extremely strong restriction on the value of the θ coupling:

$$|\theta| \leq 10^{-10}. \tag{13}$$

The explanation of this practically zero value of the coupling θ is the essence of the solution to the strong CP problem.

The presently popular solution to the problem is the famous axion solution. It assumes the addition to the QCD Lagrangian of the term with the new axion field $a(x)$, which, in fact, reduces the shift of the coupling θ in the QCD Lagrangian $\theta \to a(x)/f_a + \theta$. So, the corresponding term ΔL_θ of Equation (2) in the QCD Lagrangian becomes as follows:

$$\Delta L_\theta \to \left(\frac{a(x)}{f_a} + \theta\right) \frac{1}{32\pi^2} G^b_{\mu\nu} \tilde{G}^b_{\mu\nu}. \tag{14}$$

After the spontaneous symmetry breaking of the global Peccei Quinn symmetry [2,3], one calculates the effective potential for the axion field $a(x)$. Then, one finds that when the axion rests at the minimum of this potential, the CP violating term (14) nullifies. This is the known axion solution to the strong CP problem.

There are different types of axions suggested in the literature. Let us consider some of them. As the scalar Higgs fields produce vacuum expectation values, the electroweak local symmetry group is broken spontaneously. This develops masses of the gauge W and Z intermediate vector bosons. At the same time, the global Peccei Quinn U(1) group is also spontaneously broken. This spontaneous breaking of the U(1) global symmetry leads to the appearance of the massless Goldstone boson, which is called the Weinberg–Wilczek (WW) axion in this case [4,5]. In the Standard Model, including two Higgs doublets, this axion is presented as the following superposition:

$a = 1/v(v_\phi Im\phi_0 - v_\chi Im\chi_0)$.

Here, ϕ_0 and χ_0 are neutral components of the two Higgs doublets. In addition, $v = \sqrt{v_\phi^2 + v_\chi^2} \approx 250$ GeV, where v_ϕ and v_χ denote vacuum expectation values of the fields ϕ and χ, correspondingly. Within the considered approximation, the WW axion is massless. But, as mentioned above, the nonperturbative effects of Quantum Chromodynamics (for example instantons) can generate the potential for the WW axion. In this way, the WW axion obtains the nonzero mass value, which is estimated according to [4,5] as follows:

$m_a \approx f_\pi m_\pi / v \approx 100$ KeV.

In addition, the decay constant of the WW axion is $1/v$. Hence, it is clear that the mass and the decay constant of the WW axion are connected to the breaking scale v of electroweak symmetry. This constraint turns out to be too strong and, correspondingly, the WW axion turns out to be excluded by the existing experimental data.

If the scale of the breaking of the Peccei Quinn symmetry is much more than the electroweak scale v, then, according to the above formula, the axion is essentially lighter and the decay constant of the axion is much smaller. This type of light axion could be in agreement with the existing experimental data.

A solution with the light "invisible" axion was first suggested in [18,19] (the so-called KSVZ axion). In Ref. [19], this type of axion is named the phantom axion. It should be underlined that in order to uncouple the "phantom" from the electroweak scale, it is necessary to decouple the proper scalar fields from the standard quarks and couple these fields to very heavy hypothetical fermions carrying color.

To be more precise, one should introduce a complex scalar Φ that is coupled to the hypothetical quark field Q, which is the electroweak singlet in the fundamental representation of the SU(3) color group.

Then, the modulus of the scalar Φ is supposed to produce the large vacuum expectation value $f/\sqrt{2}$, and the argument of the field Φ is just the axion field a up to normalization:

$$a(x) = f\alpha(x), \quad \alpha(x) = Arg\Phi(x), \quad f >> \Lambda.$$

Further, the low energy coupling of this axion to the gluons is as follows:

$$\Delta L = \frac{1}{f} a \frac{1}{32\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}.$$

In this way, the Lagrangian of QCD depends on the expression $\theta + \alpha(x)$.

More generally, it is possible to introduce more than one quark field Q or to introduce these fields in a higher representation of the SU(3) color group. In this case, the coupling of the axion to gluons obtains an integer number N:

$$\Delta L = \frac{1}{f} aN \frac{1}{32\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}.$$

This multiplier N (which should not be confused with the color number N_c) is usually called the axion index. Hence, in general, the Lagrangian of Quantum Chromodynamics depends on the sum $\theta + N\alpha(x)$. It can be assumed that the nonperturbative QCD effects produce the potential for $\theta + N\alpha(x)$. The latter sum is minimized at the point $\theta + N\alpha_{vac} = 0$. Hence, the strong CP problem is solved in this way.

One more way to produce an "invisible" axion is suggested in [20,21] (the so-called ZDFS axion). In this case, one keeps the Peccei Quinn symmetry of the two doublet Standard Model but splits the scales of the Peccei Quinn and electroweak breaking. For this purpose, the Lagrangian of the Standard Model is extended, and one adds the scalar Standard Model singlet field Σ.

Then, one notes that this Lagrangian is invariant under the axial transformations

$$q_L \to e^{i\alpha} q_L, \;\; q_R \to e^{i\alpha} q_R, \;\; \phi \to e^{2i\alpha} \phi, \;\; \chi \to e^{-2i\alpha} \chi, \;\; \Sigma \to e^{2i\alpha} \Sigma.$$

After the spontaneous breaking of this axial symmetry, the Goldstone particle, the axion appears as the superposition

$$a = \frac{1}{V}(v_\phi Im\phi_0 - v_\chi Im\chi_0 + v_\Sigma Im\Sigma).$$

Here, $V = \sqrt{v_\phi^2 + v_\chi^2 + v_\Sigma^2}$, and v_ϕ, v_{chi}, and v_Σ represent vacuum expectation values of the fields ϕ, χ, and Σ, correspondingly. The vacuum expectation value of the field Σ is not necessarily connected to the scale of the electroweak symmetry breaking. Actually, it can be chosen as large as the Grand Unification scale. In this case, the considered axion is very light and its decay constant is small.

But, first of all, the term with the axion field $a(x)$ in (14) has the dynamical dimension (which is the dimensions of the fields plus the dimensions of the derivatives of the fields) of five instead of four necessary for renormalizability. Hence, the term with the axion field $a(x)$ violates the renormalizability of the Lagrangian. As we already have mentioned in the introduction, the renormalizability of the Lagrangian is a rather important principle of Quantum Field Theory. This principle turned out to be very successful phenomenologically as is demonstrated, for example, by the famous case of the anomalous magnetic moment of the electron. Therefore, it is quite important to preserve the principle of renormalizability when solving the strong CP problem. Secondly, the axion is not found experimentally, in spite of numerous experimental attempts, as we also have underlined in the introduction.

Therefore, we find it necessary to suggest a new solution to the strong CP problem that preserves the renormalizability of the theory and does not involve new exotic particles like axions.

Let us now, again, consider the QCD-generating functional (1). It is well-known that this integral is not defined yet completely if only the QCD Lagrangian is defined with the corresponding gauge condition. At least within perturbation theory, one should impose on the Lagrangian fields the proper boundary conditions.

In perturbation theory, one has well-known boundary conditions. For example, for the gluon fields, one has the following conditions:

$$A^a_\mu(\vec{x}, t \to -\infty) \to A^a_{\mu,in}(x), \qquad (15)$$
$$A^a_\mu(\vec{x}, t \to +\infty) \to A^{a,out}_{\mu}(x).$$

Here, the incoming asymptotic gluon fields $A^a_{\mu,in}(x)$ contain only the positive frequency part and the outgoing gluon fields $A^{a,out}_{\mu,}(x)$ contain only the negative frequency part:

$$A^a_{\mu,in}(x) = \frac{1}{(2\pi)^{3/2}} \int d^3k\, e^{i(\vec{k}\vec{x}-\omega t)} v^i_\mu(k) a_i(k)/\sqrt{2w}, \qquad (16)$$
$$A^{a,out}_{\mu,}(x) = \frac{1}{(2\pi)^{3/2}} \int d^3k\, e^{i(\vec{k}\vec{x}+\omega t)} v^i_\mu(k) a^*_i(k)/\sqrt{2w},$$

where $\omega = \sqrt{\vec{k}^2}$ and $v^i_\mu(k)$ are polarization vectors of the gluons. Here, the sums over the gluon polarizations $i = 1, 2$ are assumed.

These are the known Feynman boundary conditions. They are necessary to obtain the correct form of the perturbative propagators of the fields of the type $1/(k^2 + i\epsilon)$ with the correct plus $i\epsilon$ prescription. Thus, with these boundary conditions, the gluon fields (the quark fields also) oscillate at the time infinities. After transitioning to the Euclidean space by means of the Wick rotation $t \to ix_4$, the fields decrease at the time infinities, so in the Euclidean space, it is easy to see that in perturbation theory, total derivatives in the Lagrangian are zero.

Hence, one can write perturbative boundary conditions (15) for all fields Φ_i of the QCD Lagrangian symbolically as follows:

$$\Phi(t \to \pm\infty) \to \Phi^{out}_{in}(x). \qquad (17)$$

This helpful notation will be used below to formulate the QCD-generating functional integral as a compact formula.

3. Results

Let us now formulate our solution to the strong CP problem. As already underlined above, the solution is based on the proper use of the boundary conditions for all Lagrangian fields in the QCD-generating functional integral. So, let us now again consider the QCD-generating functional integral (1). In our opinion, it is necessary and natural to generalize the boundary conditions (17) for the perturbative fields to all fields Φ_i of the QCD Lagrangian, which are integrated into the generating functional integral (1). Then, all Lagrangian fields will decrease in the Euclidean space at the time infinities. Hence, such a definition of the boundary conditions nullifies all the total derivatives in the Lagrangian both for the perturbative contributions and the nonperturbative contributions. Thus, the CP odd term in the QCD Lagrangian will be nullified and it solves the strong CP problem. In addition, this definition allows us to formulate exactly the complete (the perturbative part plus the nonperturbative part) QCD-generating functional integral as one compact mathematical formula:

$$Z(J) = \int_{\Phi(t\to\pm\infty)\to\Phi^{out}_{in}} d\Phi\, exp\Big(i \int d^4x \big(L_{QCD} + J_k \cdot \Phi_k\big)\Big). \qquad (18)$$

4. Discussions

The strong CP problem for a long time is in fact considered as the still unsolved (or not completely adequately solved) prominent problem of the Quantum Field Theory and Elementary Particle Physics. For an excellent review of the CP problem and related topics, see the first reference of this present paper, where one can discover different aspects of this subject. The presently most popular axion solution [2,3] to the strong CP problem introduces new exotic elementary particles—axions. Axions are now so popular that they are presently considered as the real candidates for the dark matter of the universe. A lot of advanced experiments are performed to discover some sorts of these axions. But, presently, only some kinds of restrictions on their possible properties are obtained in spite of the numerous huge experimental efforts to find such exotic elementary particles; see, for example [6–8]. In addition, the axion solution of the strong CP problem is in contradiction with the fundamental, in our opinion, principle of renormalizability of the Quantum Field Theory. This basic, in our opinion, principle is, presently, one of the most experimentally successful principles of the Quantum Field Theory and Elementary Particle Physics. For example, this principle produced, for the famous anomalous magnetic moment of the electron within renormalized Quantum Electrodynamics, the outstanding agreement between the theoretical value and the experimental value within ten decimal points. This prominent agreement between the theory and the experiment convinces us that renormalizable Quantum Electrodynamics and, more generally, renormalizable Quantum Field Theory are the proper physical theories.

Therefore, in our opinion, it seems to be important to have a solution to the strong CP problem, which is in agreement with the principle of renormalizability of the Quantum Field Theory. In addition, we suppose that it is desirable to find a solution to the strong CP problem that does not involve new exotic elementary particles like axions or something similar. Therefore, the goal of this present paper is to find the solution that satisfies these two conditions (renormalizability and the absence of new exotic particles).

To find such a solution, we have used in this paper, in a proper way, the boundary conditions for the Lagrangian fields in the generating functional of the Green functions of Quantum Chromodynamics (a proper way in our opinion). It is well understood what kind of boundary conditions should be imposed on the fields of the theory in the generating functional integral within the perturbation approach. These are the known boundary conditions that generate the necessary form of the perturbative propagators of the fields of the Lagrangian of Quantum Chromodynamics. These boundary conditions produce the correct "$+i\epsilon$" prescription for the perturbative propagators of the fields. We have suggested that the same boundary conditions should be valid for all Lagrangian fields integrated into the QCD-generating functional integral, i.e., for both perturbative and nonperturbative contributions. We have supposed that perturbation theory calibrates the whole nonperturbative functional integral, i.e., it calibrates both the perturbative and the nonperturbative parts of the generating functional integral.

Our solution satisfies two important criteria. The solution obeys the principle of renormalizability of the Quantum Field Theory and does not involve new exotic particles like axions.

Let us make now, here, necessary remarks concerning the famous $U(1)$ problem. The essence of this problem is that the mass of the flavor singlet pseudo-scalar η' meson $m_{\eta'} \approx 958$ MeV is surprisingly heavier than the masses of the flavor octet pseudo-scalar mesons. One can argue that it is not possible to extend the perturbative boundary conditions for the fields discussed above, which are well-established within perturbation theory, to all the fields that are integrated into the QCD-generating functional integral. Such an extension excludes from the generating functional integral, for example, the instanton contributions not obeying the perturbative boundary conditions. In particular, there is the well-known statement [22,23] that instantons solve the $U(1)$ problem. Hence, it seems that they should not be excluded from the theory. But, one can note that there is also the well-known solution [24,25] to the $U(1)$ problem using the axial anomaly, which was suggested before the discovery of instantons. Therefore, one can argue that the $U(1)$ problem can be solved without involving the instantons. Thus, the extension of the perturbative boundary conditions to all the fields of the generating functional integral is well-allowed.

5. Conclusions

We have suggested a new solution to the strong CP problem. To find such a solution, we use, in a new way, the boundary conditions in the Quantum Chromodynamics generating functional of Green functions. It is well-established what kind of boundary conditions are imposed on the fields of the QCD Lagrangian in the functional integral within perturbation theory. We assume that the same boundary conditions are valid for all Lagrangian fields of the functional integral, i.e., for both perturbative and nonperturbative fields. This allows us to nullify the total derivatives in the QCD action, in particular the CP odd term, which can be presented as the total derivative. Hence, it solves the strong CP problem. Thus, we suppose that perturbation theory calibrates the complete nonperturbative functional integral.

Maybe it is worthwhile also to mention that our solution does not violate any symmetries of Quantum Chromodynamics. We would like to underline once more that our solution to the strong CP problem obeys the principle of renormalizability of the Quantum Field Theory and does not involve new exotic particles like axions.

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