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A Two-Factor Autoregressive Moving Average Model Based on Fuzzy Fluctuation Logical Relationships

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Abstract: Many of the existing autoregressive moving average (ARMA) forecast models are based on one main factor. In this paper, we proposed a new two-factor first-order ARMA forecast model based on fuzzy fluctuation logical relationships of both a main factor and a secondary factor of a historical training time series. Firstly, we generated a fluctuation time series (FTS) for two factors by calculating the difference of each data point with its previous day, then finding the absolute means of the two FTSs. We then constructed a fuzzy fluctuation time series (FFTS) according to the defined linguistic sets. The next step was establishing fuzzy fluctuation logical relation groups (FFLRGs) for a two-factor first-order autoregressive (AR(1)) model and forecasting the training data with the AR(1) model. Then we built FFLRGs for a two-factor first-order autoregressive moving average (ARMA(1,m)) model. Lastly, we forecasted test data with the ARMA(1,m) model. To illustrate the performance of our model, we used real Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) and Dow Jones datasets as a secondary factor to forecast TAIEX. The experiment results indicate that the proposed two-factor fluctuation ARMA method outperformed the one-factor method based on real historic data. The secondary factor may have some effects on the main factor and thereby impact the forecasting results. Using fuzzified fluctuations rather than fuzzified real data could avoid the influence of extreme values in historic data, which performs negatively while forecasting. To verify the accuracy and effectiveness of the model, we also employed our method to forecast the Shanghai Stock Exchange Composite Index (SHSECI) from 2001 to 2015 and the international gold price from 2000 to 2010.

Keywords: fuzzy fluctuation logical relationships; fuzzy forecasting; fuzzy fluctuation time series; fuzzy logical relationships; two-factor autoregressive moving average (ARMA) model

1. Introduction

A historic time series can show the rules and patterns of some phenomena and can be applied to forecast the same event in the future [1]. Many researchers have described time series models to predict the future of a given system, including regression analysis [2], artificial neural networks (ANN) [3], evolutionary computation [4], support vector machines (SVM) [5], and immune systems [6]. However, although these models satisfy the constraints, they might overemphasize the randomness of the dataset and distort the internal evolutionary rules, and may not perform optimally. To solve this problem, Song and Chissom proposed the fuzzy time series forecasting model [7] which introduced the fuzzy set theory by Zadeh [8] into a time series. Chen [9] developed a first order fuzzy time series to simplify the fuzzy relationships in Song and Chissom's model [7,10,11], described by complex matrix operations. Chen's method [9] has been the basis for the future research of fuzzy logic groups because of its universality and level of performance. For the selection of the length of the intervals, Huarng [12] proposed two methods: based on averages and on distribution. Since then, the fuzzy time series model has been widely used for forecasting in many nonlinear and complex forecasting problems. In order to forecast the fluctuation of the stock market, Chen [13] proposed a hybrid first order fuzzy time series model using granular computing as the partitioning method. Many studies [14–16] used a second-order fuzzy time series model to create the rules for the forecasting of future trends. The biggest differences between these fuzzy time series models are the detailed partitioning method and the trend rules. Efendi et al. [17] used a fuzzy time series model to forecast daily electricity load demand. Sadaei et al. [18] proposed a short-term load forecasting model based on the seasonality memory process and fuzzy time series model. These fuzzy time series models are all autoregressive (AR) models. With fuzzy lagged variables of a time series, these models can be represented as AR(n). Such models are also used for project cost forecasting [19] and the enrollment forecasting at Alabama University [20,21].

In order to improve the accuracy of fuzzy time series models, many researchers have proposed other models on the basis of Chen's model. For example, an unequal interval length method was proposed by Huarng and Yu [22] based on the ratios of data in which the length of interval was exponentially variable. In addition to determining the intervals, the definition of the universe of discourse also plays an effective role in the forecasting accuracy. To establish a suitable universe of discourse, in addition to the maximum and minimum values of the historical data of the main factor, the models need two proper real numbers to cover the noise.

Another essential step when creating fuzzy time series models is the establishment of fuzzy logical relationships (FLR). In this realm, the research by Egrioglu et al. [23] is regarded as a basic high-order method for forecasting based on artificial neural networks. Moreover, Egrioglu [24] employed generic algorithms to establish fuzzy relations. Some other soft computing techniques have been used to forecast in many studies [25–27]. In fact, fuzzy time series forecasting studies are frequently based on fuzzy autoregressive (AR) structures [28–32]. To further improve the performance of fuzzy AR models, an adaptive fuzzy inference system (ANFIS) [33] has been used in time series prediction [34–37]. However, only using an AR structure for some of the time series may lead to unsatisfactory and flawed results. To address this, we combined moving average (MA) structures and produced an ARMA-type fuzzy time series forecasting model that includes both AR and MA structures. Because of the excellent performance of the ARMA model, it has been widely mentioned in the. For example, Kocak [38] and Kocak el al. [39] researched first-order ARMA fuzzy time series models based on fuzzy logical relation tables and an artificial neural network, respectively. Kocak [40] also studied a high-order ARMA fuzzy time series model.

Most of the existing fuzzy time series models first fuzzify the exact values of the time series, then use AR models of the dataset itself to forecast its future. Such methods usually improve the performance by using extra solution steps, such as the use of artificial neural networks. In this paper, we propose a new first-order ARMA model based on two-factor fuzzy logical relationships. The advantages of this model are that it uses the fluctuation values rather than the exact values of the time series, and a secondary factor is used to help forecast the main factor with ARMA fuzzy time series models. Since the fluctuation orientations, including up, equal, and down, and the extent to which the trends would be realized, are the crucial ingredients for financial forecasting. Because of this, using a fluctuation time series for further rules generation would be more reasonable. Although internal rules determine future changes, we could not ignore the effects of relative external changes. Therefore, we chose an external element as the secondary factor to generate the logical rules. The experiment results indicate that the proposed two-factor fluctuation method outperforms the one-factor method, based on real historic data, because the secondary factor may have some effects on the main factor and thereby impact the forecasting results. Using fuzzified fluctuations, rather than fuzzified real data, could avoid the influence of extreme values in the historic data which negatively affects forecasting.

The remainder of the paper is organized as follows. The next section presents the basic preliminaries of fuzzy-fluctuation time series. The third section introduces the procedure used to build the ARMA(1,m) model. Next, the proposed model is used to forecast the stock market using TAIEX datasets from 1997 to 2005, SHSECI from 2001 to 2015, and internal gold prices from 2000 to 2010. Finally, we discuss the conclusions and potential future research.

2. Preliminaries

In this section, the general definitions of a fuzzy fluctuation time series in ARMA(1,m) models are outlined.

Definition 1. *Let $A(t)$, $(t = 1, 2, 3, \ldots, T)$ be a time series of real numbers, where T is the number of the time series, and can be defined as the universe of discourse of the fuzzy sets $L = \{L_1, L_2, \ldots, L_g\}$. According to the membership function, $\mu_L : A(t) \to [0, 1]$, each element of the time series $A(t)$, $(t = 1, 2, 3, \ldots, T)$ can be represented by a fuzzy number $Z(t) = L_i$, $(t = 1, 2, 3, \ldots, T,\ i = 1, 2, \ldots, g)$. We called $Z(t)$, $(t = 1, 2, 3, \ldots, T)$ a fuzzy time series.*

Definition 2. *For a time series $G(t)$, $(t = 1,\ 2,\ 3,\ \ldots,\ T)$, $Y(t)$ is defined as a fluctuation time series where $Y(t) = G(t) - G(t-1)$, $(t = 2, 3, \cdots, T)$. As described in Definition 1, $t = 2,\ 3,\ \ldots, T$, $Y(t)$ could be represented by a fuzzy time series $H(t)$, $(t = 2,\ 3,\ \ldots, T)$. Thereby, the time series $Y(t)$ is fuzzified into a fuzzy-fluctuation time series (FFTS) $H(t)$.*

Definition 3. *Let $H(t)$ be a FFTS $(t = 2,\ 3, \ldots, T)$. If the "next status" of $H(t)$ is caused by the "current status" of $H(t-1)$, the first order fuzzy-fluctuation AR(1) is represented by [10,11]:*

$$H(t-1) \to H(t) \tag{1}$$

Similarily, let $Q(t)$ and *P(t)* be two FFTS*s* $(t = 2,\ 3, \ldots, T)$. If the next status of $Q(t)$ is caused by the current status of $Q(t-1)$ and $P(t-1)$, the two-factor first order fuzzy-fluctuation AR(1) is represented by:

$$Q(t-1), P(t-1) \to Q(t) \tag{2}$$

This is called the two-factor first order fuzzy-fluctuation logical relationship (FFLR). $\mathrm{Q(t-1)}$, $\mathrm{P(t-1)}$ is the left-hand side (LHS) and $\mathrm{Q(t)}$ is the right-hand side (RHS) of the FFLR. A forecasting model based on these relationships is called a two-factor first order time series forecasting model.

Definition 4. *Let $F(t)$ be a fuzzy time series and $\varepsilon(t)$ be a fuzzy error series obtained from the $F(t)$. If $F(t)$ is affected by both the lagged fuzzy time series $(F(t-1), F(t-2), \ldots, F(t-n))$ and the lagged fuzzy error series $(\varepsilon(t-1), \varepsilon(t-2), \ldots, \varepsilon(t-m))$, the fuzzy logical relationship can be represented by [40]:*

$$F(t-1), F(t-2), \ldots, F(t-n), \varepsilon(t-1), \varepsilon(t-2), \ldots, \varepsilon(t-m) \to F(t) \tag{3}$$

Similarly, let $Q(t)$ and *P(t)* be two FFTSs $(t = 2,\ 3, \ldots, T)$ and $\varepsilon(t)$ be a fuzzy error series obtained from $Q(t)$. If $Q(t)$ is affected by both the lagged fuzzy time series $(Q(t-1), P(t-1))$ and the lagged fuzzy error series $(\varepsilon(t-1), \varepsilon(t-2), \ldots, \varepsilon(t-m))$, the fuzzy logical relationship can be represented by:

$$Q(t-1), P(t-1), \varepsilon(t-1), \varepsilon(t-2), \ldots, \varepsilon(t-m) \to Q(t) \tag{4}$$

This is called a two-factor ARMA(1,m) fuzzy-fluctuation time series forecasting model, where $m \leq$ T. In this expression, m gives the order of the MA model.

3. New Forecasting Model Based on Two-Factor ARMA(1,m) FFLRs

In this paper, we propose a new forecasting model with two-factor first-order fuzzy fluctuation logical relationships ARMA model. To make a comparison with the forecasting results of other researchers' work [29,30,41,42], we used the real Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) to show the forecasting procedure. We used the data from January to October of the given year as a training time series and the data from November to December of the same year as the testing dataset. The basic steps of the proposed model are shown in Figure 1.

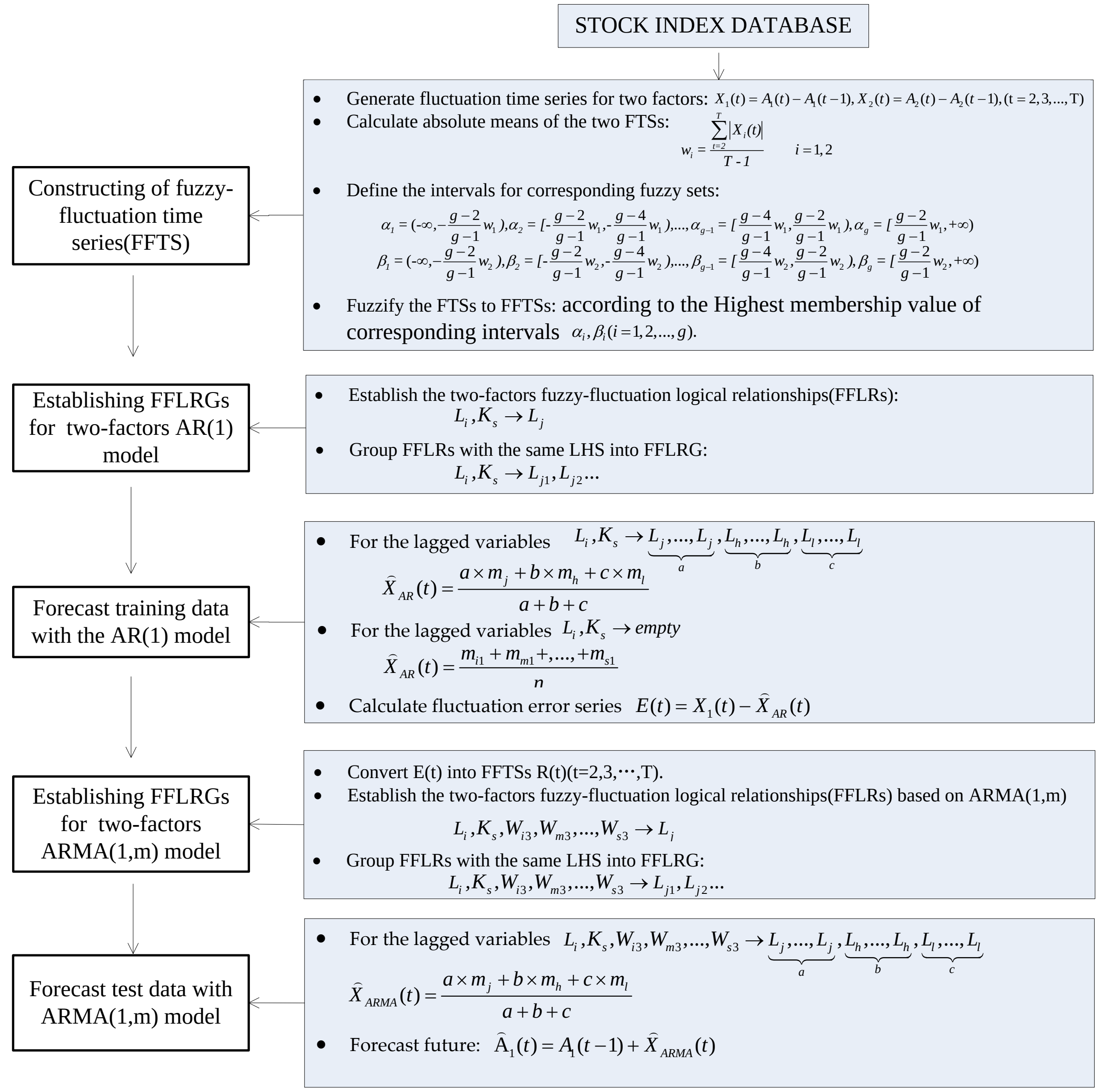

Figure 1. Flowchart of the proposed forecasting model.

Step 1:

The first step was to construct a FFTS for the historical main and secondary factor training data. For each element $A_1(t)$, $(t = 1, 2, \ldots, T)$ in the historical training time series of the main factor, its fluctuation trend was determined by $X_1(t) = A_1(t) - A_1(t-1)$, where $t = 2, 3, 4, \ldots, T$. By the values and directions of fluctuation, we fuzzified $X_1(t)$ into a linguistic set {down, equal, up}. We assumed w_1 and w_2 are the absolute means of all elements in the fluctuation time series $X_1(t)$, $X_2(t)(t = 2, 3, 4, \ldots T)$, respectively. Then, we had $\alpha_1 = (-\infty, -\frac{w_1}{2})$, $\alpha_2 = [-\frac{w_1}{2}, \frac{w_1}{2})$, and $\alpha_3 = [\frac{w_1}{2}, \infty)$. Similarly we divided $X_1(t)$ into 5 intervals such that $\alpha_1 = \left(-\infty, -\frac{3w_1}{4}\right)$, $\alpha_2 = \left[-\frac{3w_1}{4}, -\frac{w_1}{4}\right)$, $\alpha_3 = [-\frac{w_1}{4}, \frac{w_1}{4})$, $\alpha_4 = \left[\frac{w_1}{4}, \frac{3w_1}{4}\right)$, and $\alpha_5 = \left[\frac{3w_1}{4}, \infty\right)$. We divided $X_1(t)$ into any $g = 2l + 1$ intervals, where l is an integer. Next, for each element $A_2(t)$, $(t = 1, 2, \ldots, T)$ in the historical training time series of the secondary factor, its fluctuation trend was determined by $X_2(t) = A_2(t) - A_2(t-1)$, where $t = 2, 3, 4, \ldots, T$. According to Definition 1, $X_2(t)$ can also be divided into g intervals, namely β_i, $(i = 1, 2, \ldots, g)$. Then we fuzzified $X_1(t)$ and $X_2(t)$ into FFTSs $Q_1(t)$ and $Q_2(t)$, $(t = 2, 3, \ldots, T)$, respectively, where $\mathrm{Q_1(t)} = L_i$ and $\mathrm{Q_2(t)} = K_j$ both have the highest membership value of corresponding intervals α_i and β_j $(i, j = 1, 2, \ldots, g)$, respectively, and $\{L_1, L_2, \ldots, L_g\}$, $\{K_1, K_2, \ldots, K_g\}$ are fuzzy sets.

Step 2:

The second step was to determine the two-factors fuzzy-fluctuation logic relationships for the AR(1) model. In this step, we determined the two-factor fuzzy-fluctuation logical relationships for the AR(1) model as outlined in Definition 3. Let the lagged variables $Q_1(t-1) = L_i$, $Q_2(t-1) = K_s$, and $Q_1(t) = L_j$; the FFLR of this two-factor AR(1) model is $L_i, K_s \rightarrow L_j$. Then, the FFLRs with the same LHS were grouped into a fuzzy-fluctuation logical relationship group (FFLRG) by putting all their RHSs together, as on the RHS of the FFLRG. For example, when the FFLRs for a two-factor AR(1) model are $L_1, K_2 \rightarrow L_2$ and $L_1, K_2 \rightarrow L_3$, then the FFLRG would be $L_1, K_2 \rightarrow L_2, L_3$.

Step 3:

The next step was to obtain the fuzzy fluctuation forecast result from AR(n) model. We assumed the lagged variables $Q_1(t-1) = L_i$, $Q_2(t-1) = K_s$, and we defined the following conditions:RHS Conditions: If $L_i, K_s \rightarrow L_j, \ldots, L_j, L_h, \ldots, L_h, L_l, \ldots, L_l$ exists and assuming the numbers of L_j, L_h , and L_l from the previous equation are a, b, and c respectively, then the fuzzy fluctuation forecast result would be $L_j, \ldots, L_j, L_h, \ldots, L_h, L_l, \ldots, L_l$. Null RHS Condition: If $L_i, K_s \rightarrow empty$ exists on the FFLRG, then the fuzzy forecast is L_i, K_s.

Step 4:

Next, we defuzzified the fluctuation forecast result for the AR(1) model. We used the centralization method to defuzzify the forecast results. For example, assuming m_j, m_h , and m_l are the middle points of corresponding sub-intervals of L_j, L_h , and L_l respectively, the defuzzified fluctuation forecast result is represented by:

$$X\hat{_{AR}}(t) = \frac{a \times\ m_j + b \times\ m_h + c \times\ m_l}{a + b + c} \tag{5}$$

Step 5:

Next, we calculated the fluctuation error series $E(t)$:

$$E(t) = X_1(t) - X\hat{_{AR}}(t) \tag{6}$$

where $X_1(t)$ is the time series of the fluctuation numbers of main factor, and $\hat{X}_1(t)$ is calculated result from Step 4.

Step 6:

The next step was to construct fuzzy fluctuation time series for the error series $E(t)$. In the same manner as described in Step 1, we fuzzified $E(t)$ into FFTSs $R(t)$. We assumed h is the absolute mean of all elements in the time series $E(t)$, $(t = 2, 3, 4, \ldots T)$, g is the number of intervals of the fuzzy sets, ε_1, $\varepsilon_2, \ldots, \varepsilon_g$ are corresponding intervals, $\mathrm{R(t)} = W_i$ has the highest membership value of corresponding intervals ε_i $(i = 1, 2, \ldots, g)$, and $\{W_1, W_2, \ldots, W_g\}$ are the corresponding fuzzy sets.

Step 7:

Next, we determined the two-factor fuzzy logical relationships for the ARMA(n,m) model. In this step, we determined the fuzzy logical relationships for ARMA(n,m) model as outlined in

Definition 4. Let the lagged variables $Q_1(t-1) = L_i$, $Q_2(t-1) = K_s$, $Q_1(t) = L_j$, $R(t-m) = W_{i3}$, $R(t-(m-1)) = W_{m2}$, ... , $R(t-1) = W_{s2}$, and the FFLR of this two-factor ARMA(1,m) model is $L_i, K_s, W_{i2}, W_{m2} \ldots, W_{s2} \to L_j$. Then, as described in Step 2, the FFLRs with the same LHS were grouped into a FFLRG for the ARMA(1,m) model.

Step 8:

Next, we obtained the fuzzy fluctuation forecast result from the ARMA(1,m) model. In the same manner as described in Step 3, we forecasted the future based on the two-factor FFLRG and the lagged variables. Assuming the lagged variables $Q_1(t-1) = L_i$, $Q_2(t-1) = K_s$, and the lagged error variables $R(t-m) = W_{i3}$, $R(t-(m-1)) = W_{m2}$, ... , and $R(t-1) = W_{s2}$, we defined the following conditions:

RHS Condition: If $L_i, K_s, W_{i3}, W_{m3}, \ldots, W_{s3} \to L_j, \ldots, L_j, L_h, \ldots, L_h, L_l, \ldots, L_l$ exists and assume the number of L_j L_h and L_l from the previous equation is a, b, and c, respectively, then the fuzzy fluctuation forecast result would be $L_j, \ldots,$ $L_j, L_h, \ldots, L_h, L_l, \ldots, L_l$.

Null RHS Condition: If $L_i, K_s, W_{i3}, W_{m3}, \ldots, W_{s3} \to empty$ exists on the FFLRG, then it was replaced with the FFLRG of its corresponding AR(1) model of L_i, K_s.

Step 9:

In the final step, we defuzzified the forecast fluctuation and obtained forecast results. As described in Step 4, we defuzzified the obtained new forecast fluctuation:

$$X_{A\hat{R}MA}(t) = \frac{a \times\ m_j + b \times\ m_h + c \times\ m_l}{a+b+c} \quad (7)$$

Then, we obtained the forecasting value with:

$$\hat{A_1}(t) = A_1(t-1) + X_{A\hat{R}MA}(t)\ (6) \quad (8)$$

4. Applications

4.1. Forecasting TAIEX 2004

We used the 2004 TAIEX data as an example to illustrate our method. As the secondary factor, the 2004 Dow Jones data was used.

Step 1: Construct FFTS for historical main and secondary factor training data.

Firstly, the absolute mean of the fluctuation historical dataset of TAIEX 2004 from January to October was 66.87and the absolute mean of the fluctuation of the Dow Jones was 55.58. Then we divided both TAIEX 2004 and Dow Jones 2004 from January to October into 5 intervals according to their absolute means. Therefore, $\alpha_1 = (-\infty, -50.15)$, $\alpha_2 = [-50.15, -16.72)$, $\alpha_3 = [-16.72, 16.72)$, $\alpha_4 = [16.72, 50.15)$ and $\alpha_5 = [50.15,\ \infty)$, $\beta_1 = (-\infty, -41.69)$, $\beta_2 = [-41.69, -13.90)$, $\beta_3 = [-13.90, 13.90)$, $\beta_4 = [13.90, 41.69)$, and $\beta_5 = [41.69,\ \infty)$. In this way, the historical training dataset was represented by a fuzzified fluctuation dataset (Appendix A).

Step 2: Determine the fuzzy logical relationships (FFLRs) for two-factor AR(1) model.

Step 3: Obtain fuzzy fluctuation forecast result for time series.

Based on the results obtained from Step 2, the two-factor AR(1) FFLRs are shown in Table 1.

Step 4: Defuzzify the fluctuation forecast result.

The fluctuation forecast result was defuzzified according to Equation (3); the results are shown in Table 1.

Table 1. Fuzzy two-factor first-order autoregressive(AR(1)) solution.

Fuzzy Value of Main Factor	Fuzzy Value of Secondary Factor	Fuzzy Forecast	Defuzzified Forecast
1	1	2,1,5,5,1,1,5,5,3,1,5,3,1,5,3,5,2,1,	0
1	2	4,3,1,	−11.17
1	3	1,1,5,4,1,3,2,3,5,5,	0
1	4	5,1,4,4,3,	13.4
1	5	3,2,1,5,1,1,5,5,3,	−3.72
2	1	1,4,5,1,5,2,3,2,	−4.19
2	2	4,3,1,1,	−25.13
2	3	5,5,3,3,5,3,3,3,2,5,5,	27.41
2	4	1,4,	−16.75
2	5	4,4,3,2,5,3,	16.75
3	1	2,3,2,1,1,5,4,1,5,4,5,1,5,5,1,3,	0
3	2	3,4,3,2,2,	−6.7
3	3	2,4,3,5,5,1,4,2,1,	0
3	4	3,5,1,5,2,3,3,2,3,1,4,	−3.05
3	5	1,5,5,2,5,5,4,	28.71
4	1	3,1,1,3,5,2,5,1,2,	−14.89
4	2	5,3,1,1,5,4,	5.58
4	3	3,1,2,3,	−25.13
4	4	2,5,	16.75
4	5	5,3,5,5,4,4,	44.67
5	1	1,1,3,2,2,4,5,2,3,3,3,	−12.18
5	2	4,2,1,1,	−33.5
5	3	3,3,3,1,1,3,1,1,1,3,5,4,	−19.54
5	4	2,3,3,4,5,2,4,	9.57
5	5	2,4,2,2,3,5,5,3,3,3,2,5,4,4,1,5,5,5,5,	19.39

Step 5: Calculate the fluctuation error series $E(t)$ of the historic training data.

We first added the forecast fluctuation to the previous day and obtained our forecast results. Then we calculated the difference between our forecast values and actual values.

Step 6: Fuzzify the fluctuation error series.

Based on the results of Step 5, we fuzzified the fluctuation error series $E(t)$. as we did in Step 1. The absolute mean of the fluctuation error series was 64.32. Then we divided the fluctuation error series $E(t)$ into 5 intervals according to their absolute mean. The results are shown in Appendix B.

Step 7: Determine the fuzzy logical relationships for the ARMA(1,m) model.

In this case, to obtain optimal results, we used m = 3 to build our model.

Step 8: Obtain fuzzy fluctuation forecast result for the time series based on the FFLRGs of the ARMA(1,m) model.

Based on the results obtained in Step 2, the two-factor ARMA(1,3) fuzzy logic relationships are shown in Appendix C.

Step 9: Defuzzify the fluctuation forecast result.

We defuzzified the fluctuation forecast result according to Equation (3). The results are shown in Appendix C.

Then we used the fuzzy two-factor ARMA(1,3) solution to forecast the test dataset, which is the TAIEX 2004 from November to December. The forecast result is shown in Table 2. The forecast values were obtained by adding the fluctuation values to the current values. The forecast results are shown in Table 2.

Table 2. Forecasting results from 1 November 2004 to 31 December 2004.

Date (MM/DD/YYYY)	Actual	Forecast	(Forecast–Actual)^2	Date (MM/DD/YYYY)	Actual	Forecast	(Forecast–Actual)^2
11/05/2004	5931.31	5889.44	1753.10	12/06/2004	5919.17	5868.14	2604.06
11/08/2004	5937.46	5950.70	175.30	12/07/2004	5925.28	5904.28	441.00
11/09/2004	5945.20	5937.46	59.91	12/08/2004	5892.51	5925.28	1073.87
11/10/2004	5948.49	5945.20	10.82	12/09/2004	5913.97	5909.26	22.18
11/11/2004	5874.52	5948.49	5471.56	12/10/2004	5911.63	5958.64	2209.94
11/12/2004	5917.16	5870.80	2149.25	12/13/2004	5878.89	5911.63	1071.91
11/15/2004	5906.69	5961.83	3040.42	12/14/2004	5909.65	5895.64	196.28
11/16/2004	5910.85	5906.69	17.31	12/15/2004	6002.58	5926.40	5803.39
11/17/2004	6028.68	5910.85	13883.91	12/16/2004	6019.23	6012.15	50.13
11/18/2004	6049.49	6048.07	2.02	12/17/2004	6009.32	6019.23	98.21
11/19/2004	6026.55	6066.24	1575.30	12/20/2004	5985.94	6026.07	1610.42
11/22/2004	5838.42	5993.05	23910.44	12/21/2004	5987.85	6013.35	650.25
11/23/2004	5851.10	5851.82	0.52	12/22/2004	6001.52	6016.56	226.20
11/24/2004	5911.31	5851.10	3625.24	12/23/2004	5997.67	6030.23	1060.15
11/25/2004	5855.24	5920.88	4308.61	12/24/2004	6019.42	5997.67	473.06
11/26/2004	5778.65	5855.24	5866.03	12/27/2004	5985.94	6036.17	2523.05
11/29/2004	5785.26	5711.65	5418.43	12/28/2004	6000.57	5981.75	354.19
11/30/2004	5844.76	5785.26	3540.25	12/29/2004	6088.49	6029.28	3505.82
12/01/2004	5798.62	5832.58	1153.28	12/30/2004	6100.86	6054.99	2104.06
12/02/2004	5867.95	5815.37	2764.66	12/31/2004	6139.69	6094.16	2072.98
12/03/2004	5893.27	5800.95	8522.98		RMSE	53.05	

We assessed the forecast performance by comparing the difference between the forecast values and the actual values. The widely used indicators in time series model comparisons are the mean squared error (MSE), root of the mean squared error (RMSE), mean absolute error (MAE), and mean percentage error (MPE),. To compare the performance of different forecasting methods, the Diebold-Mariano test statistic (S) is also used. These formulas are defined by Equations (9)–(13):

$$\text{MSE} = \frac{\sum_{t=1}^{n} (\text{forecast}(t) - \text{actual}(t))^2}{n} \tag{9}$$

$$\text{RMSE} = \sqrt{\frac{\sum_{t=1}^{n} (\text{forecast}(t) - \text{actual}(t))^2}{n}} \tag{10}$$

$$\text{MAE} = \frac{\sum_{t=1}^{n} |(\text{forecast}(t) - \text{actual}(t))|}{n} \tag{11}$$

$$\text{MPE} = \frac{\sum_{t=1}^{n} |(\text{forecast}(t) - \text{actual}(t))|/\text{actual}(t)}{n} \tag{12}$$

$$S = \frac{\overline{d}}{(\text{Variance}(\overline{d}))^{1/2}},\ \overline{d} = \frac{\sum_{t=1}^{n} (\text{error} \quad of \quad Forecast1)_t^2 - \sum_{t=1}^{n} (\text{error} \quad of \quad Forecast2)_t^2}{n} \tag{13}$$

where n denotes the number of values forecasted, and *forecast(t)* and *actual(t)* denote the predicted value and actual value at time t, respectively. S is a test statistic of the Diebold method, that is used to compare the predictive accuracy of two forecasts obtained by different methods. *Forecast1* represents the dataset obtained by Method 1, and *Forecast2* represents another dataset from Method 2. If $S > 0$ and $|S| > Z = 1.64$, at the 0.05 significance level, then *Forecast2* has better predictive accuracy than *Forecast1*. With respect to the proposed method for two-factor ARMA(1,3), the MSE, RMSE, MAE, and MPE were 2814.65, 53.05, 42.09, and 0.0071, respectively.

To compare the forecasting results with different parameters, such as the number m of the two-factor ARMA(1,m) model and the element number g of linguistic sets, used in the fluctuation fuzzifying process, we completed different experiments and calculated the results. The forecasting errors of the averages for the experiments are shown in Tables 3 and 4.

Table 3. Comparison of forecasting errors for different two-factor first-order autoregressive moving average (ARMA(1,m)) model (g = 5).

m	**None**	**1**	**2**	**3**	**4**	**5**
RMSE	57.59	59.32	61.74	53.05	60.84	63.22

Table 4. Comparison of forecasting errors for different linguistic sets (m = 3).

g	**3**	**5**	**7**	**9**
RMSE	57.25	53.05	58.99	65.8

In Table 4, g = 3 means the linguistic set is *{down, equal, up}*, g = 5 means *{greatly down, slightly down, equal, slightly up, greatly up}*, g = 7 means *{very greatly down, greatly down, slightly down, equal, slightly up, greatly up, very greatly up}*, etc. "*None*" means that the model only used the AR(1) method to forecast.

We employed the proposed method to forecast the TAIEX from 1997 to 2005. The forecast results and errors are shown in Figure 2 and Table 5.

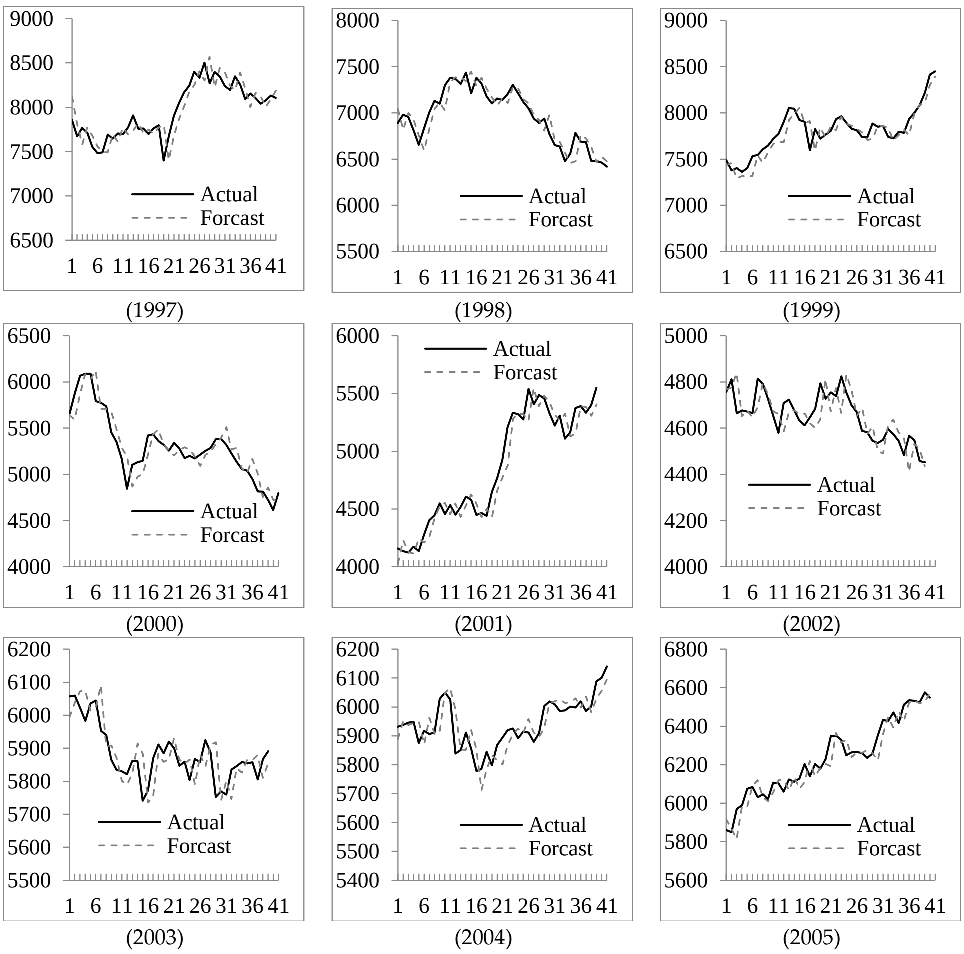

Figure 2. Comparison of actual and forecast results for Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) test dataset (1997–2005). (X coordinate is the TAIEX and Y coordinate is the time series number remarked by "time(s)".)

Table 5. RMSEs of forecast errors for TAIEX 1997 to 2005.

Year	1997	1998	1999	2000	2001	2002	2003	2004	2005
RMSE	130.9	111.95	101.11	127.47	114.19	61.92	53.05	53.07	52.27

Table 6 shows a comparison of the RMSEs for the different methods when forecasting the TAIEX 2004. From this table, the performance of the proposed method is excellent. Though some of the other methods have better RMSEs results, they often need to build complex discretization partitioning rules or employ adaptive expectation models to modify the final forecast results. The method proposed in this paper is easily achieved by a computer program.

Table 6. A comparison of RMSEs for different methods for forecasting the TAIEX 2004.

Methods	RMSE						
	1999	2000	2001	2002	2003	2004	Average
Huarng et al.'s Method [41]	N/A	158.7 **	136.49 **	95.15 **	65.51 **	73.57 **	105.88
Chen and Chang's Method [29]	123.64 **	131.1	115.08	73.06 **	66.36 **	60.48 **	94.95
Chen and Chen's Method [30]	119.32 **	129.87	123.12	71.01	65.14 **	61.94 **	95.07
Chen et al.'s Method [42]	102.34	131.25	113.62	65.77	52.23	56.16	86.89
Cheng et al.'s method [43]	100.74	125.62	113.04	62.94	51.46	54.24	84.68
Chen and Kao's method [44]	87.63	125.34	114.57	76.86 **	54.29	58.17	86.14
Yu and Huarng's method [45]	N/A	149.59 **	98.91	78.71 **	58.78	55.91	88.38
The Proposed Method	101.11	127.47	114.19	61.92	53.05	53.07	85.14

** Use Diebold-Mariano test statistic (S), the proposed method has better accuracy than other methods at 5% significance level at least.

4.2. Forecasting SHSECI

The SHSECI (Shanghai Stock Exchange Composite Index) is the most influential stock market index in China. We chose Dow Jones as a secondary factor to build our model. For each year, the authentic datasets of historical daily SHSECI closing prices from January to October were used as the training data, and the datasets from November to December were employed as the testing data. The RMSEs of forecast errors are shown in Table 7. The proposed model accurately forecasted the SHSECI stock market.

Table 7. Root of the mean squared error (RMSE)s of forecast errors for Shanghai Stock Exchange Composite Index (SHSECI) from 2007 to 2015.

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
RMSE	24.86	21.75	26.57	19.07	9.63	28.98	129.22	79.77	59.96	49.48	29.7	23.14	22.13	44.11	58.89

4.3. Forecasting Gold Price

We also applied the proposed method to forecast the international gold price in USD from 2000 to 2010. We chose the COMEX gold price as a secondary factor. For each year, the authentic datasets of the historical daily closing prices from January to October were used as the training data, and the datasets from November to December were the testing data. The RMSEs of the forecast errors are shown in Table 8. Taking the 2010 gold price as an example, the forecast results are shown in Figure 3.

Table 8. RMSEs of forecast errors for gold price from 2000 to 2010.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
RMSE	1.27	1.52	2.33	2.81	3.34	6.65	5.44	11.48	19.81	14.61	14.33

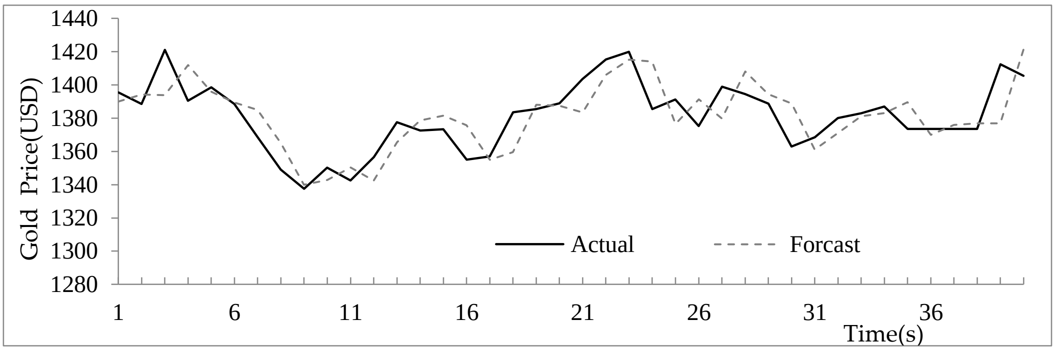

Figure 3. Comparison of actual and forecast results for gold prices in 2010.

We can see that the proposed model can accurately forecast the international gold price.

5. Conclusions

In this paper, a new forecasting model is proposed based on a first-order two-factor ARMA(1,m) model. The proposed method is based on the fluctuations of two time series. The secondary factor was used to modify the forecast performance of the main factor. The experiments showed that the fuzzy logic relations of the main and secondary factors obtained from the two training datasets can successfully predict the testing dataset of the main factor. To compare the performance with other methods, we employed TAIEX 2004 as an example to illustrate our process. We also forecasted TAIEX 1997–2005, SHSECI 2001–2015, and the international gold price 2000–2010 to show its accuracy and versatility. For future research, we may consider additional aspects of the stock markets such as volumes, ending prices, opening prices, etc. A third factor, or more, could be used to modify the forecasting process.

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Author Contributions: Shuang Guan designed the experiments and wrote the paper; Aiwu Zhao conceived the main idea of the method.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

The historical training dataset can be represented by a fuzzified fluctuation dataset as shown in Tables A1 and A2.

Table A1. Historical training data and fuzzified fluctuation data of TAIEX2004.

Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified
01/02/2004	6041.56	-	-	04/16/2004	6818.20	81.41	5	07/23/2004	5373.85	−14.11	3
01/05/2004	6125.42	83.86	5	04/19/2004	6779.18	−39.02	2	07/26/2004	5331.71	−42.14	2
01/06/2004	6144.01	18.59	4	04/20/2004	6799.97	20.79	4	07/27/2004	5398.61	66.90	5
01/07/2004	6141.25	−2.76	3	04/21/2004	6810.25	10.28	3	07/28/2004	5383.57	−15.04	3
01/08/2004	6169.17	27.92	4	04/22/2004	6732.09	−78.16	1	07/29/2004	5349.66	−33.91	2
01/09/2004	6226.98	57.81	5	04/23/2004	6748.10	16.01	3	07/30/2004	5420.57	70.91	5
01/12/2004	6219.71	−7.27	3	04/26/2004	6710.70	−37.40	2	08/02/2004	5350.40	−70.17	1
01/13/2004	6210.22	−9.49	3	04/27/2004	6646.80	−63.90	1	08/03/2004	5367.22	16.82	4
01/14/2004	6274.97	64.75	5	04/28/2004	6574.75	−72.05	1	08/04/2004	5316.87	−50.35	1
01/15/2004	6264.37	−10.60	3	04/29/2004	6402.21	−172.54	1	08/05/2004	5427.61	110.74	5
01/16/2004	6269.71	5.34	3	04/30/2004	6117.81	−284.40	1	08/06/2004	5399.16	−28.45	2
01/27/2004	6384.63	114.92	5	05/03/2004	6029.77	−88.04	1	08/09/2004	5399.45	0.29	3
01/28/2004	6386.25	1.62	3	05/04/2004	6188.15	158.38	5	08/10/2004	5393.73	−5.72	3
01/29/2004	6312.65	−73.60	1	05/05/2004	5854.23	−333.92	1	08/11/2004	5367.34	−26.39	2
01/30/2004	6375.38	62.73	5	05/06/2004	5909.79	55.56	5	08/12/2004	5368.02	0.68	3
02/02/2004	6319.96	−55.42	1	05/07/2004	6040.26	130.47	5	08/13/2004	5389.93	21.91	4
02/03/2004	6252.23	−67.73	1	05/10/2004	5825.05	−215.21	1	08/16/2004	5352.01	−37.92	2
02/04/2004	6241.39	−10.84	3	05/11/2004	5886.36	61.31	5	08/17/2004	5342.49	−9.52	3
02/05/2004	6268.14	26.75	4	05/12/2004	5958.79	72.43	5	08/18/2004	5427.75	85.26	5
02/06/2004	6353.35	85.21	5	05/13/2004	5918.09	−40.70	2	08/19/2004	5602.99	175.24	5
02/09/2004	6463.09	109.74	5	05/14/2004	5777.32	−140.77	1	08/20/2004	5622.86	19.87	4
02/10/2004	6488.34	25.25	4	05/17/2004	5482.96	−294.36	1	08/23/2004	5660.97	38.11	4
02/11/2004	6454.39	−33.95	2	05/18/2004	5557.68	74.72	5	08/26/2004	5813.39	152.42	5
02/12/2004	6436.95	−17.44	2	05/19/2004	5860.58	302.90	5	08/27/2004	5797.71	−15.68	3
02/13/2004	6549.18	112.23	5	05/20/2004	5815.33	−45.25	2	08/30/2004	5788.94	−8.77	3
02/16/2004	6565.37	16.19	3	05/21/2004	5964.94	149.61	5	08/31/2004	5765.54	−23.40	2
02/17/2004	6600.47	35.10	4	05/24/2004	5942.08	−22.86	2	09/01/2004	5858.14	92.60	5
02/18/2004	6605.85	5.38	3	05/25/2004	5958.38	16.30	3	09/02/2004	5852.85	−5.29	3
02/19/2004	6681.52	75.67	5	05/26/2004	6027.27	68.89	5	09/03/2004	5761.14	−91.71	1
02/20/2004	6665.54	−15.98	3	05/27/2004	6033.05	5.78	3	09/06/2004	5775.99	14.85	3
02/23/2004	6665.89	0.35	3	05/28/2004	6137.26	104.21	5	09/07/2004	5846.83	70.84	5
02/24/2004	6589.23	−76.66	1	05/31/2004	5977.84	−159.42	1	09/08/2004	5846.02	−0.81	3
02/25/2004	6644.28	55.05	5	06/01/2004	5986.20	8.36	3	09/09/2004	5842.93	−3.09	3
02/26/2004	6693.25	48.97	4	06/02/2004	5875.67	−110.53	1	09/10/2004	5846.19	3.26	3
02/27/2004	6750.54	57.29	5	06/03/2004	5671.45	−204.22	1	09/13/2004	5928.22	82.03	5
03/01/2004	6888.43	137.89	5	06/04/2004	5724.89	53.44	5	09/14/2004	5919.77	−8.45	3
03/02/2004	6975.26	86.83	5	06/07/2004	5935.82	210.93	5	09/15/2004	5871.07	−48.70	2
03/03/2004	6932.17	−43.09	2	06/08/2004	5986.76	50.94	5	09/16/2004	5891.05	19.98	4
03/04/2004	7034.10	101.93	5	06/09/2004	5965.70	−21.06	2	09/17/2004	5818.39	−72.66	1
03/05/2004	6943.68	−90.42	1	06/10/2004	5867.51	−98.19	1	09/20/2004	5864.54	46.15	4

Table A1. *Cont.*

Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified
03/08/2004	6901.48	−42.20	2	06/11/2004	5735.07	−132.44	1	09/21/2004	5949.26	84.72	5
03/09/2004	6973.90	72.42	5	06/14/2004	5574.08	−160.99	1	09/22/2004	5970.18	20.92	4
03/10/2004	6874.91	−98.99	1	06/15/2004	5646.49	72.41	5	09/23/2004	5937.25	−32.93	2
03/11/2004	6879.11	4.20	3	06/16/2004	5560.16	−86.33	1	09/24/2004	5892.21	−45.04	2
03/12/2004	6800.24	−78.87	1	06/17/2004	5664.35	104.19	5	09/27/2004	5849.22	−42.99	2
03/15/2004	6635.98	−164.26	1	06/18/2004	5569.29	−95.06	1	09/29/2004	5809.75	−39.47	2
03/16/2004	6589.72	−46.26	2	06/21/2004	5556.54	−12.75	3	09/30/2004	5845.69	35.94	4
03/17/2004	6577.98	−11.74	3	06/23/2004	5729.30	172.76	5	10/01/2004	5945.35	99.66	5
03/18/2004	6787.03	209.05	5	06/24/2004	5779.09	49.79	4	10/04/2004	6077.96	132.61	5
03/19/2004	6815.09	28.06	4	06/25/2004	5802.55	23.46	4	10/05/2004	6081.01	3.05	3
03/22/2004	6359.92	−455.17	1	06/28/2004	5709.84	−92.71	1	10/06/2004	6060.61	−20.40	2
03/23/2004	6172.89	−187.03	1	06/29/2004	5741.52	31.68	4	10/07/2004	6103.00	42.39	4
03/24/2004	6213.56	40.67	4	06/30/2004	5839.44	97.92	5	10/08/2004	6102.16	−0.84	3
03/25/2004	6156.73	−56.83	1	07/01/2004	5836.91	−2.53	3	10/11/2004	6089.28	−12.88	3
03/26/2004	6132.62	−24.11	2	07/02/2004	5746.70	−90.21	1	10/12/2004	5979.56	−109.72	1
03/29/2004	6474.11	341.49	5	07/05/2004	5659.78	−86.92	1	10/13/2004	5963.07	−16.49	3
03/30/2004	6494.71	20.60	4	07/06/2004	5733.57	73.79	5	10/14/2004	5831.07	−132.00	1
03/31/2004	6522.19	27.48	4	07/07/2004	5727.78	−5.79	3	10/15/2004	5820.82	−10.25	3
04/01/2004	6523.49	1.30	3	07/08/2004	5713.39	−14.39	3	10/18/2004	5772.12	−48.70	2
04/02/2004	6545.54	22.05	4	07/09/2004	5777.72	64.33	5	10/19/2004	5807.79	35.67	4
04/05/2004	6682.73	137.19	5	07/12/2004	5758.74	−18.98	2	10/20/2004	5788.34	−19.45	2
04/06/2004	6635.54	−47.19	2	07/13/2004	5685.57	−73.17	1	10/21/2004	5797.24	8.90	3
04/07/2004	6646.74	11.20	3	07/14/2004	5623.65	−61.92	1	10/22/2004	5774.67	−22.57	2
04/08/2004	6672.86	26.12	4	07/15/2004	5542.80	−80.85	1	10/26/2004	5662.88	−111.79	1
04/09/2004	6620.36	−52.50	1	07/16/2004	5502.14	−40.66	2	10/27/2004	5650.97	−11.91	3
04/12/2004	6777.78	157.42	5	07/19/2004	5489.10	−13.04	3	10/28/2004	5695.56	44.59	4
04/13/2004	6794.33	16.55	3	07/20/2004	5325.68	−163.42	1	10/29/2004	5705.93	10.37	3
04/14/2004	6880.18	85.85	5	07/21/2004	5409.13	83.45	5				
04/15/2004	6736.79	−143.39	1	07/22/2004	5387.96	−21.17	2				

Table A2. Historical training data and fuzzified fluctuation data of Dow Jones 2004.

Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified
01/02/2004	10409.85	-	-	04/14/2004	10377.95	−3.33	3	07/26/2004	9961.92	−0.30	3
01/05/2004	10544.07	134.22	5	04/15/2004	10397.46	19.51	4	07/27/2004	10085.14	123.22	5
01/06/2004	10538.66	−5.41	3	04/16/2004	10451.97	54.51	5	07/28/2004	10117.07	31.93	4
01/07/2004	10529.03	−9.63	3	04/19/2004	10437.85	−14.12	2	07/29/2004	10129.24	12.17	3
01/08/2004	10592.44	63.41	5	04/20/2004	10314.50	−123.35	1	07/30/2004	10139.71	10.47	3
01/09/2004	10458.89	−133.55	1	04/21/2004	10317.27	2.77	3	08/02/2004	10179.16	39.45	4
01/12/2004	10485.18	26.29	4	04/22/2004	10461.20	143.93	5	08/03/2004	10120.24	−58.92	1
01/13/2004	10427.18	−58.00	1	04/23/2004	10472.84	11.64	3	08/04/2004	10126.51	6.27	3
01/14/2004	10538.37	111.19	5	04/26/2004	10444.73	−28.11	2	08/05/2004	9963.03	−163.48	1
01/15/2004	10553.85	15.48	4	04/27/2004	10478.16	33.43	4	08/06/2004	9815.33	−147.70	1
01/16/2004	10600.51	46.66	5	04/28/2004	10342.60	−135.56	1	08/09/2004	9814.66	−0.67	3
01/20/2004	10528.66	−71.85	1	04/29/2004	10272.27	−70.33	1	08/10/2004	9944.67	130.01	5
01/21/2004	10623.62	94.96	5	04/30/2004	10225.57	−46.70	1	08/11/2004	9938.32	−6.35	3
01/22/2004	10623.18	−0.44	3	05/03/2004	10314.00	88.43	5	08/12/2004	9814.59	−123.73	1
01/23/2004	10568.29	−54.89	1	05/04/2004	10317.20	3.20	3	08/13/2004	9825.35	10.76	3
01/26/2004	10702.51	134.22	5	05/05/2004	10310.95	−6.25	3	08/16/2004	9954.55	129.20	5
01/27/2004	10609.92	−92.59	1	05/06/2004	10241.26	−69.69	1	08/17/2004	9972.83	18.28	4
01/28/2004	10468.37	−141.55	1	05/07/2004	10117.34	−123.92	1	08/18/2004	10083.15	110.32	5
01/29/2004	10510.29	41.92	5	05/10/2004	9990.02	−127.32	1	08/19/2004	10040.82	−42.33	1
01/30/2004	10488.07	−22.22	2	05/11/2004	10019.47	29.45	4	08/20/2004	10110.14	69.32	5
02/02/2004	10499.18	11.11	3	05/12/2004	10045.16	25.69	4	08/23/2004	10073.05	−37.09	2
02/03/2004	10505.18	6.00	3	05/13/2004	10010.74	−34.42	2	08/24/2004	10098.63	25.58	4
02/04/2004	10470.74	−34.44	2	05/14/2004	10012.87	2.13	3	08/25/2004	10181.74	83.11	5
02/05/2004	10495.55	24.81	4	05/17/2004	9906.91	−105.96	1	08/26/2004	10173.41	−8.33	3
02/06/2004	10593.03	97.48	5	05/18/2004	9968.51	61.60	5	08/27/2004	10195.01	21.60	4
02/09/2004	10579.03	−14.00	2	05/19/2004	9937.71	−30.80	2	08/30/2004	10122.52	−72.49	1
02/10/2004	10613.85	34.82	4	05/20/2004	9937.64	−0.07	3	08/31/2004	10173.92	51.40	5
02/11/2004	10737.70	123.85	5	05/21/2004	9966.74	29.10	4	09/01/2004	10168.46	−5.46	3
02/12/2004	10694.07	−43.63	1	05/24/2004	9958.43	−8.31	3	09/02/2004	10290.28	121.82	5
02/13/2004	10627.85	−66.22	1	05/25/2004	10117.62	159.19	5	09/03/2004	10260.20	−30.08	2
02/17/2004	10714.88	87.03	5	05/26/2004	10109.89	−7.73	3	09/07/2004	10341.16	80.96	5
02/18/2004	10671.99	−42.89	1	05/27/2004	10205.20	95.31	5	09/08/2004	10313.36	−27.80	2
02/19/2004	10664.73	−7.26	3	05/28/2004	10188.45	−16.75	2	09/09/2004	10289.10	−24.26	2
02/20/2004	10619.03	−45.70	1	06/01/2004	10202.65	14.20	4	09/10/2004	10313.07	23.97	4
02/23/2004	10609.62	−9.41	3	06/02/2004	10262.97	60.32	5	09/13/2004	10314.76	1.69	3

Table A2. *Cont.*

Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified
02/24/2004	10566.37	−43.25	1	06/03/2004	10195.91	−67.06	1	09/14/2004	10318.16	3.40	3
02/25/2004	10601.62	35.25	4	06/04/2004	10242.82	46.91	5	09/15/2004	10231.36	−86.80	1
02/26/2004	10580.14	−21.48	2	06/07/2004	10391.08	148.26	5	09/16/2004	10244.49	13.13	3
02/27/2004	10583.92	3.78	3	06/08/2004	10432.52	41.44	5	09/17/2004	10284.46	39.97	4
03/01/2004	10678.14	94.22	5	06/09/2004	10368.44	−64.08	1	09/20/2004	10204.89	−79.57	1
03/02/2004	10591.48	−86.66	1	06/10/2004	10410.10	41.66	5	09/21/2004	10244.93	40.04	4
03/03/2004	10593.11	1.63	3	06/14/2004	10334.73	−75.37	1	09/22/2004	10109.18	−135.75	1
03/04/2004	10588.00	−5.11	3	06/15/2004	10380.43	45.70	5	09/23/2004	10038.90	−70.28	1
03/05/2004	10595.55	7.55	3	06/16/2004	10379.58	−0.85	3	09/24/2004	10047.24	8.34	3
03/08/2004	10529.48	−66.07	1	06/17/2004	10377.52	−2.06	3	09/27/2004	9988.54	−58.70	1
03/09/2004	10456.96	−72.52	1	06/18/2004	10416.41	38.89	4	09/28/2004	10077.40	88.86	5
03/10/2004	10296.89	−160.07	1	06/21/2004	10371.47	−44.94	1	09/29/2004	10136.24	58.84	5
03/11/2004	10128.38	−168.51	1	06/22/2004	10395.07	23.60	4	09/30/2004	10080.27	−55.97	1
03/12/2004	10240.08	111.70	5	06/23/2004	10479.57	84.50	5	10/01/2004	10192.65	112.38	5
03/15/2004	10102.89	−137.19	1	06/24/2004	10443.81	−35.76	2	10/04/2004	10216.54	23.89	4
03/16/2004	10184.67	81.78	5	06/25/2004	10371.84	−71.97	1	10/05/2004	10177.68	−38.86	2
03/17/2004	10300.30	115.63	5	06/28/2004	10357.09	−14.75	2	10/06/2004	10239.92	62.24	5
03/18/2004	10295.78	−4.52	3	06/29/2004	10413.43	56.34	5	10/07/2004	10125.40	−114.52	1
03/19/2004	10186.60	−109.18	1	06/30/2004	10435.48	22.05	4	10/08/2004	10055.20	−70.20	1
03/22/2004	10064.75	−121.85	1	07/01/2004	10334.16	−101.32	1	10/11/2004	10081.97	26.77	4
03/23/2004	10063.64	−1.11	3	07/02/2004	10282.83	−51.33	1	10/12/2004	10077.18	−4.79	3
03/24/2004	10048.23	−15.41	2	07/06/2004	10219.34	−63.49	1	10/13/2004	10002.33	−74.85	1
03/25/2004	10218.82	170.59	5	07/07/2004	10240.29	20.95	4	10/14/2004	9894.45	−107.88	1
03/26/2004	10212.97	−5.85	3	07/08/2004	10171.56	−68.73	1	10/15/2004	9933.38	38.93	4
03/29/2004	10329.63	116.66	5	07/09/2004	10213.22	41.66	5	10/18/2004	9956.32	22.94	4
03/30/2004	10381.70	52.07	5	07/12/2004	10238.22	25.00	4	10/19/2004	9897.62	−58.70	1
03/31/2004	10357.70	−24.00	2	07/13/2004	10247.59	9.37	3	10/20/2004	9886.93	−10.69	3
04/01/2004	10373.33	15.63	4	07/14/2004	10208.80	−38.79	2	10/21/2004	9865.76	−21.17	2
04/02/2004	10470.59	97.26	5	07/15/2004	10163.16	−45.64	1	10/22/2004	9757.81	−107.95	1
04/05/2004	10558.37	87.78	5	07/16/2004	10139.78	−23.38	2	10/25/2004	9749.99	−7.82	3
04/06/2004	10570.81	12.44	3	07/19/2004	10094.06	−45.72	1	10/26/2004	9888.48	138.49	5
04/07/2004	10480.15	−90.66	1	07/20/2004	10149.07	55.01	5	10/27/2004	10002.03	113.55	5
04/08/2004	10442.03	−38.12	2	07/21/2004	10046.13	−102.94	1	10/28/2004	10004.54	2.51	3
04/12/2004	10515.56	73.53	5	07/22/2004	10050.33	4.20	3	10/29/2004	10027.47	22.93	4
04/13/2004	10381.28	−134.28	1	07/23/2004	9962.22	−88.11	1				

Appendix B

The fluctuation error series of training data is shown in Table A3.

Table A3. The Fluctuation Error Series.

Date	TAIEX Group	Dow Jones Group	Actual TAIEX	Forecast TAIEX	Fluctuation	Fuzzified Group of Fluctuation	Date	TAIEX Group	Dow Jones Group	Actual TAIEX	Forecast TAIEX	Fluctuation	Fuzzified Group of Fluctuation
01/05/2004	3	3	6125.42	6041.56	83.86	5	06/03/2004	1	5	5671.45	5871.95	−200.50	1
01/06/2004	5	5	6144.01	6144.81	−0.80	3	06/04/2004	1	1	5724.89	5671.45	53.44	5
01/07/2004	4	3	6141.25	6118.88	22.37	4	06/07/2004	5	5	5935.82	5744.28	191.54	5
01/08/2004	3	3	6169.17	6141.25	27.92	4	06/08/2004	5	5	5986.76	5955.21	31.55	4
01/09/2004	4	5	6226.98	6213.84	13.14	3	06/09/2004	5	5	5965.70	6006.15	−40.45	2
01/12/2004	5	1	6219.71	6214.80	4.91	3	06/10/2004	2	1	5867.51	5961.51	−94.00	1
01/13/2004	3	4	6210.22	6216.66	−6.44	3	06/11/2004	1	5	5735.07	5863.79	−128.72	1
01/14/2004	3	1	6274.97	6210.22	64.75	5	06/14/2004	1	1	5574.08	5735.07	−160.99	1
01/15/2004	5	5	6264.37	6294.36	−29.99	2	06/15/2004	1	1	5646.49	5574.08	72.41	5
01/16/2004	3	4	6269.71	6261.32	8.39	3	06/16/2004	5	5	5560.16	5665.88	−105.72	1
01/27/2004	3	5	6384.63	6298.42	86.21	5	06/17/2004	1	3	5664.35	5560.16	104.19	5
01/28/2004	5	1	6386.25	6372.45	13.80	3	06/18/2004	5	3	5569.29	5644.81	−75.52	1
01/29/2004	3	1	6312.65	6386.25	−73.60	1	06/21/2004	1	4	5556.54	5582.69	−26.15	2
01/30/2004	1	5	6375.38	6308.93	66.45	5	06/23/2004	3	1	5729.30	5556.54	172.76	5
02/02/2004	5	2	6319.96	6341.88	−21.92	2	06/24/2004	5	5	5779.09	5748.69	30.40	4
02/03/2004	1	3	6252.23	6319.96	−67.73	1	06/25/2004	4	2	5802.55	5784.67	17.88	4
02/04/2004	1	3	6241.39	6252.23	−10.84	3	06/28/2004	4	1	5709.84	5787.66	−77.82	1
02/05/2004	3	2	6268.14	6234.69	33.45	4	06/29/2004	1	2	5741.52	5698.67	42.85	4
02/06/2004	4	4	6353.35	6284.89	68.46	5	06/30/2004	4	5	5839.44	5786.19	53.25	5
02/09/2004	5	5	6463.09	6372.74	90.35	5	07/01/2004	5	4	5836.91	5849.01	−12.10	3
02/10/2004	5	2	6488.34	6429.59	58.75	5	07/02/2004	3	1	5746.70	5836.91	−90.21	1
02/11/2004	4	4	6454.39	6505.09	−50.70	1	07/05/2004	1	1	5659.78	5746.70	−86.92	1
02/12/2004	2	5	6436.95	6471.14	−34.19	2	07/06/2004	1	1	5733.57	5659.78	73.79	5
02/13/2004	2	1	6549.18	6432.76	116.42	5	07/07/2004	5	1	5727.78	5721.39	6.39	3
02/16/2004	5	1	6565.37	6537.00	28.37	4	07/08/2004	3	4	5713.39	5724.73	−11.34	3
02/17/2004	3	3	6600.47	6565.37	35.10	4	07/09/2004	3	1	5777.72	5713.39	64.33	5
02/18/2004	4	5	6605.85	6645.14	−39.29	2	07/12/2004	5	5	5758.74	5797.11	−38.37	2
02/19/2004	3	1	6681.52	6605.85	75.67	5	07/13/2004	2	4	5685.57	5741.99	−56.42	1
02/20/2004	5	3	6665.54	6661.98	3.56	3	07/14/2004	1	3	5623.65	5685.57	−61.92	1
02/23/2004	3	1	6665.89	6665.54	0.35	3	07/15/2004	1	2	5542.80	5612.48	−69.68	1
02/24/2004	3	3	6589.23	6665.89	−76.66	1	07/16/2004	1	1	5502.14	5542.80	−40.66	2
02/25/2004	1	1	6644.28	6589.23	55.05	5	07/19/2004	2	2	5489.10	5477.01	12.09	3
02/26/2004	5	4	6693.25	6653.85	39.40	4	07/20/2004	3	1	5325.68	5489.10	−163.42	1

Table A3. *Cont.*

Date	TAIEX Group	Dow Jones Group	Actual TAIEX	Forecast TAIEX	Fluctuation	Fuzzified Group of Fluctuation	Date	TAIEX Group	Dow Jones Group	Actual TAIEX	Forecast TAIEX	Fluctuation	Fuzzified Group of Fluctuation
02/27/2004	4	2	6750.54	6698.83	51.71	5	07/21/2004	1	5	5409.13	5321.96	87.17	5
03/01/2004	5	3	6888.43	6731.00	157.43	5	07/22/2004	5	1	5387.96	5396.95	−8.99	3
03/02/2004	5	5	6975.26	6907.82	67.44	5	07/23/2004	2	3	5373.85	5415.37	−41.52	2
03/03/2004	5	1	6932.17	6963.08	−30.91	2	07/26/2004	3	1	5331.71	5373.85	−42.14	2
03/04/2004	2	3	7034.10	6959.58	74.52	5	07/27/2004	2	3	5398.61	5359.12	39.49	4
03/05/2004	5	3	6943.68	7014.56	−70.88	1	07/28/2004	5	5	5383.57	5418.00	−34.43	2
03/08/2004	1	3	6901.48	6943.68	−42.20	2	07/29/2004	3	4	5349.66	5380.52	−30.86	2
03/09/2004	2	1	6973.90	6897.29	76.61	5	07/30/2004	2	3	5420.57	5377.07	43.50	4
03/10/2004	5	1	6874.91	6961.72	−86.81	1	08/02/2004	5	3	5350.40	5401.03	−50.63	1
03/11/2004	1	1	6879.11	6874.91	4.20	3	08/03/2004	1	4	5367.22	5363.80	3.42	3
03/12/2004	3	1	6800.24	6879.11	−78.87	1	08/04/2004	4	1	5316.87	5352.33	−35.46	2
03/15/2004	1	5	6635.98	6796.52	−160.54	1	08/05/2004	1	3	5427.61	5316.87	110.74	5
03/16/2004	1	1	6589.72	6635.98	−46.26	2	08/06/2004	5	1	5399.16	5415.43	−16.27	2
03/17/2004	2	5	6577.98	6606.47	−28.49	2	08/09/2004	2	1	5399.45	5394.97	4.48	3
03/18/2004	3	5	6787.03	6606.69	180.34	5	08/10/2004	3	3	5393.73	5399.45	−5.72	3
03/19/2004	5	3	6815.09	6767.49	47.60	4	08/11/2004	3	5	5367.34	5422.44	−55.10	1
03/22/2004	4	1	6359.92	6800.20	−440.28	1	08/12/2004	2	3	5368.02	5394.75	−26.73	2
03/23/2004	1	1	6172.89	6359.92	−187.03	1	08/13/2004	3	1	5389.93	5368.02	21.91	4
03/24/2004	1	3	6213.56	6172.89	40.67	4	08/16/2004	4	3	5352.01	5364.80	−12.79	3
03/25/2004	4	2	6156.73	6219.14	−62.41	1	08/17/2004	2	5	5342.49	5368.76	−26.27	2
03/26/2004	1	5	6132.62	6153.01	−20.39	2	08/18/2004	3	4	5427.75	5339.44	88.31	5
03/29/2004	2	3	6474.11	6160.03	314.08	5	08/19/2004	5	5	5602.99	5447.14	155.85	5
03/30/2004	5	5	6494.71	6493.50	1.21	3	08/20/2004	5	1	5622.86	5590.81	32.05	4
03/31/2004	4	5	6522.19	6539.38	−17.19	2	08/23/2004	4	5	5660.97	5667.53	−6.56	3
04/01/2004	4	2	6523.49	6527.77	−4.28	3	08/26/2004	4	2	5813.39	5666.55	146.84	5
04/02/2004	3	4	6545.54	6520.44	25.10	4	08/27/2004	5	3	5797.71	5793.85	3.86	3
04/05/2004	4	5	6682.73	6590.21	92.52	5	08/30/2004	3	4	5788.94	5794.66	−5.72	3
04/06/2004	5	5	6635.54	6702.12	−66.58	1	08/31/2004	3	1	5765.54	5788.94	−23.40	2
04/07/2004	2	3	6646.74	6662.95	−16.21	2	09/01/2004	2	5	5858.14	5782.29	75.85	5
04/08/2004	3	1	6672.86	6646.74	26.12	4	09/02/2004	5	3	5852.85	5838.60	14.25	3
04/09/2004	4	2	6620.36	6678.44	−58.08	1	09/03/2004	3	5	5761.14	5881.56	−120.42	1
04/12/2004	1	1	6777.78	6620.36	157.42	5	09/06/2004	1	2	5775.99	5749.97	26.02	4
04/13/2004	5	5	6794.33	6797.17	−2.84	3	09/07/2004	3	3	5846.83	5775.99	70.84	5
04/14/2004	3	1	6880.18	6794.33	85.85	5	09/08/2004	5	5	5846.02	5866.22	−20.20	2
04/15/2004	5	3	6736.79	6860.64	−123.85	1	09/09/2004	3	2	5842.93	5839.32	3.61	3
04/16/2004	1	4	6818.20	6750.19	68.01	5	09/10/2004	3	2	5846.19	5836.23	9.96	3
04/19/2004	5	5	6779.18	6837.59	−58.41	1	09/13/2004	3	4	5928.22	5843.14	85.08	5

Table A3. *Cont.*

Date	TAIEX Group	Dow Jones Group	Actual TAIEX	Forecast TAIEX	Fluctuation	Fuzzified Group of Fluctuation	Date	TAIEX Group	Dow Jones Group	Actual TAIEX	Forecast TAIEX	Fluctuation	Fuzzified Group of Fluctuation
04/20/2004	2	2	6799.97	6754.05	45.92	4	09/14/2004	5	3	5919.77	5908.68	11.09	3
04/21/2004	4	1	6810.25	6785.08	25.17	4	09/15/2004	3	3	5871.07	5919.77	−48.70	1
04/22/2004	3	3	6732.09	6810.25	−78.16	1	09/16/2004	2	1	5891.05	5866.88	24.17	4
04/23/2004	1	5	6748.10	6728.37	19.73	4	09/17/2004	4	3	5818.39	5865.92	−47.53	2
04/26/2004	3	3	6710.70	6748.10	−37.40	2	09/20/2004	1	4	5864.54	5831.79	32.75	4
04/27/2004	2	2	6646.80	6685.57	−38.77	2	09/21/2004	4	1	5949.26	5849.65	99.61	5
04/28/2004	1	4	6574.75	6660.20	−85.45	1	09/22/2004	5	4	5970.18	5958.83	11.35	3
04/29/2004	1	1	6402.21	6574.75	−172.54	1	09/23/2004	4	1	5937.25	5955.29	−18.04	2
04/30/2004	1	1	6117.81	6402.21	−284.40	1	09/24/2004	2	1	5892.21	5933.06	−40.85	2
05/03/2004	1	1	6029.77	6117.81	−88.04	1	09/27/2004	2	3	5849.22	5919.62	−70.40	1
05/04/2004	1	5	6188.15	6026.05	162.10	5	09/29/2004	2	1	5809.75	5845.03	−35.28	2
05/05/2004	5	3	5854.23	6168.61	−314.38	1	09/30/2004	2	5	5845.69	5826.50	19.19	4
05/06/2004	1	3	5909.79	5854.23	55.56	5	10/01/2004	4	1	5945.35	5830.80	114.55	5
05/07/2004	5	1	6040.26	5897.61	142.65	5	10/04/2004	5	5	6077.96	5964.74	113.22	5
05/10/2004	5	1	5825.05	6028.08	−203.03	1	10/05/2004	5	4	6081.01	6087.53	−6.52	3
05/11/2004	1	1	5886.36	5825.05	61.31	5	10/06/2004	3	2	6060.61	6074.31	−13.70	3
05/12/2004	5	4	5958.79	5895.93	62.86	5	10/07/2004	2	5	6103.00	6077.36	25.64	4
05/13/2004	5	4	5918.09	5968.36	−50.27	1	10/08/2004	4	1	6102.16	6088.11	14.05	3
05/14/2004	2	2	5777.32	5892.96	−115.64	1	10/11/2004	3	1	6089.28	6102.16	−12.88	3
05/17/2004	1	3	5482.96	5777.32	−294.36	1	10/12/2004	3	4	5979.56	6086.23	−106.67	1
05/18/2004	1	1	5557.68	5482.96	74.72	5	10/13/2004	1	3	5963.07	5979.56	−16.49	2
05/19/2004	5	5	5860.58	5577.07	283.51	5	10/14/2004	3	1	5831.07	5963.07	−132.00	1
05/20/2004	5	2	5815.33	5827.08	−11.75	3	10/15/2004	1	1	5820.82	5831.07	−10.25	3
05/21/2004	2	3	5964.94	5842.74	122.20	5	10/18/2004	3	4	5772.12	5817.77	−45.65	2
05/24/2004	5	4	5942.08	5974.51	−32.43	2	10/19/2004	2	4	5807.79	5755.37	52.42	5
05/25/2004	2	3	5958.38	5969.49	−11.11	3	10/20/2004	4	1	5788.34	5792.90	−4.56	3
05/26/2004	3	5	6027.27	5987.09	40.18	4	10/21/2004	2	3	5797.24	5815.75	−18.51	2
05/27/2004	5	3	6033.05	6007.73	25.32	4	10/22/2004	3	2	5774.67	5790.54	−15.87	3
05/28/2004	3	5	6137.26	6061.76	75.50	5	10/26/2004	2	1	5662.88	5770.48	−107.60	1
05/31/2004	5	2	5977.84	6103.76	−125.92	1	10/27/2004	1	5	5650.97	5659.16	−8.19	3
06/01/2004	1	1	5986.20	5977.84	8.36	3	10/28/2004	3	5	5695.56	5679.68	15.88	3
06/02/2004	3	4	5875.67	5983.15	−107.48	1	10/29/2004	4	3	5705.93	5670.43	35.50	4

Appendix C

The fuzzy two-factor ARMA (1,3) solution is shown in Table A4.

Table A4. Fuzzy two-factor AR (1,3) solution.

Fuzzy Value of Main Factor	Fuzzy Value of Secondary Factor	Fuzzy Value of Lagged Errors			Fuzzy Forecast	Defuzzified Forecast	Fuzzy Value of Main Factor	Fuzzy Value of Secondary Factor	Fuzzy Value of Lagged Errors			Fuzzy Forecast	Defuzzified Forecast
		1	2	3					1	2	3		
1	1	1	1	1	2,1,5,5,	8.38	3	3	5	3	3	1,	−67
1	1	1	2	1	3,	0	3	4	1	5	3	3,	0
1	1	2	1	1	1,1,	−67	3	4	2	1	3	2,	−33.5
1	1	2	2	1	1,	−67	3	4	2	3	3	5,	67
1	1	2	4	1	5,	67	3	4	2	4	2	2,	−33.5
1	1	2	5	1	3,	0	3	4	3	2	3	4,	33.5
1	1	3	1	1	5,2,5,	33.5	3	4	3	5	3	3,	0
1	1	3	3	1	5,	67	3	4	3	5	2	3,	0
1	1	4	5	1	3,	0	3	4	4	3	3	3,	0
1	1	5	3	1	1,	−67	3	4	4	3	2	5,	67
1	1	5	4	1	1,	−67	3	4	4	3	3	1,	−67
1	1	5	5	1	5,	67	3	4	5	1	3	1,	−67
1	2	2	1	1	1,	−67	3	5	1	2	2	5,	67
1	2	4	4	1	4,	33.5	3	5	2	3	3	2,	−33.5
1	2	5	3	1	3,	0	3	5	2	5	3	1,	−67
1	3	1	3	2	5,	67	3	5	3	1	3	4,	33.5
1	3	1	5	1	5,	67	3	5	3	4	4	5,	67
1	3	1	5	2	1,	−67	3	5	5	2	3	5,5,	67
1	3	1	5	1	5,	67	4	1	1	2	4	5,	67
1	3	2	5	1	2,	−33.5	4	1	2	5	4	1,	−67
1	3	3	3	1	3,	0	4	1	3	2	5	2,	−33.5
1	3	4	1	1	4,	33.5	4	1	3	3	4	3,	0
1	3	5	1	1	1,	−67	4	1	4	1	3	1,	−67
1	3	5	2	1	1,3,	−33.5	4	1	4	2	4	5,	67
1	4	1	4	2	4,	33.5	4	1	4	5	3	2,	−33.5
1	4	1	5	1	3,	0	4	1	5	1	4	3,	0
1	4	2	4	1	4,	33.5	4	1	5	4	4	1,	−67
1	4	3	5	1	5,	67	4	2	1	1	4	1,	−67
1	4	4	2	2	1,	−67	4	2	1	2	4	1,	−67
1	5	1	1	1	5,	67	4	2	1	5	4	5,	67
1	5	1	3	1	1,1,	−67	4	2	2	5	4	4,	33.5
1	5	1	4	1	2,	−33.5	4	2	5	3	2	3,	0
1	5	2	3	1	3,5,	33.5	4	2	5	4	3	5,	67
1	5	4	2	1	1,	−67	4	3	1	2	4	2,	−33.5

Table A4. *Cont.*

Fuzzy Value of Main Factor	**Fuzzy Value of Secondary Factor**	**Fuzzy Value of Lagged Errors**			**Fuzzy Forecast**	**Defuzzified Forecast**	**Fuzzy Value of Main Factor**	**Fuzzy Value of Secondary Factor**	**Fuzzy Value of Lagged Errors**			**Fuzzy Forecast**	**Defuzzified Forecast**
		1	**2**	**3**					**1**	**2**	**3**		
1	5	4	4	1	3,	0	4	3	1	3	3	3,	0
1	5	5	3	1	5,	67	4	3	3	1	4	1,	−67
2	1	2	2	1	2,	−33.5	4	4	1	3	4	5,	67
2	1	2	5	2	3,	0	4	4	5	5	5	2,	−33.5
2	1	3	2	3	1,	−67	4	5	2	3	4	5,	67
2	1	5	1	2	5,5,	67	4	5	2	5	3	4,	33.5
2	1	5	3	2	2,	−33.5	4	5	3	4	4	5,	67
2	1	5	3	1	4,	33.5	4	5	4	1	4	5,	67
2	1	5	4	2	1,	−67	4	5	5	4	4	3,	0
2	2	1	1	2	3,	0	4	5	5	5	4	4,	33.5
2	2	1	4	2	1,	−67	5	1	1	1	5	3,	0
2	2	1	5	1	4,	33.5	5	1	1	2	5	3,1,	−33.5
2	2	5	5	1	1,	−67	5	1	1	5	5	1,	−67
2	3	1	5	3	3,	0	5	1	2	3	5	3,	0
2	3	2	5	3	3,	0	5	1	2	5	5	4,	33.5
2	3	3	2	2	5,2,	16.75	5	1	3	1	5	2,	−33.5
2	3	3	3	1	3,	0	5	1	3	2	5	2,	−33.5
2	3	3	5	2	3,	0	5	1	4	4	3	3,	0
2	3	4	1	2	5,	67	5	1	5	1	5	5,	67
2	3	4	2	2	5,	67	5	1	5	5	5	2,	−33.5
2	3	4	5	1	3,	0	5	2	1	5	5	2,	−33.5
2	3	5	5	2	5,	67	5	2	3	1	5	1,	−67
2	3	5	5	3	5,	67	5	2	4	4	5	1,	−67
2	4	1	3	2	4,	33.5	5	2	4	5	5	4,	33.5
2	4	3	5	2	1,	−67	5	3	1	1	5	1,	−67
2	5	1	1	2	3,	0	5	3	2	2	4	1,	−67
2	5	2	1	2	4,	33.5	5	3	2	2	5	4,	33.5
2	5	2	4	3	3,	0	5	3	2	3	4	3,	0
2	5	3	3	2	5,	67	5	3	3	2	5	3,	0
2	5	5	3	3	4,	33.5	5	3	3	3	5	3,	0
2	5	5	5	1	2,	−33.5	5	3	4	2	5	3,	0
3	1	1	2	3	1,	−67	5	3	4	3	5	3,	0
3	1	1	5	3	5,	67	5	3	5	1	5	1,	−67
3	1	2	5	3	3,	0	5	3	5	2	5	1,	−67
3	1	3	1	2	4,1,	−16.75	5	3	5	3	5	1,	−67
3	1	3	3	3	5,	67	5	3	5	4	5	5,	67
3	1	3	4	3	3,	0	5	4	1	4	5	3,	0
3	1	3	5	3	1,	−67	5	4	1	5	5	2,	−33.5
3	1	4	4	2	5,	67	5	4	2	4	5	4,	33.5
3	1	4	5	3	1,	−67	5	4	3	1	5	4,	33.5

Table A4. *Cont.*

Fuzzy Value of Main Factor	Fuzzy Value of Secondary Factor	Fuzzy Value of Lagged Errors			Fuzzy Forecast	Defuzzified Forecast	Fuzzy Value of Main Factor	Fuzzy Value of Secondary Factor	Fuzzy Value of Lagged Errors			Fuzzy Forecast	Defuzzified Forecast
		1	2	3					1	2	3		
3	1	5	1	2	5,	67	5	4	4	5	5	3,	0
3	1	5	1	3	1,	−67	5	4	5	1	5	5,	67
3	1	5	1	2	4,	33.5	5	4	5	3	5	2,	−33.5
3	1	5	3	3	2,5,	16.75	5	5	1	1	5	1,5,5,	22.33
3	1	5	3	2	2,	−33.5	5	5	1	2	5	4,4,	33.5
3	2	2	1	3	4,	33.5	5	5	1	4	5	3,	0
3	2	4	5	2	3,	0	5	5	1	5	5	5,	67
3	2	5	2	3	3,	0	5	5	2	2	4	3,	0
3	2	5	3	2	2,	−33.5	5	5	2	4	5	5,	67
3	2	5	5	3	2,	−33.5	5	5	3	2	5	5,	67
3	3	1	4	4	1,	−67	5	5	3	3	5	2,3,	−16.75
3	3	2	5	4	4,	33.5	5	5	3	4	5	2,5,	16.75
3	3	3	1	4	5,	67	5	5	4	1	5	3,	0
3	3	3	5	3	2,	−33.5	5	5	4	5	5	5,	67
3	3	4	1	4	2,	−33.5	5	5	5	1	5	2,	−33.5
3	3	5	2	3	3,	0	5	5	5	5	4	2,	−33.5
3	3	5	3	4	4,	33.5							

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