axioms

Article

Prediction of the Share of Solar Power in China Based on FGM (1,1) Model

Yuhan Li, Shuya Wang, Wei Dai and Liusan Wu *

College of Information Management, Nanjing Agricultural University, Nanjing 210031, China
* Correspondence: wuls@njau.edu.cn

Abstract: In recent years, fossil energy reserves have decreased year by year, and the development and use of renewable energy has attracted great attention of governments all over the world. China continues to promote the high-quality development of renewable energy such as solar power generation. Accurate prediction of the share of solar power in China is beneficial to implementing the goals of carbon peaking and carbon neutralization. According to the website of China's National Bureau of statistics, the earliest annual data of China's solar power generation is 2017, which leads to there being very few data on the share of China's solar power generation. Therefore, the prediction accuracy of most prediction methods is low, and the advantages of the grey prediction model are shown. Based on the share of solar power in China from 2017 to 2020, this paper constructs an FGM (1,1) model, calculates r using the Particle Swarm Optimization (PSO) algorithm, and predicts the share of solar power in China in the next few years. $r = 0.3858$ and $MAPE = 0.20\%$ were obtained by calculation of the model. The prediction results show that the share of solar power generation in China will increase year by year, and it will reach about 4.2301% by 2030. In addition, it is found that the share of China's solar power generation in 2021 is 2.1520%, and the predicted value is 2.1906%. It can be seen that the prediction error is small. Finally, the limitations and future research directions are elucidated. The prediction results presented in this paper will help to guide the development of solar power generation in China, and are of great significance in speeding up the pace of energy structural adjustment, accelerating the construction of a clean, low-carbon, safe and efficient energy system, and promoting sustainable development.

Keywords: share of solar power; FGM (1,1) model; particle swarm optimization algorithm

Citation: Li, Y.; Wang, S.; Dai, W.; Wu, L. Prediction of the Share of Solar Power in China Based on FGM (1,1) Model. *Axioms* **2022**, *11*, 581. https://doi.org/10.3390/axioms11110581

Academic Editors: Lifeng Wu and Ricardo Almeida

Received: 20 September 2022
Accepted: 20 October 2022
Published: 22 October 2022

1. Introduction

With the deepening of the greenhouse effect [1] and the sharp reduction in fossil energy reserves [2], the vigorous development and use of nonrenewable resources is no longer in line with the current energy development trend. Therefore, the efficient use of renewable resources has attracted the attention of countries all over the world. To implement the goals of carbon peaking and carbon neutralization, president Xi of China mentioned in his speech that "we should accelerate the development of new energy such as wind energy, solar energy, biomass energy, geothermal energy, marine energy, hydrogen energy, nuclear power and so on". In addition, the Chinese government has put forward key project objectives such as "accelerating the application of renewable energy such as wind energy, solar energy and biomass energy in agricultural production and rural life".

In August 2022, a large-scale and long-term power outage occurred in some areas of Sichuan province, China. The main reason for this was that the proportion of hydropower generation in Sichuan province exceeds 80%, and the lower amount of rainfall and high temperatures in 2022 lead to a sharp reduction in hydropower generation capacity. Therefore, in addition to hydroelectric power generation, the development of other sources of renewable energy, such as solar power generation, should be accelerated and the proportion of renewable energy should be increased. Solar power generation can be divided into photovoltaic power generation and photothermal power generation, and photovoltaic power generation is the main use of solar power generation technology. At present, the share of solar power generation in total power generation in China is still very low, and there is no literature predicting its development trend. Many scholars have studied the influence factors of solar power generation, and have used traditional statistical models and intelligent algorithms to study solar power generation in recent years. Chang et al. predicted the solar power generation based on deep learning [3]. Spyros predicted day-ahead PV generation based on machine learning and statistical post-processing [4]. Zeng and Jang et al. predicted solar power using a support vector machine [5,6]. Erduman predicted the smart short-term solar power output using artificial neural network [7]. Hua et al. used radial basis function neural network (RBFN) to select the optimal input parameters to predict the power output of 20 kW solar panel [8]. Wang et al. estimated the potential of photovoltaic power generation in 2020 and 2030 on the basis of land resource change [9]. Veysel et al. analyzed the solar energy generation capacity using hesitant fuzzy cognitive maps [10]. Prema et al. studies the development of statistical time series models for solar power prediction [11]. Jafarzadeh et al. proposes type-1 and interval type-2 Takagi-Sugeno-Kang (TSK) fuzzy systems for the modeling and prediction of solar power plants [12]. Most of the above-mentioned prediction methods require more historical data.

However, although solar power generation forecasting is a hot topic, the statistical work of solar power generation data began in May 2016 in China, and the earliest annual data of China's solar power generation is in 2017, which leads to there being very few data on the share of China's solar power generation. Due to there being fewer data, the prediction accuracy of most prediction methods is low, and the advantages of the grey prediction model are shown. The grey prediction model does not require a large number of data samples [13], and offers a good short-term prediction effect and a simple operation process. Therefore, this paper employs the grey prediction model for prediction and analysis.

Since Deng et al. put forward the grey system theory, various models represented by GM (1,1) have been widely used in energy [14,15], medical [16,17], finance [18,19], agriculture [20,21] and other fields. Compared with general prediction methods, the grey GM (1,1) model is characterized by the small amount of original data required for prediction, high prediction accuracy, and no empirical coefficient. When the GM (1,1) model is faced with a higher cumulative order, the disturbance bound of the model solution is larger, leading to a greater error in the prediction results. To improve the prediction accuracy of the GM (1,1) model, many scholars have improved the model, mainly by including the optimization of the original data [22–24], the correction of the model residuals [25], improving the accumulation method [26,27], and so on. In addition, Wu, Jiang and Chen et al. proposed the fractional FGM (1,1) model [28–30]. In the FGM (1,1) model, each sequence is multiplied by a different fractional order and then accumulated. At the same time, the FGM (1,1) model considers uncertain factors such as data acquisition environment and acquisition method as grey level, and weakens the grey level of the system through the operation process of accumulation and subtraction. The operation of the process can eliminate the noise of the data itself, and through the fractional r optimization accumulation method, the randomness and volatility of the original data are smoothed, and the stability of the model operation is improved. At present, the FGM (1,1) model has been used for air quality prediction [31], natural gas consumption [32], express business volume [33], high-tech industry value-added [34], total social electricity consumption [35], the total output value of China's construction industry [36], and so on. Compared with the GM (1,1) model, the FGM (1,1) model improves prediction accuracy by applying fractional order.

To understand the development law of the share of solar power generation in China, this paper constructs the FGM (1,1) model, calculates r using the particle swarm optimization (PSO) algorithm, and forecasts the share of solar power generation in China in the next few years based on the share of solar power generation in China from 2017 to 2020. These results will help to guide the development of the solar power generation in China.

Furthermore, it is of great significance to speed up the structural adjustment of the energy and promote sustainable development.

2. Modeling Process of the FGM (1,1) Model and PSO Algorithm

2.1. Modeling Process of the FGM (1,1) Model

With the aim of solving the shortcomings of the GM (1,1) model, the FGM (1,1) model further subdivides the integer order into fractional order to improve the accuracy of the model.

The modeling process of the FGM (1,1) model is as follows.

Step 1: Construct r-order accumulation sequence.

The original non-negative sequence of the original data was written as $X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right\}$, and the cumulative sequence of order r was obtained by calculation as $X^{(r)} = \left\{x^{(r)}(1), x^{(r)}(2), \cdots, x^{(r)}(n)\right\}$, where

$$x^{(r)}(k) = \sum_{i=1}^{k} C_{k-i+r-1}^{k-i} x^{(0)}(i),\ C_{k-i+r-1}^{k-i} = \frac{(k-i+r-1)(k-i+r-2)\cdots(r+1)r}{(k-i)!},\ C_{r-1}^{0} = 1,\ C_{k}^{k+1} = 0 \tag{1}$$

Step 2: The whitening differential equation is established as follows:

$$\frac{dx^{(r)}(t)}{dt} + ax^{(r)}(t) = b \tag{2}$$

where a and b are called the developmental grey number and endogenous control grey number respectively. The solution of the above equation is as follows:

$$x^{(r)}(t+1) = [x^{(0)}(1) - \frac{b}{a}]e^{-at} + \frac{b}{a} \tag{3}$$

Using the least squares method, the numerical solutions of parameters $\hat{a}$ and $\hat{b}$ are $\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \left(B^T B\right)^{-1} B^T Y$, where

$$B = \begin{pmatrix} -0.5\left(x^{(r)}(1) + x^{(r)}(2)\right) & 1 \\ -0.5\left(x^{(r)}(2) + x^{(r)}(3)\right) & 1 \\ \vdots & \vdots \\ -0.5\left(x^{(r)}(n-1) + x^{(r)}(n)\right) & 1 \end{pmatrix}, Y = \begin{pmatrix} \left(x^{(r)}(2) - x^{(r)}(1)\right) \\ \left(x^{(r)}(3) - x^{(r)}(2)\right) \\ \vdots \\ \left(x^{(r)}(n) - x^{(r)}(n-1)\right) \end{pmatrix} \tag{4}$$

Step 3: The time response function is as follows:

$$\hat{x}^{(r)}(k+1) = [x^{(0)}(1) - \frac{\hat{b}}{\hat{a}}]e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}} \tag{5}$$

where $\hat{x}^{(r)}(k+1)$ is the value at time $k+1$.

Step 4: The reduction sequence of $\hat{X}^{(r)} = \left\{\hat{x}^{(r)}(1), \hat{x}^{(r)}(2), \cdots, \hat{x}^{(r)}(n)\right\}$ is as follows:

$$\alpha^{(r)}\hat{X}^{(r)} = \left\{\alpha^{(1)}\hat{x}^{(r)(1-r)}(1), \alpha^{(1)}\hat{x}^{(r)(1-r)}(2), \cdots, \alpha^{(1)}\hat{x}^{(r)(1-r)}(n)\right\}$$

where $\alpha^{(1)}\hat{x}^{(r)(1-r)}(k) = \hat{x}^{(r)(1-r)}(k) - \hat{x}^{(r)(1-r)}(k-1)$. So the prediction sequence is $\hat{X}^{(0)} = \left\{\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \cdots, \hat{x}^{(0)}(n)\right\}$.

Step 5: Model evaluation.

The Mean Absolute Percentage Error ($MAPE$) is used to evaluate the accuracy of the model.

$$MAPE = \frac{1}{n} \times \sum_{k=1}^{n} \left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \times 100\% \quad (6)$$

The accuracy standards are shown in Table 1.

Table 1. Accuracy class.

MAPE Values	Accuracy Class
<10%	Excellent Class
10–20%	Good Class
20–50%	General Class
>50%	Poor Class

2.2. PSO Algorithm

Particle swarm optimization (PSO) is an evolutionary computing technology that originates from the study of bird predation behavior. The idea of the particle swarm optimization algorithm is to find the optimal solution through cooperation and information sharing among individuals in the group. The PSO algorithm is easy to implement, and does not require the adjustment of many parameters. Therefore, the solution concept of particle swarm optimization algorithm is matched with the FGM model, which provides the optimal r value for FGM model through local optimization and global optimization.

Particles in the particle swarm optimization algorithm have two attributes: speed and position. Each particle searches for the optimal solution separately in the search space, records it as the current individual extreme value, shares the individual extreme value with other particles throughout the whole particle swarm, and finds the optimal individual extreme value as the current global optimal solution of the whole particle swarm. All particles in the particle swarm adjust their speed and position according to the current global optimal solution, and approach the optimal solution until the end of optimization.

The PSO is initialized as a group of random particles (random solutions), and then the optimal solution is found through iteration. Suppose there are N particles forming a community in the D-dimensional target search space, where the i-particle represents a vector of D dimensions, it is denoted by $X_i = (x_{i1}, \ x_{i2}, \ \cdots, \ x_{iD})$. A particle's velocity is also a vector of D dimensions, denoted by $V_i = (v_{i1}, \ v_{i2}, \ \cdots, \ v_{iD})$. The optimal position of the i-particle and the entire particle group searched so far are individual extreme value $P_{Best} = (p_{i1}, \ p_{i2}, \ \cdots, \ p_{iD})$ and global extreme value $G_{Best} = (g_{i1}, \ g_{i2}, \ \cdots, \ g_{iD})$. The particle updates its velocity and position by using the formula below.

$$\begin{cases} v_{ij}(t+1) = \omega^{(t)} v_{ij}(t) + c_1 r_1(t)[p_{ij}(t) - x_{ij}(t)] + c_2 r_2(t)[g_{ij}(t) - x_{ij}(t)] \\ x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \end{cases} \quad (7)$$

where $\omega^{(t)} \in (0,1)$ is the inertia factor, c_1 and c_2 are the individual learning factor and social learning factor of each particle, $r_1(t)$ and $r_2(t)$ represent random numbers in [0,1]. In addition, $\omega^{(t)} = (\omega_{ini} - \omega_{end})(G_k - g)/G_k + \omega_{end}$, where G_k is the maximum number of iterations, ω_{ini} is the initial inertia weight, and ω_{end} is the inertia value when iterating to the maximum evolutionary generation.

The algorithm flow chart of PSO is shown in Figure 1, below.

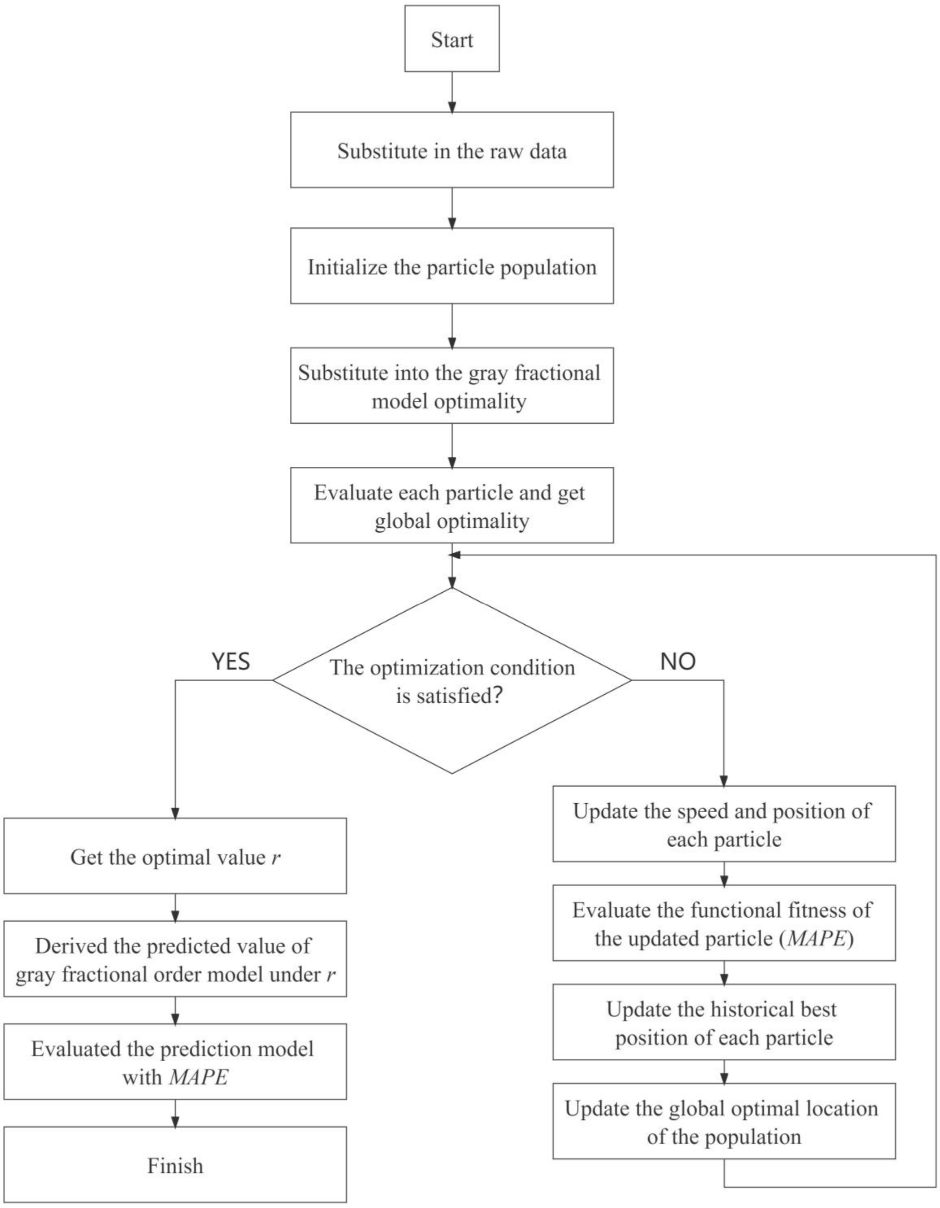

Figure 1. Algorithm flow chart.

3. Empirical Research

Solar power generation is an emerging field in China, and the life of solar systems is generally around 25 years, so the impact of external uncertainties such as battery life and equipment depreciation is not considered. In addition, the sample size of existing data is small, and there is no obvious rule that is suitable for the FGM (1,1) model. The share of China's solar power from 2017 to 2020 is shown in Table 2 (data from the website of China's National Bureau of Statistics).

Table 2. Share of China's solar power from 2017 to 2020.

Year	Solar Power Generation (Billion KWh)	Total Power Generation (Billion KWh)	The Share of China's Solar Power Generation (%)
2017	64.75	6275.82	1.03
2018	89.45	6791.42	1.32
2019	117.22	7142.21	1.64
2020	142.1	7417.04	1.92

The share of China's solar power in the years from 2017 to 2020 was 1.03, 1.32, 1.64 and 1.92, respectively (%). Using the above data as the original series, the FGM (1,1) model was established, and the annual solar power share from 2021 to 2030 was predicted. The calculation process was as follows.

(1) Construct the original series of annual solar power share in China from 2017 to 2020.

$$X^{(0)} = \{1.03, 1.32, 1.64, 1.92\} \tag{8}$$

(2) Take the minimum MAPE of the FGM (1,1) model as the objective function, and obtain the value of r through particle swarm optimization algorithm.

(3) Let N = 50, d = 1, ω = 0.8, $c_1 = c_2$ =2, the maximum number of iterations is 200. The stop criterion is the satisfactory solution eps = 10^{-6}; according to the concept of fractional order, r is between 0 and 1.

The particle swarm optimization algorithm is iterated using MATLAB, and the convergence process of r is shown in Figure 2. It is found that r eventually tends to 0.3858.

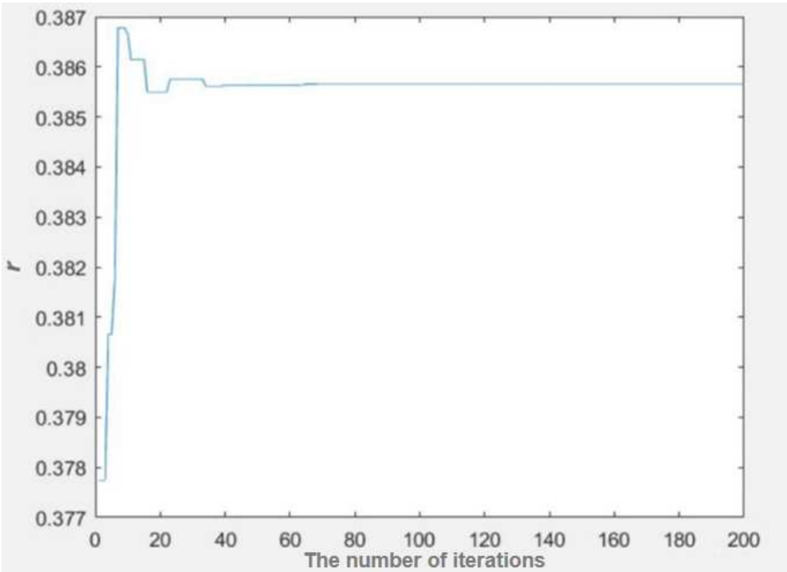

Figure 2. The convergence process of r.

The 0.3858-order accumulation sequence is as follows.

$$\begin{aligned} X^{(0.3858)} &= \left\{x^{(0.3858)}(1), x^{(0.3858)}(2), x^{(0.3858)}(3), x^{(0.3858)}(4)\right\} \\ &= \{1.0300, 1.7174, 2.4246, 3.1245\} \end{aligned} \tag{9}$$

The parameters $\hat{a}$ and $\hat{b}$ can be obtained using the following formula.

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = (B^T B)^{-1} B^T Y = \begin{pmatrix} -0.0089 \\ 0.6796 \end{pmatrix}$$

where $B = \begin{pmatrix} -1.3737 & 1 \\ -2.0710 & 1 \\ -2.7746 & 1 \end{pmatrix}$ and $Y = \begin{pmatrix} 0.6874 \\ 0.7072 \\ 0.6999 \end{pmatrix}$.

The time response function is as follows.

$$\hat{x}^{(0.3858)}(k+1) = [1.03 - \frac{0.6796}{-0.0089}]e^{-(-0.0089)k} + \frac{0.6796}{-0.0089} \tag{10}$$

So

$$\begin{aligned}\hat{X}^{(0.3858)} &= \left\{\hat{x}^{(0.3858)}(1), \hat{x}^{(0.3858)}(2), \hat{x}^{(0.3858)}(3), \cdots, \hat{x}^{(0.3858)}(8), \hat{x}^{(0.3858)}(9)\right\} \\ &= \{1.0300, 1.7219, 2.4201, \ldots, 6.0061, 6.7428\}\end{aligned}$$

The reduction sequence is as follows.

$$\begin{aligned}\hat{X}^{(1)} &= \left\{\hat{x}^{(0.3858)(0.6142)}(1), \hat{x}^{(0.3858)(0.6142)}(2), \cdots, \hat{x}^{(0.3858)(0.6142)}(8), \hat{x}^{(0.3858)(0.6142)}(9)\right\} \\ &= \{1.0300, 2.3546, 3.9883, \ldots, 16.1597, 19.3123\}\end{aligned}$$

Finally, the simulation results of China's annual solar power share in 2017–2020 are $\hat{X}^{(1)} = \left\{\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \hat{x}^{(0)}(3), \hat{x}^{(0)}(4)\right\} = \{1.0300, 1.3246, 1.6337, 1.9212\}$. When $r = 1$, the FGM (1,1) model is the same as the traditional GM (1,1) model. The convergence process of *MAPE* is shown in Figure 3, and the simulation results are shown in Table 3.

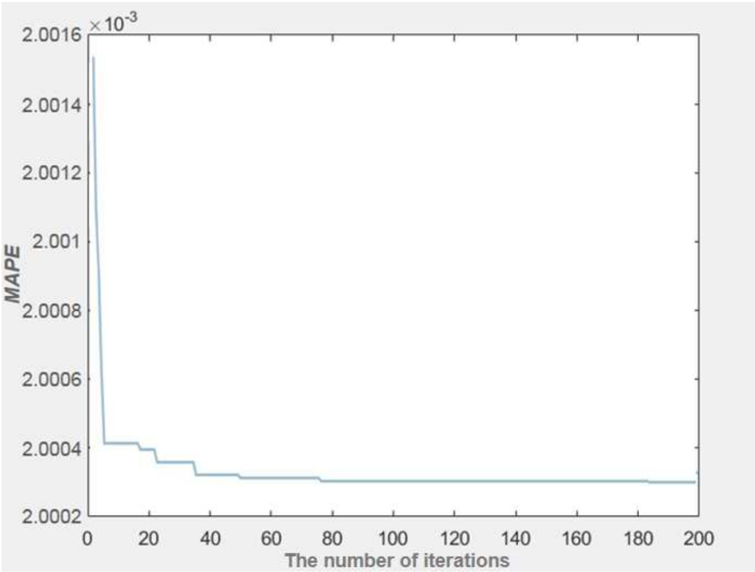

Figure 3. The convergence process of *MAPE*.

Table 3. The *MAPE* of the GM (1,1) model and the FGM (1,1) model (%).

Year	Original Data	GM (1,1)	FGM (1,1) (r = 0.3858)
2017	1.03	1.0300	1.0300
2018	1.32	1.3349	1.3246
2019	1.64	1.6032	1.6337
2020	1.92	1.9255	1.9212
MAPE		0.91%	0.20%

As can be seen from Table 3, the *MAPE* of the FGM (1,1) model and GM (1,1) model are 0.20% and 0.91%, which are all in the excellent class. Meanwhile, the *MAPE* of the FGM (1,1) model is less than the GM (1,1) model's, which verifies the superiority of the FGM (1,1) model.

The prediction results of China's solar power share in 2021–2030 based on the FGM (1,1) model are shown in Table 4. The results show that China's annual solar power share will increase year by year, and will reach 4.2301% in 2030.

Table 4. The prediction results of China's annual solar power share in 2021–2030 (%).

Year	FGM (1,1) (r = 0.3858)	Year	FGM (1,1) (r = 0.3858)
2021	2.1906	2026	3.3784
2022	2.4460	2027	3.5967
2023	2.6906	2028	3.8110
2024	2.9265	2029	4.0220
2025	3.1553	2030	4.2301

4. Results

In this paper, the FGM (1,1) model and the particle swarm optimization algorithm were combined to forecast the share of solar power generation in China in the next few years. It can be seen that the *MAPE* of the FGM (1,1) (r = 0.3858) model and the GM (1,1) model are 0.20% and 0.91%, respectively; thus, the *MAPE* of the FGM (1,1) model is less than that of the GM (1,1) model. This shows that the prediction accuracy of the FGM (1,1) model is higher than that of the GM (1,1) model. At the same time, the share of solar power generation in China will increase year by year, and it will reach from 2.1906% in 2021 to 4.2301% in 2030.

5. Conclusions

It can be seen that the FGM (1,1) model has more advantages than the GM (1,1) model. In addition, the relevant data from 2021 were obtained by querying the website of the National Bureau of statistics of China. It is found that the solar power generation and the total power generation are 183.66 billion KWh and 8112.18 billion KWh, respectively, so the share of China's solar power generation in 2021 is 2.2640%, while the predicted result is 2.1906%. It can be seen that the prediction error is small. This further illustrates the practical value of this paper. At the same time, while the share of solar power generation in China will increase year by year, the growth rate is relatively slow. Therefore, the Chinese government needs to take more effective measures to ensure the growth of solar power generation.

6. Discussion

Although the share of solar power generation in China will increase year by year, there is still a gap between the prediction results of this paper and the expectations of the Chinese government. The main reason may be that China's energy structure is not rational enough. The data shows that the share of thermal power generation in China exceeds 60% of the total power generation, which has indirectly led to the slow development of solar power generation. Therefore, the prediction results presented in this paper will help to guide the development of solar power generation in China and attract interest from the Chinese government in paying more attention to solar power generation, as well as being of great significance in speeding up the pace of energy structural adjustment, accelerating the construction of a clean, low-carbon, safe and efficient energy system, and promoting sustainable development.

However, in energy forecasting, ten-year prediction are regarded as short-term predictions; when performing long-term predictions the FGM (1,1) model, the error will increase. Therefore, the existing algorithm and model can be optimized to expand the prediction time range as much as possible on the premise of ensuring prediction accuracy in future. In addition, the prediction results presented in this paper were not compared with those obtained using other model, which will also be a key research direction in the next step.

Author Contributions: Conceptualization, Y.L.; data curation, S.W.; methodology, L.W. and W.D.; resources, Y.L.; software, S.W.; supervision, L.W.; visualization, W.D.; writing—original draft, Y.L.; writing—review and editing, S.W. and L.W. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Social Science Foundation of Jiangsu Province, China (21GLC003).

Data Availability Statement: Not applicable.

Acknowledgments: The authors also gratefully acknowledge the editor and anonymous reviewers for their helpful comments and suggestions which improved the paper.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Dong, K.; Dong, X.; Jiang, Q.; Zhao, J. Valuing the greenhouse effect of political risks: The global case. *Appl. Econ.* **2021**, *53*, 3604–3618. [CrossRef]
2. Yang, Z. Analysis of Paris Agreement-compliant fossil fuel burnable upper limit. *Environ. Sci. Technol.* **2020**, *43*, 111–118.
3. Chang, R.; Bai, L.; Hsu, C. Solar power generation prediction based on deep learning. *Sustain. Energy Technol. Assess.* **2021**, *47*, 101354. [CrossRef]
4. Spyros, T.; George, M.; Andreas, L.; Marios, T.; Paris, K.; George, E. Day-ahead photovoltaic power production forecasting methodology based on machine learning and statistical post-processing. *Appl. Energy* **2020**, *268*, 115023.
5. Zeng, J.; Qiao, W. Short-term solar power prediction using a support vector machine. *Renew. Energy* **2013**, *52*, 118–127. [CrossRef]
6. Jang, H.; Bae, K.; Park, H.; Sung, D. Solar power prediction based on satellite images and support vector machine. *IEEE Trans. Sustain. Energy* **2016**, *7*, 1255–1263. [CrossRef]
7. Erduman, A. A smart short-term solar power output prediction by artificial neural network. *Electr. Eng.* **2020**, *102*, 1441–1449. [CrossRef]
8. Hua, L.; Qi, C.; Liu, X. Solar Power Prediction Based on RBF Neural Network. *Sci. Technol. Ind.* **2022**, *22*, 375–380.
9. Wang, P.; Zhang, S.; Pu, Y.; Cao, S.; Zhang, Y. Estimation of photovoltaic power generation potential in 2020 and 2030 using land resource changes: An empirical study from China. *Energy* **2021**, *219*, 119611. [CrossRef]
10. Veysel, Ç.; Sezi, Ç. Analysis of solar energy generation capacity using hesitant fuzzy cognitive maps. *Int. J. Comput. Intell. Syst.* **2017**, *10*, 1149.
11. Prema, V.; Rao, K. Development of statistical time series models for solar power prediction. *Renew. Energy* **2015**, *83*, 100–109. [CrossRef]
12. Jafarzadeh, S.; Fadali, M.; Evrenosoglu, C. Solar power prediction using interval type-2 TSK modeling. *IEEE Trans. Sustain. Energy* **2013**, *4*, 333–339. [CrossRef]
13. Deng, J. *Basis of Grey Theory*; Huazhong University of Science and Technology Press: Wuhan, China, 2002.
14. You, K.; Kang, N.; Fu, J.; Gao, Y.; Qian, W.; Wang, J. Research on forecast of water demand in Jinzhou based on grey model. *IOP Conf. Ser. Earth Environ. Sci.* **2021**, *770*, 012–037. [CrossRef]
15. Wu, Z.; Shi, J. The Influencing Factor Analysis and trend forecasting of Beijing energy carbon emission based on STIRPAT and GM (1,1) model's. *Chin. J. Manag. Sci.* **2012**, *20*, 803–809.
16. Fan, M.; Gu, S.; Jin, Y.; Ding, L.; Ghonaem, E.; Mohamed, A.; Arbab, H. Big data-based grey forecast mathematical model to evaluate the effect of Escherichia coli infection on patients with lupus nephritis. *Results Phys.* **2021**, *26*, 104339. [CrossRef]
17. Liang, B.; Huang, L. Prediction and analysis of China's medical and health resources based on GM (1,1) grey prediction model. *Mod. Prev. Med.* **2021**, *48*, 3655–3659.
18. Xia, H. Empirical analysis and prediction of logistics economic development scale in Henan province based on grey prediction GM (1,1) model. *Logist. Technol.* **2014**, *33*, 263–266.
19. Liang, F.; Sha, Y. Research on the grey forecast model GM (1,1) of loans of financial institutions. *Econ. Probl.* **2014**, *1*, 58–61.
20. Nguyen, H. Applications optimal math model to solve difficult problems for businesses producing and processing agricultural products in Vietnam. *Axioms* **2021**, *10*, 90. [CrossRef]
21. Yang, W.; Li, B. Prediction of grain supply and demand structural balance in China based on grey models. *Grey Syst. Theory Appl.* **2021**, *11*, 253–264. [CrossRef]
22. Zhang, H.; Liu, X. Application of improved multivariable grey model in grain yield forecasting of Shandong province. *J. Ludong Univ. (Nat. Sci. Ed.)* **2018**, *34*, 199–207.
23. Xie, M.; Wu, L. Application of GM (1, n) power model combined with rough set in short-term traffic flow prediction. *Math. Pract. Theory* **2021**, *51*, 241–249.
24. Gou, X.; Zeng, B.; Gong, Y. An improved multi-variable grey model for forecasting China's finished products from comprehensive waste utilization. *Environ. Sci. Pollut. Res.* **2021**, *28*, 42901–42915. [CrossRef] [PubMed]
25. Zhou, P.; Ang, B.W.; Poh, K.L. A trigonometric grey prediction approach to forecasting electricity demand. *Energy* **2006**, *31*, 2839–2847. [CrossRef]
26. Wu, L.; Liu, S.; Fang, Z.; Xu, H. Properties of the GM(1,1) with fractional order accumulation. *Appl. Math. Comput.* **2015**, *252*, 287–293. [CrossRef]
27. Pei, L.; Li, Q. Forecasting quarterly sales volume of the new energy vehicles industry in China using a data grouping approach-based nonlinear grey bernoulli model. *Sustainability* **2019**, *11*, 1247. [CrossRef]

28. Wu, L.; Liu, S.; Yao, L. Discrete grey model based on fractional order accumulate. *Syst. Eng.-Theory Pract.* **2014**, *34*, 1822–1827.
29. Jiang, J.; Wu, W.; Luo, D.; Fan, D. Conformable fractional order optimization grey model and its application. *Stat. Decis.* **2022**, *1*, 43–46.
30. Chen, Y.; Wu, L.; Liu, L.; Zhang, K. Fractional Hausdorff grey model and its properties. *Chaos Solitons Fractals* **2020**, *138*, 109915. [CrossRef]
31. Zhang, Y.; Wang, Z.; Yu, N.; Zhang, M.; Tang, S. Research on airport air quality forecast based on wavelet period and FGM (1,1) model. *Math. Pract. Theory* **2021**, *51*, 82–92.
32. Tong, Y.; Chen, H.; Zhang, X.; Wu, L. Forecast of Beijing natural gas consumption based on FGM (1,1) model. *Math. Pract. Theory* **2020**, *50*, 79–83.
33. Xiong, C.; Wu, L. Prediction of China's express business volume based on FGM(1,1) model. *J. Math.* **2021**, *2021*, 8585238. [CrossRef]
34. Zhao, G. Prediction of added value of high-tech industry in Hebei province based on FGM (1,1) model. *Math. Pract. Theory* **2021**, *51*, 313–317.
35. Zhang, S.; Wu, L.; Cheng, M.; Zhang, D. Prediction of whole social electricity consumption in Jiangsu province based on metabolic FGM (1,1) model. *Mathematics* **2022**, *10*, 1791. [CrossRef]
36. Zhang, X.; Wang, J.; Wu, L.; Cheng, M.; Zhang, D. Prediction of the total output value of China's construction industry based on FGM(1,1) model. *Axioms* **2022**, *11*, 450. [CrossRef]