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Fixed Points of Some Asymptotically Regular Multivalued Mappings Satisfying a Kannan-Type Condition

Pradip Debnath [1,*], Zoran D. Mitrović [2] and Hari Mohan Srivastava [3,4,5,*]

[1] Department of Applied Science and Humanities, Assam University, Silchar Cachar, Assam 788011, India
[2] Faculty of Electrical Engineering, University of Banja Luka, Patre 5, 78 000 Banja Luka, Bosnia and Herzegovina; zoran.mitrovic@etf.unibl.org
[3] Department of Mathematics and Statistics, University of Victoria, Victoria, BC V8W 3R4, Canada
[4] Department of Medical Research, China Medical University Hospital, China Medical University, Taichung 40402, Taiwan
[5] Department of Mathematics and Informatics, Azerbaijan University, 71 Jeyhun Hajibeyli Street, AZ1007 Baku, Azerbaijan
* Correspondence: debnath.pradip@yahoo.com (P.D.); harimsri@math.uvic.ca (H.M.S.)

Abstract: In this paper, we establish some existence of fixed-point results for some asymptotically regular multivalued mappings satisfying Kannan-type contractive condition without assuming compactness of the underlying metric space or continuity of the mapping.

Keywords: fixed point; multivalued map; Kannan-type contraction; complete metric space; boundedly compact; orbitally continuous mapping

MSC: 47H10; 54H25; 54E50

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1. Preliminaries

Kannan's famous generalization of Banach's contraction principle is as follows.

Theorem 1. *[1] If Ψ is a self-map on a complete metric space (MS) $(\Im, \rho)$ satisfying*

$$\rho(\Psi\theta, \Psi\xi) \leq \tau\big[\rho(\theta, \Psi\theta) + \rho(\xi, \Psi\xi)\big],$$

where $\theta, \xi \in \Im$ and $0 < \tau < \frac{1}{2}$, then Ψ has a unique fixed point in $\Im$.

Such a mapping Ψ is said to be a Kannan map and it is not necessarily continuous. In [2], Kannan proved the above theorem by omitting the completeness criterion of the space and by assuming continuity of the map at a point.

Reich [3] generalized Banach and Kannan's fixed-point results as given below.

Theorem 2. *Let $(\Im, \rho)$ be a complete MS and $\Psi : \Im \to \Im$ be a self-map. Suppose there exist nonnegative constants a, b, c satisfying $a + b + c < 1$ such that*

$$\rho(\Psi\theta, \Psi\xi) \leq a\rho(\theta, \xi) + b\rho(\Psi\theta, \theta) + c\rho(\Psi\xi, \xi)$$

for all $\theta, \xi \in \Im$. Then Ψ has a unique fixed point.

In Reich's theorem, $b = c = 0$ yields Banach's result, whereas $b = c, a = 0$ produces Kannan's theorem.

Subhramanyam [4] used Kannan's theorem to characterize metric completeness. Kannan's theorem was further extended by many authors in different directions over the decades [5–10]. The concepts of continuity and compactness play significant roles is the discussion of Kannan-type results. In this note, we rather try to present our results using the concepts of boundedly compact an orbitally compact MSs which are weaker properties than compactness. An MS is said to be boundedly compact if every bounded sequence in it has a convergent subsequence (see [11]).

Let $(\Im, \rho)$ be a complete MS and let $CB(\Im)$ denote the class of all nonempty closed and bounded subsets of the nonempty set $\Im$. For $\mathcal{A}, \mathcal{B} \in CB(\Im)$, the function $\mathcal{H} : CB(\Im) \times CB(\Im) \to [0, +\infty)$ defined by

$$\mathcal{H}(\mathcal{A}, \mathcal{B}) = \max\{\sup_{\xi \in \mathcal{B}} \Delta(\xi, \mathcal{A}), \sup_{\delta \in \mathcal{A}} \Delta(\delta, \mathcal{B})\},$$

where $\Delta(\delta, \mathcal{B}) = \inf_{\xi \in \mathcal{B}} \rho(\delta, \xi)$, is a metric on $CB(\Im)$.

$v \in \Im$ is called a fixed point of the multivalued map $\Upsilon : \Im \to CB(\Im)$ if $v \in \Upsilon v$. For $\theta_0 \in \Im$, if the sequence $\{\theta_n\}$ is constructed in such a way that $\theta_{n+1} \in \Upsilon\theta_n$, then $O(\Upsilon, \theta_0) = \{\theta_0, \theta_1, \theta_2, \ldots\}$ is called an orbit of Υ at θ_0. A function $\psi : \Im \to \mathbb{R}$ is called Υ-orbitally lower semi-continuous if for any sequence $\{\xi_n\} \subset O(\Upsilon, \theta_0)$ with $\xi_n \to \xi$ implies $\psi(\xi) \leq \liminf_{n\to\infty} \psi(\xi_n)$ (see [12]).

A multivalued mapping $\Upsilon : \Im \to CB(\Im)$ is said to be asymptotically regular (AR, in short) at $\theta_0 \in \Im$, if for any sequence $\{\xi_n\} \subset O(\Upsilon, \theta_0)$, we have $\lim_{n\to\infty} \rho(\xi_n, \xi_{n+1}) = 0$ (see e.g., [13]). The mapping $\Upsilon : \Im \to CB(\Im)$ is said to be orbitally continuous (OC, in short) at a point $\theta_0 \in \Im$, if for any sequence $\{\xi_n\} \subset O(\Upsilon, \theta_0)$, we have $\xi_n \to \xi$ (for some $\xi \in \Im$) implies that $\Upsilon\xi_n \to \Upsilon\xi$ (see [14]). When Υ is OC at all points of its domain, then it is called OC.

The following lemmas are significant in the present context.

Lemma 1 ([15,16])**.** *Let $(\Im, \rho)$ be an MS and $\mathcal{A}, \mathcal{B} \in CB(\Im)$. Then*

(i) $\Delta(\theta, \mathcal{B}) \leq \rho(\theta, \gamma)$ *for any* $\gamma \in \mathcal{B}$ *and* $\theta \in \Im$*;*

(ii) $\Delta(\theta, \mathcal{B}) \leq \mathcal{H}(\mathcal{A}, \mathcal{B})$ *for any* $\theta \in \mathcal{A}$*.*

Lemma 2 ([17])**.** *Let $\mathcal{A}, \mathcal{B} \in CB(\Im)$ and let $\theta \in \mathcal{A}$. If $p > 0$, then there exists $\xi \in \mathcal{B}$ such that*

$$\rho(\theta, \xi) \leq \mathcal{H}(\mathcal{A}, \mathcal{B}) + p.$$

In general, we may not obtain a point $\xi \in \mathcal{B}$ such that

$$\rho(\theta, \xi) \leq \mathcal{H}(\mathcal{A}, \mathcal{B}).$$

But when $\mathcal{B}$ is compact, then such a point ξ exists, i.e., $\rho(\theta, \xi) \leq \mathcal{H}(\mathcal{A}, \mathcal{B})$.

Lemma 3 ([17])**.** *Let $\{U_n\}$ be a sequence in $CB(\Im)$ and $\lim_{n\to\infty} \mathcal{H}(U_n, U) = 0$ for some $U \in CB(\Im)$. If $\mu_n \in U_n$ and $\lim_{n\to\infty} \rho(\mu_n, \mu) = 0$ for some $\mu \in \Im$, then $\mu \in U$.*

Some significant developments in fixed points results for AR multivalued mappings may be found in [13,14,18–20].

Reich [21] proved some fixed-point theorems for multivalued maps using the concept of δ-distance instead of Pompeiu–Hausdorff metric, which is defined as follows: for $\mathcal{A}, \mathcal{B} \in CB(\Im)$,

$$\delta(\mathcal{A}, \mathcal{B}) = \sup\{\rho(\theta, \xi) : \theta \in \mathcal{A}, \xi \in \mathcal{B}\}.$$

Srivastava et al. [22] presented Krasnosel'skii type hybrid fixed-point theorems. Xu et al. [23] proved Schwarz lemma related to boundary fixed points. Very recently, Debnath and Srivastava [24] investigated common best proximity points for multivalued contractive pairs of mappings in connection with global optimization. Debnath and Srivastava [25] also proved new extensions of Kannan's and Reich's theorems in the context of multivalued mappings using Wardowski's technique. Furthermore, an important use of fixed points of $F(\psi, \varphi)$-contractions to fractional differential equations was recently established by Srivastava et al. [26].

In the current paper, we present some fixed-point theorems for AR multivalued maps satisfying a Kannan-type condition in an MS. We assume that the MS is either boundedly compact or Υ-orbitally compact. The orbital continuity of the mapping under consideration or orbital lower semi-continuity of Δ has been assumed as well. In Section 2, we present the results considering the Pompeiu–Hausdorff metric. Furthermore, in Section 3, we present alternate versions of these results considering the δ-distance, where some stronger conditions from Section 2 can be dropped.

2. Results with Respect to Pompeiu–Hausdorff Metric

First, we present a result where boundedly compactness of the MS is assumed.

Theorem 3. *Let $(\Im, \rho)$ be a boundedly compact MS and the multivalued mapping $\Upsilon : \Im \to CB(\Im)$ be AR at a point $\theta_0 \in \Im$ satisfying*

$$\mathcal{H}(\Upsilon\theta, \Upsilon\xi) < \frac{1}{2}\{\Delta(\theta, \Upsilon\theta) + \Delta(\xi, \Upsilon\xi)\}$$

for all $\theta, \xi \in \Im$ with $\Delta(\theta, \Upsilon\theta) > 0$ and $\Delta(\xi, \Upsilon\xi) > 0$. Also let $\rho(u, v) \leq \mathcal{H}(\Upsilon\theta, \Upsilon\xi)$ for all $u \in \Upsilon\theta$ and $v \in \Upsilon\xi$.

If Υ is OC or Δ is Υ-orbitally lower semi-continuous, then $Fix(\Upsilon) \neq \phi$.

Proof. We construct the orbit of Υ at θ_0 as $O(\Upsilon, \theta_0)$ and consider the sequence $\{\xi_n\} \subset O(\Upsilon, \theta_0)$. Let $r_n = \rho(\xi_n, \xi_{n+1}) > 0$.

Since Υ is AR, we have $r_n \to 0$. Now we have

$$\begin{aligned}
\rho(\xi_n, \xi_m) &\leq \mathcal{H}(\Upsilon\xi_{n-1}, \Upsilon\xi_{m-1}) \\
&< \frac{1}{2}\{\Delta(\xi_{n-1}, \Upsilon\xi_{n-1}) + \Delta(\xi_{m-1}, \Upsilon\xi_{m-1})\} \\
&\leq \frac{1}{2}\{\rho(\xi_{n-1}, \xi_n) + \rho(\xi_{m-1}, \xi_m)\} \\
&= \frac{1}{2}(r_{n-1}, r_{m-1}) \\
&\to 0 \text{ as } m, n \to \infty.
\end{aligned}$$

Therefore, the sequence $\{\xi_n\}$ is Cauchy and hence it is bounded. Since $(\Im, \rho)$ is boundedly compact, $\{\xi_n\}$ has a convergent subsequence $\{\xi_{n_k}\}$ which converges to $\xi \in \Im$.

Since $\{\xi_n\}$ is Cauchy and its subsequence $\{\xi_{n_k}\}$ converges to $\xi \in \Im$, we have that $\xi_n \to \xi$ as $n \to \infty$.

Let Υ be OC. Thus, we have that $\Upsilon\xi_n \to \Upsilon\xi$. But $\xi_{n+1} \in \Upsilon\xi_n$ for all $n \in \mathbb{N}$ and $\xi_{n+1} \to \xi$ as $n \to \infty$. Hence, using Lemma 3, we conclude that $\xi \in \Upsilon\xi$.

Furthermore, if Δ is Υ-orbitally lower semi-continuous, we have that

$$\Delta(\xi, \Upsilon\xi) \leq \liminf_{k \to \infty} \Delta(\xi_{n_k}, \Upsilon\xi_{n_k}) = 0.$$

Finally, the closedness of $\Upsilon\xi$ implies that $\xi \in \Upsilon\xi$. □

The next result is in connection with Υ-orbitally compactness.

Definition 1. *Let $(\Im, \rho)$ be an MS and $\Upsilon : \Im \to CB(\Im)$ be a multivalued mapping. $\Im$ is said to be Υ-orbitally compact if every sequence in the orbit $O(\Upsilon, \theta)$ has a convergent subsequence for all $\theta \in \Im$.*

Theorem 4. *Let $(\Im, \rho)$ be a Υ-orbitally compact MS and the multivalued mapping $\Upsilon : \Im \to CB(\Im)$ be AR at a point $\theta_0 \in \Im$ satisfying*

$$\mathcal{H}(\Upsilon\theta, \Upsilon\xi) < \frac{1}{2}\{\Delta(\theta, \Upsilon\theta) + \Delta(\xi, \Upsilon\xi)\}$$

for all $\theta, \xi \in \Im$ with $\Delta(\theta, \Upsilon\theta) > 0$ and $\Delta(\xi, \Upsilon\xi) > 0$. Also let $\rho(u, v) \leq \mathcal{H}(\Upsilon\theta, \Upsilon\xi)$ for all $u \in \Upsilon\theta$ and $v \in \Upsilon\xi$.

If Υ is orbitally continuous or Δ is Υ-orbitally lower semi-continuous, then $Fix(\Upsilon) \neq \phi$.

Proof. Like earlier, we construct the orbit of Υ at θ_0 as $O(\Upsilon, \theta_0)$ and consider the sequence $\{\xi_n\} \subset O(\Upsilon, \theta_0)$.

Since $\Im$ is Υ-orbitally compact, $\{\xi_n\}$ has a convergent subsequence $\{\xi_{n_k}\}$ which converges to $\alpha \in \Im$.

Now, for all $m, n \in \mathbb{N}$ we have

$$\begin{aligned}
\rho(\xi_n, \xi_m) &\leq \mathcal{H}(\Upsilon\xi_{n-1}, \Upsilon\xi_{m-1}) \\
&< \frac{1}{2}\{\Delta(\xi_{n-1}, \Upsilon\xi_{n-1}) + \Delta(\xi_{m-1}, \Upsilon\xi_{m-1})\} \\
&\leq \frac{1}{2}\{\rho(\xi_{n-1}, \xi_n) + \rho(\xi_{m-1}, \xi_m)\} \\
&\to 0 \text{ as } m, n \to \infty \text{ (since } \Upsilon \text{ is AR).}
\end{aligned}$$

Therefore, the sequence $\{\xi_n\}$ is Cauchy and since its subsequence $\{\xi_{n_k}\}$ converges to $\xi \in \Im$, we have that $\xi_n \to \xi$ as $n \to \infty$.

Let Υ be OC. Thus, we have that $\Upsilon\xi_n \to \Upsilon\xi$. But $\xi_{n+1} \in \Upsilon\xi_n$ for all $n \in \mathbb{N}$ and $\xi_{n+1} \to \xi$ as $n \to \infty$. Hence, using Lemma 3, we conclude that $\xi \in \Upsilon\xi$.

Next, we assume that Δ is Υ-orbitally lower semi-continuous. Since $\xi_n \to \xi$ as $n \to \infty$, we have

$$\Delta(\xi, \Upsilon\xi) \leq \liminf_{k\to\infty} \Delta(\xi_n, \Upsilon\xi_n) = 0,$$

because Υ is AR implies that $\lim_{n\to\infty} \rho(\xi_n, \xi_{n+1}) = 0$ and $\lim_{n\to\infty} \Delta(\xi_n, \Upsilon\xi_n) = 0$. Finally, since $\Upsilon\xi$ is closed, we have $\xi \in \Upsilon\xi$. □

3. Multivalued Versions with Respect to δ-Distance

In this section, we present multivalued versions of the results presented in the previous section with respect to δ-distance instead of Pompeiu–Hausdorff metric. We observe that here we can drop the additional condition $\rho(u, v) \leq \mathcal{H}(\Upsilon\theta, \Upsilon\xi)$ for all $u \in \Upsilon\theta$ and $v \in \Upsilon\xi$.

Theorem 5. *Let $(\Im, \rho)$ be a boundedly compact MS and the multivalued mapping $\Upsilon : \Im \to CB(\Im)$ be AR at a point $\theta_0 \in \Im$ satisfying*

$$\delta(\Upsilon\theta, \Upsilon\xi) < \frac{1}{2}\{\Delta(\theta, \Upsilon\theta) + \Delta(\xi, \Upsilon\xi)\}$$

for all $\theta, \xi \in \Im$ with $\Delta(\theta, \Upsilon\theta) > 0$ and $\Delta(\xi, \Upsilon\xi) > 0$.

If Δ is Υ-orbitally lower semi-continuous, then $Fix(\Upsilon) \neq \phi$.

Proof. We construct the orbit of Υ at θ_0 as $O(\Upsilon, \theta_0)$ and consider the sequence $\{\xi_n\} \subset O(\Upsilon, \theta_0)$. Let $r_n = \rho(\xi_n, \xi_{n+1}) > 0$.

Since Υ is AR, we have $r_n \to 0$. Now we have

$$\begin{aligned}
\rho(\xi_n, \xi_m) &\leq \delta(\Upsilon\xi_{n-1}, \Upsilon\xi_{m-1}), \text{(using the definition of } \delta) \\
&< \frac{1}{2}\{\Delta(\xi_{n-1}, \Upsilon\xi_{n-1}) + \Delta(\xi_{m-1}, \Upsilon\xi_{m-1})\} \\
&\leq \frac{1}{2}\{\rho(\xi_{n-1}, \xi_n) + \rho(\xi_{m-1}, \xi_m)\}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2}(r_{n-1}, r_{m-1}) \\
&\to 0 \text{ as } m, n \to \infty.
\end{aligned}
$$

Therefore, the sequence $\{\xi_n\}$ is Cauchy and hence it is bounded. Since $(\Im, \rho)$ is boundedly compact, $\{\xi_n\}$ has a convergent subsequence $\{\xi_{n_k}\}$ which converges to $\xi \in \Im$.

Now, since Δ is Υ-orbitally lower semi-continuous, we have that

$$\Delta(\xi, \Upsilon\xi) \leq \liminf_{k\to\infty} \Delta(\xi_{n_k}, \Upsilon\xi_{n_k}) = 0 \text{ (for } \Upsilon \text{ is AR).}$$

Finally, the closedness of $\Upsilon\xi$ implies that $\xi \in \Upsilon\xi$. □

Theorem 6. *Let $(\Im, \rho)$ be a Υ-orbitally compact MS and the multivalued mapping $\Upsilon : \Im \to CB(\Im)$ be AR at a point $\theta_0 \in \Im$ satisfying*

$$\delta(\Upsilon\theta, \Upsilon\xi) < \frac{1}{2}\{\Delta(\theta, \Upsilon\theta) + \Delta(\xi, \Upsilon\xi)\}$$

for all $\theta, \xi \in \Im$ with $\Delta(\theta, \Upsilon\theta) > 0$ and $\Delta(\xi, \Upsilon\xi) > 0$.

If Δ is Υ-orbitally lower semi-continuous, then $Fix(\Upsilon) \neq \phi$.

Proof. Consider the orbit of Υ at θ_0 as $O(\Upsilon, \theta_0)$ and let $\{\xi_n\} \subset O(\Upsilon, \theta_0)$.

Since $\Im$ is Υ-orbitally compact, $\{\xi_n\}$ has a convergent subsequence $\{\xi_{n_k}\}$ which converges to $\alpha \in \Im$.

Now, for all $m, n \in \mathbb{N}$ we have

$$
\begin{aligned}
\rho(\xi_n, \xi_m) &\leq \delta(\Upsilon\xi_{n-1}, \Upsilon\xi_{m-1}), \text{(using the definition of } \delta) \\
&< \frac{1}{2}\{\Delta(\xi_{n-1}, \Upsilon\xi_{n-1}) + \Delta(\xi_{m-1}, \Upsilon\xi_{m-1})\} \\
&\leq \frac{1}{2}\{\rho(\xi_{n-1}, \xi_n) + \rho(\xi_{m-1}, \xi_m)\} \\
&\to 0 \text{ as } m, n \to \infty \text{ (since } \Upsilon \text{ is AR).}
\end{aligned}
$$

Therefore, the sequence $\{\xi_n\}$ is Cauchy and since its subsequence $\{\xi_{n_k}\}$ converges to $\xi \in \Im$, we have that $\xi_n \to \xi$ as $n \to \infty$.

Since Δ is Υ-orbitally lower semi-continuous, we have

$$\Delta(\xi, \Upsilon\xi) \leq \liminf_{k\to\infty} \Delta(\xi_n, \Upsilon\xi_n) = 0,$$

because Υ is AR implies that $\lim_{n\to\infty} \rho(\xi_n, \xi_{n+1}) = 0$ and $\lim_{n\to\infty} \Delta(\xi_n, \Upsilon\xi_n) = 0$. Finally, since $\Upsilon\xi$ is closed, we have $\xi \in \Upsilon\xi$. □

Finally, we provide an example to validate Theorem 6. All other results may be validated in a similar manner.

Example 1. *Consider $\Im = (0, \infty)$ with the usual metric $\rho(\theta, \xi) = |\theta - \xi|$, for all $\theta, \xi \in \Im$. Define $\Upsilon : \Im \to CB(\Im)$ by*

$$\Upsilon\theta = \begin{cases} \{0\}, \text{ if } \theta \in (0, 7) \\ \{\theta, \theta + 1\}, \text{ if } \theta \geq 7. \end{cases}$$

Let $\theta, \xi \in \Im$ with $\Delta(\theta, \Upsilon\theta) > 0$ and $\Delta(\xi, \Upsilon\xi) > 0$. Then $\delta(\Upsilon\theta, \Upsilon\xi) = \delta(\{0\}, \{0\}) = 0$. Also, it is easy to check that Υ is AR and Δ is Υ-orbitally lower semi-continuous.

Here $(\Im, \rho)$ is not complete but it is Υ-orbitally compact. Thus, all conditions of Theorem 6 are satisfied and hence $Fix(\Upsilon) \neq \phi$. In fact, $Fix(\Upsilon) = \{n \in \mathbb{N} : n \geq 7\}$.

4. Conclusions

Some fixed-point theorems have been established for AR multivalued maps satisfying a Kannan-type condition in an MS. Boundedly compactness or Υ-orbitally compactness of the MS has been assumed. The results have been established considering the Pompeiu–Hausdorff metric as well as the δ-distance. In the latter case, some stronger conditions from Pompeiu–Hausdorff metric (such as $\rho(u,v) \leq \mathcal{H}(\Upsilon\theta, \Upsilon\xi)$ for all $u \in \Upsilon\theta$ and $v \in \Upsilon\xi$) have been dropped. Proof of the results of Section 2 without assuming this condition would be an interesting future study.

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