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A New Multi-Attribute Decision-Making Method Based on m-Polar Fuzzy Soft Rough Sets

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Abstract: We introduce notions of soft rough m-polar fuzzy sets and m-polar fuzzy soft rough sets as novel hybrid models for soft computing, and investigate some of their fundamental properties. We discuss the relationship between m-polar fuzzy soft rough approximation operators and crisp soft rough approximation operators. We also present applications of m-polar fuzzy soft rough sets to decision-making.

Keywords: soft rough m-polar fuzzy sets; m-polar fuzzy soft rough sets; m-polar fuzzy soft rough approximation operators; decision-making

1. Introduction

The notion of bipolar fuzzy sets was generalized to m-polar fuzzy sets by Chen et al. [1] in 2014. Chen et al. [1] proved that bipolar fuzzy sets and 2-polar fuzzy sets are cryptomorphic mathematical tools. In many real life complicated problems, data sometimes comes from n agents $(n \geq 2)$, that is, multipolar information (not just bipolar information, which corresponds to two-valued logic) exists. There are many applications of m-polar fuzzy sets to decision-making problems when it is compulsory to make assessments with a group of agreements. For example, similarity degrees of two logic formulas that are based on n logic implication operators $(n \geq 2)$, ordering results of a magazine, a group of friends wants to plan to visit a country, ordering results of a university. Akram et al. [2–5] promoted the work on m-polar fuzzy graphs and introduced many new concepts. Li et al. [6] considered different algebraic operations on m-polar fuzzy graphs. In 1982, Pawlak [7] introduced the idea of rough set theory, which is an important mathematical tool to handle imprecise, vague and incomplete information. In fuzzy set theory [8], membership function plays the vital role. However, the selection of membership function is uncertain. The fuzzy set theory is an uncertain tool to solve the uncertain problems, but, in rough set theory, two precise boundary lines are established to describe the vague concepts. Consequently, the rough set theory is a mathematical tool to solve uncertain problems. Dubois and Prade [9] introduced the ideas of rough fuzzy sets and fuzzy rough sets by combining fuzzy sets and rough sets. Recently, works on granular computing are progressing rapidly. Xu and Gou [10] described an overview of interval-valued intuitionistic fuzzy information aggregation techniques, and their applications in different fields such as decision-making, entropy measure and supplier selection. Das et al. [11] introduced a robust decision-making approach using intuitionistic trapezoidal fuzzy number. Cai et al. [12] defined dynamic fuzzy sets by means of shadowed sets and proposed an analytic solution to computing the pair of thresholds by searching for a balance of uncertainty in the framework of shadowed sets. Pedrycz and Chen [13] provided various methods of fuzzy sets and granular computing, brings new concepts, architectures and practice of fuzzy decision-making with various applications.

Many real-world problems in different domains, including social sciences, physical sciences, applied sciences and life sciences contain vague and imprecise information. The classical mathematical tools and theories are unfit to handle the difficulties of the data having uncertainties, whereas a lot of theories including probability theory and fuzzy set theory [8] are very helpful mathematical tools for dealing with different types of uncertain data. Molodtsov [14] indicated the drawbacks of these theories. In order to overcome these difficulties, Molodtsov [14] introduced the concept of soft set theory. Maji et al. [15] proposed some fundamental algebraic operations for soft sets. Maji et al. [16] generalized the idea of soft sets and presented a hybrid model fuzzy soft sets. Alcantud [17–19] gave a novel approach to the problems of fuzzy soft sets based decision-making. Alcantud and Santos-Garcia [20,21] produced a completely new approach to soft set based decision-making problems when information is incomplete. They also proposed and compared an algorithmic solution with previous approaches in the literature in [20]. Feng et al. [22] gave the novel idea of rough soft sets by combining the Pawlak rough sets and soft sets. In 2011, Feng et al. [23] introduced the idea of soft rough sets. All mathematical models, including fuzzy sets, rough sets, soft sets and fuzzy soft sets have their advantages and drawbacks. One of the crucial drawbacks of all of these models is that they have a lack of a sufficient number of parameters to handle the uncertain data. In order to overcome this problem, we combine rough sets, soft sets with m-polar fuzzy sets and propose the concepts of new hybrid models called soft rough m-polar fuzzy sets and m-polar fuzzy soft rough sets. We define the lower and upper soft approximations of an m-polar fuzzy set. The idea of m-polar fuzzy soft rough sets can be utilized to solve different real-life problems. Thus, we present a new method to decision-making based on m-polar fuzzy soft rough sets.

2. Soft Rough *m*-Polar Fuzzy Sets

Definition 1. *An m-polar fuzzy set (mF set, for short) on a universe Y is a function $Q = (p_1 \circ Q(z), p_2 \circ Q(z), \ldots, p_m \circ Q(z)) : Y \to [0,1]^m$, where the i-th projection mapping is defined as $p_i \circ Q : [0,1]^m \to [0,1]$. Denote $\mathbf{0} = (0,0,\cdots,0)$ is the smallest element in $[0,1]^m$ and $\mathbf{1} = (1,1,\cdots,1)$ is the largest element in $[0,1]^m$ [1].*

Definition 2. *([14]) Let Y be a nonempty set called universe, T a set of parameters. A pair (η, T) is called a soft set over Y if η is a mapping given by $\eta : T \to P(Y)$, where $P(Y)$ is the collection of all subsets of Y.*

Definition 3. *([24]) Let Y be an initial universe, (η, T) a soft set on Y. For any $N \subseteq Y \times T$, the crisp soft relation N over $Y \times T$ is given by*

$$N = \{\langle (v,w), \varrho_N(v,w)\rangle \mid (v,w) \in Y \times T\},$$

where $\varrho_N : Y \times T \to \{0,1\}$, $\varrho_N(v,w) = \begin{cases} 1 & if \ \ (v,w) \in N, \\ 0 & if \ \ (v,w) \notin N. \end{cases}$

Definition 4. *([25]) Let Y be the universe of discourse and let T be a set of parameters. For any crisp soft relation $\xi \subseteq Y \times T$, a set-valued function $\xi_s : Y \to P(T)$ is given by*

$$\xi_s(v) = \{w \in T \mid (v,w) \in \xi\}, \quad v \in Y.$$

ξ is referred to as serial if $\forall\, v \in Y, \xi_s(v) \neq \emptyset$. The pair (Y, T, ξ) is said to be a crisp soft approximation space. For any $Q \subseteq T$, the lower and upper soft approximations of Q about (Y, T, ξ), denoted by $\underline{\xi}(Q)$ and $\overline{\xi}(Q)$, respectively, are defined as

$$\underline{\xi}(Q) = \{v \in Y \mid \xi_s(v) \cap Q \neq \phi\},$$

$$\overline{\xi}(Q) = \{v \in Y \mid \xi_s(v) \subseteq Q\}.$$

The pair $(\underline{\xi}(Q), \overline{\xi}(Q))$ is said to be a crisp soft rough set and $\underline{\xi}, \overline{\xi} : P(T) \to P(Y)$ are, respectively, called lower and upper crisp soft rough approximation operators. Furthermore, if $\underline{\xi}(Q) = \overline{\xi}(Q)$, then Q is called a definable set.

We now define soft rough m-polar fuzzy sets.

Definition 5. *Let Y be an initial universe and T a universe of parameters. For any crisp soft relation ξ over $Y \times T$, the pair (Y, T, ξ) is called a crisp soft approximation space. For an arbitrary $Q \in m(T)$, the lower and upper soft approximations of Q about (Y, T, ξ), denoted by $\underline{\xi}(Q)$ and $\overline{\xi}(Q)$, respectively, are defined by*

$$\underline{\xi}(Q) = \big\{\langle v, Q_{\underline{\xi}}(v)\rangle \mid v \in Y\big\},$$
$$\overline{\xi}(Q) = \big\{\langle v, Q_{\overline{\xi}}(v)\rangle \mid v \in Y\big\},$$

where

$$Q_{\underline{\xi}}(v) = \bigwedge_{w \in \xi_s(v)} p_i \circ Q(w), \qquad Q_{\overline{\xi}}(v) = \bigvee_{w \in \xi_s(v)} p_i \circ Q(w).$$

The pair $(\underline{\xi}(Q), \overline{\xi}(Q))$ is called the soft rough mF set of Q about (Y, T, ξ), and $\underline{\xi}$, $\overline{\xi} : m(T) \to m(Y)$ are, respectively, said to be lower and upper soft rough mF approximation operators. Moreover, if $\underline{\xi}(Q) = \overline{\xi}(Q)$, then Q is referred to as definable.

Example 1. *Let $Y = \{y_1, y_2, y_3, y_4, y_5, y_6\}$ be a universe of discourse, $T = \{k_1, k_2, k_3, k_4\}$ a set of parameters. Assume that a soft set on Y is defined by*

$$\eta(k_1) = \{y_1, y_2, y_5\}, \qquad \eta(k_2) = \{y_3, y_4, y_5\},$$
$$\eta(k_3) = \emptyset, \qquad \eta(k_4) = Y.$$

Then, a crisp soft relation ξ over $Y \times T$ is given by

$$\xi = \big\{(y_1, k_1), (y_2, k_1), (y_5, k_1), (y_3, k_2), (y_4, k_2), (y_5, k_2), (y_1, k_4), (y_2, k_4), (y_3, k_4), (y_4, k_4), (y_5, k_4), (y_6, k_4)\big\}.$$

By Definition 4, we have

$$\xi_s(y_1) = \{k_1, k_4\}, \qquad \xi_s(y_2) = \{k_1, k_4\},$$
$$\xi_s(y_3) = \{k_2, k_4\}, \qquad \xi_s(y_4) = \{k_2, k_4\},$$
$$\xi_s(y_5) = \{k_1, k_2, k_4\}, \qquad \xi_s(y_6) = \{k_4\}.$$

Consider a 3-polar fuzzy set $Q \in m(T)$ as follows:

$$Q = \big\{(k_1, 0.75, 0.25, 0.13), (k_2, 0.12, 0.7, 0.4), (k_3, 0.3, 0.85, 0.6), (k_4, 0.1, 0.3, 0.5)\big\}.$$

By Definition 5, we have lower and upper soft approximations:

$$Q_{\underline{\xi}}(y_1) = (0.1, 0.25, 0.13), \qquad Q_{\overline{\xi}}(y_1) = (0.75, 0.3, 0.5),$$
$$Q_{\underline{\xi}}(y_2) = (0.1, 0.25, 0.13), \qquad Q_{\overline{\xi}}(y_2) = (0.75, 0.3, 0.5),$$
$$Q_{\underline{\xi}}(y_3) = (0.1, 0.3, 0.4), \qquad Q_{\overline{\xi}}(y_3) = (0.12, 0.7, 0.5),$$
$$Q_{\underline{\xi}}(y_4) = (0.1, 0.3, 0.4), \qquad Q_{\overline{\xi}}(y_4) = (0.12, 0.7, 0.5),$$
$$Q_{\underline{\xi}}(y_5) = (0.1, 0.25, 0.13), \qquad Q_{\overline{\xi}}(y_5) = (0.75, 0.7, 0.5),$$
$$Q_{\underline{\xi}}(y_6) = (0.1, 0.3, 0.5), \qquad Q_{\overline{\xi}}(y_6) = (0.1, 0.3, 0.5).$$

Thus,

$$\underline{\xi}(Q) = \{(y_1, 0.1, 0.25, 0.13), (y_2, 0.1, 0.25, 0.13), (y_3, 0.1, 0.3, 0.4), (y_4, 0.1, 0.3, 0.4), (y_5, 0.1, 0.25, 0.13), (y_6, 0.1, 0.3, 0.5)\},$$
$$\overline{\xi}(Q) = \{(y_1, 0.75, 0.3, 0.5), (y_2, 0.75, 0.3, 0.5), (y_3, 0.12, 0.7, 0.5), (y_4, 0.12, 0.7, 0.5), (y_5, 0.75, 0.7, 0.5), (y_6, 0.1, 0.3, 0.5)\}.$$

Hence, the pair $(\underline{\xi}(Q), \overline{\xi}(Q))$ *is said to be a soft rough* 3*-polar fuzzy set.*

We now present properties of soft rough mF sets.

Theorem 1. *Let* (Y, T, ξ) *be a crisp soft approximation space. Then, the lower and upper soft rough mF approximation operators* $\underline{\xi}(Q)$ *and* $\overline{\xi}(Q)$*, respectively, satisfy the following properties, for any* $Q, R \in m(T)$*:*

1. $\underline{\xi}(Q) = \sim \overline{\xi}(\sim Q)$,
2. $Q \subseteq R \Rightarrow \underline{\xi}(Q) \subseteq \underline{\xi}(R)$,
3. $\underline{\xi}(Q \cap R) = \underline{\xi}(Q) \cap \underline{\xi}(R)$,
4. $\underline{\xi}(Q \cup R) \supseteq \underline{\xi}(Q) \cup \underline{\xi}(R)$,
5. $\overline{\xi}(Q) = \sim \underline{\xi}(\sim Q)$,
6. $Q \subseteq R \Rightarrow \overline{\xi}(Q) \subseteq \overline{\xi}(R)$,
7. $\overline{\xi}(Q \cup R) = \overline{\xi}(Q) \cup \overline{\xi}(R)$,
8. $\overline{\xi}(Q \cap R) \subseteq \overline{\xi}(Q) \cap \overline{\xi}(R)$,

where $\sim Q$ *denotes the compliment of* Q*.*

Proof. 1. From Definition 5, we have

$$\begin{aligned}
\sim \overline{\xi}(\sim Q) &= \Big\{\Big\langle v, (\mathbf{1} - (\sim Q)_{\overline{\xi}}(v))\Big\rangle \mid v \in Y\Big\}, \\
&= \Big\{\Big\langle v, \Big(\mathbf{1} - \bigvee_{w \in \xi_s(v)} p_i \circ (\sim Q)(w)\Big)\Big\rangle \mid v \in Y\Big\}, \\
&= \Big\{\Big\langle v, \Big(\mathbf{1} \wedge \bigwedge_{w \in \xi_s(v)} p_i \circ Q(w)\Big)\Big\rangle \mid v \in Y\Big\}, \\
&= \Big\{\Big\langle v, \bigwedge_{w \in \xi_s(v)} p_i \circ Q(w)\Big\rangle \mid v \in Y\Big\}, \\
&= \Big\{\Big\langle v, Q_{\underline{\xi}}(v)\Big\rangle \mid v \in Y\Big\}, \\
&= \underline{\xi}(Q).
\end{aligned}$$

It follows that $\underline{\xi}(Q) = \sim \overline{\xi}(\sim Q)$.

2. It can be easily proved by Definition 5.

3. By Definition 5,

$$\begin{aligned}
\underline{\xi}(Q\cap R) &= \Big\{\Big\langle v, (Q\cap R)_{\underline{\xi}}(v)\Big\rangle \mid v\in Y\Big\},\\
&= \Big\{\Big\langle v, \bigwedge_{w\in\xi_s(v)} p_i\circ(Q\cap R)(w)\Big\rangle \mid v\in Y\Big\},\\
&= \Big\{\Big\langle v, \bigwedge_{w\in\xi_s(v)} \Big(p_i\circ Q(w)\wedge p_i\circ R(w)\Big)\Big\rangle \mid v\in Y\Big\},\\
&= \Big\{\Big\langle v, \bigwedge_{w\in\xi_s(v)} \Big(p_i\circ Q(w)\Big)\wedge \bigwedge_{w\in\xi_s(v)} \Big(p_i\circ R(w)\Big)\Big\rangle \mid v\in Y\Big\},\\
&= \Big\{\Big\langle v, Q_{\underline{\xi}}(v)\wedge R_{\underline{\xi}}(v)\Big\rangle \mid v\in Y\Big\},\\
&= \underline{\xi}(Q)\cap\underline{\xi}(R).
\end{aligned}$$

Hence, $\underline{\xi}(Q\cap R)=\underline{\xi}(Q)\cap\underline{\xi}(R)$.

4. From Definition 5,

$$\begin{aligned}
\underline{\xi}(Q\cup R) &= \Big\{\Big\langle v, (Q\cup R)_{\underline{\xi}}(v)\Big\rangle \mid v\in Y\Big\},\\
&= \Big\{\Big\langle v, \bigwedge_{w\in\xi_s(v)} p_i\circ(Q\cup R)(w)\Big\rangle \mid v\in Y\Big\},\\
&= \Big\{\Big\langle v, \bigwedge_{w\in\xi_s(v)} \Big(p_i\circ Q(w)\vee p_i\circ R(w)\Big)\Big\rangle \mid v\in Y\Big\},\\
&\supseteq \Big\{\Big\langle v, \bigwedge_{w\in\xi_s(v)} \Big(p_i\circ Q(w)\Big)\vee \bigwedge_{w\in\xi_s(v)} \Big(p_i\circ R(w)\Big)\Big\rangle \mid v\in Y\Big\},\\
&= \Big\{\Big\langle v, Q_{\underline{\xi}}(v)\vee R_{\underline{\xi}}(v)\Big\rangle \mid v\in Y\Big\},\\
&= \underline{\xi}(Q)\cup\underline{\xi}(R).
\end{aligned}$$

Hence, $\underline{\xi}(Q\cup R)\supseteq\underline{\xi}(Q)\cup\underline{\xi}(R)$.

Similarly, properties (5–8) of the upper soft rough mF approximation operator $\overline{\xi}(Q)$ can be proved by using the above arguments. □

Example 2. *Let $Y=\{g_1,g_2,g_3,g_4\}$ be a universe and let $T=\{n_1,n_2,n_3\}$ be a set of parameters. Consider a soft set (η,T) over Y is defined as*

$$\eta(n_1)=\{g_1,g_2,g_4\},\qquad \eta(n_2)=\{g_3\},\qquad \eta(n_3)=Y.$$

Then, a crisp soft relation ξ on $Y\times T$ is given by

$$\xi=\big\{(g_1,n_1),(g_2,n_1),(g_4,n_1),(g_3,n_2),(g_1,n_3),(g_2,n_3),(g_3,n_3),(g_4,n_3)\big\}.$$

By Definition 4,

$$\begin{aligned}
&\xi_s(g_1)=\{n_1,n_3\}, && \xi_s(g_2)=\{n_1,n_3\},\\
&\xi_s(g_3)=\{n_2,n_3\}, && \xi_s(g_4)=\{n_1,n_3\}.
\end{aligned}$$

Consider 3-polar fuzzy sets $Q,R\in m(T)$ as follows:

$$Q=\big\{(n_1,0.5,0.2,0.3),(n_2,0.2,0.6,0.5),(n_3,0.5,0.9,0.1)\big\},$$

$$R=\big\{(n_1,0.7,0.5,0.1),(n_2,0.1,0.7,0.4),(n_3,0.3,0.8,0.6)\big\}.$$

Then,

$$\begin{aligned}
\sim Q &= \{(n_1, 0.5, 0.8, 0.7), (n_2, 0.8, 0.4, 0.5), (n_3, 0.5, 0.1, 0.9)\},\\
Q \cup R &= \{(n_1, 0.7, 0.5, 0.3), (n_2, 0.2, 0.7, 0.5), (n_3, 0.5, 0.9, 0.6)\},\\
Q \cap R &= \{(n_1, 0.5, 0.2, 0.1), (n_2, 0.1, 0.6, 0.4), (n_3, 0.3, 0.8, 0.1)\}.
\end{aligned}$$

By Definition 5, we have

$$\begin{aligned}
\underline{\xi}(Q) &= \{(g_1, 0.5, 0.2, 0.1), (g_2, 0.5, 0.2, 0.1), (g_3, 0.2, 0.6, 0.1), (g_4, 0.5, 0.2, 0.1)\},\\
\overline{\xi}(Q) &= \{(g_1, 0.5, 0.9, 0.3), (g_2, 0.5, 0.9, 0.3), (g_3, 0.5, 0.9, 0.5), (g_4, 0.5, 0.9, 0.3)\},\\
\underline{\xi}(R) &= \{(g_1, 0.3, 0.5, 0.1), (g_2, 0.3, 0.5, 0.1), (g_3, 0.1, 0.7, 0.4), (g_4, 0.3, 0.5, 0.1)\},\\
\overline{\xi}(R) &= \{(g_1, 0.7, 0.8, 0.6), (g_2, 0.7, 0.8, 0.6), (g_3, 0.3, 0.8, 0.6), (g_4, 0.7, 0.8, 0.6)\},\\
\underline{\xi}(\sim Q) &= \{(g_1, 0.5, 0.1, 0.7), (g_2, 0.5, 0.1, 0.7), (g_3, 0.5, 0.1, 0.5), (g_4, 0.5, 0.1, 0.7)\},\\
\overline{\xi}(\sim Q) &= \{(g_1, 0.5, 0.8, 0.9), (g_2, 0.5, 0.8, 0.9), (g_3, 0.8, 0.4, 0.9), (g_4, 0.5, 0.8, 0.9)\},\\
\sim \underline{\xi}(\sim Q) &= \{(g_1, 0.5, 0.9, 0.3), (g_2, 0.5, 0.9, 0.3), (g_3, 0.5, 0.9, 0.5), (g_4, 0.5, 0.9, 0.3)\},\\
\sim \overline{\xi}(\sim Q) &= \{(g_1, 0.5, 0.2, 0.1), (g_2, 0.5, 0.2, 0.1), (g_3, 0.2, 0.6, 0.1), (g_4, 0.5, 0.2, 0.1)\},\\
\underline{\xi}(Q \cup R) &= \{(g_1, 0.5, 0.5, 0.3), (g_2, 0.5, 0.5, 0.3), (g_3, 0.2, 0.7, 0.5), (g_4, 0.5, 0.5, 0.3)\},\\
\overline{\xi}(Q \cup R) &= \{(g_1, 0.7, 0.9, 0.6), (g_2, 0.7, 0.9, 0.6), (g_3, 0.5, 0.9, 0.6), (g_4, 0.7, 0.9, 0.6)\},\\
\underline{\xi}(Q \cap R) &= \{(g_1, 0.3, 0.2, 0.1), (g_2, 0.3, 0.2, 0.1), (g_3, 0.1, 0.6, 0.1), (g_4, 0.3, 0.2, 0.1)\},\\
\overline{\xi}(Q \cap R) &= \{(g_1, 0.5, 0.8, 0.1), (g_2, 0.5, 0.8, 0.1), (g_3, 0.3, 0.8, 0.4), (g_4, 0.5, 0.8, 0.1)\}.
\end{aligned}$$

Now,

$$\begin{aligned}
\underline{\xi}(Q) \cup \underline{\xi}(R) &= \{(g_1, 0.5, 0.5, 0.1), (g_2, 0.5, 0.5, 0.1), (g_3, 0.2, 0.7, 0.4), (g_4, 0.5, 0.5, 0.1)\},\\
\overline{\xi}(Q) \cup \overline{\xi}(R) &= \{(g_1, 0.7, 0.9, 0.6), (g_2, 0.7, 0.9, 0.6), (g_3, 0.5, 0.9, 0.6), (g_4, 0.7, 0.9, 0.6)\},\\
\underline{\xi}(Q) \cap \underline{\xi}(R) &= \{(g_1, 0.3, 0.2, 0.1), (g_2, 0.3, 0.2, 0.1), (g_3, 0.1, 0.6, 0.1), (g_4, 0.3, 0.2, 0.1)\},\\
\overline{\xi}(Q) \cap \overline{\xi}(R) &= \{(g_1, 0.5, 0.8, 0.3), (g_2, 0.5, 0.8, 0.3), (g_3, 0.3, 0.8, 0.5), (g_4, 0.5, 0.8, 0.3)\}.
\end{aligned}$$

From the above calculations, we observe that the following properties are satisfied:

$$\begin{array}{ll}
\sim \underline{\xi}(\sim Q) = \overline{\xi}(Q), & \sim \overline{\xi}(\sim Q) = \underline{\xi}(Q),\\
\underline{\xi}(Q \cap R) = \underline{\xi}(Q) \cap \underline{\xi}(R), & \underline{\xi}(Q \cup R) \supseteq \underline{\xi}(Q) \cup \underline{\xi}(R),\\
\overline{\xi}(Q \cup R) = \overline{\xi}(Q) \cup \overline{\xi}(R), & \overline{\xi}(Q \cap R) \subseteq \overline{\xi}(Q) \cap \overline{\xi}(R).
\end{array}$$

Proposition 1. *Let (Y, T, ξ) be a crisp soft approximation space. Then, lower and upper soft rough approximations of mF sets Q and R satisfies the following laws:*

1. $\sim \left(\underline{\xi}(Q) \cup \underline{\xi}(R)\right) = \overline{\xi}(\sim Q) \cap \overline{\xi}(\sim R),$
2. $\sim \left(\underline{\xi}(Q) \cup \overline{\xi}(R)\right) = \overline{\xi}(\sim Q) \cap \underline{\xi}(\sim R),$
3. $\sim \left(\overline{\xi}(Q) \cup \underline{\xi}(R)\right) = \underline{\xi}(\sim Q) \cap \overline{\xi}(\sim R),$
4. $\sim \left(\overline{\xi}(Q) \cup \overline{\xi}(R)\right) = \underline{\xi}(\sim Q) \cap \underline{\xi}(\sim R),$
5. $\sim \left(\underline{\xi}(Q) \cap \underline{\xi}(R)\right) = \overline{\xi}(\sim Q) \cup \overline{\xi}(\sim R),$
6. $\sim \left(\underline{\xi}(Q) \cap \overline{\xi}(R)\right) = \overline{\xi}(\sim Q) \cup \underline{\xi}(\sim R),$
7. $\sim \left(\overline{\xi}(Q) \cap \underline{\xi}(R)\right) = \underline{\xi}(\sim Q) \cup \overline{\xi}(\sim R),$

8. $\sim\left(\overline{\zeta}(Q)\cap\overline{\zeta}(R)\right)=\underline{\zeta}(\sim Q)\cup\underline{\zeta}(\sim R)$.

Proof. Its proof follows immediately from the Definition 5. □

3. *m*F Soft Rough Sets

Definition 6. *Let Y be a universe of discourse, T a set of parameters and $V\subseteq T$. A pair (τ,V) is referred to as an mF soft set on Y if τ is a mapping $\tau:T\to m(Y)$.*

Definition 7. *Let (τ,V) be an mF soft set over Y. Then, an mF subset ζ of $Y\times T$ is referred to as an mF soft relation from Y to T is given by*

$$\zeta=\{\langle(x,t),p_i\circ\zeta(x,t)\rangle\mid(x,t)\in Y\times T\},$$

where $\zeta:Y\times T\to[0,1]^m$.

If $Y=\{x_1,x_2,\cdots,x_n\}$, $T=\{t_1,t_2,\cdots,t_n\}$, then an mF soft relation ζ over $Y\times T$ can be presented as follows:

ζ	t_1	t_2	$\cdots$	t_n
x_1	$p_i\circ(x_1,t_1)$	$p_i\circ(x_1,t_2)$	$\cdots$	$p_i\circ(x_1,t_n)$
x_2	$p_i\circ(x_2,t_1)$	$p_i\circ(x_2,t_2)$	$\cdots$	$p_i\circ(x_2,t_n)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_n	$p_i\circ(x_n,t_1)$	$p_i\circ(x_n,t_2)$	$\cdots$	$p_i\circ(x_n,t_n)$.

Example 3. *Let $Y=\{x_1,x_2,x_3\}$ be a universe, $T=\{t_1,t_2,t_3\}$ a set of parameters. A 3-polar fuzzy soft relation $\zeta:Y\to T$ of the universe $Y\times T$ is given by*

ζ	t_1	t_2	t_3
x_1	(0.6, 0.3, 0.1)	(0.4, 0.7, 0.6)	(0.4, 0.6, 0.2)
x_2	(0.5, 0.3, 0.2)	(0.5, 0.2, 0.8)	(0.6, 0.9, 0.6)
x_3	(0.3, 0.2, 0.1)	(0.3, 0.4, 0.8)	(0.7, 0.3, 0.5).

We now define m-polar fuzzy soft rough sets.

Definition 8. *Let Y be a nonempty set called universe, T a universe of parameters. For any mF soft relation ζ on $Y\times T$, the pair (Y,T,ζ) is referred to as an mF soft approximation space. For an arbitrary $Q\in m(T)$, the lower and upper soft approximations of Q about (Y,T,ζ), denoted by $\underline{\zeta}(Q)$ and $\overline{\zeta}(Q)$, respectively, are defined as follows:*

$$\underline{\zeta}(Q)=\{\langle v,Q_{\underline{\zeta}}(v)\rangle\mid v\in Y\},$$
$$\overline{\zeta}(Q)=\{\langle v,Q_{\overline{\zeta}}(v)\rangle\mid v\in Y\},$$

where

$$Q_{\underline{\zeta}}(v)=\bigwedge_{w\in T}\left[(\mathbf{1}-p_i\circ Q_\zeta(v,w))\vee p_i\circ Q(w)\right],$$
$$Q_{\overline{\zeta}}(v)=\bigvee_{w\in T}\left(p_i\circ Q_\zeta(v,w)\wedge p_i\circ Q(w)\right).$$

The pair $(\underline{\zeta}(Q),\overline{\zeta}(Q))$ is called mF soft rough set of Q about (Y,T,ζ), and $\underline{\zeta}$, $\overline{\zeta}:m(T)\to m(Y)$ are, respectively, said to be lower and upper mF soft rough approximation operators. Moreover, if $\underline{\zeta}(Q)=\overline{\zeta}(Q)$, then Q is said to be definable.

Example 4. *Let* $Y = \{x_1, x_2, x_3, x_4, x_5\}$ *be the set of five laptops and let* $T = \{t_1 = size, t_2 = beautiful, t_3 = technology,\ t_4 = price\}$ *be the set of parameters. Consider a* 3*-polar fuzzy soft relation* $\zeta : Y \to T$ *is given by*

ζ	t_1	t_2	t_3	t_4
x_1	$(0.6, 0.3, 0.1)$	$(0.4, 0.7, 0.6)$	$(0.4, 0.6, 0.2)$	$(0.4, 0.6, 0.2)$
x_2	$(0.5, 0.3, 0.2)$	$(0.5, 0.2, 0.8)$	$(0.6, 0.9, 0.6)$	$(0.7, 0.3, 0.6)$
x_3	$(0.3, 0.2, 0.1)$	$(0.3, 0.4, 0.8)$	$(0.7, 0.3, 0.5)$	$(0.2, 0.9, 0.9)$
x_4	$(0.4, 0.3, 0.6)$	$(0.5, 0.1, 0.4)$	$(0.3, 0.1, 0.0)$	$(0.6, 0.4, 0.4)$
x_5	$(0.2, 0.7, 0.3)$	$(0.4, 0.8, 0.1)$	$(0.4, 0.0, 0.7)$	$(0.8, 0.9, 0.0)$.

Consider a 3*-polar fuzzy subset* Q *of* T *as follows:*

$$Q = \{(t_1, 0.3, 0.1, 0.7), (t_2, 0.3, 0.6, 0.4), (t_3, 0.5, 0.6, 0.1), (t_4, 0.9, 0.1, 0.4)\}.$$

From Definition 8, the lower and upper soft approximations are given by

$$\begin{array}{ll}
Q_{\underline{\zeta}}(x_1) = (0.4, 0.4, 0.4), & Q_{\overline{\zeta}}(x_1) = (0.4, 0.6, 0.4), \\
Q_{\underline{\zeta}}(x_2) = (0.5, 0.6, 0.4), & Q_{\overline{\zeta}}(x_2) = (0.7, 0.6, 0.4), \\
Q_{\underline{\zeta}}(x_3) = (0.5, 0.1, 0.4), & Q_{\overline{\zeta}}(x_3) = (0.5, 0.4, 0.4), \\
Q_{\underline{\zeta}}(x_4) = (0.5, 0.6, 0.6), & Q_{\overline{\zeta}}(x_4) = (0.6, 0.1, 0.6), \\
Q_{\underline{\zeta}}(x_5) = (0.6, 0.1, 0.3), & Q_{\overline{\zeta}}(x_5) = (0.8, 0.6, 0.3).
\end{array}$$

Now,

$$\begin{aligned}
\underline{\zeta}(Q) = \{&(x_1, 0.4, 0.4, 0.4), (x_2, 0.5, 0.6, 0.4), (x_3, 0.5, 0.1, 0.4), (x_4, 0.5, 0.6, 0.6), \\
&(x_5, 0.6, 0.1, 0.3)\}, \\
\overline{\zeta}(Q) = \{&(x_1, 0.4, 0.6, 0.4), (x_2, 0.7, 0.6, 0.4), (x_3, 0.5, 0.4, 0.4), (x_4, 0.6, 0.1, 0.6), \\
&(x_5, 0.8, 0.6, 0.3)\}.
\end{aligned}$$

Hence, the pair $(\underline{\zeta}(Q), \overline{\zeta}(Q))$ *is called a* 3*-polar fuzzy soft rough set.*

We now present properties of mF soft rough sets.

Theorem 2. *Let* (Y, T, ζ) *be an mF soft approximation space. Then, the lower and upper soft rough mF approximation operators* $\underline{\zeta}(Q)$ *and* $\overline{\zeta}(Q)$*, respectively, satisfy the following properties, for any* $Q, R \in m(T)$*:*

1. $\underline{\zeta}(Q) = \sim \overline{\zeta}(\sim Q)$,
2. $Q \subseteq R \Rightarrow \underline{\zeta}(Q) \subseteq \underline{\zeta}(R)$,
3. $\underline{\zeta}(Q \cap R) = \underline{\zeta}(Q) \cap \underline{\zeta}(R)$,
4. $\underline{\zeta}(Q \cup R) \supseteq \underline{\zeta}(Q) \cup \underline{\zeta}(R)$,
5. $\overline{\zeta}(Q) = \sim \underline{\zeta}(\sim Q)$,
6. $Q \subseteq R \Rightarrow \overline{\zeta}(Q) \subseteq \overline{\zeta}(R)$,
7. $\overline{\zeta}(Q \cup R) = \overline{\zeta}(Q) \cup \overline{\zeta}(R)$,
8. $\overline{\zeta}(Q \cap R) \subseteq \overline{\zeta}(Q) \cap \overline{\zeta}(R)$,

where $\sim Q$ *denotes the compliment of* Q*.*

Proof. 1. From Definition 8,

$$
\begin{aligned}
\sim \overline{\zeta}(\sim Q) &= \Big\{\Big\langle v, (\mathbf{1} - (\sim Q)_{\overline{\zeta}}(v))\Big\rangle \mid v \in Y\Big\}, \\
&= \Big\{\Big\langle v, \mathbf{1} - \bigvee_{w \in T} (p_i \circ (\sim Q)_{\zeta}(v,w) \wedge p_i \circ (\sim Q)(w))\Big\rangle \mid v \in Y\Big\}, \\
&= \Big\{\Big\langle v, \mathbf{1} \wedge \bigwedge_{w \in T} (\mathbf{1} - p_i \circ Q_{\zeta}(v,w)) \vee p_i \circ Q(w)\Big\rangle \mid v \in Y\Big\}, \\
&= \Big\{\Big\langle v, \bigwedge_{w \in T} (\mathbf{1} - p_i \circ Q_{\zeta}(v,w)) \vee p_i \circ Q(w)\Big\rangle \mid v \in Y\Big\}, \\
&= \Big\{\Big\langle v, Q_{\underline{\zeta}}(v)\Big\rangle \mid v \in Y\Big\}, \\
&= \underline{\zeta}(Q).
\end{aligned}
$$

Thus, $\underline{\zeta}(Q) = \sim \overline{\zeta}(\sim Q)$.

2. It can be proved directly by Definition 8.
3. By Definition 8,

$$
\begin{aligned}
\underline{\zeta}(Q \cap R) &= \Big\{\Big\langle v, (Q \cap R)_{\underline{\zeta}}(v)\Big\rangle \mid v \in Y\Big\}, \\
&= \Big\{\Big\langle v, \bigwedge_{w \in T} (\mathbf{1} - p_i \circ (Q \cap R)(v,w)) \vee p_i \circ (Q \cap R)(w)\Big\rangle \mid v \in Y\Big\}, \\
&= \Big\{\Big\langle v, \bigwedge_{w \in T} \Big(\mathbf{1} - p_i \circ \big(Q(v,w) \wedge R(v,w)\big)\Big) \vee p_i \circ \big(Q(w) \wedge R(w)\big)\Big\rangle \mid v \in Y\Big\}, \\
&= \Big\{\Big\langle v, Q_{\underline{\zeta}}(v) \wedge R_{\underline{\zeta}}(v)\Big\rangle \mid v \in Y\Big\}, \\
&= \underline{\zeta}(Q) \cap \underline{\zeta}(R).
\end{aligned}
$$

Hence, $\underline{\zeta}(Q \cap R) = \underline{\zeta}(Q) \cap \underline{\zeta}(R)$.

4. Using Definition 8,

$$
\begin{aligned}
\underline{\zeta}(Q \cup R) &= \Big\{\Big\langle v, (Q \cup R)_{\underline{\zeta}}(v)\Big\rangle \mid v \in Y\Big\}, \\
&= \Big\{\Big\langle v, \bigwedge_{w \in T} (\mathbf{1} - p_i \circ (Q \cup R)(v,w)) \vee p_i \circ (Q \cup R)(w)\Big\rangle \mid v \in Y\Big\}, \\
&\supseteq \Big\{\Big\langle v, \bigwedge_{w \in T} \Big(\mathbf{1} - p_i \circ \big(Q(v,w) \vee R(v,w)\big)\Big) \vee p_i \circ \big(Q(w) \vee R(w)\big)\Big\rangle \mid v \in Y\Big\}, \\
&= \Big\{\Big\langle v, Q_{\underline{\zeta}}(v) \vee R_{\underline{\zeta}}(v)\Big\rangle \mid v \in Y\Big\}, \\
&= \underline{\zeta}(Q) \cup \underline{\zeta}(R).
\end{aligned}
$$

Thus, $\underline{\zeta}(Q \cup R) \supseteq \underline{\zeta}(Q) \cup \underline{\zeta}(R)$.

The properties (5–8) can be proved by using similar arguments. □

Example 5. *Let $Y = \{w_1, w_2, w_3, w_4\}$ be the set of four cars and let $T = \{v_1, v_2, v_3\}$ be the set of parameters, where*

- *v_1 denotes the Fuel efficiency,*
- *v_2 denotes the Price,*
- *v_3 denotes the Technology.*

Consider a 3*-polar fuzzy soft relation* $\zeta : Y \to T$ *is given by*

ζ	v_1	v_2	v_3
w_1	$(0.6, 0.3, 0.1)$	$(0.4, 0.7, 0.6)$	$(0.4, 0.6, 0.2)$
w_2	$(0.5, 0.3, 0.2)$	$(0.5, 0.2, 0.8)$	$(0.6, 0.9, 0.6)$
w_3	$(0.3, 0.2, 0.1)$	$(0.3, 0.4, 0.8)$	$(0.7, 0.3, 0.5)$
w_4	$(0.4, 0.3, 0.6)$	$(0.5, 0.1, 0.4)$	$(0.3, 0.1, 0.0)$.

Consider 3*-polar fuzzy subsets* Q, R *of* T *as follows:*

$$Q = \{(v_1, 0.2, 0.1, 0.9), (v_2, 0.7, 0.5, 0.3), (v_3, 0.5, 0.6, 0.1)\},$$

$$R = \{(v_1, 0.4, 0.2, 0.5), (v_2, 0.6, 0.7, 0.3), (v_3, 0.4, 0.7, 0.8)\}.$$

Then,

$$\begin{aligned}
\sim Q &= \{(v_1, 0.8, 0.9, 0.1), (v_2, 0.3, 0.5, 0.7), (v_3, 0.5, 0.4, 0.9)\},\\
Q \cup R &= \{(v_1, 0.4, 0.2, 0.9), (v_2, 0.7, 0.7, 0.3), (v_3, 0.5, 0.7, 0.8)\},\\
Q \cap R &= \{(v_1, 0.2, 0.1, 0.5), (v_2, 0.6, 0.5, 0.3), (v_3, 0.4, 0.6, 0.1)\}.
\end{aligned}$$

By Definition 8, we have

$$\begin{aligned}
\underline{\zeta}(Q) &= \{(w_1, 0.4, 0.5, 0.4), (w_2, 0.5, 0.6, 0.3), (w_3, 0.5, 0.6, 0.3), (w_4, 0.6, 0.7, 0.6)\},\\
\overline{\zeta}(Q) &= \{(w_1, 0.4, 0.6, 0.3), (w_2, 0.5, 0.6, 0.3), (w_3, 0.5, 0.4, 0.3), (w_4, 0.5, 0.1, 0.6)\},\\
\underline{\zeta}(R) &= \{(w_1, 0.4, 0.7, 0.4), (w_2, 0.4, 0.7, 0.3), (w_3, 0.4, 0.7, 0.3), (w_4, 0.6, 0.7, 0.5)\},\\
\overline{\zeta}(R) &= \{(w_1, 0.4, 0.7, 0.3), (w_2, 0.5, 0.7, 0.6), (w_3, 0.4, 0.4, 0.5), (w_4, 0.5, 0.2, 0.5)\},\\
\sim \underline{\zeta}(\sim Q) &= \{(w_1, 0.4, 0.6, 0.3), (w_2, 0.5, 0.6, 0.3), (w_3, 0.5, 0.4, 0.3), (w_4, 0.5, 0.1, 0.6)\},\\
\sim \overline{\zeta}(\sim Q) &= \{(w_1, 0.4, 0.5, 0.4), (w_2, 0.5, 0.6, 0.3), (w_3, 0.5, 0.6, 0.3), (w_4, 0.6, 0.7, 0.6)\},\\
\underline{\zeta}(Q \cup R) &= \{(w_1, 0.4, 0.7, 0.4), (w_2, 0.5, 0.7, 0.3), (w_3, 0.5, 0.7, 0.3), (w_4, 0.6, 0.7, 0.6)\},\\
\overline{\zeta}(Q \cup R) &= \{(w_1, 0.4, 0.7, 0.3), (w_2, 0.5, 0.7, 0.6), (w_3, 0.5, 0.4, 0.5), (w_4, 0.5, 0.2, 0.6)\},\\
\underline{\zeta}(Q \cap R) &= \{(w_1, 0.4, 0.5, 0.4), (w_2, 0.4, 0.6, 0.3), (w_3, 0.4, 0.6, 0.3), (w_4, 0.6, 0.7, 0.5)\},\\
\overline{\zeta}(Q \cap R) &= \{(w_1, 0.4, 0.6, 0.3), (w_2, 0.5, 0.6, 0.3), (w_3, 0.4, 0.4, 0.3), (w_4, 0.5, 0.1, 0.5)\}.
\end{aligned}$$

Now,

$$\begin{aligned}
\underline{\zeta}(Q) \cup \underline{\zeta}(R) &= \{(w_1, 0.4, 0.7, 0.4), (w_2, 0.5, 0.7, 0.3), (w_3, 0.5, 0.7, 0.3), (w_4, 0.6, 0.7, 0.6)\},\\
\overline{\zeta}(Q) \cup \overline{\zeta}(R) &= \{(w_1, 0.4, 0.7, 0.3), (w_2, 0.5, 0.7, 0.6), (w_3, 0.5, 0.4, 0.5), (w_4, 0.5, 0.2, 0.6)\},\\
\underline{\zeta}(Q) \cap \underline{\zeta}(R) &= \{(w_1, 0.4, 0.5, 0.4), (w_2, 0.4, 0.6, 0.3), (w_3, 0.4, 0.6, 0.3), (w_4, 0.6, 0.7, 0.5)\},\\
\overline{\zeta}(Q) \cap \overline{\zeta}(R) &= \{(w_1, 0.4, 0.6, 0.3), (w_2, 0.5, 0.6, 0.3), (w_3, 0.4, 0.4, 0.3), (w_4, 0.5, 0.1, 0.5)\}.
\end{aligned}$$

From the above calculations,

$$\begin{aligned}
&\sim \underline{\zeta}(\sim Q) = \overline{\zeta}(Q), && \sim \overline{\zeta}(\sim Q) = \underline{\zeta}(Q),\\
&\underline{\zeta}(Q \cap R) = \underline{\zeta}(Q) \cap \underline{\zeta}(R), && \underline{\zeta}(Q \cup R) \supseteq \underline{\zeta}(Q) \cup \underline{\zeta}(R),\\
&\overline{\overline{\zeta}}(Q \cup R) = \overline{\overline{\zeta}}(Q) \cup \overline{\overline{\zeta}}(R), && \overline{\overline{\zeta}}(Q \cap R) \subseteq \overline{\overline{\zeta}}(Q) \cap \overline{\overline{\zeta}}(R).
\end{aligned}$$

Remark 1. *In Theorem 2, properties* (1) *and* (5) *show that the lower and upper mF soft rough approximations operators* $\underline{\zeta}$ *and* $\overline{\zeta}$*, respectively, are dual to one another.*

Proposition 2. *Let (Y, T, ζ) be an mF soft approximation space. Then, the lower and upper soft rough approximations of mF sets Q and R satisfy the following laws:*

1. $\sim \Big(\underline{\zeta}(Q) \cup \underline{\zeta}(R)\Big) = \overline{\zeta}(\sim Q) \cap \overline{\zeta}(\sim R)$,
2. $\sim \Big(\underline{\zeta}(Q) \cup \overline{\zeta}(R)\Big) = \overline{\zeta}(\sim Q) \cap \underline{\zeta}(\sim R)$,
3. $\sim \Big(\overline{\zeta}(Q) \cup \underline{\zeta}(R)\Big) = \underline{\zeta}(\sim Q) \cap \overline{\zeta}(\sim R)$,
4. $\sim \Big(\overline{\zeta}(Q) \cup \overline{\zeta}(R)\Big) = \underline{\zeta}(\sim Q) \cap \underline{\zeta}(\sim R)$,
5. $\sim \Big(\underline{\zeta}(Q) \cap \underline{\zeta}(R)\Big) = \overline{\zeta}(\sim Q) \cup \overline{\zeta}(\sim R)$,
6. $\sim \Big(\underline{\zeta}(Q) \cap \overline{\zeta}(R)\Big) = \overline{\zeta}(\sim Q) \cup \underline{\zeta}(\sim R)$,
7. $\sim \Big(\overline{\zeta}(Q) \cap \underline{\zeta}(R)\Big) = \underline{\zeta}(\sim Q) \cup \overline{\zeta}(\sim R)$,
8. $\sim \Big(\overline{\zeta}(Q) \cap \overline{\zeta}(R)\Big) = \underline{\zeta}(\sim Q) \cup \underline{\zeta}(\sim R)$.

Proof. Its proof follows immediately from Definition 8. □

Definition 9. *Let Y be a universe, $Q = \{(v, p_i \circ Q(v)) \mid v \in Y\} \in m(Y)$, and $\sigma \in [0,1]^m$. The σ-level cut set of Q and the strong σ-level cut set of Q, denoted by Q_σ and $Q_{\sigma+}$, respectively, are defined as follows:*

$$Q_\sigma = \{v \in Y \mid p_i \circ Q(v) \geq \sigma\},$$
$$Q_{\sigma+} = \{v \in Y \mid p_i \circ Q(v) > \sigma\}.$$

Definition 10. *Let ζ be an mF soft relation on $Y \times T$, we define*

$$\begin{aligned} \zeta_\sigma &= \{(v,w) \in Y \times T \mid p_i \circ \zeta(v,w) \geq \sigma\}, \\ \zeta_\sigma(v) &= \{w \in T \mid p_i \circ \zeta(v,w) \geq \sigma\}, \\ \zeta_{\sigma+} &= \{(v,w) \in Y \times T \mid p_i \circ \zeta(v,w) > \sigma\}, \\ \zeta_{\sigma+}(v) &= \{w \in T \mid p_i \circ \zeta(v,w) > \sigma\}. \end{aligned}$$

Then, ζ_σ and $\zeta_{\sigma+}$ are two crisp soft relations on $Y \times T$.

We now prove that the mF soft rough approximation operators can be described by crisp soft rough approximation operators.

Theorem 3. *Let (Y, T, ζ) be an mF soft approximation space and $Q \in m(T)$. Then, the upper mF soft rough approximation operator can be described as follows, $\forall\, v \in Y$:*

1.
$$\begin{aligned} Q_{\overline{\zeta}}(v) &= \bigvee_{\sigma \in [0,1]^m} \Big(\sigma \wedge \overline{\zeta}_\sigma(Q_\sigma)(v)\Big) = \bigvee_{\sigma \in [0,1]^m} \Big(\sigma \wedge \overline{\zeta}_\sigma(Q_{\sigma+})(v)\Big), \\ &= \bigvee_{\sigma \in [0,1]^m} \Big(\sigma \wedge \overline{\zeta}_{\sigma+}(Q_\sigma)(v)\Big) = \bigvee_{\sigma \in [0,1]^m} \Big(\sigma \wedge \overline{\zeta}_{\sigma+}(Q_{\sigma+})(v)\Big). \end{aligned}$$

2. $[\overline{\zeta}(Q)]_{\sigma+} \subseteq \overline{\zeta}_{\sigma+}(Q_{\sigma+}) \subseteq \overline{\zeta}_{\sigma+}(Q_\sigma) \subseteq \overline{\zeta}_\sigma(Q_\sigma) \subseteq [\overline{\zeta}(Q)]_\sigma$.

Proof. 1. For all $v \in Y$,

$$\begin{aligned}\bigvee_{\sigma \in [0,1]^m} \left(\sigma \wedge \overline{\zeta}_\sigma(Q_\sigma)(v)\right) &= \sup\{\sigma \in [0,1]^m \mid v \in \overline{\zeta}_\sigma(Q_\sigma)\},\\ &= \sup\{\sigma \in [0,1]^m \mid \zeta_\sigma(v) \cap Q_\sigma\},\\ &= \sup\{\sigma \in [0,1]^m \mid \exists\, w \in T[w \in \zeta_\sigma(v), w \in Q_\sigma]\},\\ &= \sup\{\sigma \in [0,1]^m \mid \exists\, w \in T[p_i \circ Q_\zeta(v,w) \geq \sigma, p_i \circ Q(w) \geq \sigma]\},\\ &= \bigvee_{w \in T} \left(p_i \circ Q_\zeta(v,w) \wedge p_i \circ Q(w)\right),\\ &= Q_{\overline{\zeta}}(v).\end{aligned}$$

By similar arguments, we can compute

$$Q_{\overline{\zeta}}(v) = \bigvee_{\sigma \in [0,1]^m} \left(\sigma \wedge \overline{\zeta}_\sigma(Q_{\sigma+})(v)\right) = \bigvee_{\sigma \in [0,1]^m} \left(\sigma \wedge \overline{\zeta}_{\sigma+}(Q_\sigma)(v)\right) = \bigvee_{\sigma \in [0,1]^m} \left(\sigma \wedge \overline{\zeta}_{\sigma+}(Q_{\sigma+})(v)\right).$$

2. By Definitions 9 and 10, we directly verified that $\overline{\zeta}_{\sigma+}(Q_{\sigma+}) \subseteq \overline{\zeta}_{\sigma+}(Q_\sigma) \subseteq \overline{\zeta}_\sigma(Q_\sigma)$. Now, it is sufficient to show that $[\overline{\zeta}(Q)]_{\sigma+} \subseteq \overline{\zeta}_{\sigma+}(Q_{\sigma+})$ and $\overline{\zeta}_\sigma(Q_\sigma) \subseteq [\overline{\zeta}(Q)]_\sigma$.
For all $v \in [\overline{\zeta}(Q)]_{\sigma+}$, we have $Q_{\overline{\zeta}}(v) > \sigma$. By Definition 8, $\bigvee_{w \in T} \left(p_i \circ Q_\zeta(v,w) \wedge p_i \circ Q(w)\right) > \sigma$. Then, there exists $w_0 \in T$, such that $p_i \circ Q_\zeta(v,w_0) \wedge p_i \circ Q(w_0) > \sigma$, that is, $p_i \circ Q_\zeta(v,w_0) > \sigma$ and $p_i \circ Q(w_0) > \sigma$. Thus, $w_0 \in \zeta_{\sigma+}(v)$ and $w_0 \in Q_\sigma$. It follows that $\zeta_{\sigma+}(v) \cap Q_\sigma \neq \emptyset$. By Definition 4, we have $v \in \overline{\zeta}_{\sigma+}(Q_{\sigma+})$. Hence, $[\overline{\zeta}(Q)]_{\sigma+} \subseteq \overline{\zeta}_{\sigma+}(Q_{\sigma+})$.

To prove $\overline{\zeta}_\sigma(Q_\sigma) \subseteq [\overline{\zeta}(Q)]_\sigma$, let an arbitrary $v \in \overline{\zeta}_\sigma(Q_\sigma)$, we have $\overline{\zeta}_\sigma(Q_\sigma)(v) = \mathbf{1}$. Since $Q_{\overline{\zeta}}(v) = \bigvee_{\sigma \in [0,1]^m} [\overline{\zeta}_\sigma(Q_\sigma)(v)] \geq \sigma \wedge \overline{\zeta}_\sigma(Q_\sigma)(v) = \sigma$, we obtain $v \in [\overline{\zeta}(Q)]_\sigma$. Hence, $\overline{\zeta}_\sigma(Q_\sigma) \subseteq [\overline{\zeta}(Q)]_\sigma$.
□

Theorem 4. *Let (Y, T, ζ) be an mF soft approximation apace. If ζ is serial, then the lower and upper mF soft rough approximation operators $\underline{\zeta}(Q)$ and $\overline{\zeta}(Q)$, respectively, satisfy the following:*

1. $\underline{\zeta}(\emptyset) = \emptyset. \overline{\zeta}(T) = Y,$
2. $\underline{\zeta}(Q) \subseteq \overline{\zeta}(Q)$, *for all* $Q \in m(T)$.

Proof. Its proof follows directly by Definition 8. □

Definition 11. *Let Q be an mF set of the universe set Y and let $\underline{\zeta}(Q)$, $\overline{\zeta}(Q)$ be the lower and upper soft rough approximation operators. Then, ring sum operation about mF sets $\underline{\zeta}(Q)$ and $\overline{\zeta}(Q)$ is defined by*

$$\underline{\zeta}(Q) \oplus \overline{\zeta}(Q) = \left\{\left(v, p_i \circ Q_{\underline{\zeta}}(v) + p_i \circ Q_{\overline{\zeta}}(v) - p_i \circ Q_{\underline{\zeta}}(v) \times p_i \circ Q_{\overline{\zeta}}(v)\right) \mid v \in Y\right\}.$$

4. Applications to Decision-Making

4.1. Selection of a Hotel

The selection of the right hotel to stay is always a difficult task. Since every person has different needs when searching for a hotel. The location of the hotel is something that is very important for an enjoyable stay. There are a number of factors to take into consideration for selecting the right hotel, whether we are looking for a great location, a great meal option or a great service. Suppose a person (Mr. Adeel) wants to stay in a hotel for a long period. There are four alternatives in his mind.

The alternatives are y_1, y_2, y_3, y_4. He wants to select the most suitable hotel. The location, meal options and services are the main parameters for the selection of a hotel.
Let $Y = \{y_1, y_2, y_3, y_4\}$ be the set of four hotels under consideration and let $T = \{z_1, z_2, z_3\}$ be the set of parameters related to the hotels in Y, where,

'z_1' represents the Location,
'z_2' represents the Meal Options,
'z_3' represents the Services.

We give more features of these parameters as follows:

- The "Location" of the hotel include close to main road, in the green surroundings, in the city center.
- The "Meal options" of the hotel include fast food, fast casual, casual dining.
- The "Services" of the hotel include Wi-Fi connectivity, fitness center, room service.

Suppose that Adeel explains the "attractiveness of the hotel" by forming a 3-polar fuzzy soft relation $\zeta : Y \to T$, which is given by

ζ	z_1	z_2	z_3
y_1	$(0.2, 0.6, 0.1)$	$(0.3, 0.4, 0.7)$	$(0.7, 0.3, 0.2)$
y_2	$(0.4, 0.5, 0.7)$	$(0.4, 0.5, 0.5)$	$(0.7, 0.4, 0.1)$
y_3	$(0.7, 0.8, 0.3)$	$(0.8, 0.9, 0.4)$	$(0.6, 0.2, 0.6)$
y_4	$(0.5, 0.6, 0.4)$	$(0.6, 0.7, 0.1)$	$(0.8, 0.5, 0.3)$.

Thus, ζ over $Y \times T$ is the 3-polar fuzzy soft relation in which location, meal option and price of the hotels are considered. For example, if we consider "Location" of the hotel, $((y_1, z_1), 0.2, 0.6, 0.1)$ means that the hotel y_1 is 20% close to the main road, 60% in the green surroundings and 10% in the city center.

We now assume that Adeel gives the optimal normal decision object Q, which is a 3-polar fuzzy subset of T as follows:

$$Q = \{(z_1, 0.5, 0.6, 0.7), (z_2, 0.7, 0.6, 0.9), (z_3, 0.9, 0.6, 0.8)\}.$$

By Definition 8,

$$\begin{aligned}
Q_{\underline{\zeta}}(y_1) &= (0.7, 0.6, 0.8), & Q_{\overline{\zeta}}(y_1) &= (0.7, 0.6, 0.7),\\
Q_{\underline{\zeta}}(y_2) &= (0.6, 0.6, 0.7), & Q_{\overline{\zeta}}(y_2) &= (0.7, 0.5, 0.7),\\
Q_{\underline{\zeta}}(y_3) &= (0.5, 0.6, 0.7), & Q_{\overline{\zeta}}(y_3) &= (0.7, 0.6, 0.6),\\
Q_{\underline{\zeta}}(y_4) &= (0.5, 0.6, 0.7), & Q_{\overline{\zeta}}(y_4) &= (0.8, 0.6, 0.4).
\end{aligned}$$

Now, 3-polar fuzzy soft rough approximation operators $\underline{\zeta}(Q)$, $\overline{\zeta}(Q)$, respectively, are given by

$$\begin{aligned}
\underline{\zeta}(Q) &= \{(y_1, 0.7, 0.6, 0.8), (y_2, 0.6, 0.6, 0.7), (y_3, 0.5, 0.6, 0.7), (y_4, 0.5, 0.6, 0.7)\},\\
\overline{\zeta}(Q) &= \{(y_1, 0.7, 0.6, 0.7), (y_2, 0.7, 0.5, 0.7), (y_3, 0.7, 0.6, 0.6), (y_4, 0.8, 0.6, 0.4)\}.
\end{aligned}$$

These operators are very close to the decision alternatives $y_n,\ n = 1, 2, 3, 4$.
By Definition 11, we have the choice set as follows:

$$\underline{\zeta}(Q) \oplus \overline{\zeta}(Q) = \{(y_1, 0.91, 0.84, 0.94), (y_2, 0.88, 0.8, 0.91), (y_3, 0.85, 0.84, 0.88), (y_4, 0.9, 0.84, 0.82)\}.$$

Thus, Mr. Adeel will select the hotel y_1 to stay because the optimal decision in the choice set $\underline{\zeta}(Q) \oplus \overline{\zeta}(Q)$ is y_1.

The method of selecting a suitable hotel is explained in the following Algorithm 1.

Algorithm 1: Selection of a suitable hotel

1. Input Y as universe of discourse.
2. Input T as a set of parameters.
3. Construct an mF soft relation $\zeta : Y \to T$ according to the different needs of the decision maker.
4. Give an mF subset Q over T, which is an optimal normal decision object according to the various requirements of decision maker.
5. Compute the mF soft rough approximation operators $\underline{\zeta}(Q)$ and $\overline{\zeta}(Q)$ by Definition 8.
6. Find the choice set $S = \underline{\zeta}(Q) \oplus \overline{\zeta}(Q)$ by Definition 11.
7. Select the optimal decision y_k. If $p_i \circ S(y_k) \geq M$, where $M = \bigvee\limits_{1 \leq k \leq n} p_i \circ S(y_k)$, n is equal to the number of objects in Y, then the optimal decision will be y_k.

If there exists more than one optimal choice in step 7 of the Algorithm 1, that is, $y_{k_i} = y_{k_j}$, where $1 \leq k_i \neq k_j \leq n$, one may go back and change the optimal normal decision object Q and repeat the Algorithm 1 so that the final decision is only one.

4.2. Selection of a Place

Choosing a place to go when some people have the opportunity to travel can sometimes be very difficult task. Suppose that a group of ten peoples plan a tour to a suitable place in a country Z. There are four alternatives in their mind. The alternatives are q_1, q_2, q_3, q_4. They want to select the best place for the tour. It is a challenge to find advice in one place. The environment and cost are the main parameters for the selection of a suitable place. In the environment of the place, they want to check whether the place has availability of built environment, natural environment and social environment. The term built environment refers to the man-made surroundings. Built environment of the place includes buildings, parks and every other things that are made by human beings. Natural environment of the place includes forests, oceans, rivers, lakes, atmosphere, climate, weather, etc. The social environment includes the culture and lifestyle of the human beings. Lastly, the tour cost is an important criteria for the place selection. It includes low, medium and high.

Let $Y = \{q_1, q_2, q_3, q_4\}$ be the set of four places and $T = \{a_1, a_2\}$ be the set of parameters, where

'a_1' represents the Environment,
'a_2' represents the Tour Cost.

We give more characteristics of these parameters.

- The "Environment" of the place includes built environment, natural environment, and social environment.
- The "Tour Cost" of the place may be low, medium, or high.

Suppose that they describe the "attractiveness of the place" by constructing a 3-polar fuzzy soft relation ζ over $Y \times T$, which is given by

ζ	a_1	a_2
q_1	$(0.8, 0.8, 0.9)$	$(0.4, 0.7, 0.6)$
q_2	$(0.5, 0.7, 0.6)$	$(0.5, 0.7, 0.8)$
q_3	$(0.8, 0.6, 0.7)$	$(0.8, 0.9, 0.4)$
q_4	$(0.7, 0.9, 0.6)$	$(0.6, 0.7, 0.8)$

Thus, $\zeta : Y \rightarrow T$ is the 3-polar fuzzy soft relation in which environment and tour cost of the places are considered. For example, if we consider "Environment" of the place, $((q_1, a_1), 0.8, 0.8, 0.9)$ means that the place q_1 include 80% built environment, 80% natural environment and 90% social environment.

We now assume that they give the optimal normal decision object Q, which is a 3-polar fuzzy subset of T as follows:

$$Q = \{(a_1, 0.8, 0.7, 0.9), (a_2, 0.7, 0.6, 0.8)\}.$$

From Definition 8,

$$\begin{aligned} Q_{\underline{\zeta}}(q_1) &= (0.7, 0.6, 0.8), & Q_{\overline{\zeta}}(q_1) &= (0.8, 0.7, 0.9), \\ Q_{\underline{\zeta}}(q_2) &= (0.7, 0.6, 0.8), & Q_{\overline{\zeta}}(q_2) &= (0.5, 0.7, 0.8), \\ Q_{\underline{\zeta}}(q_3) &= (0.7, 0.6, 0.8), & Q_{\overline{\zeta}}(q_3) &= (0.8, 0.6, 0.7), \\ Q_{\underline{\zeta}}(q_4) &= (0.7, 0.6, 0.8), & Q_{\overline{\zeta}}(q_4) &= (0.7, 0.7, 0.8). \end{aligned}$$

We now have 3-polar fuzzy soft rough approximation operators $\underline{\zeta}(Q), \overline{\zeta}(Q)$, respectively, as follows:

$$\underline{\zeta}(Q) = \{(q_1, 0.7, 0.6, 0.8), (q_2, 0.7, 0.6, 0.8), (q_3, 0.7, 0.6, 0.8), (q_4, 0.7, 0.6, 0.8)\},$$
$$\overline{\zeta}(Q) = \{(q_1, 0.8, 0.7, 0.9), (q_2, 0.5, 0.7, 0.8), (q_3, 0.8, 0.6, 0.7), (q_4, 0.7, 0.7, 0.8)\}.$$

These operators are very close to the decision alternatives q_n, $n = 1, 2, 3, 4$.

By Definition 11,

$$\underline{\zeta}(Q) \oplus \overline{\zeta}(Q) = \{(q_1, 0.94, 0.88, 0.98), (q_2, 0.85, 0.88, 0.96), (q_3, 0.94, 0.84, 0.94), (q_4, 0.91, 0.88, 0.96)\}.$$

Thus, the optimal decision in the choice set $\underline{\zeta}(Q) \oplus \overline{\zeta}(Q)$ is q_1. Therefore, they will select the place q_1 for the tour.

The method of selecting a suitable place for tour is explained in the following Algorithm 2.

Algorithm 2: Selection of a suitable place

1. Input Y as universe of discourse.
2. Input T as a set of parameters.
3. Construct an mF soft relation ζ over $Y \times T$ according to the different needs of the decision makers.
4. Give an mF subset Q of T, which is an optimal normal decision object according to the various requirements of decision makers.
5. Compute the mF soft rough approximation operators $\underline{\zeta}(Q)$ and $\overline{\zeta}(Q)$ by Definition 8.
6. Find the choice set $S = \underline{\zeta}(Q) \oplus \overline{\zeta}(Q)$ by Definition 11.
7. Select the optimal decision q_k. If $p_i \circ S(q_k) \geq M$, where $M = \bigvee_{1 \leq k \leq n} p_i \circ S(q_k)$, n is equal to the number of objects in Y, and then the optimal decision will be q_k.

If there exists more than one optimal choice in step 7 of the Algorithm 2, that is, $q_{k_i} = q_{k_j}$ where $1 \leq k_i \neq k_j \leq n$, one may go back and change the optimal normal decision object Q and repeat the Algorithm 2 so that the final decision is only one.

4.3. Selection of a House

Buying a house is an exhilarating time in many people lives, but it is also a very difficult task to those who are not particularly real estate savvy. There are a number of factors to take into consideration for buying the house such as location of the house, size of the house and price of the house. These factors among many others influence house buyers before they even get to start thinking about buying a new house. Suppose a person (Mr. Ali) wants to buy a house. The alternatives in his mind are u_1, u_2, u_3. The size, location and price are the main parameters for the selection of a suitable house.

Let $Y = \{u_1, u_2, u_3\}$ be the set of three houses and let $T = \{t_1, t_2, t_3\}$ be the set of parameters related to the houses in Y, where

't_1' represents the Size,
't_2' represents the Location,
't_3' represents the Price.

We give further characteristics of these parameters.

- The "Size" of the house include small , large, and very large.
- The "Location" of the house include close to the main road, in the green surroundings, and in the city center.
- The "Price" of the house includes low, medium, and high.

Suppose that Ali describes the "attractiveness of the house" by forming a 3-polar fuzzy soft relation $\zeta : Y \to T$, which is given by

ζ	t_1	t_2	t_3
u_1	$(0.5, 0.7, 0.9)$	$(0.7, 0.6, 0.8)$	$(0.5, 0.6, 0.9)$
u_2	$(0.8, 0.9, 0.1)$	$(0.6, 0.8, 0.9)$	$(0.8, 0.4, 0.2)$
u_3	$(0.9, 0.7, 0.6)$	$(0.9, 0.8, 0.9)$	$(0.4, 0.6, 0.3)$.

Thus, ζ over $Y \times T$ is the 3-polar fuzzy soft relation in which size, location and price of the houses are considered. For example, if we consider "Location" of the house, $((u_2, t_1), 0.8, 0.9, 0.1)$ means that the house u_1 is, 80% close to the main road, 90% in the green surroundings and 10% in the city center.

We now assume that Ali gives the optimal normal decision object Q, which is a 3-polar fuzzy subset of T as follows:

$$Q = \{(t_1, 0.6, 0.8, 0.7), (t_2, 0.5, 0.8, 0.8), (t_3, 0.9, 0.8, 0.7)\}.$$

By Definition 8,

$$\begin{aligned} Q_{\underline{\zeta}}(u_1) &= (0.5, 0.8, 0.7), & Q_{\overline{\zeta}}(u_1) &= (0.5, 0.7, 0.8), \\ Q_{\underline{\zeta}}(u_2) &= (0.5, 0.8, 0.8), & Q_{\overline{\zeta}}(u_2) &= (0.8, 0.8, 0.8), \\ Q_{\underline{\zeta}}(u_3) &= (0.5, 0.8, 0.7), & Q_{\overline{\zeta}}(u_3) &= (0.6, 0.8, 0.8). \end{aligned}$$

Now, 3-polar fuzzy soft rough approximation operators $\underline{\zeta}(Q)$, $\overline{\zeta}(Q)$, respectively, are given by

$$\underline{\zeta}(Q) = \{(u_1, 0.5, 0.8, 0.7), (u_2, 0.5, 0.8, 0.8), (u_3, 0.5, 0.8, 0.7)\},$$
$$\overline{\zeta}(Q) = \{(u_1, 0.5, 0.7, 0.8), (u_2, 0.8, 0.8, 0.8), (u_3, 0.6, 0.8, 0.8)\}.$$

These operators are very close to the decision alternatives $u_n,\ n = 1, 2, 3$.

Using Definition 11,

$$\underline{\zeta}(Q) \oplus \overline{\zeta}(Q) = \{(u_1, 0.75, 0.94, 0.94), (u_2, 0.9, 0.96, 0.96), (u_3, 0.8, 0.96, 0.94)\}.$$

Hence, Ali will buy the house u_2 because the optimal decision in the choice set $\underline{\zeta}(Q) \oplus \overline{\zeta}(Q)$ is u_2. The method of selecting a suitable house is explained in the following Algorithm 3.

Algorithm 3: Selection of a suitable house

1. Input Y as universe of discourse.
2. Input T as a set of parameters.
3. Construct an mF soft relation $\zeta : Y \to T$ according to the different needs of the decision maker.
4. Give an mF subset Q over T, which is an optimal normal decision object according to the various requirements of the decision maker.
5. Compute the mF soft rough approximation operators $\underline{\zeta}(Q)$ and $\overline{\zeta}(Q)$ by Definition 8.
6. Find the choice set $S = \underline{\zeta}(Q) \oplus \overline{\zeta}(Q)$ by Definition 11.
7. Select the optimal decision u_k. If $p_i \circ S(u_k) \geq M$, where $M = \bigvee\limits_{1 \leq k \leq n} p_i \circ S(u_k)$, n is equal to the number of objects in Y, and then the optimal decision will be u_k.

If there exist too many optimal choices in step 7 of Algorithm 3, that is, $u_{k_i} = u_{k_j}$, where $1 \leq k_i \neq k_j \leq n$, change the optimal normal decision object Q and repeat the Algorithm 3 so that the final decision is only one.

5. Conclusions

The theory of mF sets plays a vital role in decision-making problems, when multiple information is given. An mF soft rough set is a combination of an mF set, soft set and rough set. In this paper, we have presented the concepts of two new hybrid models called soft rough mF sets and mF soft rough sets, which provide more exactness and compatibility with a system when compared with other hybrid mathematical models. We have discussed the properties of both hybrid models. We have examined the relationship between mF soft rough approximation operators and crisp soft rough approximation operators. We have discussed some applications of mF soft rough sets in real-life decision-making problems. We are expanding our research work to (1) soft rough mF graphs; (2) soft rough mF hypergraphs; (3) mF soft rough graphs; (4) mF soft rough hypergraphs; and a (5) decision support system based on mF soft rough hypergraphs.

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