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Choquet Integral-Based Aczel–Alsina Aggregation Operators for Interval-Valued Intuitionistic Fuzzy Information and Their Application to Human Activity Recognition

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Abstract: Human activity recognition (HAR) is the process of interpreting human activities with the help of electronic devices such as computer and machine version technology. Humans can be explained or clarified as gestures, behavior, and activities that are recorded by sensors. In this manuscript, we concentrate on studying the problem of HAR; for this, we use the proposed theory of Aczel and Alsina, such as Aczel–Alsina (AA) norms, and the derived theory of Choquet, such as the Choquet integral in the presence of Atanassov interval-valued intuitionistic fuzzy (AIVIF) set theory for evaluating the novel concept of AIVIF Choquet integral AA averaging (AIVIFC-IAAA), AIVIF Choquet integral AA ordered averaging (AIVIFC-IAAOA), AIVIF Choquet integral AA hybrid averaging (AIVIFC-IAAHA), AIVIF Choquet integral AA geometric (AIVIFC-IAAG), AIVIF Choquet integral AA ordered geometric (AIVIFC-IAAOG), and AIVIF Choquet integral AA hybrid geometric (AIVIFC-IAAHG) operators. Many essential characteristics of the presented techniques are shown, and we also identify their properties with some results. Additionally, we take advantage of the above techniques to produce a technique to evaluate the HAR multiattribute decision-making complications. We derive a functional model for HAR problems to justify the evaluated approaches and to demonstrate their supremacy and practicality. Finally, we conduct a comparison between the proposed and prevailing techniques for the legitimacy of the invented methodologies.

Keywords: interval-valued intuitionistic fuzzy sets; Choquet integral; Aczel–Alsina; human activity recognition; decision making; fuzzy logic

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1. Introduction

Human activity recognition (HAR) [1] is a diverse and universal field of investigation and attention that engages the classification and identification of human activities in the presence of information arranged from different sources, such as computers, the Internet, cameras, sensors, and wearable devices [2,3]. The main theme of HAR is to detect and find human actions, activities, or behaviors in order to know and employ human actions in various contexts directly or automatically. Mostly, it involves working in pattern recognition procedures and machine learning to evaluate and explore the information arranged from sensors or other sources. The arranged information will be given in the shape of motion, orientation, acceleration, audio, video, or physiological signals [4,5]. The practical applications of HAR are diverse and many and can be discovered in various fields, for instance, healthcare, sports and fitness, human–computer interactions, surveillance and security, and ambient-assisted living. Overall, HAR plays a valuable and essential role in considering human performance, qualifying context-conscious systems, and smoothing relevance that needs automated interpretation and reaction to social activities [6,7].

A decision-making technique [8] or procedure signifies a logical approach or technique used to find data, address decisions, and talk about decisions in many complicated situations. In practical applications, managing the symmetry or asymmetry that exists among several sources of information is the main objective of decision-making procedures. There are various decision-making procedures, each with its benefits and advantages, for instance, rational decision making; strength, weakness, opportunities, and threats (SWOT) analysis; cost-benefit analysis; the decision matrix; Pareto analysis; and the Delphi technique [9,10]. These techniques have been utilized by different scholars in various fields in the presence of classical information. But, during the decision-making procedure, we lose a lot of information because of classical information due to limited opinions, such as zero or one. To improve the range of classical set theory, the major concept of fuzzy set (FS) was addressed by Zadeh [11]. The numerical form of the truth grade in FS is given by $\Gamma^{\mathcal{I}} : \mathcal{X} \to [0,1]$, where $\Gamma^{\mathcal{I}}(\nu) \in [0,1], \nu \in \mathcal{X}$. FS theory and classical set theory have the same structure but different features because the range of FS theory is wider compared to crisp set theory. FS theory also has many limitations and restrictions because, in some cases, the experts are faced with two types of information, such as truth and falsity, supporting and opposing, and membership and non-membership. For FS theory, it is not possible to evaluate such a kind of problem; therefore, Atanassov [12] explained the novel concept of intuitionistic FS (IFS) that allows for the show of uncertainty and hesitation in various decision-making techniques. Derived by Atanassov in 1983, IFS aims to depict not only the truth grade but also the falsity of information and neutrality or hesitation between these two data types. In traditional FS, an element can represent the grade of truth between unit intervals. In IFS, however, an element is explained by two types of data called the truth grade "$\Gamma^{\mathcal{I}}(\nu)$" and the falsity grade "$\Lambda^{\mathcal{I}}(\nu)$", with $0 \leq \Gamma^{\mathcal{I}}(\nu) + \Lambda^{\mathcal{I}}(\nu) \leq 1$. In many research articles, FS theory has been employed in the form of combination, modification, and utilization, such as a tribute to Zadeh's extension principle [13], an extension principle of Zadeh's and fuzzy numbers [14], fuzzy superior Mandelbrot sets [15], continuous fuzzy differential equations [16], Cauchy fuzzy differential equations [17], numerical solutions of fuzzy differential and integral equations [18,19], the analysis of the stability of fuzzy differential equations [20], and the analysis and classification of the rise and fall of fuzzy fidelity in Europe [21].

IFS has a lot of applications in the shape of modification, utilization, and combination with other information; for instance, Atanassov [22] invented the interval-valued IFS (IVIFS), Ejegwa and Agbetayo [23] exposed the similarity and distance measures, Tripathi et al. [24] evaluated the divergence measures, Sharma et al. [25] presented the analytical hierarchical process, Rani and Garg [26] derived the trigonometric operators, Hezam et al. [27] examined the MAIRCA techniques, Gong and Wang [28] evaluated different types of inequalities, Jana and Pal [29] presented the decision-making problems, Garg et al. [30] derived the Schweizer–Sklar prioritized operators, Mahmood et al. [31] exposed the power aggregation operators, and finally, Shi et al. [32] examined the Aczel–Alsina power operators.

A lot of scholars have been working on fuzzy set theory, and many of them have utilized different types of operators, techniques, and measures based on fuzzy set theory and its extensions. Additionally, the Choquet integral (CI) operator is one of them, and it was invented by Choquet [33] in 1954. Choquet integral operators have been utilized in fuzzy set theory; for instance, Meyer and Roubens [34] derived Choquet operators based on fuzzy numbers. Moreover, Tan and Chen [35] evaluated CI operators for IFS theory, where the weighted CI operators for IFS theory were exposed by Xu [36]. All these techniques were derived based on algebraic norms. Here, we also talk about the modified version of algebraic norms, such as the Aczel–Alsina norms, which were derived by Aczel and Alsina [37] in 1982. Further, the Aczel–Alsina norms have received a lot of attention from different scholars; for example, Senapati et al. [38] evaluated the Aczel–Alsina averaging operators for IFS theory. Senapati et al. [39] also examined the Aczel–Alsina geometric operators for IFS theory. Ahmad et al. [40] derived the Aczel–Alsina operators for intuitionistic fuzzy rough set theory. Furthermore, Xu [41] exposed the averaging operators for AIFSs, Xu and Yager [42] derived the geometric operators for AIFSs, and Wang et al. [43] presented the aggregation operators for AIVIFSs. Wang and Liu [44] exposed the geometric operators for AIVIFSs. Garg et al. [45] derived the CI operators for AIVIFSs, Meng et al. [46] proposed the geometric CI operators for AIVIFSs, Senapati et al. [47] evaluated the Aczel–Alsina operators for AIVIFSs, and Senapati et al. [48] exposed the Aczel–Alsina geometric operators for AIVIFSs. From the brief analysis above, we noticed that every operator has its own merits, but they have some limitations as well. For instance, the existing operational laws are restricted in nature, and also consider the feature that all the interacting features are independent of each other.

In this paper, we utilize the concept of AIVIFS to model the uncertainties of information and the generalized operational laws based on Aczel–Alsina to determine the membership and nonmembership degrees of AIVIFNs. Also, we state the generalized aggregation operators based on these laws and hence state an MADM algorithm to solve the decision-making problems. The applicability of the proposed algorithm is demonstrated through a numerical example.

The proposed operational laws based on Aczel–Alsina norms are considered as the generalization of several existing laws. In the current study, we considered the AIVIFNs to represent the uncertainties in the data, while the Choquet integral-based Aczel–Alsina norms were used to define their basic operational laws. The Aczel–Alsina explored the theory of triangular norms in the form of Aczel–Alsina triangular norms. Aczel–Alsina triangular norms are robust tools utilized to overcome the loss of information during the aggregation of information. The major advantage of defining these operational laws is that it generalizes the algebraic and Einstein t-norm operations. Another advantage of the proposed method is the utilization of the Choquet integral (CI) to consider an inter-relationship between the attribute information. This CI considers the fuzzy measures during the measurement using ordered position. Finally, based on these proposed laws, we summarized the various generalized aggregation operators, namely, weighted averaging or geometric. The theory of Aczel–Alsina Choquet integral operators has many benefits compared to existing operators because the averaging operators, geometric operators, Choquet integral operators, averaging Aczel–Alsina operators, and geometric Aczel–Alsina operators are the special cases of the proposed theory. Also, the proposed Aczel–Alsina averaging and geometric aggregation operators based on FSs, IFSs, interval-valued FSs, and interval-valued IFSs are the special cases of the derived theory. In addition to this, the derived Aczel–Alsina Choquet integral-based aggregation operators under the existing theories are also considered as a special case.

In a nutshell, the main objectives of this work are listed below:

(1) To define some weighted operators using Choquet integral Aczel–Alsina norm operations;
(2) To investigate the basic fundamental features of the proposed operators;
(3) To develop an MADM algorithm based on the stated operators to address the decision-making problems;
(4) To illustrate the stated algorithm, using a numerical example related to HAR problems to demonstrate its supremacy and practicality;
(5) To conduct a comparison between the proposed and prevailing techniques for the legitimacy of the invented methodologies.

The rest of the manuscript is summarized as follows: In Section 2, we recall the idea of fuzzy measure (FM), the Choquet integral (CI), Aczel–Alsina norms and their special cases, AIVIFS, and their operational laws. In Section 3, we derive the AIVIFC-IAAA, AIVIFC-IAAOA, AIVIFC-IAAHA, AIVIFC-IAAG, AIVIFC-IAAOG, and AIVIFC-IAAHG operators. Many essential characteristics of the presented techniques are shown, and we also identify their properties with some results. In Section 4, we take advantage of the above techniques to produce a technique to evaluate the HAR-MADM complications. In Section 5, we derive a functional model for HAR problems to justify the evaluated approaches and to demonstrate their supremacy and practicality. In Section 6, we conduct a comparison between the proposed and prevailing techniques for the legitimacy of the invented methodologies. Some final and concluding remarks are stated in Section 7.

2. Preliminaries

In this section, we revise the FM, CI, Aczel–Alsina norms, AIVIFS, and their operational laws, as this information is very valuable and beneficial for us to evaluate the proposed techniques. Furthermore, all the symbols and their meanings that are used in this manuscript are stated in Table 1.

Table 1. Meaning of different symbols used in this manuscript.

Symbol	Meanings	Symbol	Meanings
$\blacksquare$	Function	z	Ordered of $\boldsymbol{\phi}$, where $\boldsymbol{\phi} = 1, 2, \ldots z$
$\boldsymbol{\mathcal{X}}$	Universal sets	$\varpi \geq 1$	Scaler
$\boldsymbol{\mathcal{P}}(\boldsymbol{\mathcal{X}})$	Power sets	$\zeta \geq 1$	Scaler
$\mathcal{C}_{\blacksquare}(\mathfrak{R})$	Choquet integral		

Definition 1. [33] *A function* $\blacksquare : \boldsymbol{\mathcal{P}}(\boldsymbol{\mathcal{X}}) \rightarrow [\mathbf{0}, \mathbf{1}]$ *is called a fuzzy measure if* $\blacksquare$ *is justified by the following conditions:*

$$\blacksquare(\boldsymbol{\phi}) = \mathbf{0}, \blacksquare(\boldsymbol{\mathcal{X}}) = \mathbf{1}, (\textit{Boundary Condition}) \tag{1}$$

$$\textbf{If } \Xi\rho, \boldsymbol{\mathcal{B}} \in \boldsymbol{\mathcal{P}}(\boldsymbol{\mathcal{X}}) \textbf{ and } \Xi\rho \subseteq \boldsymbol{\mathcal{B}}, \textbf{then } \blacksquare(\Xi\rho) \leq \blacksquare(\boldsymbol{\mathcal{B}}), (\textit{Monotonicity}) \tag{2}$$

where $\blacksquare$ *is a universal set,* $\mathcal{P}(\mathcal{X})$ *is a power set, and* ϕ *represents an empty set.*

Definition 2. [33] *Let* $\blacksquare$ *be a fuzzy measure, and for any* $H, J \in \mathcal{P}(\mathcal{X})$*, we can define*

$$\blacksquare(H \cup J) = \blacksquare(H) + \blacksquare(J) + \lambda\blacksquare(H \cap J)$$

where λ *represents the interaction between the parameters. The basis features of* $\blacksquare$ *are additive, subadditive, supermodular, and submodular.*

Definition 3. [33] *A positive real-valued function* $\mathfrak{R}$ *on fixed set* $\mathcal{X}$ *with a fuzzy measure* $\blacksquare$*. The discrete CI of* $\mathfrak{R}$ *based on* $\blacksquare$ *is demonstrated by:*

$$\mathcal{C}_{\blacksquare}(\mathfrak{R}) = \sum_{\varphi=1}^{z} \mathfrak{R}_{(\varphi)} \left(\blacksquare\left(\Xi\rho_{(\varphi)}\right) - \blacksquare\left(\Xi\rho_{(\varphi+1)}\right) \right) \tag{3}$$

Notice that the $(.)$*is stated the permutation on* $\mathcal{X}$*, such as* $\mathfrak{R}_{(1)} \leq \mathfrak{R}_{(2)} \leq \cdots \leq \mathfrak{R}_{(z)}$*, where* $\Xi\varrho_{(\varphi)} = \{\varphi = 1, 2, \ldots, z\}, \Xi\varrho_{(z+1)} = \phi$.

Definition 4. [31] *A mapping* $\mathcal{T}_{TN} : [0, 1] \times [0, 1] \rightarrow [0, 1]$ *is called a t-norm if:*

1. $\mathcal{T}_{TN}(\Xi, \Theta) = \mathcal{T}_{TN}(\Theta, \Xi)$;
2. $\mathcal{T}_{TN}(\Xi, \mathcal{T}_{TN}(\Xi', \Theta)) = \mathcal{T}_{TN}(\mathcal{T}_{TN}(\Xi, \Xi'), \Theta)$;
3. *If* $\Xi \leq \Xi'$*and* $\Theta \leq \Theta'$*, then* $\mathcal{T}_{TN}(\Xi, \Theta) \leq \mathcal{T}_{TN}(\Xi', \Theta')$;
4. $\mathcal{T}_{TN}(\Theta.1) = \Theta$.

Definition 5. [31] *A mapping* $\mathcal{S}_{TCN} : [0, 1] \times [0, 1] \rightarrow [0, 1]$ *is called a t-conorm if:*

1. $\mathcal{S}_{TCN}(\Xi,\ \Theta) = \mathcal{S}_{TCN}(\Theta,\Xi)$;
2. $\mathcal{S}_{TCN}(\Xi,\ \mathcal{S}_{TCN}(\Xi',\ \Theta)) = \mathcal{S}_{TCN}(\mathcal{S}_{TCN}(\Xi,\ \Xi'),\Theta)$;
3. *If* $\Xi \leq \Xi'$ *and* $\Theta \leq \Theta'$, *then* $\mathcal{S}_{TCN}(\Xi,\ \Theta) \leq \mathcal{S}_{TCN}(\Xi',\ \Theta')$;
4. $\mathcal{S}_{TCN}(\Theta.1) = \Theta$.

Definition 6. [37] *The mathematical and theoretical shape of an Aczel–Alsina t-norm and t-conorm is stated by:*

$$\mathcal{T}_{TN}^{\varpi}(\Xi,\ \Theta) = \begin{cases} \mathcal{T}_{TN}^{D}(\Xi,\ \Theta) & \varpi = 0 \\ min(\Xi,\ \Theta) & \varpi = \infty \\ e^{-((-\log\Xi)^{\varpi}+(-\log\Theta)^{\varpi})^{\frac{1}{\varpi}}} & otherwise \end{cases} \tag{4}$$

$$\mathcal{S}_{TCN}^{\varpi}(\Xi,\ \Theta) = \begin{cases} \mathcal{S}_{TCN}^{D}(\Xi,\ \Theta) & \varpi = 0 \\ max(\Xi,\ \Theta) & \varpi = \infty \\ 1 - e^{-((-\log(1-\Xi))^{\varpi}+(-\log(1-\Theta))^{\varpi})^{\frac{1}{\varpi}}} & otherwise \end{cases} \tag{5}$$

where $\varpi > 1$, $\mathcal{T}_{TN}^{D}$ *and* $\mathcal{S}_{TCN}^{D}$ *are represented by the discrete t-norm and t-conorm, such as*

$$\mathcal{T}_{TN}^{D}(\Xi,\ \Theta) = \begin{cases} \Xi & \Theta = 1 \\ \Theta & \Xi = 1 \\ 0 & otherwise \end{cases} \tag{6}$$

$$\mathcal{S}_{TCN}^{D}(\Xi,\ \Theta) = \begin{cases} \Xi & \Theta = 0 \\ \Theta & \Xi = 0 \\ 1 & otherwise \end{cases} \tag{7}$$

Notice that $\Xi,\Theta \in [0,1]$.

Definition 7. [22] *The expression of the AIVIFS* $\mathcal{I}^{if}$ *is constructed by:*

$$\mathcal{I}^{if} = \left\{ \left(\left[\Gamma^{\mathcal{I}^-}(\nu), \Gamma^{\mathcal{I}^+}(\nu) \right], \left[\Lambda^{\mathcal{I}^-}(\nu), \Lambda^{\mathcal{I}^+}(\nu) \right] \right) : \nu \in \mathcal{X} \right\} \tag{8}$$

The information in the pair $\left(\left[\Gamma^{\mathcal{I}^-}(\nu), \Gamma^{\mathcal{I}^+}(\nu) \right], \left[\Lambda^{\mathcal{I}^-}(\nu), \Lambda^{\mathcal{I}^+}(\nu) \right] \right)$ *represents the membership and nonmembership degrees with a suitable condition:* $0 \leq \Gamma^{\mathcal{I}^+}(\nu) + \Lambda^{\mathcal{I}^+}(\nu) \leq 1$. *Moreover, we construct the neutral/refusal information, such as* $\eta^{\mathcal{I}}(\nu) = \left[\eta^{\mathcal{I}^-}(\nu), \eta^{\mathcal{I}^+}(\nu) \right] = \left[1 - \left(\Gamma^{\mathcal{I}^+}(\nu) + \Lambda^{\mathcal{I}^+}(\nu) \right), 1 - \left(\Gamma^{\mathcal{I}^-}(\nu) + \Lambda^{\mathcal{I}^-}(\nu) \right) \right]$.

Finally, the AIVIFV is constructed in the form of

$$\mathcal{I}_{\varphi}^{if} = \left(\left[\Gamma_{\varphi}^{\mathcal{I}^-}, \Gamma_{\varphi}^{\mathcal{I}^+} \right], \left[\Lambda_{\varphi}^{\mathcal{I}^-}, \Lambda_{\varphi}^{\mathcal{I}^+} \right] \right), \varphi = 1, 2, \ldots, z$$

Definition 8. [22] *For any two AIVIFVs, the basic operational laws are stated as*

$$\mathcal{I}_1^{if} \oplus \mathcal{I}_2^{if} = \left(\left[\Gamma_1^{\mathcal{I}^-} + \Gamma_2^{\mathcal{I}^-} - \Gamma_1^{\mathcal{I}^-}\Gamma_2^{\mathcal{I}^-}, \Gamma_1^{\mathcal{I}^+} + \Gamma_2^{\mathcal{I}^+} - \Gamma_1^{\mathcal{I}^+}\Gamma_2^{\mathcal{I}^+} \right], \left[\Lambda_1^{\mathcal{I}^-}\Lambda_2^{\mathcal{I}^-}, \Lambda_1^{\mathcal{I}^+}\Lambda_2^{\mathcal{I}^+} \right] \right) \tag{9}$$

$$\mathcal{I}_1^{if} \otimes \mathcal{I}_2^{if} = \left(\left[\Gamma_1^{\mathcal{I}^-}\Gamma_2^{\mathcal{I}^-}, \Gamma_1^{\mathcal{I}^+}\Gamma_2^{\mathcal{I}^+} \right], \left[\Lambda_1^{\mathcal{I}^-} + \Lambda_2^{\mathcal{I}^-} - \Lambda_1^{\mathcal{I}^-}\Lambda_2^{\mathcal{I}^-}, \Lambda_1^{\mathcal{I}^+} + \Lambda_2^{\mathcal{I}^+} - \Lambda_1^{\mathcal{I}^+}\Lambda_2^{\mathcal{I}^+} \right] \right) \tag{10}$$

$$\zeta\mathcal{I}_{\varphi}^{if} = \left(\left[1 - \left(1 - \Gamma_{\varphi}^{\mathcal{I}^-} \right)^{\zeta}, 1 - \left(1 - \Gamma_{\varphi}^{\mathcal{I}^+} \right)^{\zeta} \right], \left[\left(\Lambda_{\varphi}^{\mathcal{I}^-} \right)^{\zeta}, \left(\Lambda_{\varphi}^{\mathcal{I}^+} \right)^{\zeta} \right] \right) \tag{11}$$

$$\left(\mathcal{I}_{\varphi}^{if} \right)^{\zeta} = \left(\left[\left(\Gamma_{\varphi}^{\mathcal{I}^-} \right)^{\zeta}, \left(\Gamma_{\varphi}^{\mathcal{I}^+} \right)^{\zeta} \right], \left[1 - \left(1 - \Lambda_{\varphi}^{\mathcal{I}^-} \right)^{\zeta}, 1 - \left(1 - \Lambda_{\varphi}^{\mathcal{I}^+} \right)^{\zeta} \right] \right) \tag{12}$$

Using Equations (9)–(12) and using the idea in Equation (3), Garg et al. *[45] exposed the following theory:*

$$\begin{aligned}
&AIVIFCI\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right)\\
&=\mathcal{I}_1^{if}\left(\blacksquare\left(\Xi\rho_{(1)}\right)-\blacksquare\left(\Xi\rho_{(2)}\right)\right)\oplus\mathcal{I}_2^{if}\left(\blacksquare\left(\Xi\rho_{(2)}\right)-\blacksquare\left(\Xi\rho_{(3)}\right)\right)\oplus\ldots\\
&\oplus\mathcal{I}_z^{if}\left(\blacksquare\left(\Xi\rho_{(z)}\right)-\blacksquare\left(\Xi\rho_{(z+1)}\right)\right)=\sum_{\varphi=1}^{z}\mathcal{I}_\varphi^{if}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\\
&=\left(\begin{array}{c}\left[\begin{array}{c}1-\prod_{\varphi=1}^{z}\left(1-\Gamma_\varphi^{\mathcal{I}-}\right)^{(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))},\\1-\prod_{\varphi=1}^{z}\left(1-\Gamma_\varphi^{\mathcal{I}+}\right)^{(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))}\end{array}\right],\\\left[\prod_{\varphi=1}^{z}\left(\Lambda_\varphi^{\mathcal{I}-}\right)^{(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))},\prod_{\varphi=1}^{z}\left(\Lambda_\varphi^{\mathcal{I}+}\right)^{(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))}\right]\end{array}\right)
\end{aligned}\tag{13}$$

Our aim is to construct the CI in consideration of the Aczel–Alsina t-norm and t-conorm instead of the algebraic norms; therefore, for this, we used the proposed work of Senapati et al. *[47] for the AIVIFVs. The operational laws are defined as the following:*

Definition 9. *For AIVIFVs $\mathcal{I}_1^{if}=\left(\left[\Gamma_1^{\mathcal{I}-},\Gamma_1^{\mathcal{I}+}\right],\left[\Lambda_1^{\mathcal{I}-},\Lambda_1^{\mathcal{I}+}\right]\right)$ and $\mathcal{I}_2^{if}=\left(\left[\Gamma_2^{\mathcal{I}-},\Gamma_2^{\mathcal{I}+}\right],\left[\Lambda_2^{\mathcal{I}-},\Lambda_2^{\mathcal{I}+}\right]\right)$ and$\zeta>0$ be a real number, then*

$$\mathcal{I}_1^{if}\oplus\mathcal{I}_2^{if}=\left(\begin{array}{c}\left[\begin{array}{c}1-e^{-\left(\left(-\log\left(1-\Gamma_1^{\mathcal{I}-}\right)\right)^{\varpi}+\left(-\log\left(1-\Gamma_2^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\1-e^{-\left(\left(-\log\left(1-\Gamma_1^{\mathcal{I}+}\right)\right)^{\varpi}+\left(-\log\left(1-\Gamma_2^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right],\\\left[e^{-\left(\left(-\log\Lambda_1^{\mathcal{I}-}\right)^{\varpi}+\left(-\log\Lambda_2^{\mathcal{I}-}\right)^{\varpi}\right)^{\frac{1}{\varpi}}},e^{-\left(\left(-\log\Lambda_1^{\mathcal{I}+}\right)^{\varpi}+\left(-\log\Lambda_2^{\mathcal{I}+}\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\right]\end{array}\right)\tag{14}$$

$$\mathcal{I}_1^{if}\otimes\mathcal{I}_2^{if}=\left(\begin{array}{c}\left[\begin{array}{c}e^{-\left(\left(-\log\left(\Gamma_1^{\mathcal{I}-}\right)\right)^{\varpi}+\left(-\log\left(\Gamma_2^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\e^{-\left(\left(-\log\left(\Gamma_1^{\mathcal{I}+}\right)\right)^{\varpi}+\left(-\log\left(\Gamma_2^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right],\\\left[\begin{array}{c}e^{-\left(\left(-\log\left(1-\Lambda_1^{\mathcal{I}-}\right)\right)^{\varpi}+\left(-\log\left(1-\Lambda_2^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\e^{-\left(\left(-\log\left(1-\Lambda_1^{\mathcal{I}+}\right)\right)^{\varpi}+\left(-\log\left(1-\Lambda_2^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)\tag{15}$$

$$\zeta\mathcal{I}_1^{if}=\left(\begin{array}{c}\left[1-e^{-\left(\zeta\left(-\log\left(1-\Gamma_1^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},1-e^{-\left(\zeta\left(-\log\left(1-\Gamma_1^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\right],\\\left[e^{-\left(\zeta\left(-\log\Lambda_1^{\mathcal{I}-}\right)^{\varpi}\right)^{\frac{1}{\varpi}}},e^{-\left(\zeta\left(-\log\Lambda_1^{\mathcal{I}+}\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\right]\end{array}\right)\tag{16}$$

$$\left(\mathcal{I}_\varphi^{if}\right)^{\zeta}=\left(\begin{array}{c}\left[e^{-\left(\zeta\left(-\log\left(\Gamma_1^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},e^{-\left(\zeta\left(-\log\left(\Gamma_1^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\right],\\\left[1-e^{-\left(\zeta\left(-\log\left(1-\Lambda_1^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},1-e^{-\left(\zeta\left(-\log\left(1-\Lambda_1^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\right]\end{array}\right)\tag{17}$$

Definition 10. *For any AIVIFV $\mathcal{I}_\varphi^{if}=\left(\left[\Gamma_\varphi^{\mathcal{I}-},\Gamma_\varphi^{\mathcal{I}+}\right],\left[\Lambda_\varphi^{\mathcal{I}-},\Lambda_\varphi^{\mathcal{I}+}\right]\right)$, the score and accuracy functions are stated as:*

$$\sqsupset^{SV}\left(\mathcal{I}_{\varphi}^{if}\right) = \frac{1}{2}\left(\Gamma_{\varphi}^{\mathcal{I}-} + \Gamma_{\varphi}^{\mathcal{I}+} - \Lambda_{\varphi}^{\mathcal{I}-} - \Lambda_{\varphi}^{\mathcal{I}+}\right) \in [-1, 1] \tag{18}$$

$$\sqsupset^{AV}\left(\mathcal{I}_{\varphi}^{if}\right) = \frac{1}{2}\left(\Gamma_{\varphi}^{\mathcal{I}-} + \Gamma_{\varphi}^{\mathcal{I}+} + \Lambda_{\varphi}^{\mathcal{I}-} + \Lambda_{\varphi}^{\mathcal{I}+}\right) \in [0, 1] \tag{19}$$

Definition 11. *To compare the two different AIVIFNs $\mathcal{I}_1^{if}$ and $\mathcal{I}_2^{if}$, an order comparison between them is stated below:*

(a) *If $\sqsupset^{SV}\left(\mathcal{I}_1^{if}\right) > \sqsupset^{SV}\left(\mathcal{I}_2^{if}\right)$, then $\mathcal{I}_1^{if} > \mathcal{I}_2^{if}$,*
(b) *If $\sqsupset^{SV}\left(\mathcal{I}_1^{if}\right) = \sqsupset^{SV}\left(\mathcal{I}_2^{if}\right)$, and $\sqsupset^{AV}\left(\mathcal{I}_1^{if}\right) > \sqsupset^{AV}\left(\mathcal{I}_2^{if}\right)$, then $\mathcal{I}_1^{if} > \mathcal{I}_2^{if}$.*

3. Proposed C-IAA Aggregation Operators for AIVIFSs

In this section, we aim to evaluate the theory of AIVIFC-IAAA, AIVIFC-IAAOA, AIVIFC-IAAHA, AIVIFC-IAAG, AIVIFC-IAAOG, and AIVIFC-IAAHG operators. Many essential characteristics of the presented techniques are shown, and we also identify their properties with some results. Throughout this paper, we used the family of AIVIFSs $\mathcal{I}_{\varphi}^{if} = \left(\left[\Gamma_{\varphi}^{\mathcal{I}-}, \Gamma_{\varphi}^{\mathcal{I}+}\right], \left[\Lambda_{\varphi}^{\mathcal{I}-}, \Lambda_{\varphi}^{\mathcal{I}+}\right]\right), \varphi = 1, 2, \ldots, z.$

Definition 12. *For the family of AIVIFSs $\mathcal{I}_{\varphi}^{if}$, an AIVIFC-IAAA operator is demonstrated by:*

$$\begin{aligned}
&AIVIFC - IAAA\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right)\\
&= \mathcal{I}_1^{if}\left(\blacksquare\left(\Xi\rho_{(1)}\right) - \blacksquare\left(\Xi\rho_{(2)}\right)\right) \oplus \mathcal{I}_2^{if}\left(\blacksquare\left(\Xi\rho_{(2)}\right) - \blacksquare\left(\Xi\rho_{(3)}\right)\right)\\
&\oplus \ldots \oplus \mathcal{I}_z^{if}\left(\blacksquare\left(\Xi\rho_{(z)}\right) - \blacksquare\left(\Xi\rho_{(z+1)}\right)\right)\\
&= \sum_{\varphi=1}^{z} \mathcal{I}_{\varphi}^{if}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right) - \blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\\
&= \oplus_{\varphi=1}^{z} \mathcal{I}_{\varphi}^{if}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right) - \blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)
\end{aligned} \tag{20}$$

Theorem 1. *For the family of AIVIFSs, the aggregated value by the$AIVIFC - IAAA$operator is also AIVIFV and given as*

$$\begin{aligned}
&AIVIFC - IAAA\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right)\\
&= \left(\begin{array}{c}
\left[\begin{array}{c}
1 - e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1 - \Gamma_{\varphi}^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\
1 - e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1 - \Gamma_{\varphi}^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}
\end{array}\right],\\
\left[\begin{array}{c}
e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Lambda_{\varphi}^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\
e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Lambda_{\varphi}^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}
\end{array}\right]
\end{array}\right)
\end{aligned} \tag{21}$$

The proof of Theorem 1 is given in Appendix A.

To demonstrate the functioning of the proposed AIVIFC-IAAA operator, we consider four AIVIFNs as

$$\mathcal{I}_1^{if} = ([0.40,\ 0.41], [0.30,\ 0.31]); \qquad \mathcal{I}_2^{if} = ([0.50,\ 0.51], [0.10,\ 0.11])$$

$$\mathcal{I}_3^{if} = ([0.59,\ 0.60], [0.39,\ 0.40]); \qquad \mathcal{I}_4^{if} = ([0.70,\ 0.71], [0.20,\ 0.21])$$

To aggregate these numbers, we utilize Equation (21) and obtain the aggregated number as

$$AIVIFC-IAAA\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\mathcal{I}_3^{if},\mathcal{I}_4^{if}\right)=([0.3386,0.3463],[0.5035,0.5156]).$$

Further, the proposed operator $AIVIFC-IAAA$ satisfies certain properties, which are listed below.

Property 1. *When $\mathcal{I}_\varphi^{if}=\mathcal{I}^{if}=\left(\left[\Gamma^{\mathcal{I}^-},\Gamma^{\mathcal{I}^+}\right],\left[\Lambda^{\mathcal{I}^-},\Lambda^{\mathcal{I}^+}\right]\right),\varphi=1,2,\ldots,z$, thus*

$$AIVIFC-IAAA\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right)=\mathcal{I}^{if} \tag{22}$$

The proof of Property 1 is given in Appendix B.

Property 2. *When $\mathcal{I}_\varphi^{if}\leq\mathcal{I}^{*}{}_\varphi^{if},\varphi=1,2,\ldots,z$, then*

$$AIVIFC-IAAA\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right)\leq AIVIFC-IAAA\left(\mathcal{I}^{*}{}_1^{if},\mathcal{I}^{*}{}_2^{if},\ldots,\mathcal{I}^{*}{}_z^{if}\right) \tag{23}$$

The proof of Property 2 is given in Appendix C.

Property 3. *When $\mathcal{I}_\varphi^{-}=\left(\left[\min\limits_\varphi\Gamma_\varphi^{\mathcal{I}^-},\min\limits_\varphi\Gamma_\varphi^{\mathcal{I}^+}\right],\left[\max\limits_\varphi\Lambda_\varphi^{\mathcal{I}^-},\max\limits_\varphi\Lambda_\varphi^{\mathcal{I}^+}\right]\right)$ and $\mathcal{I}_\varphi^{+}=\left(\left[\max\limits_\varphi\Gamma_\varphi^{\mathcal{I}^-},\max\limits_\varphi\Gamma_\varphi^{\mathcal{I}^+}\right],\left[\min\limits_\varphi\Lambda_\varphi^{\mathcal{I}^-},\min\limits_\varphi\Lambda_\varphi^{\mathcal{I}^+}\right]\right),\varphi=1,2,\ldots,z$, thus*

$$\mathcal{I}_\varphi^{-}\leq AIVIFC-IAAA\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right)\leq\mathcal{I}_\varphi^{+} \tag{24}$$

The proof of Property 3 is given in Appendix D.

Definition 13. *An AIVIFC-IAAOA operator is demonstrated by*:

$$\begin{aligned} IVIFC-IAAOA\quad &\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right)\\ &=\mathcal{I}_{o(1)}^{if}\left(\blacksquare\left(\Xi\rho_{(1)}\right)-\blacksquare\left(\Xi\rho_{(2)}\right)\right)\oplus\mathcal{I}_{o(2)}^{if}\left(\blacksquare\left(\Xi\rho_{(2)}\right)-\blacksquare\left(\Xi\rho_{(3)}\right)\right)\oplus\ldots\\ &\oplus\mathcal{I}_{o(z)}^{if}\left(\blacksquare\left(\Xi\rho_{(z)}\right)-\blacksquare\left(\Xi\rho_{(z+1)}\right)\right)\\ &=\sum_{\varphi=1}^{z}\mathcal{I}_{o(\varphi)}^{if}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\\ &=\oplus_{\varphi=1}^{z}\mathcal{I}_{o(\varphi)}^{if}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right) \end{aligned} \tag{25}$$

With a dominant condition$o(\varphi)\leq o(\varphi-1)$ and with the data in Equation (18), we find their orders.

Theorem 2. *For the family of AIVIFSs, the aggregated value by the $AIVIFC-IAAOA$ operator is also AIVIFV and given as*

$$AIVIFC-IAAOA\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right)=\left(\begin{array}{c}\left[\begin{array}{c}1-e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_{o(\varphi)}^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ 1-e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_{o(\varphi)}^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right],\\ \left[\begin{array}{c}e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(\Lambda_{o(\varphi)}^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(\Lambda_{o(\varphi)}^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right) \tag{26}$$

Proof. Similar to the proof of Theorem 1. □

Property 4. *When* $\mathcal{I}_{\varphi}^{if} = \mathcal{I}^{if} = \left(\left[\Gamma^{\mathcal{I}-}, \Gamma^{\mathcal{I}+}\right], \left[\Lambda^{\mathcal{I}-}, \Lambda^{\mathcal{I}+}\right]\right), \varphi = 1, 2, \ldots, z$, *thus*

$$AIVIFC - IAAOA\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) = \mathcal{I}^{if} \tag{27}$$

Proof. Similar to the proof of Property 1.□

Property 5. *When* $\mathcal{I}_{\varphi}^{if} \leq \mathcal{I}^{*}{}_{\varphi}^{if}, \varphi = 1, 2, \ldots, z$, *then*

$$AIVIFC - IAAOA\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) \leq AIVIFC - IAAOA\left(\mathcal{I}^{*}{}_1^{if}, \mathcal{I}^{*}{}_2^{if}, \ldots, \mathcal{I}^{*}{}_z^{if}\right) \tag{28}$$

Proof. Similar to the proof of Property 2.□

Property 6. *When* $\mathcal{I}_{\varphi}^{-} = \left(\left[\min_{\varphi}\Gamma_{\varphi}^{\mathcal{I}-}, \min_{\varphi}\Gamma_{\varphi}^{\mathcal{I}+}\right], \left[\max_{\varphi}\Lambda_{\varphi}^{\mathcal{I}-}, \max_{\varphi}\Lambda_{\varphi}^{\mathcal{I}+}\right]\right)$ *and* $\mathcal{I}_{\varphi}^{+} = \left(\left[\max_{\varphi}\Gamma_{\varphi}^{\mathcal{I}-}, \max_{\varphi}\Gamma_{\varphi}^{\mathcal{I}+}\right], \left[\min_{\varphi}\Lambda_{\varphi}^{\mathcal{I}-}, \min_{\varphi}\Lambda_{\varphi}^{\mathcal{I}+}\right]\right), \varphi = 1, 2, \ldots, z$, *thus*

$$\mathcal{I}_{\varphi}^{-} \leq AIVIFC - IAAOA\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) \leq \mathcal{I}_{\varphi}^{+} \tag{29}$$

Proof. Similar to the proof of Property 3. □

Definition 14. *An AIVIFC-IAAHA operator is demonstrated by*:

$$\begin{aligned} AIVIFC - IAAHA & \left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) \\ & = \mathcal{I}_{o(1)}^{*}\left(\blacksquare\left(\Xi\rho_{(1)}\right) - \blacksquare\left(\Xi\rho_{(2)}\right)\right) \oplus \mathcal{I}_{o(2)}^{*}\left(\blacksquare\left(\Xi\rho_{(2)}\right) - \blacksquare\left(\Xi\rho_{(3)}\right)\right) \oplus \ldots \\ & \oplus \mathcal{I}_{o(z)}^{*}\left(\blacksquare\left(\Xi\rho_{(z)}\right) - \blacksquare\left(\Xi\rho_{(z+1)}\right)\right) = \sum_{\varphi=1}^{z} \mathcal{I}_{o(\varphi)}^{*}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right) - \blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right) \\ & = \oplus_{\varphi=1}^{z} \mathcal{I}_{o(\varphi)}^{*}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right) - \blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right) \end{aligned} \tag{30}$$

With a dominant condition $o(\varphi) \leq o(\varphi - 1)$ *and with the data in Equation (18), we find their order with* $\mathcal{I}_{o(\varphi)}^{*} = zw_{\varphi}\mathcal{I}_{(\varphi)}^{if}, \varphi = 1, 2, \ldots, z$, *where* w_{φ} *represents the weight vector with the rule* $\sum_{\varphi=1}^{z} w_{\varphi} = 1$.

Theorem 3. *For the family of AIVIFSs, the aggregated value by the*$AIVIFC - IAAHA$*operator is also AIVIFV and given as*

$$AIVIFC - IAAHA\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) = \left(\begin{array}{c} \left[\begin{array}{l} 1 - e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1-\Gamma_{o(\varphi)}^{\mathcal{I}-^{*}}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ 1 - e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1-\Gamma_{o(\varphi)}^{\mathcal{I}+^{*}}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \end{array}\right], \\ \left[\begin{array}{l} e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Lambda_{o(\varphi)}^{\mathcal{I}-^{*}}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Lambda_{o(\varphi)}^{\mathcal{I}+^{*}}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \end{array}\right] \end{array}\right) \tag{31}$$

Property 7. *When* $\mathcal{I}_{\varphi}^{if} = \mathcal{I}^{if} = \left(\left[\Gamma^{\mathcal{I}-}, \Gamma^{\mathcal{I}+}\right], \left[\Lambda^{\mathcal{I}-}, \Lambda^{\mathcal{I}+}\right]\right), \varphi = 1, 2, \ldots, z$, *thus*

$$AIVIFC-IAAHA\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right)=\mathcal{I}^{if} \tag{32}$$

Property 8. *When* $\mathcal{I}_\varphi^{if}\leq\mathcal{I}^{*}{}_{\varphi}^{if},\varphi=1,2,\ldots,z$, *then*

$$AIVIFC-IAAHA\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right)\leq AIVIFC-IAAHA\left(\mathcal{I}^{*}{}_1^{if},\mathcal{I}^{*}{}_2^{if},\ldots,\mathcal{I}^{*}{}_z^{if}\right) \tag{33}$$

Property 9. *When* $\mathcal{I}_\varphi^-=\left(\left[\min_\varphi\Gamma_\varphi^{\mathcal{I}^-},\min_\varphi\Gamma_\varphi^{\mathcal{I}^+}\right],\left[\max_\varphi\Lambda_\varphi^{\mathcal{I}^-},\max_\varphi\Lambda_\varphi^{\mathcal{I}^+}\right]\right)$ *and* $\mathcal{I}_\varphi^+=\left(\left[\max_\varphi\Gamma_\varphi^{\mathcal{I}^-},\max_\varphi\Gamma_\varphi^{\mathcal{I}^+}\right],\left[\max_\varphi\Gamma_\varphi^{\mathcal{I}^-},\max_\varphi\Gamma_\varphi^{\mathcal{I}^+}\right]\right),\varphi=1,2,\ldots,z$, *thus*

$$\mathcal{I}_\varphi^-\leq AIVIFC-IAAHA\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right)\leq\mathcal{I}_\varphi^+ \tag{34}$$

Definition 15. *An AIVIFC-IAAG operator is demonstrated by*:

$$\begin{aligned}
&AIVIFC-IAAG\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right)\\
&=\left(\mathcal{I}_1^{if}\right)^{(\blacksquare(\Xi\rho_{(1)})-\blacksquare(\Xi\rho_{(2)}))}\otimes\left(\mathcal{I}_2^{if}\right)^{(\blacksquare(\Xi\rho_{(2)})-\blacksquare(\Xi\rho_{(3)}))}\otimes\ldots\otimes\left(\mathcal{I}_z^{if}\right)^{(\blacksquare(\Xi\rho_{(z)})-\blacksquare(\Xi\rho_{(z+1)}))}\\
&=\prod_{\varphi=1}^{z}\left(\mathcal{I}_\varphi^{if}\right)^{(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))}\\
&=\otimes_{\varphi=1}^{z}\left(\mathcal{I}_\varphi^{if}\right)^{(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))}
\end{aligned} \tag{35}$$

Theorem 4. *For the family of AIVIFSs, the aggregated value by the* $AIVIFC-IAAG$ *operator is also AIVIFV and given as*

$$AIVIFC-IAAG\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right)=\left(\begin{array}{c}\left[\begin{array}{c}e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))\left(-\log\left(\Gamma_\varphi^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))\left(-\log\left(\Gamma_\varphi^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right],\\ \left[\begin{array}{c}1-e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))\left(-\log\left(1-\Lambda_\varphi^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ 1-e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))\left(-\log\left(1-\Lambda_\varphi^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right) \tag{36}$$

The proof of Theorem 4 is given in Appendix E.

To demonstrate the functioning of the proposed AIVIFC-IAAG operator, we consider four AIVIFNs:

$$\mathcal{I}_1^{if}=([0.40,\ 0.41],[0.30,\ 0.31])\,;\qquad\mathcal{I}_2^{if}=([0.50,\ 0.51],[0.10,\ 0.11])$$

$$\mathcal{I}_3^{if}=([0.59,\ 0.60],[0.39,\ 0.40])\,;\ \mathcal{I}_4^{if}=([0.70,\ 0.71],[0.20,\ 0.21])$$

To aggregate these numbers, we utilize Equation (36) and obtain the aggregated number

$$AIVIFC-IAAG\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\mathcal{I}_3^{if},\mathcal{I}_4^{if}\right)=([0.7763,0.7823],\quad[0.1326,0.1326]).$$

Further, the proposed operator $AIVIFC-IAAG$ *satisfies certain properties, which are listed below.*

Property 10. *When* $\mathcal{I}_\varphi^{if}=\mathcal{I}^{if}=\left(\left[\Gamma^{\mathcal{I}^-},\Gamma^{\mathcal{I}^+}\right],\left[\Lambda^{\mathcal{I}^-},\Lambda^{\mathcal{I}^+}\right]\right),\varphi=1,2,\ldots,z$, *thus*

$$AIVIFC - IAAG\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) = \mathcal{I}^{if} \tag{37}$$

Property 11. *When* $\mathcal{I}_\varphi^{if} \le \mathcal{I}*_\varphi^{if}, \varphi = 1, 2, \ldots, z$, *then*

$$AIVIFC - IAAG\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) \le AIVIFC - IAAG\left(\mathcal{I}*_1^{if}, \mathcal{I}*_2^{if}, \ldots, \mathcal{I}*_z^{if}\right) \tag{38}$$

Property 12. *When* $\mathcal{I}_\varphi^- = \left(\left[\min_\varphi \Gamma_\varphi^{\mathcal{I}^-}, \min_\varphi \Gamma_\varphi^{\mathcal{I}^+}\right], \left[\max_\varphi \Lambda_\varphi^{\mathcal{I}^-}, \max_\varphi \Lambda_\varphi^{\mathcal{I}^+}\right]\right)$ *and* $\mathcal{I}_\varphi^+ = \left(\left[\max_\varphi \Gamma_\varphi^{\mathcal{I}^-}, \max_\varphi \Gamma_\varphi^{\mathcal{I}^+}\right], \left[\min_\varphi \Lambda_\varphi^{\mathcal{I}^-}, \min_\varphi \Lambda_\varphi^{\mathcal{I}^+}\right]\right), \varphi = 1, 2, \ldots, z$, *thus*

$$\mathcal{I}_\varphi^- \le AIVIFC - IAAG\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) \le \mathcal{I}_\varphi^+ \tag{39}$$

Definition 16. *An AIVIFC-IAAOA operator is demonstrated by*:

$$\begin{aligned} AIVIFC - IAAOG & \left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) \\ &= \left(\mathcal{I}_{o(1)}^{if}\right)^{(\blacksquare(\Xi\rho_{(1)}) - \blacksquare(\Xi\rho_{(2)}))} \otimes \left(\mathcal{I}_{o(2)}^{if}\right)^{(\blacksquare(\Xi\rho_{(2)}) - \blacksquare(\Xi\rho_{(3)}))} \otimes \ldots \otimes \left(\mathcal{I}_{o(z)}^{if}\right)^{(\blacksquare(\Xi\rho_{(z)}) - \blacksquare(\Xi\rho_{(z+1)}))} \\ &= \prod_{\varphi=1}^{z} \left(\mathcal{I}_{o(\varphi)}^{if}\right)^{(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)}))} = \otimes_{\varphi=1}^{z} \left(\mathcal{I}_{o(\varphi)}^{if}\right)^{(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)}))} \end{aligned} \tag{40}$$

With a dominant condition $o(\varphi) \le o(\varphi - 1)$ *and with the data in Equation (18), we find their orders.*

Theorem 5. *For the family of AIVIFSs, the aggregated value by the* $AIVIFC - IAAOG$ *operator is also AIVIFV and given as*

$$AIVIFC - IAAOG\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) = \left(\begin{array}{c} \left[\begin{array}{c} e^{-\left(\sum_{\varphi=1}^{z} (\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(\Gamma_{o(\varphi)}^{\mathcal{I}^-}))^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ e^{-\left(\sum_{\varphi=1}^{z} (\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(\Gamma_{o(\varphi)}^{\mathcal{I}^+}))^{\varpi}\right)^{\frac{1}{\varpi}}} \end{array}\right], \\ \left[\begin{array}{c} 1 - e^{-\left(\sum_{\varphi=1}^{z} (\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(1 - \Lambda_{o(\varphi)}^{\mathcal{I}^-}))^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ 1 - e^{-\left(\sum_{\varphi=1}^{z} (\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(1 - \Lambda_{o(\varphi)}^{\mathcal{I}^+}))^{\varpi}\right)^{\frac{1}{\varpi}}} \end{array}\right] \end{array}\right) \tag{41}$$

Property 13. *When* $\mathcal{I}_\varphi^{if} = \mathcal{I}^{if} = \left(\left[\Gamma^{\mathcal{I}^-}, \Gamma^{\mathcal{I}^+}\right], \left[\Lambda^{\mathcal{I}^-}, \Lambda^{\mathcal{I}^+}\right]\right), \varphi = 1, 2, \ldots, z$, *thus*

$$AIVIFC - IAAOG\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) = \mathcal{I}^{if} \tag{42}$$

Property 14. *When* $\mathcal{I}_\varphi^{if} \le \mathcal{I}*_\varphi^{if}, \varphi = 1, 2, \ldots, z$, *then*

$$AIVIFC - IAAOG\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) \le AIVIFC - IAAOG\left(\mathcal{I}*_1^{if}, \mathcal{I}*_2^{if}, \ldots, \mathcal{I}*_z^{if}\right) \tag{43}$$

Property 15. *When* $\mathcal{I}_{\varphi}^{-} = \left(\left[\min\limits_{\varphi}\Gamma_{\varphi}^{\mathcal{I}^-}, \min\limits_{\varphi}\Gamma_{\varphi}^{\mathcal{I}^+}\right], \left[\max\limits_{\varphi}\Lambda_{\varphi}^{\mathcal{I}^-}, \max\limits_{\varphi}\Lambda_{\varphi}^{\mathcal{I}^+}\right]\right)$ *and* $\mathcal{I}_{\varphi}^{+} = \left(\left[\max\limits_{\varphi}\Gamma_{\varphi}^{\mathcal{I}^-}, \max\limits_{\varphi}\Gamma_{\varphi}^{\mathcal{I}^+}\right], \left[\min\limits_{\varphi}\Lambda_{\varphi}^{\mathcal{I}^-}, \min\limits_{\varphi}\Lambda_{\varphi}^{\mathcal{I}^+}\right]\right)$, $\varphi = 1, 2, \ldots, z$, *thus*

$$\mathcal{I}_{\varphi}^{-} \leq AIVIFC - IAAOG\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) \leq \mathcal{I}_{\varphi}^{+} \tag{44}$$

Definition 17. *An AIVIFC-IAAHG operator is demonstrated by*:

$$\begin{aligned} AIVIFC - IAAHG &\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) \\ &= \left(\mathcal{I}_{o(1)}^{*}\right)^{(\blacksquare(\Xi\rho_{(1)}) - \blacksquare(\Xi\rho_{(2)}))} \otimes \left(\mathcal{I}_{o(2)}^{*}\right)^{(\blacksquare(\Xi\rho_{(2)}) - \blacksquare(\Xi\rho_{(3)}))} \otimes \ldots \otimes \left(\mathcal{I}_{o(z)}^{*}\right)^{(\blacksquare(\Xi\rho_{(z)}) - \blacksquare(\Xi\rho_{(z+1)}))} \\ &= \prod_{\varphi=1}^{z}\left(\mathcal{I}_{o(\varphi)}^{*}\right)^{(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)}))} = \otimes_{\varphi=1}^{z}\left(\mathcal{I}_{o(\varphi)}^{*}\right)^{(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)}))} \end{aligned} \tag{45}$$

With a dominant condition $o(\varphi) \leq o(\varphi - 1)$ *and with the data in Equation (18), we find their order with* $\mathcal{I}_{o(\varphi)}^{*} = zw_{\varphi}\mathcal{I}_{(\varphi)}^{if}$, $\varphi = 1, 2, \ldots, z$, *where* w_{φ} *represents the weight vector with the rule* $\sum\limits_{\varphi=1}^{z} w_{\varphi} = 1$.

Theorem 6. *For the family of AIVIFSs, the aggregated value by the* $AIVIFC - IAAHG$ *operator is also AIVIFV and given as*

$$AIVIFC - IAAHG\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) = \left(\begin{array}{c} \left[\begin{array}{c} e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(\Gamma_{o(\varphi)}^{\mathcal{I}^{-*}}))^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(\Gamma_{o(\varphi)}^{\mathcal{I}^{+*}}))^{\varpi}\right)^{\frac{1}{\varpi}}} \end{array}\right], \\ \left[\begin{array}{c} 1 - e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(1-\Lambda_{o(\varphi)}^{\mathcal{I}^{-*}}))^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ 1 - e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(1-\Lambda_{o(\varphi)}^{\mathcal{I}^{+*}}))^{\varpi}\right)^{\frac{1}{\varpi}}} \end{array}\right] \end{array}\right) \tag{46}$$

Property 16. *When* $\mathcal{I}_{\varphi}^{if} = \mathcal{I}^{if} = \left(\left[\Gamma^{\mathcal{I}^-}, \Gamma^{\mathcal{I}^+}\right], \left[\Lambda^{\mathcal{I}^-}, \Lambda^{\mathcal{I}^+}\right]\right)$, $\varphi = 1, 2, \ldots, z$, *thus*

$$AIVIFC - IAAHG\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) = \mathcal{I}^{if} \tag{47}$$

Property 17. *When* $\mathcal{I}_{\varphi}^{if} \leq \mathcal{I}*_{\varphi}^{if}$, $\varphi = 1, 2, \ldots, z$, *then*

$$AIVIFC - IAAHG\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) \leq AIVIFC - IAAHG\left(\mathcal{I}*_1^{if}, \mathcal{I}*_2^{if}, \ldots, \mathcal{I}*_z^{if}\right) \tag{48}$$

Property 18. *When* $\mathcal{I}_{\varphi}^{-} = \left(\left[\min\limits_{\varphi}\Gamma_{\varphi}^{\mathcal{I}^-}, \min\limits_{\varphi}\Gamma_{\varphi}^{\mathcal{I}^+}\right], \left[\max\limits_{\varphi}\Lambda_{\varphi}^{\mathcal{I}^-}, \max\limits_{\varphi}\Lambda_{\varphi}^{\mathcal{I}^+}\right]\right)$ *and* $\mathcal{I}_{\varphi}^{+} = \left(\left[\max\limits_{\varphi}\Gamma_{\varphi}^{\mathcal{I}^-}, \max\limits_{\varphi}\Gamma_{\varphi}^{\mathcal{I}^+}\right], \left[\min\limits_{\varphi}\Lambda_{\varphi}^{\mathcal{I}^-}, \min\limits_{\varphi}\Lambda_{\varphi}^{\mathcal{I}^+}\right]\right)$, $\varphi = 1, 2, \ldots, z$, *thus*

$$\mathcal{I}_{\varphi}^{-} \leq AIVIFC - IAAHG\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) \leq \mathcal{I}_{\varphi}^{+} \tag{49}$$

4. Proposed MADM Technique

The multi-attribute decision-making (MADM) technique is a procedure or technique, which is especially used, for evaluating the optimal decision in the presence of multiple criteria or attributes. It is used in situations when an expert or decision-maker needs to find the best one among a finite number of alternatives or possibilities.

In MADM, each alternative contains many criteria or attributes, and these attributes can be qualitative or quantitative, which are very important for the procedure of the decision-making technique. Every example of attributes or criteria could contain cost, quality, time, environmental impact, and many others, whereas every MADM procedure contains the following major themes, such as the identification of criteria, the weighting of criteria, the evaluation of alternatives, the aggregation of sources, and finally, the selection of the best decision. Therefore, to evaluate some real-life problems, first, we need to develop the procedure of the MADM technique. For this, we have a collection of alternatives, such as $\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}$, where each alternative covers the finite family of attributes or criteria, such as $\mathcal{I}_1^{cr}, \mathcal{I}_2^{cr}, \ldots, \mathcal{I}_y^{cr}$ with w_φ, where w_φ represents the weight vector with the rule $\sum_{\varphi=1}^{z} w_\varphi = 1$. Further, we aim to assign AIVIFN to each attribute in every alternative, where the information in the pair $\left(\left[\Gamma^{\mathcal{I}^-}(\nu), \Gamma^{\mathcal{I}^+}(\nu)\right], \left[\Lambda^{\mathcal{I}^-}(\nu), \Lambda^{\mathcal{I}^+}(\nu)\right]\right)$ represents the membership and non-membership degrees with a suitable condition: $0 \leq \Gamma^{\mathcal{I}^+}(\nu) + \Lambda^{\mathcal{I}^+}(\nu) \leq 1$. Moreover, we constructed the neutral/refusal information, such as $\eta^{\mathcal{I}}(\nu) = \left[\eta^{\mathcal{I}^-}(\nu), \eta^{\mathcal{I}^+}(\nu)\right] = \left[1 - \left(\Gamma^{\mathcal{I}^-}(\nu) + \Lambda^{\mathcal{I}^-}(\nu)\right), 1 - \left(\Gamma^{\mathcal{I}^+}(\nu) + \Lambda^{\mathcal{I}^+}(\nu)\right)\right]$. Finally, the AIVIFV was constructed in the shape $\mathcal{I}_\varphi^{if} = \left(\left[\Gamma_\varphi^{\mathcal{I}^-}, \Gamma_\varphi^{\mathcal{I}^+}\right], \left[\Lambda_\varphi^{\mathcal{I}^-}, \Lambda_\varphi^{\mathcal{I}^+}\right]\right), \varphi = 1, 2, \ldots, z$. All the information was arranged in the shape of a matrix, and then for evaluating the data in the matrix, we aimed to construct the procedure of the MADM technique; therefore, we have the following procedure:

Step 1: First, we arrange the AIVIFSs in the form of the matrix, but during the collection of information, we will face two types of data, such as benefit and cost types of information. These attributes show the positive and negative features of each alternative. For both kinds of data, we have the following rules, for instance:

Rule 1: (Benefit type of data.) We are not required to normalize the arranged data in the matrix;

Rule 2: (Cost type of data.) We are required to normalize the arranged data in a matrix; for instance,

$$N = \begin{cases} \left(\left[\Gamma_\varphi^{\mathcal{I}^-}, \Gamma_\varphi^{\mathcal{I}^+}\right], \left[\Lambda_\varphi^{\mathcal{I}^-}, \Lambda_\varphi^{\mathcal{I}^+}\right]\right) & for\ B \\ \left(\left[\Lambda_\varphi^{\mathcal{I}^-}, \Lambda_\varphi^{\mathcal{I}^+}\right], \left[\Gamma_\varphi^{\mathcal{I}^-}, \Gamma_\varphi^{\mathcal{I}^+}\right]\right) & for\ C \end{cases}$$

where B and C represent the benefit and cost types of data.

Step 2: After normalization (if necessary), we aggregate the arranged information based on the proposed AIVIFC-IAAA operator and AIVIFC-IAAG operator, such as

$$AIVIFC - IAAA\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) = \left(\begin{array}{c} \left[\begin{array}{c} 1 - e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1-\Gamma_\varphi^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ 1 - e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1-\Gamma_\varphi^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \end{array}\right], \\ \left[\begin{array}{c} e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Lambda_\varphi^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Lambda_\varphi^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \end{array}\right] \end{array}\right)$$

$$AIVIFC-IAAG\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right)=\left(\begin{array}{c}\left[\begin{array}{c}e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Gamma_{\varphi}^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Gamma_{\varphi}^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right],\\ \left[\begin{array}{c}1-e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1-\Lambda_{\varphi}^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ 1-e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1-\Lambda_{\varphi}^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

Step 3: After aggregating the normalized data, we evaluate their score values (if the score function has failed, then we will use the accuracy function), which are given in Equations (18) and (19);

Step 4: Finally, we derive the ranking results of each alternative based on their score or accuracy values and try to examine the optimal or best decision.

After the successful implementation of the MADM technique for evaluating the problems, we aimed to choose some real-life applications and tried to resolve them with the help of the proposed four techniques.

5. Numerical Example (Human Activity Recognition)

The HAR technique is used for the identification or classification of human activities based on the analysis of sensor information. There are many kinds of HAR techniques, which are used in the field, varying on the concern, problems, and their requirements. The major theme of this application is to point out the best HAR technique among the five best HAR techniques, which are represented by a family of five alternatives, whose major and brief analyses are stated below:

Rule-based Systems "$\mathcal{I}_1^{if}$": To identify certain actions, rule-based systems employ predetermined rules or heuristics. These regulations are frequently founded on professional expertise or industry-specific standards. A rule-based system, for instance, may employ thresholds on accelerometer measurements to identify movements like walking or running.

Machine Learning-based Approaches "$\mathcal{I}_2^{if}$": The identification of human behavior makes extensive use of machine learning techniques. These methods use labeled datasets for model training to discover patterns and connections between sensor data and actions. Typical machine learning algorithms include the following:

(1) Supervised learning;
(2) Unsupervised learning;
(3) Deep learning.

Sensor Fusions"$\mathcal{I}_3^{if}$": To increase the precision of activity detection, sensor fusion combines data from various sensors. The system may record a more complete description of human actions by combining data from many sensors, including accelerometers, gyroscopes, and magnetometers.

Hidden Markov Models (HMMs) "$\mathcal{I}_4^{if}$": HMMs are popular probabilistic models for recognizing activities. To estimate the most likely sequence of activities given the observed sensor data, they model the temporal relationships of the activities and employ the Viterbi algorithm or forward–backward algorithm.

Dynamic Time Warping (DTW) "$\mathcal{I}_5^{if}$": DTW is a method for aligning two-time series in a nonlinear warping manner and measuring how similar they are. It is ideal for activity detection tasks, as it is frequently utilized for contrasting and identifying temporal patterns in sensor data.

Under the presence of the above five alternatives, we aimed to find the best one based on their features, which were used as attributes or criteria, such as $\mathcal{I}_1^{cr}$: data collection, $\mathcal{I}_2^{cr}$: preprocessing and data extraction, $\mathcal{I}_3^{cr}$: classification and evaluations, and $\mathcal{I}_4^{cr}$: monitoring and facilitating. Then, the steps of the proposed approach, as shown below, were implemented, to find the best alternative(s).

Step 1: First, we arrange the AIVIFSs in the form of the matrix (see Table 2), but during the collection of information, we will face two types of data, such as benefit and cost types information. These attributes show the positive and negative features of each alternative. For both kinds of data, we have the following rules, for instance:

Rule 1: (Benefit type of data.) We are not required to normalize the arranged data in the matrix;

Rule 2: (Cost type of data.) We are required to normalize the arranged data in a matrix; for instance,

$$N = \begin{cases} \left(\left[\Gamma_{\varphi}^{\mathcal{I}^-}, \Gamma_{\varphi}^{\mathcal{I}^+}\right], \left[\Lambda_{\varphi}^{\mathcal{I}^-}, \Lambda_{\varphi}^{\mathcal{I}^+}\right]\right) & for\ B \\ \left(\left[\Lambda_{\varphi}^{\mathcal{I}^-}, \Lambda_{\varphi}^{\mathcal{I}^+}\right], \left[\Gamma_{\varphi}^{\mathcal{I}^-}, \Gamma_{\varphi}^{\mathcal{I}^+}\right]\right) & for\ C \end{cases}$$

where B and C represent the benefit and cost types of data. But we used Rule 1 because we had positive types of criteria.

Table 2. AIVIF decision matrix.

	$\mathcal{I}_1^{cr}$	$\mathcal{I}_2^{cr}$	$\mathcal{I}_3^{cr}$	$\mathcal{I}_4^{cr}$
$\mathcal{I}_1^{if}$	$\left([0.4, 0.41], [0.3, 0.31]\right)$	$\left([0.5, 0.51], [0.1, 0.11]\right)$	$\left([0.59, 0.6], [0.39, 0.4]\right)$	$\left([0.7, 0.71], [0.2, 0.21]\right)$
$\mathcal{I}_2^{if}$	$\left([0.4, 0.41], [0.2, 0.21]\right)$	$\left([0.3, 0.31], [0.1, 0.11]\right)$	$\left([0.5, 0.51], [0.3, 0.31]\right)$	$\left([0.1, 0.11], [0.2, 0.21]\right)$
$\mathcal{I}_3^{if}$	$\left([0.5, 0.51], [0.3, 0.31]\right)$	$\left([0.4, 0.41], [0.3, 0.31]\right)$	$\left([0.3, 0.31], [0.2, 0.21]\right)$	$\left([0.2, 0.21], [0.1, 0.11]\right)$
$\mathcal{I}_4^{if}$	$\left([0.4, 0.41], [0.2, 0.21]\right)$	$\left([0.6, 0.61], [0.3, 0.31]\right)$	$\left([0.5, 0.51], [0.4, 0.41]\right)$	$\left([0.5, 0.51], [0.2, 0.21]\right)$
$\mathcal{I}_5^{if}$	$\left([0.1, 0.11], [0.2, 0.21]\right)$	$\left([0.2, 0.21], [0.2, 0.21]\right)$	$\left([0.3, 0.31], [0.2, 0.21]\right)$	$\left([0.6, 0.61], [0.1, 0.11]\right)$

Step 2: After normalization (if necessary), we aggregate the arranged information based on the proposed AIVIFC-IAAA operator and AIVIFC-IAAG operator (see Table 3);

Table 3. AIVIF aggregated decision matrix.

	AIVIFC-IAAA	**AIVIFC-IAAG**
$\mathcal{I}_1^{if}$	$\left([0.3386, 0.3463], [0.5035, 0.5156]\right)$	$\left([0.7763, 0.7823], [0.1326, 0.1326]\right)$
$\mathcal{I}_2^{if}$	$\left([0.1768, 0.1821], [0.4827, 0.4948]\right)$	$\left([0.4994, 0.5129], [0.1038, 0.1038]\right)$
$\mathcal{I}_3^{if}$	$\left([0.1657, 0.1711], [0.5249, 0.4713]\right)$	$\left([0.5748, 0.5842], [0.1129, 0.1129]\right)$
$\mathcal{I}_4^{if}$	$\left([0.2692, 0.2758], [0.552, 0.5617]\right)$	$\left([0.7386, 0.745], [0.142, 0.142]\right)$
$\mathcal{I}_5^{if}$	$\left([0.2311, 0.2371], [0.4376, 0.451]\right)$	$\left([0.5616, 0.5728], [0.0781, 0.0781]\right)$

Step 3: After aggregating the normalized data, we evaluate their score values (if the score function has failed, then we will use the accuracy function), which are given in Equations (18) and (19) (see Table 4);

Table 4. AIVIF score matrix.

	AIVIFC-IAAA	AIVIFC-IAAG
$\mathcal{I}_1^{if}$	-0.1671	0.6467
$\mathcal{I}_2^{if}$	-0.3093	0.4023
$\mathcal{I}_3^{if}$	-0.3298	0.4666
$\mathcal{I}_4^{if}$	-0.2844	0.5998
$\mathcal{I}_5^{if}$	-0.2102	0.4891

Step 4: Finally, we derive the ranking results of each alternative based on their score or accuracy values and try to examine the optimal or best decision (see Table 5).

Table 5. Ranking matrix.

Methods	Ranking Values	Best One
AIVIFC-IAAA	$\mathcal{I}_1^{if} > \mathcal{I}_5^{if} > \mathcal{I}_4^{if} > \mathcal{I}_2^{if} > \mathcal{I}_4^{if}$	$\mathcal{I}_1^{if}$ (Rule-Based Systems)
AIVIFC-IAAG	$\mathcal{I}_1^{if} > \mathcal{I}_4^{if} > \mathcal{I}_5^{if} > \mathcal{I}_3^{if} > \mathcal{I}_2^{if}$	$\mathcal{I}_1^{if}$ (Rule-Based Systems)

According to the analysis in Table 5, we noticed that all the proposed techniques obtained the same ranking results, where the best decision is $\mathcal{I}_1^{if}$ (a rule-based system). After the successful evaluation of the problem of HAR with the help of the MADM technique, we further aimed to compare the proposed operators with some existing operators to show the supremacy and validity of the proposed approaches.

6. Comparative Analysis

This section shows the supremacy and validity of the proposed techniques by comparing them with some prevailing methods or techniques. Comparative analysis is the best way to prove the advantages and disadvantages between the proposed and prevailing methods. Therefore, in this section, we aim to compare the proposed techniques with some prevailing methods. For this, we have the following ideas: Wang et al. [43] presented the aggregation operators for AIVIFSs, Wang and Liu [44] exposed the geometric operators for AIVIFSs, Garg et al. [45] derived the CI operators for AIVIFSs, Meng et al. [46] proposed the geometric CI operators for AIVIFSs, Senapati et al. [47] evaluated the Aczel–Alsina operators for AIVIFSs, and Senapati et al. [48] exposed the Aczel–Alsina geometric operators for AIVIFSs. Therefore, to consider the data in Table 2, the comparison is listed in Table 6.

Table 6. Comparative analysis of the derived and existing techniques.

Approaches	Score Values					Ranking
	$\mathcal{I}_1^{if}$	$\mathcal{I}_2^{if}$	$\mathcal{I}_3^{if}$	$\mathcal{I}_4^{if}$	$\mathcal{I}_5^{if}$	
Wang et al. [43]	0.3452	0.1713	0.145	0.2499	0.0981	$\mathcal{I}_1^{if} > \mathcal{I}_4^{if} > \mathcal{I}_3^{if} > \mathcal{I}_2^{if} > \mathcal{I}_5^{if}$
Wang and Liu [44]	0.2804	0.0936	0.1002	0.2005	0.0239	$\mathcal{I}_1^{if} > \mathcal{I}_4^{if} > \mathcal{I}_3^{if} > \mathcal{I}_5^{if} > \mathcal{I}_2^{if}$
Garg et al. [45]	0.3794	0.119	0.1424	0.1919	0.1646	$\mathcal{I}_1^{if} > \mathcal{I}_4^{if} > \mathcal{I}_5^{if} > \mathcal{I}_3^{if} > \mathcal{I}_2^{if}$

Table 6. *Cont.*

Approaches	Score Values					Ranking
	$\mathcal{I}_1^{if}$	$\mathcal{I}_2^{if}$	$\mathcal{I}_3^{if}$	$\mathcal{I}_4^{if}$	$\mathcal{I}_5^{if}$	
Meng et al. [46]	0.3212	0.0264	0.0921	0.1466	0.0691	$\mathcal{I}_1^{if} > \mathcal{I}_4^{if} > \mathcal{I}_3^{if} > \mathcal{I}_5^{if} > \mathcal{I}_2^{if}$
Senapati et al. [47]	−0.1817	−0.2772	−0.3138	−0.2818	−0.3275	$\mathcal{I}_1^{if} > \mathcal{I}_2^{if} > \mathcal{I}_4^{if} > \mathcal{I}_3^{if} > \mathcal{I}_5^{if}$
Senapati et al. [48]	0.6177	0.4557	0.4933	0.5798	0.4107	$\mathcal{I}_1^{if} > \mathcal{I}_4^{if} > \mathcal{I}_3^{if} > \mathcal{I}_2^{if} > \mathcal{I}_5^{if}$
Proposed AIFC-IAAA	−0.164	−0.3059	−0.2923	−0.3256	−0.2973	$\mathcal{I}_1^{if} > \mathcal{I}_3^{if} > \mathcal{I}_5^{if} > \mathcal{I}_2^{if} > \mathcal{I}_4^{if}$
Proposed AIFC-IAAG	0.6424	0.3955	0.4728	0.5523	0.4465	$\mathcal{I}_1^{if} > \mathcal{I}_4^{if} > \mathcal{I}_3^{if} > \mathcal{I}_5^{if} > \mathcal{I}_2^{if}$

According to the analysis in Table 6, we noticed that all the proposed techniques and some existing techniques obtained the same ranking results, where the best decision is $\mathcal{I}_1^{if}$ (a rule-based system). From this table, we observe that Wang et al. [43] presented the aggregation operators for AIVIFSs, which can easily evaluate the data in Table 2; the ranking result is stated in the shape $\mathcal{I}_1^{if} > \mathcal{I}_4^{if} > \mathcal{I}_3^{if} > \mathcal{I}_2^{if} > \mathcal{I}_5^{if}$, hence the best decision is $\mathcal{I}_1^{if}$. On the other hand, Wang and Liu [44] exposed the geometric operators for AIVIFSs, which can easily evaluate the data in Table 2; the ranking result is stated in the shape: $\mathcal{I}_1^{if} > \mathcal{I}_4^{if} > \mathcal{I}_3^{if} > \mathcal{I}_2^{if} > \mathcal{I}_5^{if}$, hence the best decision is $\mathcal{I}_1^{if}$. Garg et al. [45] derived the CI operators for AIVIFSs, which can easily evaluate the data in Table 2; the ranking result is stated in the shape $\mathcal{I}_1^{if} > \mathcal{I}_4^{if} > \mathcal{I}_3^{if} > \mathcal{I}_2^{if} > \mathcal{I}_5^{if}$, hence the best decision is $\mathcal{I}_1^{if}$. Meng et al. [46] proposed the geometric CI operators for AIVIFSs, which can easily evaluate the data in Table 2; the ranking result is stated in the shape $\mathcal{I}_1^{if} > \mathcal{I}_4^{if} > \mathcal{I}_3^{if} > \mathcal{I}_2^{if} > \mathcal{I}_5^{if}$, hence the best decision is $\mathcal{I}_1^{if}$. Senapati et al. [47] evaluated the Aczel–Alsina operators for AIVIFSs, which can easily evaluate the data in Table 2; the ranking result is stated in the shape $\mathcal{I}_1^{if} > \mathcal{I}_4^{if} > \mathcal{I}_3^{if} > \mathcal{I}_2^{if} > \mathcal{I}_5^{if}$, hence the best decision is $\mathcal{I}_1^{if}$. Senapati et al. [48] exposed the Aczel–Alsina geometric operators for AIVIFSs, which can easily evaluate the data in Table 2; the ranking result is stated in the shape $\mathcal{I}_1^{if} > \mathcal{I}_4^{if} > \mathcal{I}_3^{if} > \mathcal{I}_2^{if} > \mathcal{I}_5^{if}$, hence the best decision is $\mathcal{I}_1^{if}$. All these existing approaches are based on algebraic norm operations; however, the proposed operators are based on the Choquet integral-based Aczel–Alsina (AA) norms. The Aczel–Alsina explored the theory of triangular norms in the form of Aczel–Alsina triangular norms. Aczel–Alsina triangular norms are robust tools utilized to overcome the loss of information during the aggregation of information. The major advantage of defining these operational laws is that it generalizes the algebraic and Einstein t-norm operations. Another advantage of the proposed method is the utilization of the Choquet integral (CI) to consider an inter-relationship between the attribute information. This CI considers the fuzzy measures during the measurement using ordered position. Therefore, the proposed techniques are more reliable and more general than existing techniques, such as the theory of Wang et al. [43], Wang and Liu [44], Garg et al. [45], Meng et al. [46], Senapati et al. [47], and Senapati et al. [48]. Hence, the proposed technique is superior to existing methodologies.

7. Conclusions

In this paper, we addressed the algorithm related to the decision-making problem using the concept of the aggregation operators under an AIVIFS environment. In this work, the uncertainties present in the information were handled with the help of AIVIFNs, and the Choquet integral-based operators were defined by incorporating the Aczel–Alsina norm operations. The defined operational laws based on Aczel–Alsina norms are considered as the generalization of several existing laws. The key merit of such operational laws is that they generalize the algebraic and Einstein t-norm operations. Also, the Choquet integral was utilized in the study to consider an inter-relationship between the attribute information. This CI considers the fuzzy measures during the measurement using ordered position. Based on these CI and operational laws, we stated some weighted averaging and geometric operators, namely, AIVIFC-IAAA, AIVIFC-IAAG, AIVIFC-IAAOA, AIVIFC-IAAOG, AIVIF AIFC-IAAHA, and AIVIFC-IAAHG operators to aggregate the information of the AIVIFNs. Finally, we addressed the MADM algorithm, and the applicability of the proposed algorithm was demonstrated through a numerical example. A comparative analysis with some of the existing studies shows the feasibility of the proposed algorithm.

In the future, on the one hand, the proposed method has the potential to be extended to several areas, such as classification-based strategy [49], heterogeneous preference information [50], consensus [51], and addressing multicriteria problems in large-scale group decision making [52]. Further, the presented idea has been extended to different extensions of the fuzzy sets, such as the T-spherical fuzzy set, the multiplicative set, the neutrosophic set, etc. On the other hand, we shall extend our approach to analyze different applications related to evidence theory [53,54], support vector machines [55], supply chains [56], and different tools of artificial intelligence, such as optimization or neural networks, etc. [57].

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Appendix A

Proof. Considering the procedure of mathematical induction, we proved that the data in Equation (21) also hold for all positive integers. For this, we have the following procedure: Step 1: Let $z = 2$, and we have

$$\mathcal{I}_1^{if}\left(\blacksquare\left(\Xi\rho_{(1)}\right) - \blacksquare\left(\Xi\rho_{(1+1)}\right)\right) = \mathcal{I}_1^{if}\left(\blacksquare\left(\Xi\rho_{(1)}\right) - \blacksquare\left(\Xi\rho_{(2)}\right)\right)$$

$$= \left(\begin{array}{c} \left[\begin{array}{l} 1 - e^{-\left(\left(\blacksquare(\Xi\rho_{(1)}) - \blacksquare(\Xi\rho_{(2)})\right)\left(-\log\left(1-\Gamma_1^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ 1 - e^{-\left(\left(\blacksquare(\Xi\rho_{(1)}) - \blacksquare(\Xi\rho_{(2)})\right)\left(-\log\left(1-\Gamma_1^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \end{array}\right], \\ \left[\begin{array}{l} e^{-\left(\left(\blacksquare(\Xi\rho_{(1)}) - \blacksquare(\Xi\rho_{(2)})\right)\left(-\log\Lambda_1^{\mathcal{I}^-}\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ e^{-\left(\left(\blacksquare(\Xi\rho_{(1)}) - \blacksquare(\Xi\rho_{(2)})\right)\left(-\log\Lambda_1^{\mathcal{I}^+}\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \end{array}\right] \end{array}\right)$$

$$\mathcal{I}_2^{if}\left(\blacksquare\left(\Xi\rho_{(2)}\right)-\blacksquare\left(\Xi\rho_{(2+1)}\right)\right)=\mathcal{I}_2^{if}\left(\blacksquare\left(\Xi\rho_{(2)}\right)-\blacksquare\left(\Xi\rho_{(3)}\right)\right)$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}1-e^{-\left(\left(\blacksquare(\Xi\rho_{(2)})-\blacksquare(\Xi\rho_{(3)})\right)\left(-\log\left(1-\Gamma_2^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ 1-e^{-\left(\left(\blacksquare(\Xi\rho_{(2)})-\blacksquare(\Xi\rho_{(3)})\right)\left(-\log\left(1-\Gamma_2^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right],\\ \left[\begin{array}{c}e^{-\left(\left(\blacksquare(\Xi\rho_{(2)})-\blacksquare(\Xi\rho_{(3)})\right)\left(-\log\Lambda_2^{\mathcal{I}-}\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ e^{-\left(\left(\blacksquare(\Xi\rho_{(2)})-\blacksquare(\Xi\rho_{(3)})\right)\left(-\log\Lambda_2^{\mathcal{I}+}\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

Then, we have

$$AIVIFC-IAAA\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if}\right)=\mathcal{I}_1^{if}\left(\blacksquare\left(\Xi\rho_{(1)}\right)-\blacksquare\left(\Xi\rho_{(2)}\right)\right)\oplus\mathcal{I}_2^{if}\left(\blacksquare\left(\Xi\rho_{(2)}\right)-\blacksquare\left(\Xi\rho_{(3)}\right)\right)$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}1-e^{-\left(\left(\blacksquare(\Xi\rho_{(1)})-\blacksquare(\Xi\rho_{(2)})\right)\left(-\log\left(1-\Gamma_1^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ 1-e^{-\left(\left(\blacksquare(\Xi\rho_{(1)})-\blacksquare(\Xi\rho_{(2)})\right)\left(-\log\left(1-\Gamma_1^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right],\\ \left[\begin{array}{c}e^{-\left(\left(\blacksquare(\Xi\rho_{(1)})-\blacksquare(\Xi\rho_{(2)})\right)\left(-\log\Lambda_1^{\mathcal{I}-}\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ e^{-\left(\left(\blacksquare(\Xi\rho_{(1)})-\blacksquare(\Xi\rho_{(2)})\right)\left(-\log\Lambda_1^{\mathcal{I}+}\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

$$\oplus\left(\begin{array}{c}\left[\begin{array}{c}1-e^{-\left(\left(\blacksquare(\Xi\rho_{(2)})-\blacksquare(\Xi\rho_{(3)})\right)\left(-\log\left(1-\Gamma_2^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ 1-e^{-\left(\left(\blacksquare(\Xi\rho_{(2)})-\blacksquare(\Xi\rho_{(3)})\right)\left(-\log\left(1-\Gamma_2^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right],\\ \left[\begin{array}{c}e^{-\left(\left(\blacksquare(\Xi\rho_{(2)})-\blacksquare(\Xi\rho_{(3)})\right)\left(-\log\Lambda_2^{\mathcal{I}-}\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ e^{-\left(\left(\blacksquare(\Xi\rho_{(2)})-\blacksquare(\Xi\rho_{(3)})\right)\left(-\log\Lambda_2^{\mathcal{I}+}\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}1-e^{-\left(\sum_{\varphi=1}^{2}\left(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1-\Gamma_\varphi^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ 1-e^{-\left(\sum_{\varphi=1}^{2}\left(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1-\Gamma_\varphi^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right],\\ \left[\begin{array}{c}e^{-\left(\sum_{\varphi=1}^{2}\left(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Lambda_\varphi^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ e^{-\left(\sum_{\varphi=1}^{2}\left(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Lambda_\varphi^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

Information in Equation (21) is fully suitable for $z=2$. Additionally, we are letting the data in Equation (21) also be suitable for $z=q$, and then we have

$$AIVIFC-IAAA\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_q^{if}\right)=\oplus_{\varphi=1}^{q}\mathcal{I}_\varphi^{if}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}1-e^{-\left(\sum_{\varphi=1}^{q}\left(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1-\Gamma_\varphi^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ 1-e^{-\left(\sum_{\varphi=1}^{q}\left(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1-\Gamma_\varphi^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right],\\ \left[\begin{array}{c}e^{-\left(\sum_{\varphi=1}^{q}\left(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Lambda_\varphi^{\mathcal{I}-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}},\\ e^{-\left(\sum_{\varphi=1}^{q}\left(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Lambda_\varphi^{\mathcal{I}+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

Thus, we prove that the data in Equation (21) are also suitable for $z = q + 1$, and we have

$$IVIFC - IAAA\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_q^{if}, \mathcal{I}_{q+1}^{if}\right)$$

$$= \oplus_{\varphi=1}^{q} \mathcal{I}_\varphi^{if}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right) - \blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right) \oplus \mathcal{I}_{q+1}^{if}\left(\blacksquare\left(\Xi\rho_{(q+1)}\right) - \blacksquare\left(\Xi\rho_{(q+2)}\right)\right)$$

$$= \left(\begin{array}{c}\left[\begin{array}{c}1 - e^{-\left(\sum_{\varphi=1}^{q}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right) - \blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_\varphi^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ 1 - e^{-\left(\sum_{\varphi=1}^{q}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right) - \blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_\varphi^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right], \\ \left[\begin{array}{c}e^{-\left(\sum_{\varphi=1}^{q}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right) - \blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(\Lambda_\varphi^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ e^{-\left(\sum_{\varphi=1}^{q}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right) - \blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(\Lambda_\varphi^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

$$\oplus \left(\begin{array}{c}\left[\begin{array}{c}1 - e^{-\left(\left(\blacksquare\left(\Xi\rho_{(q+1)}\right) - \blacksquare\left(\Xi\rho_{(q+2)}\right)\right)\left(-\log\left(1-\Gamma_{q+1}^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ 1 - e^{-\left(\left(\blacksquare\left(\Xi\rho_{(q+1)}\right) - \blacksquare\left(\Xi\rho_{(q+2)}\right)\right)\left(-\log\left(1-\Gamma_{q+1}^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right], \\ \left[\begin{array}{c}e^{-\left(\left(\blacksquare\left(\Xi\rho_{(q+1)}\right) - \blacksquare\left(\Xi\rho_{(q+2)}\right)\right)\left(-\log\Lambda_{q+1}^{\mathcal{I}^-}\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ e^{-\left(\left(\blacksquare\left(\Xi\rho_{(q+1)}\right) - \blacksquare\left(\Xi\rho_{(q+2)}\right)\right)\left(-\log\Lambda_{q+1}^{\mathcal{I}^+}\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

$$= \left(\begin{array}{c}\left[\begin{array}{c}1 - e^{-\left(\sum_{\varphi=1}^{q+1}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right) - \blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_\varphi^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ 1 - e^{-\left(\sum_{\varphi=1}^{q+1}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right) - \blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_\varphi^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right], \\ \left[\begin{array}{c}e^{-\left(\sum_{\varphi=1}^{q+1}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right) - \blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(\Lambda_\varphi^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ e^{-\left(\sum_{\varphi=1}^{q+1}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right) - \blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(\Lambda_\varphi^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

Hence, we evaluated that the data in Equation (21) are suitable for all positive values of z. □

Appendix B

Proof. Consider $\mathcal{I}_\varphi^{if} = \mathcal{I}^{if} = \left(\left[\Gamma^{\mathcal{I}^-}, \Gamma^{\mathcal{I}^+}\right], \left[\Lambda^{\mathcal{I}^-}, \Lambda^{\mathcal{I}^+}\right]\right), \varphi = 1, 2, \ldots, z$, and then using the data in Equation (21), we have

$$AIVIFC-IAAA\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right)=\left(\begin{array}{c}\left[\begin{array}{l}1-e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(1-\Gamma_\varphi^{\mathcal{I}^-}))^{\varpi}\right)^{\frac{1}{\varpi}}},\\1-e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(1-\Gamma_\varphi^{\mathcal{I}^+}))^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right],\\\left[\begin{array}{l}e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(\Lambda_\varphi^{\mathcal{I}^-}))^{\varpi}\right)^{\frac{1}{\varpi}}},\\e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(\Lambda_\varphi^{\mathcal{I}^+}))^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

$$=\left(\begin{array}{c}\left[\begin{array}{l}1-e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho)-\blacksquare(\Xi\rho_{(1)}))(-\log(1-\Gamma^{\mathcal{I}^-}))^{\varpi}\right)^{\frac{1}{\varpi}}},\\1-e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho)-\blacksquare(\Xi\rho_{(1)}))(-\log(1-\Gamma^{\mathcal{I}^+}))^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right],\\\left[\begin{array}{l}e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho)-\blacksquare(\Xi\rho_{(1)}))(-\log(\Lambda^{\mathcal{I}^-}))^{\varpi}\right)^{\frac{1}{\varpi}}},\\e^{-\left(\sum_{\varphi=1}^{z}(\blacksquare(\Xi\rho)-\blacksquare(\Xi\rho_{(1)}))(-\log(\Lambda^{\mathcal{I}^+}))^{\varpi}\right)^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

$$=\left(\begin{array}{c}\left[1-e^{-\left((-\log(1-\Gamma^{\mathcal{I}^-}))^{\varpi}\right)^{\frac{1}{\varpi}}},1-e^{-\left((-\log(1-\Gamma^{\mathcal{I}^+}))^{\varpi}\right)^{\frac{1}{\varpi}}}\right],\\\left[e^{-\left((-\log(\Lambda^{\mathcal{I}^-}))^{\varpi}\right)^{\frac{1}{\varpi}}},e^{-\left((-\log(\Lambda^{\mathcal{I}^+}))^{\varpi}\right)^{\frac{1}{\varpi}}}\right]\end{array}\right)$$

$$=\left(\left[1-e^{-(-\log(1-\Gamma^{\mathcal{I}^-}))},1-e^{-(-\log(1-\Gamma^{\mathcal{I}^+}))}\right],\left[e^{-(-\log(\Lambda^{\mathcal{I}^-}))},e^{-(-\log(\Lambda^{\mathcal{I}^+}))}\right]\right)$$

$$=\left(\left[1-e^{\log(1-\Gamma^{\mathcal{I}^-})},1-e^{\log(1-\Gamma^{\mathcal{I}^+})}\right],\left[e^{\log(\Lambda^{\mathcal{I}^-})},e^{\log(\Lambda^{\mathcal{I}^+})}\right]\right)$$

$$=\left(\left[1-\left(1-\Gamma^{\mathcal{I}^-}\right),1-\left(1-\Gamma^{\mathcal{I}^+}\right)\right],\left[\Lambda^{\mathcal{I}^-},\Lambda^{\mathcal{I}^+}\right]\right)$$

$$=\left(\left[\Gamma^{\mathcal{I}^-},\Gamma^{\mathcal{I}^+}\right],\left[\Lambda^{\mathcal{I}^-},\Lambda^{\mathcal{I}^+}\right]\right)=\mathcal{I}^{if}.$$

Hence, we successfully proved our required results

$$AIVIFC-IAAA\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right)=\mathcal{I}^{if}.$$

□

Appendix C

Proof. Suppose $\mathcal{I}_\varphi^{if} \leq \mathcal{I}^{*}{}_\varphi^{if}$, then we have $\Gamma_\varphi^{\mathcal{I}^-} \leq \Gamma_\varphi^{\mathcal{I}^{-*}}, \Gamma_\varphi^{\mathcal{I}^+} \leq \Gamma_\varphi^{\mathcal{I}^{+*}}$ and $\Lambda_\varphi^{\mathcal{I}^-} \geq \Lambda_\varphi^{\mathcal{I}^{-*}}$, $\Lambda_\varphi^{\mathcal{I}^+} \geq \Lambda_\varphi^{\mathcal{I}^{+*}}$; thus,

$$\begin{aligned}
\Gamma_{\varphi}^{\mathcal{I}^-} \leq \Gamma_{\varphi}^{\mathcal{I}^{-*}} &\Rightarrow 1-\Gamma_{\varphi}^{\mathcal{I}^-} \geq 1-\Gamma_{\varphi}^{\mathcal{I}^{-*}} \Rightarrow \log\left(1-\Gamma_{\varphi}^{\mathcal{I}^-}\right) \geq \log\left(1-\Gamma_{\varphi}^{\mathcal{I}^{-*}}\right) \\
&\Rightarrow \left(-\log\left(1-\Gamma_{\varphi}^{\mathcal{I}^-}\right)\right)^{\varpi} \leq \left(-\log\left(1-\Gamma_{\varphi}^{\mathcal{I}^{-*}}\right)\right)^{\varpi} \\
&\Rightarrow \sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_{\varphi}^{\mathcal{I}^-}\right)\right)^{\varpi} \\
&\leq \sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_{\varphi}^{\mathcal{I}^{-*}}\right)\right)^{\varpi} \\
&\Rightarrow -\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_{\varphi}^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}} \\
&\geq -\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_{\varphi}^{\mathcal{I}^{-*}}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}} \\
&\Rightarrow e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_{\varphi}^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \geq e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_{\varphi}^{\mathcal{I}^{-*}}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \\
&\Rightarrow -e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_{\varphi}^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \geq -e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_{\varphi}^{\mathcal{I}^{-*}}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \\
&\Rightarrow 1-e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_{\varphi}^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \\
&\leq 1-e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_{\varphi}^{\mathcal{I}^{-*}}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \\
&\Rightarrow 1-e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_{\varphi}^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \\
&\leq 1-e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(1-\Gamma_{\varphi}^{\mathcal{I}^{+*}}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}
\end{aligned}$$

Furthermore, we evaluated the nonmembership grade, such as

$$\begin{aligned}
\Lambda_{\varphi}^{\mathcal{I}^-} \geq \Lambda_{\varphi}^{\mathcal{I}^{-*}} &\Rightarrow -\log\left(\Lambda_{\varphi}^{\mathcal{I}^-}\right) \leq -\log\left(\Lambda_{\varphi}^{\mathcal{I}^{-*}}\right) \\
\Rightarrow \left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(\Lambda_{\varphi}^{\mathcal{I}^-}\right)\right)^{\varpi} &\leq \left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(\Lambda_{\varphi}^{\mathcal{I}^{-*}}\right)\right)^{\varpi} \\
&\Rightarrow \left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(\Lambda_{\varphi}^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}} \\
&\leq \left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(\Lambda_{\varphi}^{\mathcal{I}^{-*}}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}} \\
&\Rightarrow -\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(\Lambda_{\varphi}^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}} \\
&\geq -\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(\Lambda_{\varphi}^{\mathcal{I}^{-*}}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}} \\
\Rightarrow e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(\Lambda_{\varphi}^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} &\geq e^{-\left(\sum_{\varphi=1}^{z}\left(\blacksquare\left(\Xi\rho_{(\varphi)}\right)-\blacksquare\left(\Xi\rho_{(\varphi+1)}\right)\right)\left(-\log\left(\Lambda_{\varphi}^{\mathcal{I}^{-*}}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}
\end{aligned}$$

Finally, we used the theory in Equations (18) and (19), and we have

$$AIVIFC-IAAA\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_z^{if}\right) \leq AIVIFC-IAAA\left(\mathcal{I}^{*}{}_1^{if},\mathcal{I}^{*}{}_2^{if},\ldots,\mathcal{I}^{*}{}_z^{if}\right).$$

□

Appendix D

Proof. We combined the statement of Property 1 and Property 2, such as

$$AIVIFC - IAAA\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) \leq AIVIFC - IAAA\left(\mathcal{I}_1^{+}, \mathcal{I}_2^{+}, \ldots, \mathcal{I}_z^{+}\right) = \mathcal{I}_\varphi^{+}$$

$$AIVIFC - IAAA\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) \geq AIVIFC - IAAA\left(\mathcal{I}_1^{-}, \mathcal{I}_2^{-}, \ldots, \mathcal{I}_z^{-}\right) = \mathcal{I}_\varphi^{-}$$

Thus,

$$\mathcal{I}_\varphi^{-} \leq AIVIFC - IAAA\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_z^{if}\right) \leq \mathcal{I}_\varphi^{+}.$$

□

Appendix E

Proof. Considering the procedure of mathematical induction, we proved that the data in Equation (36) also hold for all positive integers. For this, we have the following procedure: Step 1: Let $z = 2$, and we have

$$\left(\mathcal{I}_1^{if}\right)^{(\blacksquare(\Xi\rho_{(1)}) - \blacksquare(\Xi\rho_{(1+1)}))} = \left(\mathcal{I}_1^{if}\right)^{(\blacksquare(\Xi\rho_{(1)}) - \blacksquare(\Xi\rho_{(2)}))}$$

$$= \begin{pmatrix} \begin{bmatrix} e^{-\left((\blacksquare(\Xi\rho_{(1)}) - \blacksquare(\Xi\rho_{(2)}))(-\log(\Gamma_1^{\mathcal{I}-}))^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ e^{-\left((\blacksquare(\Xi\rho_{(1)}) - \blacksquare(\Xi\rho_{(2)}))(-\log(\Gamma_1^{\mathcal{I}+}))^{\varpi}\right)^{\frac{1}{\varpi}}} \end{bmatrix}, \\ \begin{bmatrix} 1 - e^{-\left((\blacksquare(\Xi\rho_{(1)}) - \blacksquare(\Xi\rho_{(2)}))(-\log(1-\Lambda_1^{\mathcal{I}-}))^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ 1 - e^{-\left((\blacksquare(\Xi\rho_{(1)}) - \blacksquare(\Xi\rho_{(2)}))(-\log(1-\Lambda_1^{\mathcal{I}+}))^{\varpi}\right)^{\frac{1}{\varpi}}} \end{bmatrix} \end{pmatrix}$$

$$\left(\mathcal{I}_2^{if}\right)^{(\blacksquare(\Xi\rho_{(2)}) - \blacksquare(\Xi\rho_{(2+1)}))} = \left(\mathcal{I}_2^{if}\right)^{(\blacksquare(\Xi\rho_{(2)}) - \blacksquare(\Xi\rho_{(3)}))}$$

$$= \begin{pmatrix} \begin{bmatrix} e^{-\left((\blacksquare(\Xi\rho_{(2)}) - \blacksquare(\Xi\rho_{(3)}))(-\log(\Gamma_2^{\mathcal{I}-}))^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ e^{-\left((\blacksquare(\Xi\rho_{(2)}) - \blacksquare(\Xi\rho_{(3)}))(-\log(\Gamma_2^{\mathcal{I}+}))^{\varpi}\right)^{\frac{1}{\varpi}}} \end{bmatrix}, \\ \begin{bmatrix} 1 - e^{-\left((\blacksquare(\Xi\rho_{(2)}) - \blacksquare(\Xi\rho_{(3)}))(-\log(1-\Lambda_2^{\mathcal{I}-}))^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ 1 - e^{-\left((\blacksquare(\Xi\rho_{(2)}) - \blacksquare(\Xi\rho_{(3)}))(-\log(1-\Lambda_2^{\mathcal{I}+}))^{\varpi}\right)^{\frac{1}{\varpi}}} \end{bmatrix} \end{pmatrix}$$

Then, we have

$$AIVIFC-IAAG\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if}\right)=\left(\mathcal{I}_1^{if}\right)^{(\blacksquare(\Xi\rho_{(1)})-\blacksquare(\Xi\rho_{(2)}))}\otimes\left(\mathcal{I}_2^{if}\right)^{(\blacksquare(\Xi\rho_{(2)})-\blacksquare(\Xi\rho_{(3)}))}$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}e^{-((\blacksquare(\Xi\rho_{(1)})-\blacksquare(\Xi\rho_{(2)}))(-\log(\Gamma_1^{\mathcal{I}-}))^{\varpi})^{\frac{1}{\varpi}}},\\ e^{-((\blacksquare(\Xi\rho_{(1)})-\blacksquare(\Xi\rho_{(2)}))(-\log(\Gamma_1^{\mathcal{I}+}))^{\varpi})^{\frac{1}{\varpi}}}\end{array}\right],\\ \left[\begin{array}{c}1-e^{-((\blacksquare(\Xi\rho_{(1)})-\blacksquare(\Xi\rho_{(2)}))(-\log(1-\Lambda_1^{\mathcal{I}-}))^{\varpi})^{\frac{1}{\varpi}}},\\ 1-e^{-((\blacksquare(\Xi\rho_{(1)})-\blacksquare(\Xi\rho_{(2)}))(-\log(1-\Lambda_1^{\mathcal{I}+}))^{\varpi})^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

$$\oplus\left(\begin{array}{c}\left[\begin{array}{c}e^{-((\blacksquare(\Xi\rho_{(2)})-\blacksquare(\Xi\rho_{(3)}))(-\log(\Gamma_2^{\mathcal{I}-}))^{\varpi})^{\frac{1}{\varpi}}},\\ e^{-((\blacksquare(\Xi\rho_{(2)})-\blacksquare(\Xi\rho_{(3)}))(-\log(\Gamma_2^{\mathcal{I}+}))^{\varpi})^{\frac{1}{\varpi}}}\end{array}\right],\\ \left[\begin{array}{c}1-e^{-((\blacksquare(\Xi\rho_{(2)})-\blacksquare(\Xi\rho_{(3)}))(-\log(1-\Lambda_2^{\mathcal{I}-}))^{\varpi})^{\frac{1}{\varpi}}},\\ 1-e^{-((\blacksquare(\Xi\rho_{(2)})-\blacksquare(\Xi\rho_{(3)}))(-\log(1-\Lambda_2^{\mathcal{I}+}))^{\varpi})^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}e^{-(\sum_{\varphi=1}^{2}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(\Gamma_\varphi^{\mathcal{I}-}))^{\varpi})^{\frac{1}{\varpi}}},\\ e^{-(\sum_{\varphi=1}^{2}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(\Gamma_\varphi^{\mathcal{I}+}))^{\varpi})^{\frac{1}{\varpi}}}\end{array}\right],\\ \left[\begin{array}{c}1-e^{-(\sum_{\varphi=1}^{2}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(1-\Lambda_\varphi^{\mathcal{I}-}))^{\varpi})^{\frac{1}{\varpi}}},\\ 1-e^{-(\sum_{\varphi=1}^{2}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(1-\Lambda_\varphi^{\mathcal{I}+}))^{\varpi})^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

Information in Equation (36) is fully suitable for $z=2$. Additionally, we are letting the data in Equation (36) also be suitable for $z=q$, and then we have

$$AIVIFC-IAAG\left(\mathcal{I}_1^{if},\mathcal{I}_2^{if},\ldots,\mathcal{I}_q^{if}\right)=\otimes_{\varphi=1}^{q}\left(\mathcal{I}_\varphi^{if}\right)^{(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))}$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}e^{-(\sum_{\varphi=1}^{q}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(\Gamma_\varphi^{\mathcal{I}-}))^{\varpi})^{\frac{1}{\varpi}}},\\ e^{-(\sum_{\varphi=1}^{q}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(\Gamma_\varphi^{\mathcal{I}+}))^{\varpi})^{\frac{1}{\varpi}}}\end{array}\right],\\ \left[\begin{array}{c}1-e^{-(\sum_{\varphi=1}^{q}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(1-\Lambda_\varphi^{\mathcal{I}-}))^{\varpi})^{\frac{1}{\varpi}}},\\ 1-e^{-(\sum_{\varphi=1}^{a}(\blacksquare(\Xi\rho_{(\varphi)})-\blacksquare(\Xi\rho_{(\varphi+1)}))(-\log(1-\Lambda_\varphi^{\mathcal{I}+}))^{\varpi})^{\frac{1}{\varpi}}}\end{array}\right]\end{array}\right)$$

Thus, we prove that the data in Equation (36) are also suitable for $z = q + 1$, and we have

$$AIVIFC - IAAG\left(\mathcal{I}_1^{if}, \mathcal{I}_2^{if}, \ldots, \mathcal{I}_q^{if}, \mathcal{I}_{q+1}^{if}\right)$$

$$= \otimes_{\varphi=1}^{q}\left(\mathcal{I}_\varphi^{if}\right)^{\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)} \otimes \left(\mathcal{I}_{q+1}^{if}\right)^{\left(\blacksquare(\Xi\rho_{(q+1)}) - \blacksquare(\Xi\rho_{(q+2)})\right)}$$

$$= \begin{pmatrix} \begin{bmatrix} e^{-\left(\sum_{\varphi=1}^{q}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Gamma_\varphi^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ e^{-\left(\sum_{\varphi=1}^{q}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Gamma_\varphi^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \end{bmatrix}, \\ \begin{bmatrix} 1 - e^{-\left(\sum_{\varphi=1}^{q}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1-\Lambda_\varphi^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ 1 - e^{-\left(\sum_{\varphi=1}^{q}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1-\Lambda_\varphi^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \end{bmatrix} \end{pmatrix}$$

$$\oplus \begin{pmatrix} \begin{bmatrix} e^{-\left(\left(\blacksquare(\Xi\rho_{(q+1)}) - \blacksquare(\Xi\rho_{(q+2)})\right)\left(-\log\left(\Gamma_{q+1}^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ e^{-\left(\left(\blacksquare(\Xi\rho_{(q+1)}) - \blacksquare(\Xi\rho_{(q+2)})\right)\left(-\log\left(\Gamma_{q+1}^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \end{bmatrix}, \\ \begin{bmatrix} 1 - e^{-\left(\left(\blacksquare(\Xi\rho_{(q+1)}) - \blacksquare(\Xi\rho_{(q+2)})\right)\left(-\log\left(1-\Lambda_{q+1}^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ 1 - e^{-\left(\left(\blacksquare(\Xi\rho_{(q+1)}) - \blacksquare(\Xi\rho_{(q+2)})\right)\left(-\log\left(1-\Lambda_{q+1}^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \end{bmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} \begin{bmatrix} e^{-\left(\sum_{\varphi=1}^{q+1}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Gamma_\varphi^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ e^{-\left(\sum_{\varphi=1}^{q+1}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(\Gamma_\varphi^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \end{bmatrix}, \\ \begin{bmatrix} 1 - e^{-\left(\sum_{\varphi=1}^{q+1}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1-\Lambda_\varphi^{\mathcal{I}^-}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}}, \\ 1 - e^{-\left(\sum_{\varphi=1}^{q+1}\left(\blacksquare(\Xi\rho_{(\varphi)}) - \blacksquare(\Xi\rho_{(\varphi+1)})\right)\left(-\log\left(1-\Lambda_\varphi^{\mathcal{I}^+}\right)\right)^{\varpi}\right)^{\frac{1}{\varpi}}} \end{bmatrix} \end{pmatrix}$$

Hence, we evaluated that the data in Equation (36) are suitable for all positive values of z.□

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