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Exploiting the Formation of Maximal Cliques in Social Networks

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Abstract: In social networking analysis, there exists a fundamental problem called maximal cliques enumeration(MCE), which has been extensively investigated in many fields, including social networks, biological science, etc. As a matter of fact, the formation principle of maximal cliques that can help us to speed up the detection of maximal cliques from social networks is often ignored by most existing research works. Aiming to exploit the formation of maximal cliques in social networks, this paper pioneers a creative research issue on the detection of bases of maximal cliques in social networks. We propose a formal concept analysis-based approach for detecting the bases of maximal cliques and detection theorem. It is believed that our work can provide a new research solution and direction for future topological structure analysis in various complex networking systems.

Keywords: maximal cliques enumeration (MCE); bases; formal concept analysis; social networks

1. Introduction

1.1. Background

Recent years witnessed the booming development of graph data modeling and its widely used applications. In practice, many applications can be represented with graph data modeling, such as social networks, web networks, and protein interactive networks. Therefore, analyzing and mining the useful knowledge from graphs is significantly meaningful. In particular, maximal cliques enumeration (MCE) is an important research issue in graphs. In graphs, a clique refers to a complete sub-graph where any two vertices are connected to each other. Meanwhile, a maximal clique is a clique such that there is no clique with more vertices. At present, the detection of maximal cliques or MCE is mainly to identify all maximal cliques because these cliques or maximal cliques contain more valued knowledge and information. Thus, MCE is widely used in community detection, topological analysis of web networks, and so forth.

1.2. Related Work

MCE is a fundamental problem in graph theory, and has been extensively investigated by many researchers [1,2]. They mainly focus on devising an approximate algorithm (since MCE is an NP-hard problem) for extracting all maximal cliques. The existence of an algorithm for addressing MCE is categorized as: (1) sequential in-memory algorithms [3,4]; (2) sequential I/O efficient algorithms [5,6] which concentrate on reducing the high cost of random disk I/Os for processing graphs that cannot fit in main memory [2]; (3) aiming to reduce the running time—the third type approach is parallel, and distributed algorithms [7,8] are proposed. However, these existing studies usually ignore the formation principle of maximal cliques. As a matter of fact, the formation principle of maximal cliques is a kind of useful information for better MCE process. That is to say, if we can obtain the partial topological structures which can further form the maximal clique, then this issue can be addressed efficiently.

Formally, problems in graph theory have also been analyzed by formal concept analysis; for example, K-clique of dynamic social networks are mined from formal concept lattice [9,10]. From the formal concept analysis point of view, if a graph is converted into a formal context, then notions of formal concept analysis can be used to express problems in graph theory, if fact, social networks graphs are converted into a special formal context $K = (U, A, I)$ [9,10], in which $U = A = V$ are vertices of the graphs and I means the edges between two vertices; accordingly, the K-cliques problem of the social networks is expressed by K-formal concepts of the special formal context $K = (U, A, I)$. Theoretically, many interesting notions and mining methods have been proposed in formal concept analysis, such as frequent itemsets, closed frequent itemsets, maximal frequent itemsets, expressive generalized itemsets, and disjunctive closed itemsets [11–14]. A priori-inspired algorithms and frequent pattern-growth-inspired algorithms [15–22] have been provided to fast mine many kinds of itemsets. In [23], Pei et al. proposed a method based on a topology for attributes of a formal context to generate the formal concept lattice, and the topology for attributes was induced by a reflexive and transitive relation on the set of attributes; by defining an equivalent relation on the topology for attributes, it has been proved that the formal concept lattice and the quotient topology for attributes decided by the equivalent relation is isomorphic.

1.3. Contributions

Motivated by our earlier works [9,10,23], this paper pioneers the study of the bases of maximal cliques in social networks. We firstly present the concept of base of maximal clique, then we point out that the maximal clique can be formed based on the detected bases. Therefore, the main contributions of this work are twofold: (1) formalize an interesting problem about the detection of bases of maximal cliques in social networks; (2) exploit the formation procedure of maximal cliques and then present an efficient approach for obtaining the bases of maximal cliques.

1.4. Paper Organization

The remainder of this paper is structured as follows. Section 2 provides the problem statement. Then, a formal concept analysis-based detection approach for the bases of maximal clique in social networks is presented in Section 3. A case study is conducted in Section 4. Finally, Section 5 concludes this paper.

2. Problem Statement

To study the problem of the detection of bases of maximal cliques in social networks, the graph model and basic concept of maximal cliques are firstly presented, and the problem statement is then formalized.

2.1. Graph Model and Maximal Clique

This work focuses on undirected graph and managing the maximal cliques on it; hence, graph model is here defined with an undirected graph model $G = (V, E)$, where V is the set of vertices and E is the set of edges of G. Particularly, a set of vertices $C \subseteq V$ is a clique if each vertex in C are connected each other. If there is no any other set of vertices $C' \supset C$ such that C' is a clique in G, then C is a maximal clique.

2.2. Problem Descriptions

According to our previous work [9], we know that the k-clique community can be formed with skeleton sub-graphs. Similarly, finding the bases of maximal cliques could help us to detect maximal cliques quickly. Note that the base of maximal clique refers to the common sub-graph (can be line, or other sub-graph) among maximal cliques.

Problem 1. *Problem Definition: Given a social network $G = (V, E)$, this paper proposes a novel topic and the corresponding approach for finding the bases of maximal cliques from G, denoted as B (maximal_clique(G)).*

To better understand the above problem statement, an illustrative example is shown as follows.

Figure 1a shows an input of the problem (i.e., social network G, composed of seven vertices). We can easily get the maximal cliques $\{2,3,5\}$, $\{2,5,6\}$, $\{4,7\}$, $\{1\}$ from G using the existing algorithm. However, we found that the common edges $\{2,5\}$ between maximal cliques 2,3,5 and 2,5,6 as shown in Figure 1b. Actually, the edge $\{2,5\}$ is a base of maximal cliques $\{2,3,5\}$ and $\{2,5,6\}$, since they can be formed by simply adding vertex 3 or 6.

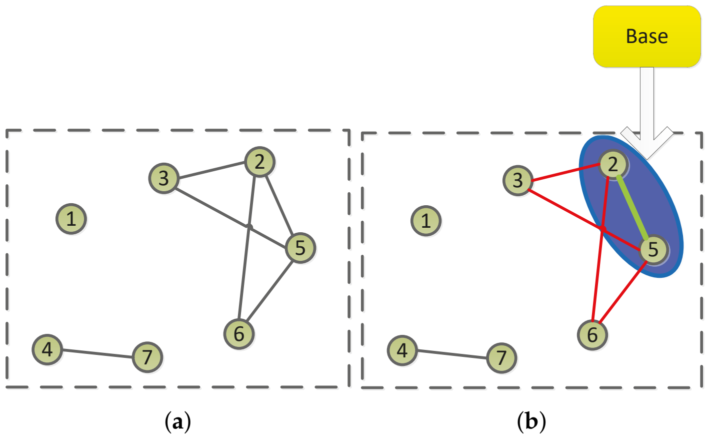

Figure 1. An illustrative example on bases of maximal cliques. (**a**) Social Network G; (**b**) Base of Maximal Cliques.

3. Detecting Bases of Maximal Cliques Based on Formal Concept Analysis

To address the above problem, this section is devoted to presenting our proposed approach for detecting the bases of maximal cliques by using formal concept analysis. In Section 3.1, we firstly analyze the reason for bases of maximal clique for interpreting how maximal cliques can be formed via their bases. Then, a new formal context and its concept lattice are generated by aggregating the attributes which have common objects. Finally, we extract the extents from the maximal cliques associated concepts and from the new formal concept lattice and then make the intersection. After that, topological structure analysis of a social graph is formally provided in Section 3.2. Based on the proposed detection approach and the corresponding topological structure analysis, a newly proposed detection theorem is presented in Section 3.3.

3.1. Detection Approach

Suppose G is a social network, denoted as $G = (V, E)$. We firstly construct the formal context $K = (V, V, I)$ using the approach presented in [9]. Obviously, $K = (V, V, I)$ is a special formal context; i.e., its objects and attributes are the same as vertices of the graph $G = (V, E)$, and I is decided by edge if it exists between two vertices. Due to the speciality of $K = (V, V, I)$, we can granulate vertices when they are as attributes; the granulation of vertices is achieved by an equivalence relation R on V which is induced by I; i.e., the granulation of vertices is formalized as follows: For any $i \in V$, we have

$$i^{\uparrow} = \{j \in V | (i,j) \in I\},\ j^{\downarrow} = \{i \in V | (i,j) \in I\}, \tag{1}$$

where $(i,j) \in I$ means there exists an edge between vertices i and j. Based on Equation (1), the equivalence relation R on V is induced by

$$\forall j_1, j_2 \in V, j_1 R j_2 \text{ if and only if } j_1^{\downarrow} = j_2^{\downarrow}, \tag{2}$$

it is easily proved that R on V is symmetrical, reflexive and transitive; i.e., R on V decided by Equation (2) is an equivalence relation, and R can be used to granulate vertices as for any $j \in V$,

$$[j]_R = \{j' \in V | jRj'\}, \tag{3}$$

Let $V_R = \{[j]_R | j \in V\}$; this makes us obtain a new formal context from $K = (V, V, I)$. That is, $K' = (V, V_R, I_R)$, where $I_R = \{(i, [j]_R) \in V \times V_R | \exists j' \in [j]_R, (i, j') \in I\}$, it is obvious that K' has the same objects with but the different attributes from the original formal context $K = (V, V, I)$. For example, in Figure 1a, as our the input is the social network G, then the converted formal context and its induced formal context are constructed as shown in Table 1 and Table 2.

Table 1. The formal context of G, K.

$V \setminus V$	1	2	3	4	5	6	7
1	X						
2		X	X		X	X	
3		X	X		X		
4				X			X
5		X	X		X	X	
6		X			X	X	
7				X			X

Table 2. The induced formal context of G, K'.

$V \setminus V$	1	[2, 5]	3	[4, 7]	6
1	X				
2		X	X		X
3		X	X		
4				X	
5		X	X		X
6		X			X
7				X	

The corresponding concept lattices are generated by the existing concept lattice generation algorithm. The relationship between the concept lattices of the original formal context K (i.e., $C(K)$) and induced formal context K' (i.e, $C(K')$) is shown in Figure 2. According to the findings about the equivalence between equiconcept and clique [9,10], it is clear that the maximal cliques in G include $\{2,3,5\}, \{2,5,6\}, \{4,7\}, \{1\}$. Interestingly, these maximal cliques are aggregated then represented as the relevant concepts in Figure 2. For example, the common attribute of maximal cliques $\{2,3,5\}$ and $\{2,5,6\}$ is $\{2,5\}$; that is to say, $\{2,5\}$ is the formation base for maximal cliques $\{2,3,5\}$ and $\{2,5,6\}$.

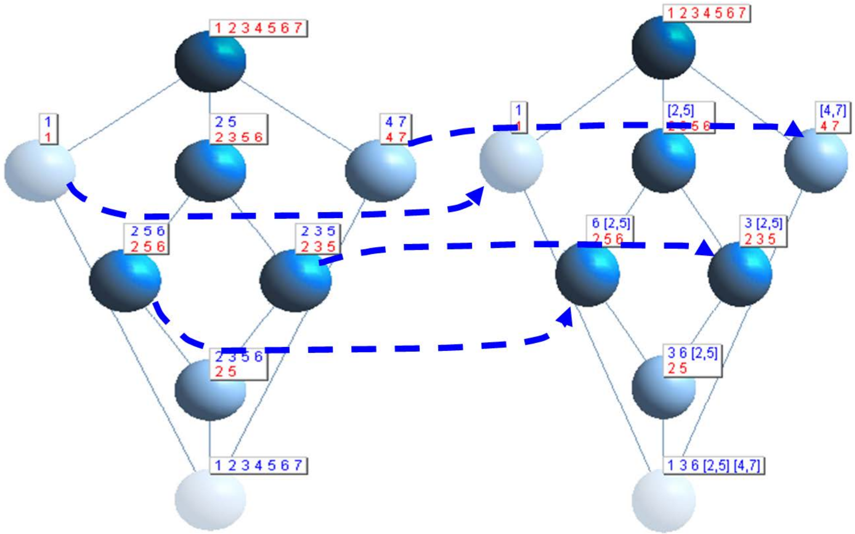

Figure 2. The relationship between $C(K)$ and $C(K^{'})$.

3.2. Topological Structure Analysis of a Social Graph

Based on the reflexive and transitive relation R on V of a social graph $G = (V, E)$, this section focuses on constructing a topological space of vertices set V. Formally, a reflexive and transitive relation on a set can be used to induce covering approximation space [24–26]. In our previous work [23], the reflexive and transitive relation on a set has been used to construct an approximation space and a topology for attributes of a formal context, respectively, and a base for the topology can be adopted to generate intensions of all formal concepts of the formal context and construct the formal concept lattice. Inspired by our previous work, a topological space of vertices set V is constructed by using the reflexive and transitive relation R that are used to represent relationships and the hierarchical structures of a social graph $G = (V, E)$, and provide several interesting results to show topological analysis of a social graph.

Property 1. *In the formal context $K = (V, V, I)$ of an undirected graph $G = (V, E)$, for any $v_i \in V$, $S_{\uparrow}(v_i)$ is a $|S_{\uparrow}(v_i)|$-clique in $G = (V, E)$.*

Definition 1. *In the formal context $K = (V, V, I)$ of a social graph $G = (V, E)$, for any subgraph $V_1 \subseteq V$, lower vertices approximations of V_1 is*

$$\underline{R}(V_1) = \{i \in V | R(i, *) \subseteq V_1\}. \tag{4}$$

According to $R(i, *) = \{j \in V | R(i, j) = 1\} = S_{\uparrow}(i)$, $\underline{R}(V_1)$ can also be rewritten by

$$\begin{aligned} \underline{R}(V_1) &= \{i \in V | R(i, *) \subseteq V_1\} \\ &= \{i \in V | S_{\uparrow}(i) \subseteq V_1\} \\ &= \bigcup_{S_{\uparrow}(i) \subseteq V_1} S_{\uparrow}(i). \end{aligned} \tag{5}$$

Theorem 1. *[23] For any formal context $FC(G) = (V, V, I)$ of undirected graph $G = (V, E)$,*

1. *$T_R = \{\underline{R}(V_1) | \forall V_1 \subseteq V\}$ is a topology for V, and (V, T_R) is a topological space for V;*
2. *$B_R = \{S_{\uparrow}(v_i) | i \in V\}$ is a base for the topology T_R.*

Theorem 1 means that any social graph $G = (V, E)$ can be represented by its topology $T_R = \{\underline{R}(V_1)|\forall V_1 \subseteq V\}$, which is induced by the binary relation R decided by Equation (2); it is more important that the topology $T_R = \{\underline{R}(V_1)|\forall V_1 \subseteq V\}$ for V can be generated by the base $B_R = \{S_\uparrow(i)|i \in V\}$, which is obtained from every vertex of V according to Equation (1).

Corollary 1. *[27] For any $i, j \in V$, if $j \in S_\uparrow(i)$ and $i \in S_\uparrow(j)$, then $S_\uparrow(i) = S_\uparrow(j)$.*

Because $S_\uparrow(j)$ is a $|S_\uparrow(i)|$-clique in $G = (V, E)$ by Property 1, Corollary 1 means that i and j generate the same k-clique in $G = (V, E)$; i.e.,

Corollary 2. *[27] If i and j are in a k-clique in $G = (V, E)$, then $S_\uparrow(i) = S_\uparrow(j)$ and they are the k-clique in $G = (V, E)$.*

According to Theorem 1 and Corollary 2, we have the following corollary.

Corollary 3. *[27] Any social network $G = (V, E)$ can be generated by its all k-clique in $G = (V, E)$, where $1 \leq k \leq |V|$.*

3.3. Detection Theorem

Therefore, the following detection theorem is derived with the above detection approach.

Theorem 2. *Given a social network $G = (V, E)$, the formal context of G is K, the concept lattice of K is denoted as $C(K)$, the bases of maximal cliques $B(maximal\_clique(G))$ can be obtained from maximal cliques associated formal concepts in concept lattice $C(K^{'})$, where $K^{'}$ is an induced formal context from K based on equivalence relation R over attributes.*

3.4. Practical Applicability

Based on the above detection theorem, our work can bring more opportunities for: (1) future topological structure analysis in various complex networking systems; (2) finding the trust/sentiment dominators by identifying the base of maximal cliques since the dominators are playing the skeleton role in trust/sentiment management and propagation in social networks [28–31] as well as the large data clustering in Internet of Things [32]; (3) providing a new solution for recommender systems incorporating the maximal cliques. We may detect the maximal cliques by virtue of base of it, then adopt the conventional approaches, such as collaborative filtering or matrix factorization for items recommendation [33]; (4) in addition, our work is also beneficial to the protein complex identification from protein–protein interaction networks [34].

4. Case Study

This section manages a case study on a real-life scientist collaboration network. Aiming to validate the feasibility and performance of the proposed approach, the largest maximal cliques (i.e, the maximal cliques including the largest number of vertices) will be detected based on our approach.

4.1. Dataset

The dataset of the case study is a network of collaborations between scientists working on "Networks" [35]. The statistics of this dataset are as follows: this collaboration social network contains 1589 scientists (vertices in the graph), and 2742 collaborations (edges in the graph).

4.2. Results

To detect the largest maximal cliques from the above network, we firstly find the base of the largest maximal cliques and exploit their formation principle. The experiments detect only one largest maximal clique formed by 20 scientists from the testing dataset. Importantly, the bases of these maximal clique $\{Hilgetag.C, Burns.G, Oneill.M, Young.M\}$, $\{Kashtan.N, Milo.R, Alon.U, Itzkovitz.S\}$, $\{Pastorsatorras.R, Vespignani.A, Moreno.Y, Vazquez.A\}$ are identified as well.

In other words, the largest maximal clique can be gradually formed based on the base of it. This evolution phenomenon is quite important and promising for later complex topological structures mining and analysis. For example, for a company who wants to promote their new product, we may suggest this company plant their ads and provide the incentives to the base of the maximal cliques (i.e, targeted seed customers). From the application point of view, the achievements of this research can be applicable to various social networking services, like social marketing, social advertising, and social recommendation.

5. Conclusions

Aiming to exploit the formation of maximal cliques in social networks, this paper formalizes a novel problem on detection of bases of maximal cliques from social networks. In order to address this problem, this paper proposed a formal concept analysis-based approach for detecting the bases of maximal cliques in social networks. We mathematically present a detection theorem according to our proposed approach. Hence, this detection theorem is ubiquitous and can be applied to various complex networks. The proposed detection approach reveals the formation principle of maximal cliques by investigating the relationship between original concept lattice and aggregated concept lattice. We believe that this work can pave the way for future topological structure analysis of social networks and other complex networking systems.

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Author Contributions: Fei Hao proposed the topic and initial idea, performed the case study and wrote the paper; Doo-Soon Park improved the idea and presentation of the paper; Zheng Pei extended and improved the Section 1 and also wrote the Section 3.2.

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