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Assessment of Non-Coplanar Maneuver Parameters and Perturbing Accelerations Using the Minimal Number of Observations

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Abstract: The algorithms for determining the active space object maneuver parameters in the conditions of near-circular orbits are presented in the paper. The right ascension and declination angles are used to determine the parameters of a single maneuver with transversal and lateral components (the application moment and the velocity impulse magnitude). Two pairs of angles are used to determine the parameters of the maneuver with only a lateral component. Two pairs of angles are needed for the determination of the parameters of the long-duration maneuver performed by a low-thrust engine (moments of the maneuver start and finish, and components of the acceleration delivered by the engine). The essential detail that makes it possible to determine the parameters of long-duration maneuvers is their symmetry relative to the center of the engine operating interval. Considerable perturbing accelerations, which are not accounted for by traditional perturbing models, affect passive objects, especially membranous objects with a big and variable area-to-mass ratio. This decreases dramatically the accuracy of these objects' motion propagation. In this paper, the magnitude of perturbing acceleration is determined with the assumption that it is constant and is active throughout the whole time interval from the moment of the last orbit determination to the moment of the new measurement used. Examples of the assessment of maneuvers performed by an object in the geostationary orbit are presented.

Keywords: maneuver; active space object; maneuver assessment; geostationary orbit; optical measurements; tracklet; perturbing accelerations; low thrust

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1. Introduction

Nowadays, the catalogs of different organizations contain information for more than 25,000 space objects (SOs) in near-Earth orbits. The majority of them move along the slightly perturbed Keplerian orbits, and their motion propagation presents no considerable difficulties. However, there are objects for which motion can be quite challenging to propagate. First of all, this concerns maneuvering space objects (MSOs). More than 5000 of such objects are already in orbit and more are going to come due to the deployment of constellations with hundreds of active satellites. These objects maneuver to maintain their orbits or with the purpose of changing their orbits in order to fulfill new mission tasks. As a rule, only MSO operators have real-time data containing information about alterations of orbit parameters. Moreover, the maneuvers of a number of satellites are being calculated on board. Hence, currently, we have to accumulate enough information for the accurate determination of the new orbit of these objects after the fulfilled maneuvers. The rapid propagation of motion of the maneuvering SO, with consideration of the performed maneuver, will allow us to calculate the collision probability between the protected and the maneuvering objects in a prompt and accurate fashion, operatively determine the MSO orbit changing goal, and assess the accuracy of the fulfillment of the maneuvers performed by our own spacecraft, i.e., assess the correctness of the engine functioning.

Due to its importance, the problem of assessing the maneuver performed by an MSO has been in the process of being solved for years by specialists of different space organizations. Many works on this topic have been published. The works differ by the measurements used (optical or radio locational) and by the types and heights of the MSO orbits. The parameters of the orbit after a maneuver are considered to be known in some papers, and the orbit is used for the assessment of the maneuver; in some works, the orbit is determined simultaneously with the maneuver parameters. Usually, only the fact that the maneuver took place is established in the papers. Paper [1] contains 55 items in bibliography list considering the maneuver assessment, paper [2]—49, and our work [3] contains 26 items. These lists differ from each other almost completely. Despite the fact that not all cited works in these three articles relate to the maneuver assessment topic, their total number exceeds 100. This list is far from being full. A review of the results from more than 100 works is worth conducting in a separate paper, which will be far more substantial than this work, not mentioning that the contents of the aforementioned papers with high quality reviews can be easily accessed. In this light, we will not repeat ourselves and consider study [1], which is one of the most recent works on the maneuver assessment topic. The choice of this work can be also explained by the fact that the problem statement in [1] practically coincides with the problem statement in this work. Hence, it can be agreed that study [1] contains the most recent accomplishments relating to the problem considered in this paper. It is not necessary to review the works considering low Earth orbits and radiolocational measurements in this paper due to the total difference between the problem of interest and these problems.

Paper [1] and this paper have the same basis. Optical measurements are used in both papers. It is assumed that there are series of close optical measurements with the distance of several seconds to several dozens of seconds. The total duration of the series is far less than the orbit period. Such series are denoted as tracklets in [1]. The basic problem of the maneuvering objects catalogue maintenance is being solved in the present paper, and it is necessary to determine the MSO orbit after the maneuver and assess the parameters of the maneuver itself with the help of the minimal number of tracklets. Although this problem is not solved in [1] (it is mentioned that it is to be solved in the future), another important problem is solved: the measurement identification problem. Since the geostationary area is heavily inhabited, several objects may occur in one limited area. It is not an easy matter to tell which new measurement corresponds to which object in the area. The identification problem is not solved in this work; it is assumed that measurements are already assigned to the specific objects. But the simplicity of the suggested maneuver assessment and hence the simplicity of the orbit determination after the maneuver allows for easy consideration of the possible variants (there are not many as only several objects may be in one GSO point) and use of the results obtained in the paper for the identification of measurements. It is mentioned several times in paper [1] that is not possible to determine the orbit by one short tracklet. This is correct if there is no information on the orbit before the maneuver. For example, Laplace and Gauss methods are now used to determine the orbit of a newly discovered object. Three short measurements on a relatively short curve are needed for these methods. However, it is shown in the present work that one measurement (two at most) is needed for the determination of the orbit after a maneuver for the maneuvering objects for which the orbits are known.

As it has been already mentioned in a number of works (including the author's works), the orbit after a maneuver is considered to be known and it is used during the maneuver assessment. However, in other works, the orbit after a maneuver is determined simultaneously with the parameters of the maneuver [4–7]. These works are noted due to the use of optical measurements in them. The next step is taken in this paper. The maneuver parameters are assessed by the minimal number of observations without an accumulation of the measurements necessary for the initial orbit determination. Then, the parameters of an orbit shaped by a maneuver are determined with the help of information about the maneuver. The methodology for the assessment of the maneuvers of active SOs in a geostationary orbit (GSO) developed in this paper will allow us to sped up the accurate modeling of the motion of these objects with the use of the minimal number of pairs α, δ (the angles of the right ascension and declination) which set the direction from the observer on the Earth's surface to the points in which MSOs are detected.

The suggested methodology to assess active SO maneuvers will allow us to determine the parameters of the single-impulse maneuvers with different engine orientations, and the maneuvers can be performed both by high- and low-thrust engines. These objects comprise approximately 20% of all objects registered in geostationary orbits. The developed approach to the solution of the problem allows us to also assess the non-modeled perturbations, which are currently not accounted for during the assessment of the orbit of some passive SOs. The motion propagation of approximately 30% of the passive objects of space debris registered in the geostationary orbit is not accurate enough for the calculation of the tolerable possibility of collision with these objects. It happens due to the fact that traditional perturbing models used for the determination of orbits of these objects do not correspond to real perturbations. An algorithm to account for the additional non-modeled perturbations is suggested in this paper. This will allow us to improve the accuracy of the motion propagation of the objects of this type. This paper contains the development of the methodology published in [3], in which only coplanar maneuvers were considered and the perturbing accelerations were not determined.

2. Materials and Methods

2.1. Variants of the Problems Solved

2.1.1. General Problem Statement

The MSO state vector $\mathbf{X} = \mathbf{X}(t_0)$ is known at time t_0. The observer's coordinates on the Earth's surface $\mathbf{Y} = \mathbf{Y}(t_i)$ for $t_i(t_i > t_0)$ and the angles α_i, δ_i, which point out the directions from the observer to MSO in this moment of time, are also known. We determine the times at which the engine starts and stops (the engine started after t_0 but before t_i), its orientation, the MSO acceleration and the maneuver total delta-v.

2.1.2. Equations Describing the Influence of the Velocity Impulses

In case the burn duration is small enough in regard to the MSO orbit period, the following assumption can be made: the orbit parameters' alteration due to the influence of the burn was instant.

Each impulse, applied at points with the angles $\varphi_i (i = 1, \ldots, N)$, causes deviations in the orbital elements at the specified point with the angle φ_f, and the sums of these deviations after N impulses can be written as follows [8]:

$$\sum_{i=1}^{N} r_0 \left(\frac{\Delta V_{r_i}}{V_0} \sin\left(\varphi_f - \varphi_i\right) + 2\frac{\Delta V_{t_i}}{V_0}\left(1 - \cos\left(\varphi_f - \varphi_i\right)\right)\right) = \Delta r, \tag{1}$$

$$\sum_{i=1}^{N} \left(\Delta V_{r_i} \cos\left(\varphi_f - \varphi_i\right) + 2\Delta V_{t_i} \sin\left(\varphi_f - \varphi_i\right)\right) = \Delta V_r, \tag{2}$$

$$\sum_{i=1}^{N} \left(-\Delta V_{r_i} \sin\left(\varphi_f - \varphi_i\right) - \Delta V_{t_i}\left(1 - 2\cos\left(\varphi_f - \varphi_i\right)\right)\right) = \Delta V_t, \tag{3}$$

$$\sum_{i=1}^{N} r_0 \left(-2\frac{\Delta V_{r_i}}{V_0}\left(1 - \cos\left(\varphi_f - \varphi_i\right)\right) - \frac{\Delta V_{t_i}}{V_0}\left(3\left(\varphi_f - \varphi_i\right) - 4\sin\left(\varphi_f - \varphi_i\right)\right)\right) = \Delta n, \tag{4}$$

$$\sum_{i=1}^{N} r_0 \frac{\Delta V_{z_i}}{V_0} \sin\left(\varphi_f - \varphi_i\right) = z, \tag{5}$$

$$\sum_{i=1}^{N} \Delta V_{z_i} \cos\left(\varphi_f - \varphi_i\right) = V_z, \tag{6}$$

where r_0, V_0 = the reference circular orbit radius and velocity,$\Delta V_{r_i}, \Delta V_{t_i}, \Delta V_{z_i}$ = the radial, transversal, and lateral components of the i-th impulse, respectively, and $\Delta r, \Delta V_r, \Delta V_t, \Delta n$ = deviations by radius, radial and transversal components of velocity and along the orbit deviation caused by the velocity impulses. The angles φ_i, φ_f are being counted from the MSO position at t_0 to the direction to the MSO movement. The motion occurs in the vicinity of the reference circular orbit.

The linearized equations of unperturbed motion in the vicinity of the circular orbit were used to obtain Equations (1)–(6) [9].

2.1.3. Determination of the Along-the-Orbit Distance and Lateral Deviations and Deviation by Radius, Using the Known Observation Angles α, δ

In order to solve the stated problem one should determine the along-the-orbit distance Δn_i, perpendicular Δz_i deviations and the deviation by radius Δr_i caused by the maneuver using the known angles α_i, δ_i in the moment t_i first. For this sake, the initial orbit $\mathbf{X}(t_0)$, which is known for t_0, is propagated from t_0 to t_i with the help of numerical integration. The orbit orientation of the uncorrected MSO orbit for t_i is determined.

Then, the point (x_i, y_i, z_i) of intersection between the beam, the direction of which is set by the position of the observer on the Earth's surface, the angles α_i, δ_i and the plane of the uncorrected orbit of the MSO, is determined. The vector $\mathbf{r}_p(t_i)$ pointed from the center of Earth to the point of intersection of the beam with the plane of the initial orbit (this vector belongs to the plane of the MSO uncorrected orbit) is determined. Then, $\Delta r_i = r_p(t_i) - r_{orbi}$ is calculated, where r_{orbi} = the magnitude of the radius vector of the uncorrected orbit, pointed along the vector $\mathbf{r}_p(t_i)$, and $r_p(t_i)$ = vector $\mathbf{r}_p(t_i)$ magnitude. The along-the-orbit distance Δn_i between the position of an MSO in the uncorrected orbit in the moment t_i (set by vector $\mathbf{r}(t_i)$) and its position in the orbit shaped by the maneuver in this moment (set by $\mathbf{r_p}(t_i)$) is calculated analogously. The angle between vectors $\mathbf{r}(t_i)$ and $\mathbf{r_p}(t_i)$ is multiplied by r_0 in order to calculate Δn_i.

In order to obtain the lateral deviation, we use the fact that the lateral velocity impulse does not change the orbit geometry. It changes only the orbital plane orientation. Hence, the distance from the Earth to the point of intersection of the beam with the new orbit plane after the maneuver coincides with the distance from the Earth to the MSO in the uncorrected orbit in this moment. Thus, the magnitude $r(t_i)$ of the vector $\mathbf{r}(t_i)$ is used for the calculation of the lateral deviation. Then, the distance L from the position of the observer on the Earth's surface to the end of the vector $\mathbf{r}(t_i)$ is found. Vector $\mathbf{R}(t_i)$ corresponds to the point on the beam, the distance from which to the center of the Earth is $r(t_i)$. This beam is set by the position of the observer and the angles α, δ. The distance from the observer to the abovementioned point is approximately L. We obtain the lateral deviation $\Delta z(t_i)$ by calculating the deviation vector $\Delta\mathbf{R} = \mathbf{R}(t_i) - \mathbf{r}(t_i)$ and by projecting $\Delta\mathbf{R}$ on the perpendicular to the initial orbit plane (effective for the moment t_i).

2.1.4. Possible Variants of the Problems Solved

Due to the existence of several possible variants regarding the realization of the maneuvers with lateral components, several problems need to be considered and solved. The methods for solving the problems and the number of the used sets of deviations $\Delta r, \Delta n, \Delta z(\alpha, \delta)$ differ too. Let us enumerate these problems.

1. The velocity impulse with only the lateral component has been performed. We determine its application angle φ and the lateral component magnitude ΔV_z. There are two unknowns in the problem. Two lateral deviations $\Delta z_1, \Delta z_2$ (two tracklets) are needed for its solution.
2. The velocity impulse with transversal and lateral components has been performed. We determine its application angle φ and the magnitude of the transversal and lateral

components $\Delta V_t, \Delta V_z$. There are three unknowns in the problem, and the deviations $\Delta r, \Delta n, \Delta z$ are used for its solution (only one tracklet is used).

3. The velocity impulse with transversal, radial, and lateral components has been performed. We determine its application angle φ and the magnitude of the transversal, radial and lateral constituents $\Delta V_t, \Delta V_r, \Delta V_z$. There are four unknowns in the problem. Hence, two sets of deviations $\Delta r, \Delta n, \Delta z$ are needed for its solution.
4. The long-duration transversal and lateral maneuver has been performed. We determine the maneuver angular duration $\Delta\varphi$, its middle φ_m, the angle between the engine's axis and the orbit plane γ and the acceleration w delivered to the MSO. There are four unknowns. Hence, two pairs of deviations $\Delta z, \Delta n$ are needed.
5. The long-duration maneuver with transversal and radial components has been performed. We determine the maneuver angular duration $\Delta\varphi$, its middle φ_m, the angle of rotation of the engine's axis in the orbit plane θ and the acceleration w delivered to the MSO. There are four unknowns in the problem. Hence, two pairs of deviations $\Delta r, \Delta n(\alpha, \delta)$ are needed for its solution. Though the accelerations in the plane of the initial orbit are calculated in this problem, the solution to the problem can be used to determine the perturbing accelerations. This problem has no direct relation to non-coplanar maneuvers, but it will be used while obtaining the non-modeled accelerations.
6. The long-duration maneuver with transversal, radial and lateral components has been performed. It is necessary to determine the maneuver angular duration $\Delta\varphi$, its middle φ_m, the angle of the engine axis's turn in the orbit plane θ, the angle between the engine's axis and the orbit plane γ and the acceleration w delivered to the MSO. There are five unknowns in the problem. Hence, two sets of deviations $\Delta r, \Delta n, \Delta z(\alpha, \delta)$ are needed to solve it.

It is worth mentioning that maneuvers in the geostationary area are usually performed for the purpose of correcting the east–west longitudinal drift and the north–south inclination drift. They are performed to maintain the set longitudinal boundaries with the minimal inclination and to resist the perturbing factors of natural Earth gravity and a third body.

2.2. Small Duration Maneuver Parameters Assessment

2.2.1. Lateral Velocity Impulse Assessment

The application angle φ_z and the magnitude of the lateral component ΔV_z of the velocity impulse are to be determined. There are two unknowns in the problem, and two deviations $\Delta z_1, \Delta z_2$ calculated for the moments t_1, t_2 are needed for its solution.

Equation (5) is used twice:

$$r_0 \frac{\Delta V_z}{V_0} \sin(\varphi_1 - \varphi_z) = \Delta z_1,$$

$$r_0 \frac{\Delta V_z}{V_0} \sin(\varphi_2 - \varphi_z) = \Delta z_2,$$

By excluding ΔV_z, we obtain

$$\sin(\varphi_1 - \varphi_z)\Delta z_2 = \Delta z_1 \sin(\varphi_2 - \varphi_z)$$

which gives us φ_z, which in turn gives us ΔV_z.

It is a widespread but simple problem with a guaranteed single solution which can be found fast. It goes without saying that the quality of the solution is affected by the accuracy of calculation of the deviations $\Delta z_1, \Delta z_2$.

2.2.2. Assessment of the Transversal and Lateral Components of the Velocity Impulse

The equations for the velocity impulse application angle φ and for the determination of the magnitudes of the transversal and lateral components $\Delta V_t, \Delta V_z$ can be written as

$$2r_0 \frac{\Delta V_t}{V_0}(1 - \cos(\varphi_f - \varphi)) = \Delta r,$$

$$r_0 \frac{\Delta V_t}{V_0}(3(\varphi_f - \varphi) - 4\sin(\varphi_f - \varphi)) = \Delta n,$$

$$r_0 \frac{\Delta V_z}{V_0} \sin(\varphi_f - \varphi) = \Delta z.$$

The angle φ_f corresponds to the point in which the deviations $\Delta r, \Delta n, \Delta z$ were calculated.

One can obtain ΔV_t from the first equation and apply it for the second one. Hence, φ is found and can be used to find ΔV_t and ΔV_z.

2.2.3. Assessment of the Impulse with Transversal, Radial and Lateral Components

There are four unknowns in the problem: the application angle φ and the magnitudes of the transversal, radial and lateral components $\Delta V_t, \Delta V_r, \Delta V_z$. Hence, two sets of the deviations $\Delta r, \Delta n, \Delta z$ calculated for the moments of time with which the angles φ_1 and φ_2 correspond are needed for its solution.

The equations for the velocity impulse application angle φ and the magnitudes of the transversal, radial and lateral components of the velocity impulse $\Delta V_t, \Delta V_r, \Delta V_z$ are as follows:

$$r_0(\frac{\Delta V_r}{V_0} \sin(\varphi_1 - \varphi) + 2\frac{\Delta V_t}{V_0}(1 - \cos(\varphi_1 - \varphi))) = \Delta r_1, \tag{7}$$

$$r_0(\frac{\Delta V_r}{V_0} \sin(\varphi_2 - \varphi) + 2\frac{\Delta V_t}{V_0}(1 - \cos(\varphi_2 - \varphi))) = \Delta r_2, \tag{8}$$

$$r_0(-2\frac{\Delta V_r}{V_0}(1 - \cos(\varphi_1 - \varphi)) - \frac{\Delta V_t}{V_0}(3(\varphi_1 - \varphi) - 4\sin(\varphi_1 - \varphi))) = \Delta n_1, \tag{9}$$

$$r_0(-2\frac{\Delta V_r}{V_0}(1 - \cos(\varphi_2 - \varphi)) - \frac{\Delta V_t}{V_0}(3(\varphi_2 - \varphi) - 4\sin(\varphi_2 - \varphi))) = \Delta n_2, \tag{10}$$

$$r_0 \frac{\Delta V_z}{V_0} \sin(\varphi_1 - \varphi) = \Delta z_1, \tag{11}$$

$$r_0 \frac{\Delta V_z}{V_0} \sin(\varphi_2 - \varphi) = \Delta z_2. \tag{12}$$

The ΔV_t and ΔV_r are obtained from Equations (7) and (8) and are later used in (10). We obtain the Equation (13) and can determine φ.

$$\begin{gathered} -\frac{2(\Delta r_1(1-\cos(\varphi_2-\varphi))-\Delta r_2(1-\cos(\varphi_1-\varphi)))}{r_0(\sin(\varphi_1-\varphi)(1-\cos(\varphi_2-\varphi)-\sin(\varphi_2-\varphi)(1-\cos(\varphi_1-\varphi))}(1-\cos(\varphi_2-\varphi))- \\ \frac{(\Delta r_1\sin(\varphi_2-\varphi)-\Delta r_2\sin(\varphi_1-\varphi))(3(\varphi_2-\varphi)-4\sin(\varphi_2-\varphi))}{2((1-\cos(\varphi_1-\varphi))\sin(\varphi_2-\varphi)-(1-\cos(\varphi_2-\varphi))\sin(\varphi_1-\varphi))}) = \Delta n_2. \end{gathered} \tag{13}$$

With the known φ, we can obtain ΔV_t and ΔV_r.

Equation (13) may have several solutions for this problem. For this case, the velocity impulse parameters for every solution are used in Equation (9). The final solution for the problem will be the solution for which Equation (9) is fulfilled with the best accuracy.

The velocity impulse of the lateral component is determined after the velocity impulse application angle. One should choose between Equation (11) or Equation (12) depending on a bigger value of Δz.

2.2.4. Shaped Orbit Determination

By altering the velocity in the initial orbit in the moment of the velocity impulse application by the magnitude of the velocity impulse components, we obtain the orbit after the maneuver.

2.3. Long-Duration Maneuver Parameters Assessment

The impulsive and the long-duration maneuvers are performed in the GSO. The type of maneuver can be distinguished by its duration in comparison to the period of the orbit in which the SO maneuvers. The maneuver will be impulsive if its duration is far less than the orbit period. When the duration of the maneuver is comparable to the orbit period, a low-thrust engine maneuver can be considered. However, the difference between two types of the maneuvers is nominal and depends on the complexity of the problem being solved. The more complex the problem is, the earlier we need to account for the maneuver duration. It was shown in [8] that a maneuver with the duration of 40° or less produces an alteration of the same orbit elements as the impulsive maneuver. The effectiveness of the long-duration maneuver decreases (especially for the eccentricity correction [8]) with a longer duration, and the maneuver duration should be accounted for.

In this problem, for the assessment of long-duration maneuvers, there is a need to determine the maneuver's start and finish and the components of the acceleration produced by the engine w_t, w_r, w_z. The symmetry relative to the center of the engine operating interval allows us to use the middle of the interval φ_m and its duration $\Delta\varphi$ instead of the beginning and end of the engine operating interval.

2.3.1. Assessment of the Influence of the Transversal and Radial Components of the Thrust Vector on the Deviations Caused by Radius and along the Orbit

Let us first determine the effect from the low-thrust maneuver on the deviation caused by radius in the moment set by the angle φ_f. The orientation of the engine is fixed by the transversal component in the orbital coordinate frame.

It is supposed that the maneuver has an angular duration, $\Delta\varphi$, the middle of the maneuver is defined by the angle φ_m, and the acceleration is delivered to the MSO w_t. By using (1), one can find the deviation caused by the radius, caused by the velocity alteration ΔV_t, equally distributed throughout the whole interval of the argument of latitude $\Delta\varphi$:

$$\Delta r_t = 2\frac{r_0}{V_0}\frac{\Delta V_t}{\Delta\varphi}\int\limits_{-\Delta\varphi/2}^{\Delta\varphi/2}(1-\cos(\varphi_f-\varphi_m-\varphi))d\varphi = 2\frac{r_0}{V_0}\frac{\Delta V_t}{\Delta\varphi}(\Delta\varphi - 2\sin\frac{\Delta\varphi}{2}\cos(\varphi_f-\varphi_m)).$$

By using the equation $\Delta\varphi = \lambda_0\Delta t = k\frac{\Delta V}{V_0} = \frac{w_c}{w}\frac{\Delta V}{V_0}$, where $\lambda_0 = \frac{V_0}{r_0}, k = \frac{mV_0^2}{Pr_0} = \frac{w_c}{w}$, w_c = centripetal acceleration of the reference circular orbit ($w_c = \frac{V_0^2}{r_0}$), and w = engine acceleration ($w = \frac{P}{m}$), we obtain the final formula for the deviation as follows:

$$\Delta r_t = 2r_0\frac{w_t}{w_c}(\Delta\varphi - 2\sin\frac{\Delta\varphi}{2}\cos(\varphi_f-\varphi_m)) \quad (14)$$

Similarly, the influence of the radial component distributed on the interval of the argument of latitude $\Delta\varphi$ on the by-radius deviation is determined as follows:

$$\Delta r_r = \frac{r_0}{V_0}\frac{\Delta V_r}{\Delta\varphi}\int\limits_{-\Delta\varphi/2}^{\Delta\varphi/2}\sin(\varphi_f-\varphi_m-\varphi)d\varphi = 2\frac{r_0}{V_0}\frac{\Delta V_r}{\Delta\varphi}\sin\frac{\Delta\varphi}{2}\sin(\varphi_f-\varphi_m)$$

$$\Delta r_r = 2r_0\frac{w_r}{w_c}\sin\frac{\Delta\varphi}{2}\sin(\varphi_f-\varphi_m) \quad (15)$$

With the help of (4), one can also find the effect of the maneuver performed by the low-thrust engine, the orientation of which is fixed by transversal component in the orbital coordinate frame, on the deviation along the orbit. Keeping in mind that the maneuver has an angular duration, $\Delta\varphi$, the middle of the maneuver is defined by the angle φ_m:

$$\Delta n_t = -\frac{r_0}{V_0}\frac{\Delta V_t}{\Delta\varphi}\int\limits_{-\Delta\varphi/2}^{\Delta\varphi/2}(3(\varphi_f-\varphi_m-\varphi)-4\sin(\varphi_f-\varphi_m-\varphi))d\varphi = \frac{r_0}{V_0}\frac{\Delta V_t}{\Delta\varphi}(3(\varphi_f-\varphi_m)\Delta\varphi-8\sin\frac{\Delta\varphi}{2}\sin(\varphi_f-\varphi_m))$$

$$\Delta n_t = r_0\frac{w_t}{w_c}(3(\varphi_f-\varphi_m)\Delta\varphi-8\sin\frac{\Delta\varphi}{2}\sin(\varphi_f-\varphi_m)) \tag{16}$$

The influence of the radial component, distributed on the interval of the latitude argument $\Delta\varphi$, on the deviation along the orbit can be found in the same fashion:

$$\Delta n_r = -2\frac{r_0}{V_0}\frac{\Delta V_r}{\Delta\varphi}\int\limits_{-\Delta\varphi/2}^{\Delta\varphi/2}(1-\cos(\varphi_f-\varphi_m-\varphi))d\varphi = -2\frac{r_0}{V_0}\frac{\Delta V_r}{\Delta\varphi}(\Delta\varphi+2\sin\frac{\Delta\varphi}{2}\cos(\varphi_f-\varphi_m))$$

$$\Delta n_r = -2r_0\frac{w_r}{w_c}(\Delta\varphi+2\sin\frac{\Delta\varphi}{2}\cos(\varphi_f-\varphi_m)), \tag{17}$$

where φ_f = the point in which the deviations are calculated.

2.3.2. Long-Duration Coplanar Maneuver Parameters Assessment

There are four unknowns in the problem of the coplanar long-duration maneuver $\varphi_m, \Delta\varphi, w_t, w_r$. Hence, two pairs of deviations $\Delta r, \Delta n(\alpha,\delta)$ should be used for its solution. By solving the equation system in which the influence of the burns is described by the Formulas (14)–(17), we obtain all the unknowns. If it is known that the object is using only the transversal orientation, only three unknowns remain, $\varphi_m, \Delta\varphi, w_t$. Only three deviations are needed: Δn for the first point and $\Delta r, \Delta n$ for the second. It is more preferable to use the deviations along the orbit, as they can be calculated more accurately, and the deviations which correspond to the more distant point from the maneuver.

2.3.3. Assessment of the Non-Modeled Accelerations in the Orbit Plane

The Formulas (14)–(17) allow us to determine the non-modeled perturbations affecting the non-maneuvering objects, the orbits of which suffer from being determined not accurately enough. It is also supposed that the non-modeled accelerations are caused not by natural causes, which is, in fact, so, but by the work of some engine. It is assumed that the perturbing acceleration is effective throughout the whole interval from the moment of the last SO orbit determination (φ_0) till the moment φ_f, for which the deviations $\Delta r, \Delta n(\alpha,\delta)$ are being calculated. Thus, we know the angular duration of the maneuver $\Delta\varphi = \varphi_f - \varphi_0$ and its middle $\varphi_m = \varphi_f - 0.5\Delta\varphi$. It can be seen that φ_f is not used anymore after the substitution of φ_m in Equations (14)–(17). Only w_t, w_r are left to determine. It is also agreed that the perturbing acceleration is constant regarding its value, does not change the orientation in the orbital coordinate frame, and is effective in the orbital plane in this problem. Thus, its action can be fully described by Equations (14)–(17). It is necessary to determine the transversal w_t and radial w_r components of the perturbing acceleration. There are two unknowns in the problem. Hence, only one pair $\Delta r, \Delta n(\alpha,\delta)$ should be used for its solution.

By using Equations (14)–(17), we obtain the system of two linear equations

$$\Delta r_t + \Delta r_r = \Delta r,$$

$$\Delta n_t + \Delta n_r = \Delta n,$$

and can obtain transversal w_t and radial w_r components of the perturbing acceleration.

The influence of the perturbations constant in the inertial coordinate frame can be calculated in a similar way. These formulae were omitted in order not to overload the understanding of the paper.

2.3.4. Assessment of the Influence of the Lateral Component of the Thrust Vector

In general, the perturbing acceleration can have the lateral component w_z, which needs to be obtained too.

Let us determine the influence of the lateral component of the perturbing acceleration w_z on the turn of the orbit plane.

Since the maneuvers in the plane and the plane-turning maneuvers do not correlate with each other, they can be considered separately.

The angular alteration of the orbit plane orientation $\Delta\gamma$, caused by the lateral velocity ΔV_z in the angular interval $\Delta\varphi$, can be calculated using the following formula:

$$\Delta\gamma = \int_{-\Delta\varphi/2}^{\Delta\varphi/2} \frac{\Delta V_z}{\Delta\varphi}\cos\varphi d\varphi = 2\frac{\sin\frac{\Delta\varphi}{2}}{\Delta\varphi}\Delta V_z,$$

where $\Delta\varphi$ = the angular magnitude of the active part, and φ = the angle between the middle of the active part and the current point. The middle of the active part is situated on the line of intersection of the orbit planes (Figure 1).

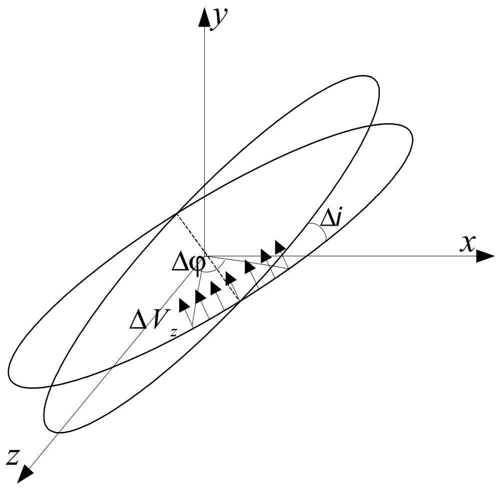

Figure 1. Orbit plane orientation alteration.

By using the equation $\Delta\varphi = \frac{w_c}{w}\Delta V$, we obtain

$$\Delta\gamma = 2\frac{w_z}{w_c}\sin(\frac{\Delta\varphi}{2}). \tag{18}$$

The line of intersection of the orbit planes, around which the turn caused by the angle $\Delta\gamma$ is performed, is set by the angle φ_m.

By using Equation (5) in the form of $r_0\frac{\Delta V_z}{V_0}\sin(\frac{\Delta\varphi}{2}) = z$, where $\frac{\Delta V_z}{V_0} \approx \Delta\gamma$, and the found equation for $\Delta\gamma$, we obtain

$$2r_0\frac{w_z}{w_c}\sin^2(\frac{\Delta\varphi}{2}) = z. \tag{19}$$

Equation (19) allows us to find the normal acceleration for the orbit plane.

2.3.5. Assessment of Influence of the Transversal and Radial Components of the Thrust Vector on Orbit Elements Deviations

Due to the burn with transversal orientation on the angular interval of the argument of latitude $\Delta\varphi$, the eccentricity and the semimajor axis will be altered by Δe and Δa, correspondingly. They can be found by using equations [8] $\Delta e = 4\frac{w_t}{w_c}\sin\frac{\Delta\varphi}{2}$ and $\Delta a = 2\frac{w_t}{w_c}\Delta\varphi$. The eccentricity alteration will be fulfilled in the direction described by the angle φ_m. The radial component of acceleration will not alter the semimajor axis. The eccentricity will be altered in the direction perpendicular to the direction of alteration of the eccentricity by the transversal component according to the magnitude $\Delta e = 2\frac{w_r}{w_c}\sin\frac{\Delta\varphi}{2}$. The alteration of the orbit plane orientation is determined by the Formula (18).

By altering the orbit elements in this point by the magnitude of the calculated deviations, we get the orbit with the accounted influence of the non-modeled perturbations. By calculating of the non-modeled accelerations on the intervals between the observations, we can form the model of additional accelerations on the whole revolution and use it for subsequent orbit determinations and motion propagations of these objects.

3. Results

The following two examples (see Tables 1–4) demonstrate the capabilities of the aforementioned method.

Table 1. Real maneuver parameters for Example 1.

Time of the initial conditions setting	t0 = 2022/12/14 09:54:51.500 (GMT + 3)
Time of the maneuver	timp 2022/12/14 18:41:51.350
Maneuver magnitude	ΔVz = 8.075 m/s
The angle between the maneuver and the initial conditions	$\Delta\varphi$ = 132.087°

Table 2. The deviations for the tracklets for Example 1.

The First Tracklet				The Second Tracklet	
t	**Δ*z***	***T***	**Δ*z***	***t***	**Δ*z***
18:47:29	−2.404681	18:48:46	−2.7938	19:25:17	−20.4987
18:47:36	−2.317569	18:48:53	2.8975	19:25:24	−20.6464
18:47:43	−2.506426	18:49:00	−2.9946	19:25:31	−20.6375
18:47:50	−2.613181	18:49:07	−3.2201	19:25:45	−20.8434
18:48:04	−2.869092	18:49:14	−3.1635	19:25:52	−20.8498
18:48:11	−2.477658	18:49:21	−3.0907	19:25:59	−20.8054
18:48:18	−2.51966	18:49:28	−3.1474	19:26:13	−20.9938

Table 3. Real maneuver parameters for Example 2.

Time of the initial conditions setting	t0 = 2022/10/25 13:14:50.000 (GMT + 3)
Time of the maneuver	timp 2022/10/26 10:12:02.550
Maneuver magnitude	ΔVz = 7.839 m/s
The angle between the maneuver and the initial conditions	$\Delta\varphi$ = 315.162°

The examples with the known realized velocity impulses were taken to assess the accuracy of the found solution.

Table 4. The deviations for the tracklets for Example 2.

The First Tracklet		**The Second Tracklet**	
T	**Δ*z*1**	***T***	**Δ*z*2**
11:15:20.366	29.166172	12:02:37.366	50.256445
11:15:34.366	29.332763	12:02:52.366	50.208965
11:15:41.366	29.304807	12:02:59.366	50.429341
11:15:48.366	29.448974	12:03:06.366	50.718814
11:15:55.366	29.529416	12:03:13.366	50.316488
11:16:02.366	29.480151	12:03:27.366	50.407611
11:16:09.366	29.499677	12:03:34.366	50.675024
11:16:16.366	29.629186		
11:16:23.366	29.687089		

3.1. Examples

3.1.1. Example 1

We have two tracklets. The optical measurements for a considered mission were obtained with errors, which do not exceed five arcseconds.

All observations of one tracklet were conducted within the intervals of less than two minutes, so several observations were replaced by one average measurement.

For the first example, the average deviations Δz are Δz1 = −2.591 km (for t = 18:48:18 $\Delta\varphi$ = 133.733°) and Δz2 = −20.754 km (for t = 19:25:45 $\Delta\varphi$ = 143.206°)

Theoretical deviations which correspond to the real velocity impulse:

Δz1 = 3.180 km $\Delta\varphi$ = 133.733° *and* Δz2 = 21.353 km $\Delta\varphi$ = 143.206°

The solution for the theoretical Δz is ΔVz = 8.075 m/s, and Δφ = 132.087° corresponds to the real velocity impulse.

The solution for the average Δz is Δz1 = −2.591 km, Δz2 = −20.754 km, ΔVz = −8.066 m/s and Δφ = 132.390°.

It can be seen that the found solution ΔVz = −8.066 m/s and Δφ = 132.390° is close to the real maneuver parameters ΔVz = 8.075 m/s and Δφ = 132.087°, despite the fact that the measurements were close to the moment of the maneuver performance, which led to the small deviation in Δz1.

3.1.2. Example 2

The average deviations for the first tracklet Δz: Δz1 = 29.453 km (for Δφ = 331.294°).

The average deviations for the second tracklet Δz: Δz2 = 50.430 km (for Δφ = 343.121°).

The solution for the theoretical Δz: Δz1 = 29.872 km, Δz2 = 50.407 km, ΔVz1 = 7.839 m/s, and Δφ = 315.162°.

The solution for the average Δz: Δz1 = 29.453 km, Δz2 = 50.430 km, ΔVz1 = 7.979 m/s, and Δφ = 315.681°.

The high accuracy of the assessment of the parameters of the real performed maneuver can be noted.

4. Conclusions

Huge sums of money in different countries are spent, nowadays, on the orbit measurements of space debris objects (SDs) and on the maintenance of the catalogues for these objects to secure the existence of owned protected spacecraft, including collision avoidance with the objects from the catalogue. However, the presence of the object in the catalogue does not guarantee that its motion is propagated with the tolerable accuracy needed for the calculation of the necessary avoidance maneuver. For example, the propagation accuracy problem arises for almost the half of SD objects from different catalogues in the geostationary orbit. Approximately 20% of 3000 objects are the maneuvering ones. Hence, a considerable amount of time will pass before their orbit after the maneuver will be determined with the necessary accuracy using the traditional methods. The orbits of approximately another 900 objects (30%) are being propagated with insufficient accuracy for the tolerable calculation of possibility of the collision with these objects and guaranteed collision avoidance. This can be explained by the fact that traditional perturbing models used for the motion propagation of these objects differ considerably from the real ones. Furthermore, the orbits of approximately 300 (10%) objects are being propagated so poorly that these objects are often lost and drop off from the catalogue. This mainly relates to the membranous objects with big varying area-to-mass ratios. This, and previously published works [3], allow us to increase the quality of motion propagation of the challenging objects, which comprise almost 50% of the objects in the GSO, registered in different catalogues. First of all, this refers to the maneuvering objects. In contrast to the traditional works in which the orbit is determined after the maneuver and the maneuver itself is assessed afterwards, or when the assessment of the maneuver occurs simultaneously with the orbit determination, in this work, the maneuvers are assessed by the measurements and the orbit of the maneuvering SO is assessed with their help. This allows us to determine the MSO orbit fast and with high accuracy as only one or two measurements are used. The given examples verify this.

The simplicity of the described maneuver assessment method ensures its simple realization and reliability.

We also hope that this work will allow us to approach the solution for the problem of propagation of motion of the membranous objects with big and variable area-to-mass ratios. By substituting the non-modeled perturbation (for example, solar radiation) with the influence of the low-thrust engine work, one can make the model of these perturbations on the intervals between the measurements and account for the influence of these perturbations during the propagation. These models are updated on each interval between the measurements.

Thus, this work, with the work [3], will allow us to improve considerably the orbit determination technology of approximately half of the objects in the geostationary orbit, currently registered in the catalogues of different organizations.

The linearized equations were used while solving this problem. This restricts the use of the suggested method to the assessment of the maneuvers performed in near-circular orbits. Since the optical measurements are used, this method fits the assessment of the maneuvers of spacecraft in the GSO and the medium Earth orbits (the navigation system orbits).

The maneuvers of the maintenance of these orbits are small enough. Hence, the linearization conditions are not violated. The use of the iteration procedure described in [8] allows for the widening the constraints of the usage of this method for the magnitude of the assessed parameters (more than 100 m/s) and for the deviations from circular orbits (several hundreds of kilometers). However, this was not the case for the practical problems of this paper.

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